The Global Environmental Multi-scale model (GEM) on the Yin-Yang grid system

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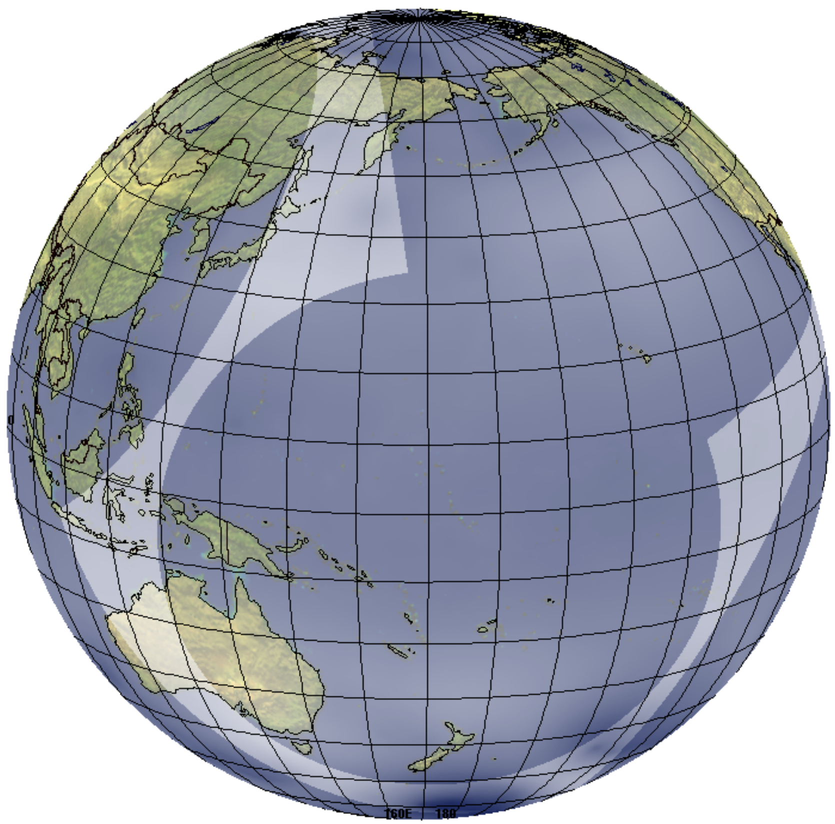
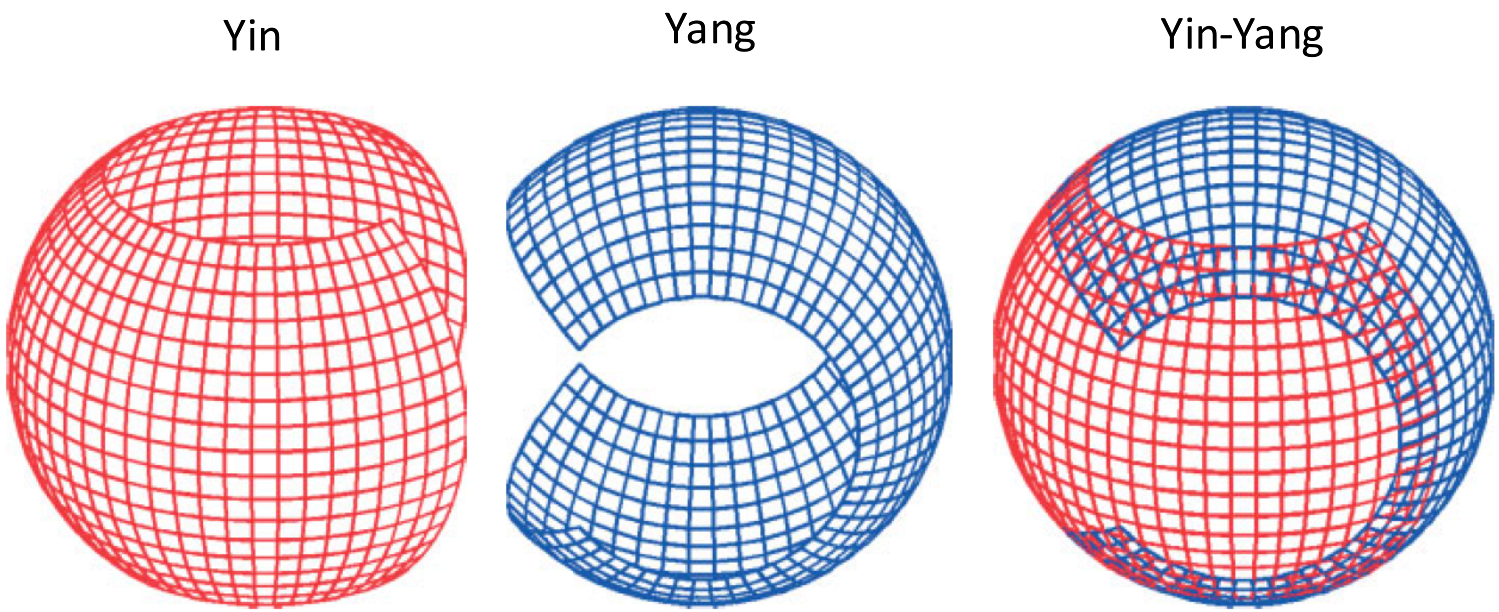


Collaboration with:
Monique Tanguay,
Claude Girard and
Jean de Grandpré

Outline

- Yin-Yang grid
- GEM model equations
- Domain decomposition method used to solve GEM model equations on Yin-Yang grid
- Superstorm Sandy

