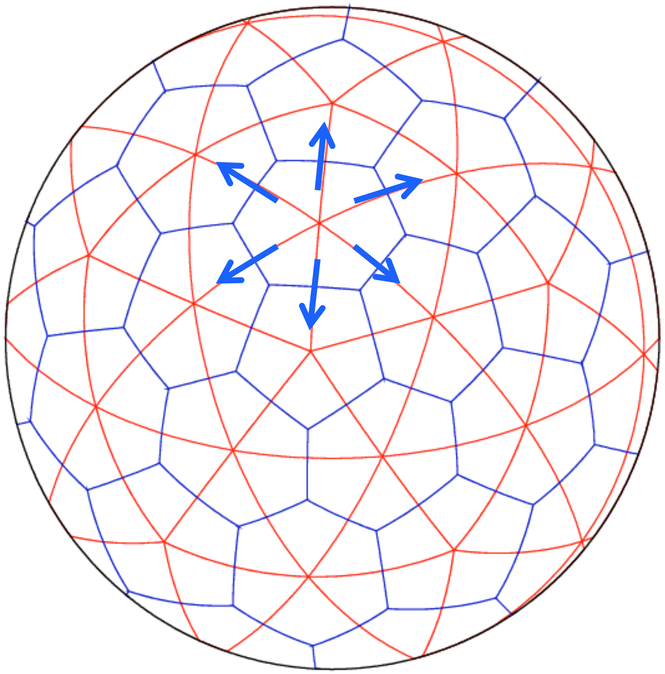
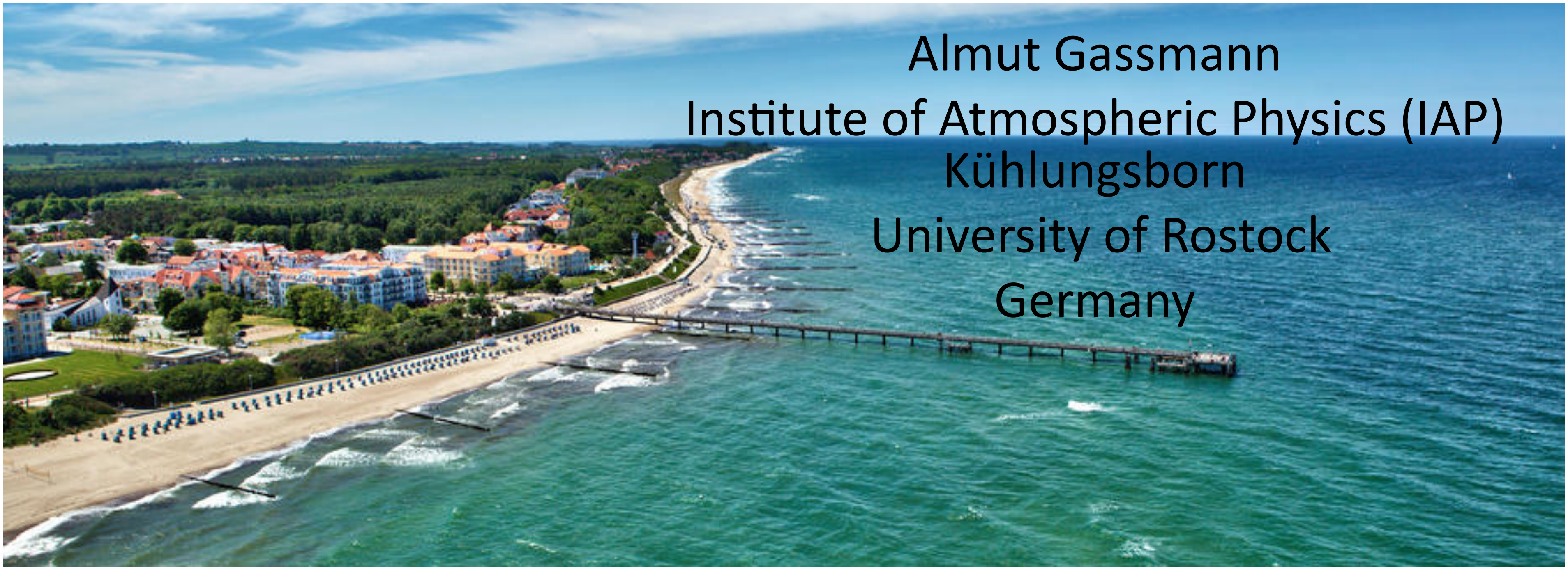