GEM Yin-Yang

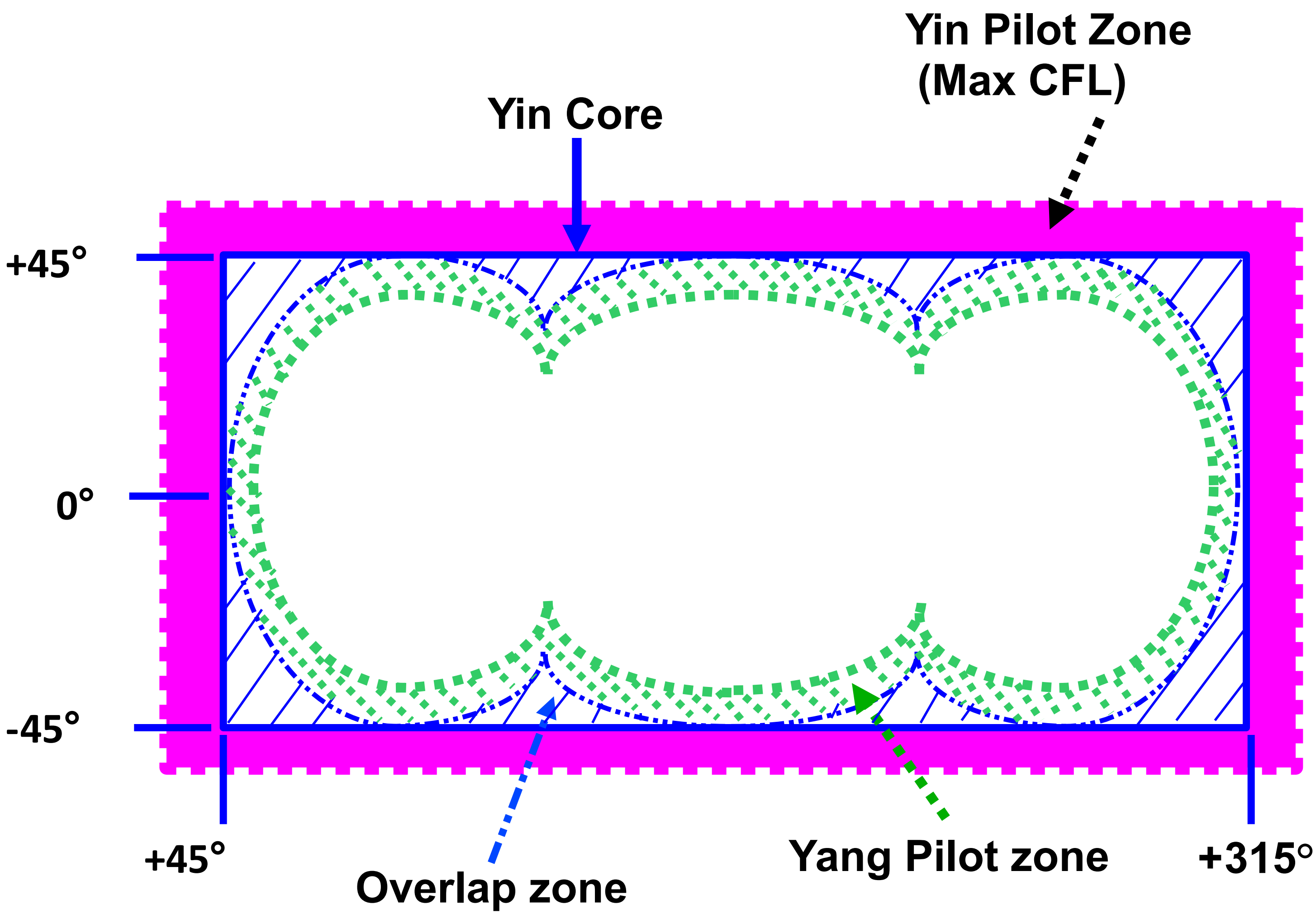


$$\Omega_1 = \left\{ 45^0 - 3\delta \le \lambda \le 315^o + 3\delta;\ -45^0 - \delta \le \theta \le 45^o + \delta \right\}; \delta = 2^0$$

$$\Omega_2 = \Omega_1$$

The global forecast is based on the two-way nesting method between 2-limited area models.

Qaddouri and Lee 2011 Q.J.R.Meteorol.Soc.137:1913-1926


Yin Pilot Zone
(Max CFL)
Yin Core
+45°
0°
-45°
+45°
Overlap zone
Yang Pilot zone
+315°

Vertical coordinate ζ: A log-hydrostatic-pressure type

$$\ln(\pi) = \zeta + B(\zeta)s \ ; \ B(\zeta) = \left(\frac{\zeta - \zeta_T}{\zeta_S - \zeta_T}\right)^{r} \ ; \ s = \ln\left(\frac{\pi_S}{p_0}\right) \ ; \ p_0 = 1000\,\mathrm{hPa}$$

Smooth transitions from terrain following levels near the ground to isobaric surfaces in the upper troposphere with appropriate choice of parameter r

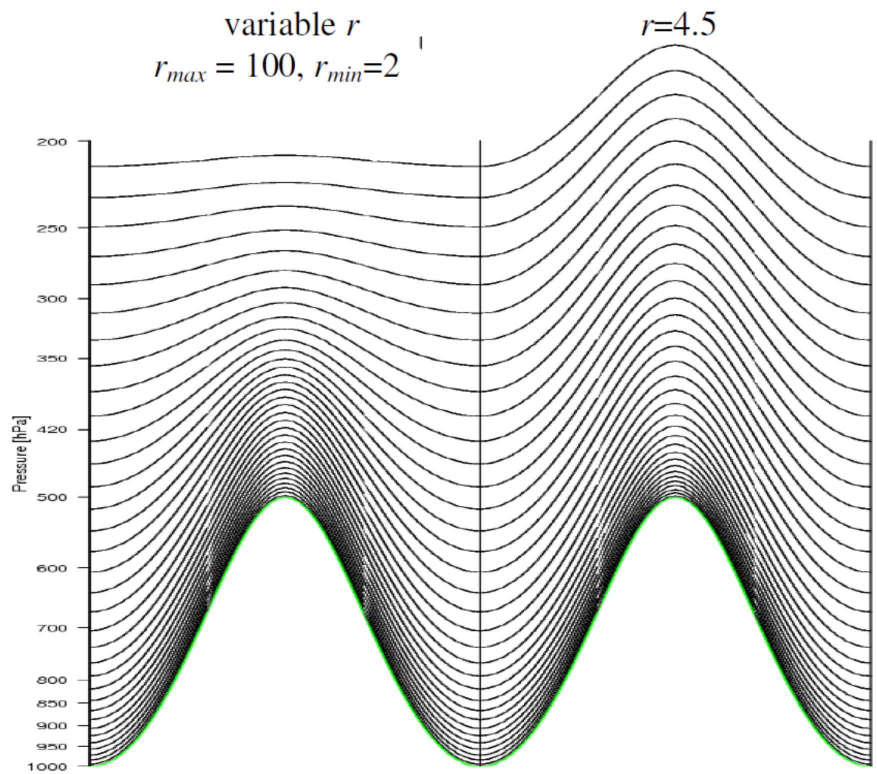


Figure 5. GEM4 with variable r (r_{max}=100, r_{min}=2) compared to GEM4 with constant r=4.5 **below 200 hPa**.

GEM non-hydrostatic equations

Momentum Eqs.

$$\frac{d\mathbf{V}_h}{dt} + f\mathbf{k}\mathrm{x}\mathbf{V}_h + R_d T_v \nabla_\zeta \ln p + (1+\mu)\nabla_\zeta \phi = 0$$

$$\frac{dw}{dt} - g\mu = 0$$

Non-hydrostatic

Continuity Eq.

$$\frac{d}{dt}\ln\left(\pi \frac{\partial \ln \pi}{\partial \zeta}\right) + \nabla_\zeta \cdot \mathbf{V}_h + \frac{\partial \dot{\zeta}}{\partial \zeta} = 0$$

Thermodynamic Eq.