Design Philosophy of ICON-IAP



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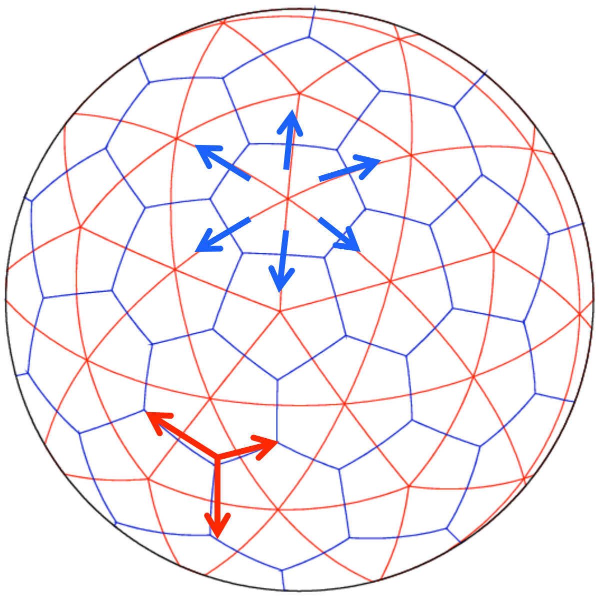
Main Properties

- non-hydrostatic shallow-atmosphere equations
- grid staggering:
  - horizontal: hexagonal C-grid
  - vertical: L-grid
- height-based terrain-following coordinates
- time stepping
  - horizontal: explicit
  - vertical: implicit
- conservation of mass, tracer-mass, energy, entropy
- 3D-vector-invariant form of the momentum equations
- higher order + positive definite tracer tranport

Hexagonal C-grid

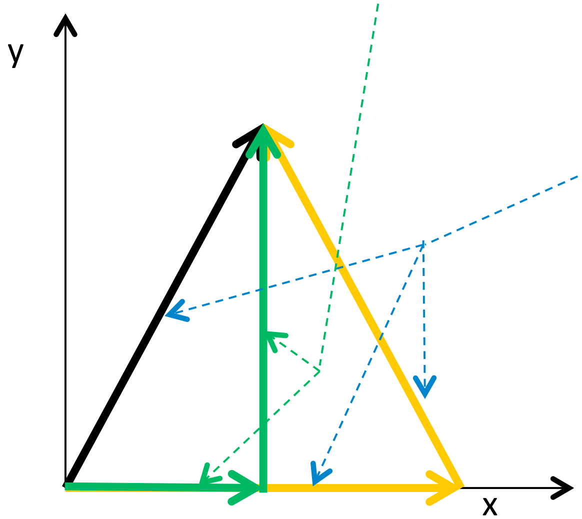


- No pole problem
- **C-grid wave propagation properties** (no stationary waves at the truncation limit)
- **Geostrophic modes and gravity wave modes** are both represented correctly
- **Laplace operator is second order accurate** for both, scalars and momentum
- Horizontal momentum diffusion is expressable as a **divergence of a turbulent stress tensor**
- Dissipated kinetic energery from horizontal diffusion can be **fed back to internal energy as a positive source**

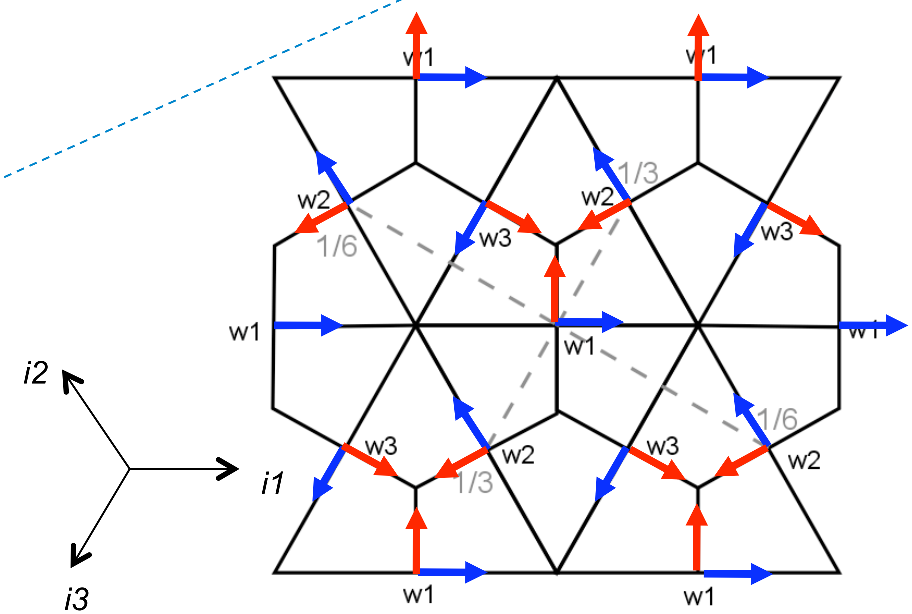
All the green properties are **not** achievable with a **triangular C-grid**.
All the green properties hold stricly only for a regular **hexagonal C-grid**.