$$\frac{d \ln T_v}{dt} - \kappa \frac{d \ln p}{dt} = 0$$

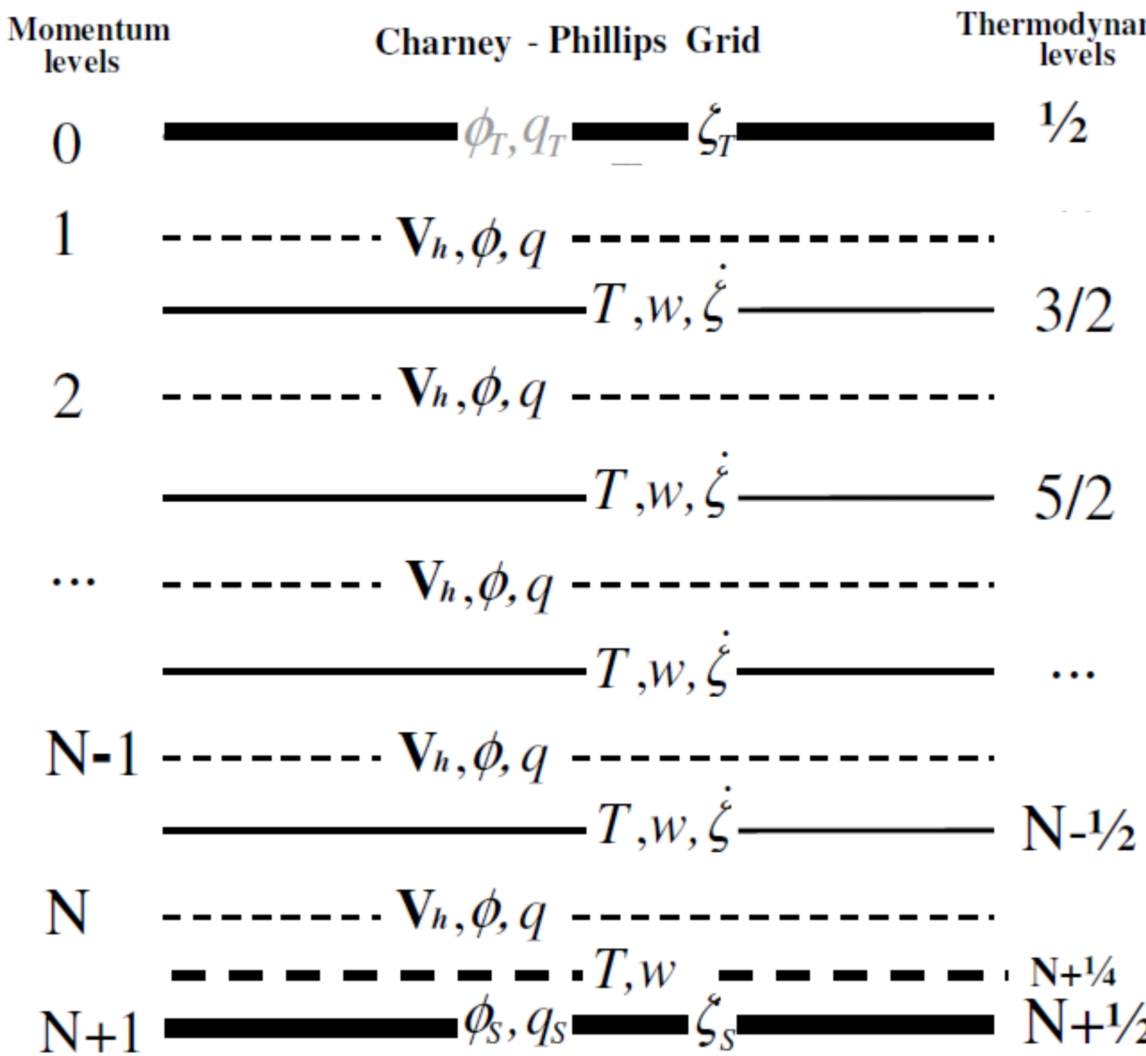
$$\frac{d\phi}{dt} - gw = 0$$

$$R_d T_v + \frac{p}{\pi}\frac{\partial \phi}{\partial \ln \pi} = 0$$

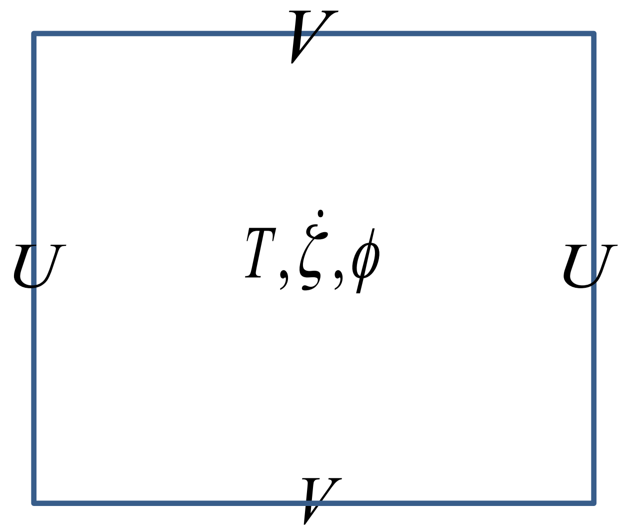
$$1 + \mu - \frac{p}{\pi}\frac{\partial \ln p}{\partial \ln \pi} = 0$$

$$\ln \pi \equiv \zeta + Bs$$

Vertical discretization: Charney-Phillips



Horizontal discretization: Arakawa-C grid



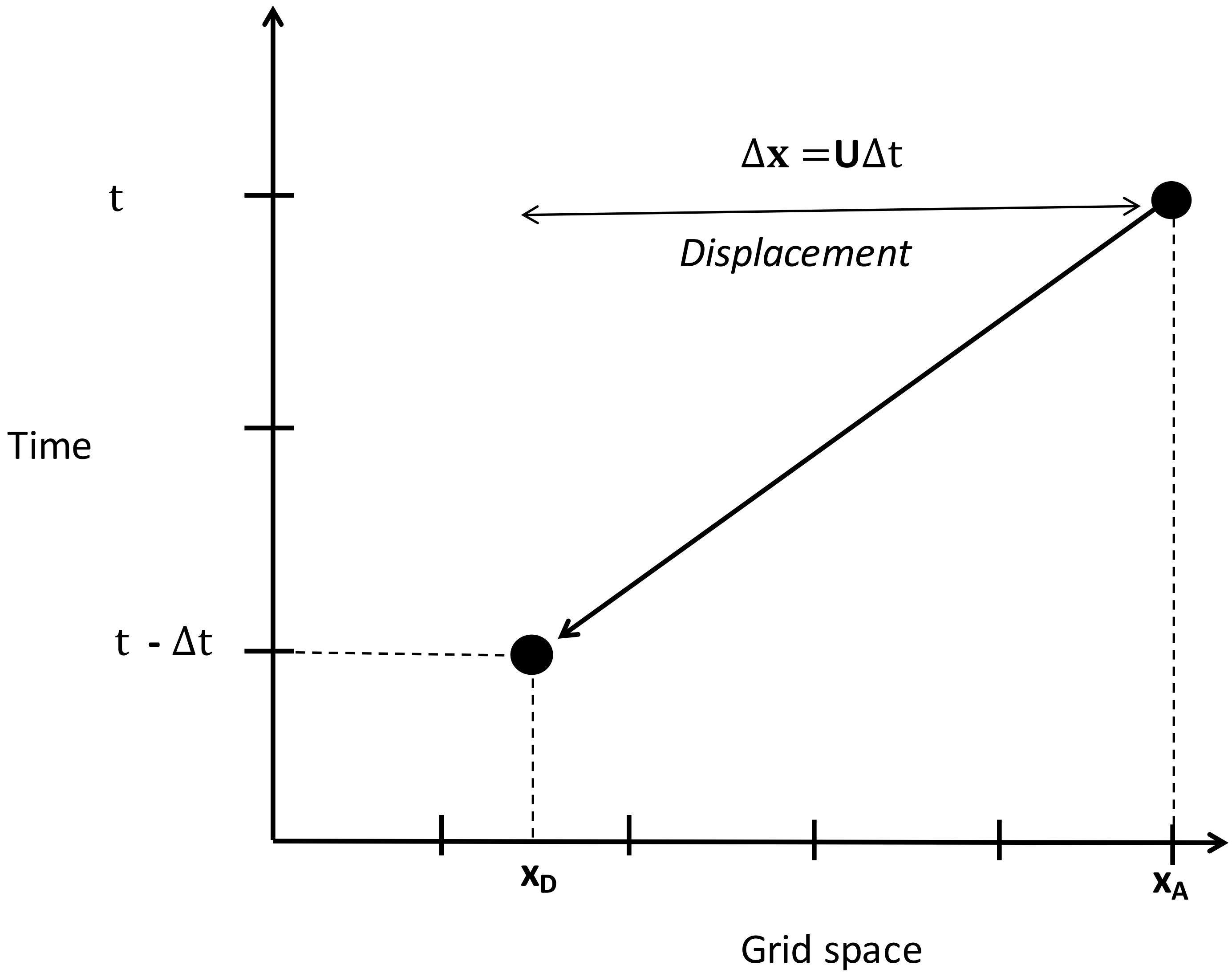
Semi-Lagrangian advection

t - Future time

Δt - Time step

$\mathbf{x_D}$ - Departure

$\mathbf{x_A}$ - Arrival



Two time-level semi-Lagrangian implicit scheme (1)

$$\Delta \mathbf{r}^{i} = \Delta t\left[b_{A}\mathbf{v}(\mathbf{r},t) + \left(1-b_{A}\right)\mathbf{v}\left(\mathbf{r}-\Delta\mathbf{r}^{i-1},t-\Delta t\right)\right]$$

Calculated Iteratively

Estimate Displacement

Also Crank-Nicholson iterations needed to estimate $\mathbf{v}(\mathbf{r},t)$

Two time-level semi-Lagrangian implicit scheme (2)

$$\boxed{\frac{dF_i}{dt} + G_i = 0}$$

$$\frac{dF_i}{dt} \approx \frac{F_i^A - F_i^D}{\Delta t}$$

Semi-Lagrangian Advection

$$G_i \approx b^A G_i^A + \left(1 - b^A\right) G_i^D$$

Average

$$0.5 \le b^A \le 0.6$$

Off-centering for stability

$A:$	$(\mathbf{r}, t)$	Arrival
$D:$	$(\mathbf{r} - \Delta\mathbf{r}, t - \Delta t)$	Departure

$$\boxed{\frac{F_i^A - F_i^D}{\Delta t} + b^A G_i^A + \left(1 - b^A\right) G_i^D = 0}$$

Two time-level semi-Lagrangian implicit scheme (3)

Separating the time levels

$$\frac{F_i^A}{\tau} + G_i^A = \frac{F_i^D}{\tau} - \beta G_i^D \equiv R_i$$

LEFT-HAND side
Time t (Unknown)

RIGHT-HAND side
Time t-Δt (Known)
Interpolated at D

$$(\tau = \Delta t b^A ; \beta = (1 - b^A)/b^A)$$

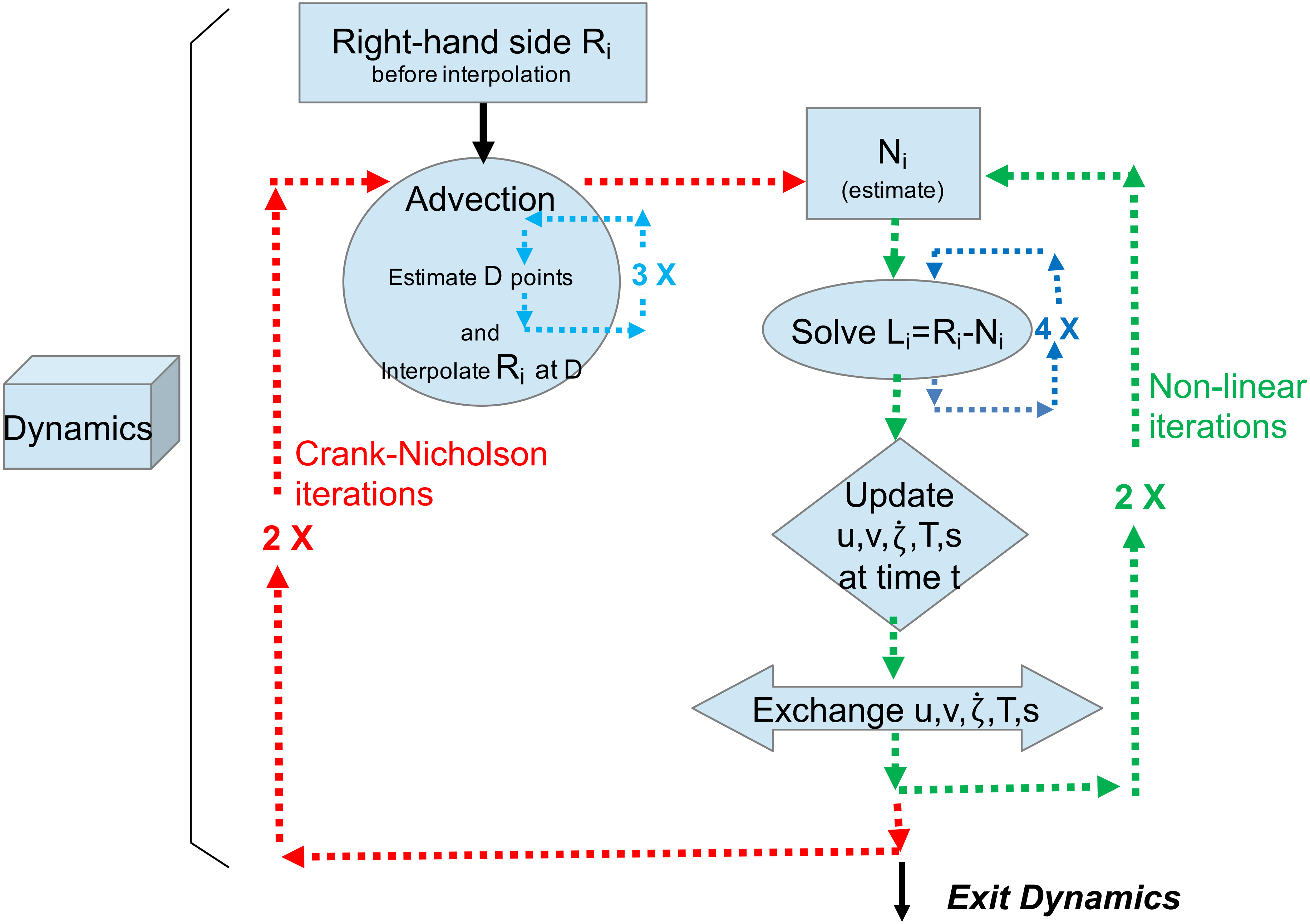
Since LEFT-HAND side is too difficult to solve,
we decompose it into L (Linear) and N (residual Non-linear) parts

$$\frac{F_i^A}{\tau} + G_i^A = L_i + N_i = R_i$$

Solve the LEFT-HAND side $L_i = R_i - N_i$

Non-linear iterations
since N_i is estimated

Flow of GEM Yin-Yang Dynamic core



Solve $L_i = R_i - N_i$

Elliptic problem on Yin-Yang grid with Schwarz method

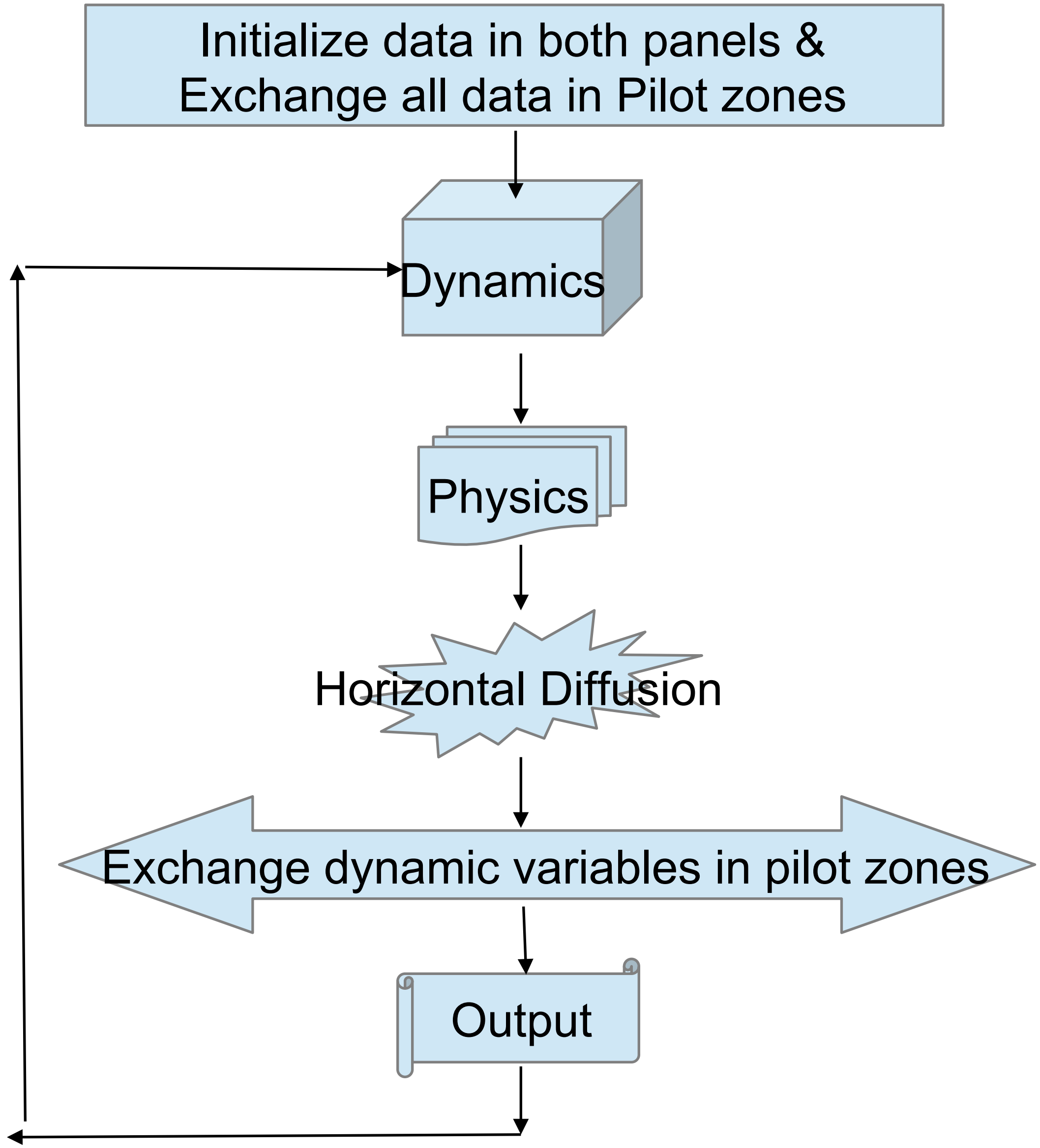
Global solution is obtained by solving iteratively 2 elliptic sub-problems (Yin/Yang)

- 1) receive Boundary conditions (BCs) from the other panel; solve local elliptic problem.
- 2) if boundary conditions converge, stop; else send BCs to other panel ; goto 1

eg: GEM Yin-Yang 25km needs only 4 iterations for convergence with 2 degrees overlap

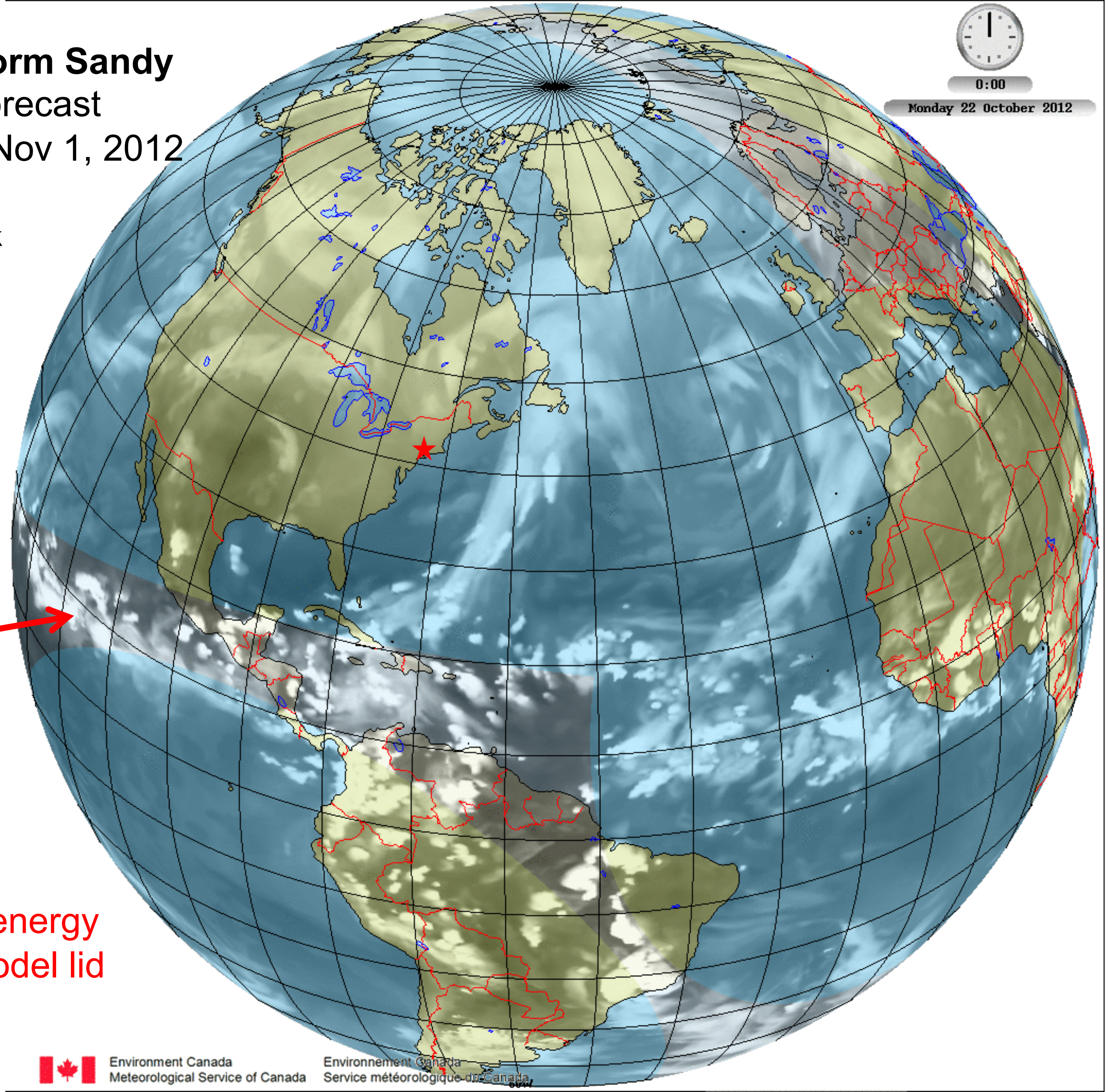
Qaddouri et al. 2008 Appl.Numerical.Math.,58,4,459-471

Flow of GEM Yin-Yang



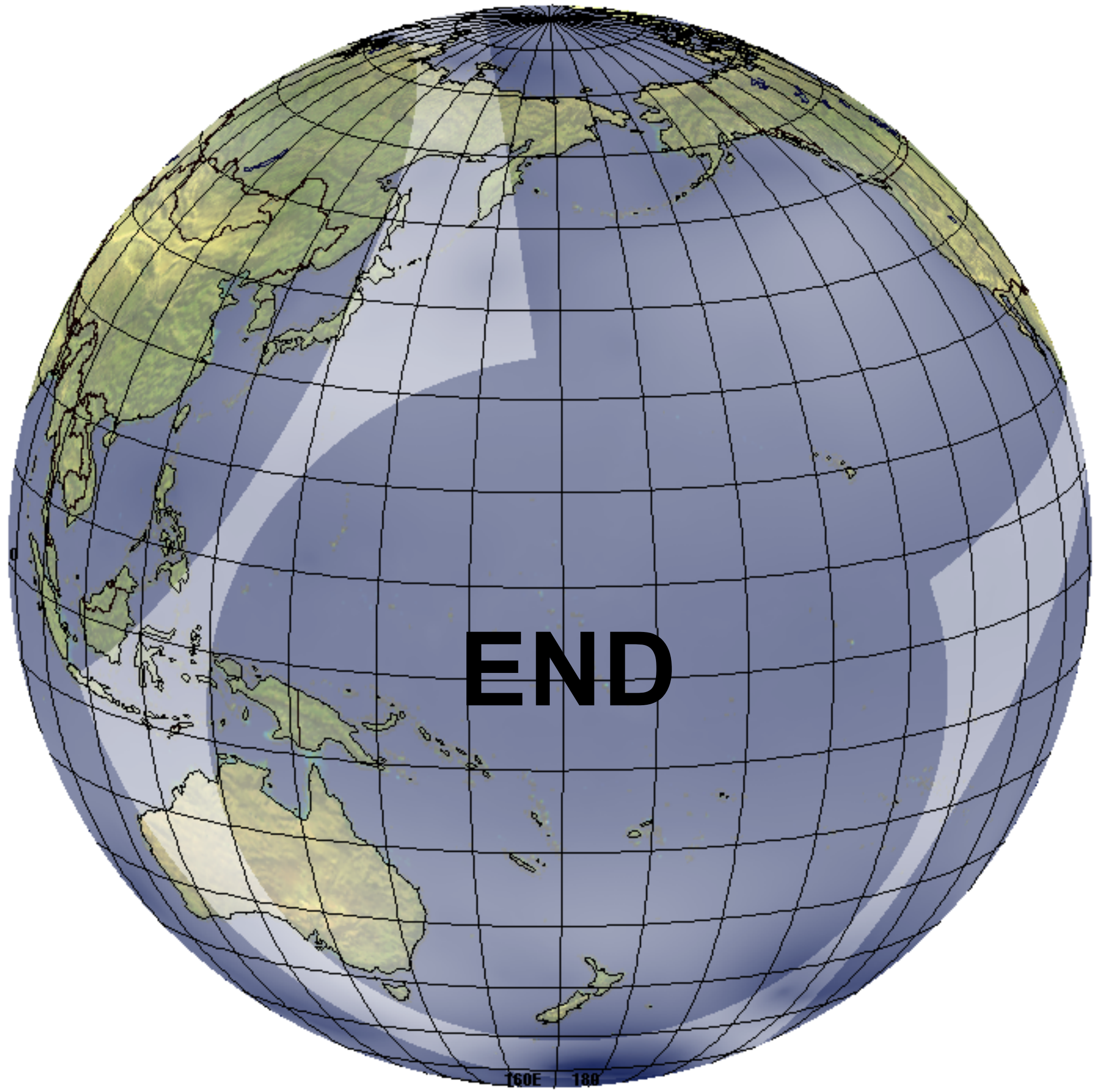
Superstorm Sandy
10-day forecast
Oct 22 –Nov 1, 2012

★ New York



Overlap region

Outgoing Infrared energy flux at model lid


END
160E
180

REFERENCES

- Côté, J., S.Gravel, A.Méthot, A.Patoine, M.Roch, A.Staniforth, 1998:
The Operational CMC–MRB Global Environmental Multiscale (GEM) Model. Part I:
Design Considerations and Formulation. Mon. Wea. Rev., 126, 1373–1395,
http://journals.ametsoc.org/doi/pdf/10.1175/1520-0493%281998%29126%3C1373%3ATOCMGE%3E2.0.CO%3B2

- Girard, C., A.Plante, M.Desgagné, R.McTaggart-Cowan, J.Côté, M.Charron, S.Gravel, V.Lee,
A.Patoine, A.Qaddouri, M.Roch, L.Spacek, M.Tanguay, P.A.Vaillancourt, and A.Zadra, 2014:
Staggered Vertical Discretization of the Canadian Environmental Multiscale (GEM) Model Using a
Coordinate of the Log-Hydrostatic-Pressure Type. Mon.Wea.Rev, 142, 1183-1196,
http://journals.ametsoc.org/doi/pdf/10.1175/MWR-D-13-00255.1

- Girard, C., A.Plante, S.Gravel, A.Qaddouri, S.Chamberland, L.Spacek, V.Lee, and M.Desgagné, 2013:
GEM4.2: A non-hydrostatic atmospheric model (Euler equations). RPN document,
http://collaboration.cmc.ec.gc.ca/science/rpn/publications/pdf/GEM4.2.pdf

- Kageyama, A., and T.Sato, 2004: The ‘Yin-Yang grid’: An overset grid in spherical geometry.
Geochem. Geophys. Geosystems 5: Q09005. DOI:10.1029/2004GC000734,
http://www.agu.org/journals/gc/gc0409/2004GC000734/2004GC000734.pdf

- Qaddouri, A., and V.Lee, 2011:
The Canadian Global Environmental Multiscale model on the Yin-Yang grid system.
Quart. J. Roy. Meteor. Soc., 137, 660, 1913-1926, DOI: 10.1002/qj.873,
http://onlinelibrary.wiley.com/doi/10.1002/qj.873/pdf

- Qaddouri, A., L. Laayouni, S. Loisel, J. Côté, and M. J. Gander, 2008: Optimized Schwarz methods with an overset grid for the Shallow-Water Equations: Preliminary results. IMACS Journal of Applied Numerical Mathematics, 58, 459-471.