How to describe a vector on C-grid triangles/hexagons?

On a **quadrilateral C-grid**, we have **2 velocity components (u,v)** and the **i and j unit vectors**.



On a **trivariate C-grid**, we have **3 velocity components (u_i or v_i)** and the **$\mathbf{i}_i$ or $\mathbf{j}_i$ unit vectors**.



The three unit vectors are linearly dependent and therefore $u_1+u_2+u_3=0$ must hold.
We call this the "linear dependency constraint".
The computational mode will be controlled.

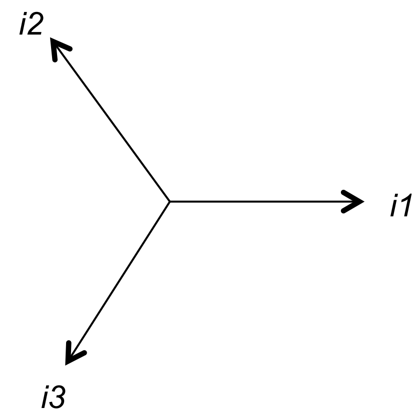
How to prove that a discretisation obeys the linear dependency constraint?

A velocity field on a plane is alternatively defined
by the streamfunction and the velocity potential.

The linear dependency constraint can be proven for gradients (as we shall see).

$$\mathbf{v} = \mathbf{k} \times \nabla\psi + \nabla\chi$$

$$\boxed{\begin{aligned}
u_1 &= -\frac{1}{\sqrt{3}}\left(\frac{\partial\psi}{\partial x_2} - \frac{\partial\psi}{\partial x_3}\right) + \frac{\partial\chi}{\partial x_1} \\
u_2 &= -\frac{1}{\sqrt{3}}\left(\frac{\partial\psi}{\partial x_3} - \frac{\partial\psi}{\partial x_1}\right) + \frac{\partial\chi}{\partial x_2} \\
u_3 &= -\frac{1}{\sqrt{3}}\left(\frac{\partial\psi}{\partial x_1} - \frac{\partial\psi}{\partial x_2}\right) + \frac{\partial\chi}{\partial x_3}
\end{aligned}}$$



Take the example of the **hexagonal C-grid**.

On the **triangular C-grid**, velocity potential and streamfunction change place.

How to prove that a discretisation obeys the linear dependency constraint?

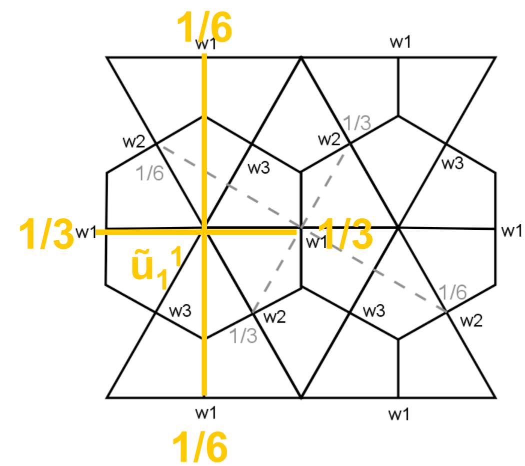
$$\widetilde{u}_1^1 = -\frac{1}{\sqrt{3}}\left(\widetilde{\delta_2\widetilde{\psi}^3}^1 - \widetilde{\delta_3\widetilde{\psi}^2}^1\right) + \widetilde{\delta_1\chi}^1$$

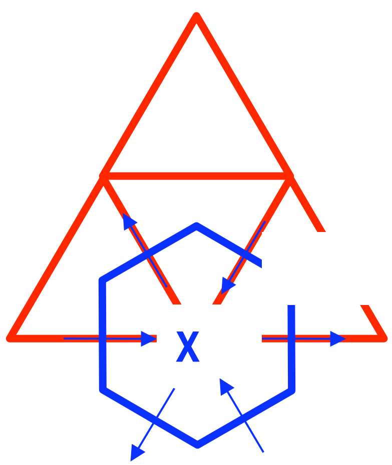
$$\widetilde{u}_2^2 = -\frac{1}{\sqrt{3}}\left(\widetilde{\delta_3\widetilde{\psi}^1}^2 - \widetilde{\delta_1\widetilde{\psi}^3}^2\right) + \widetilde{\delta_2\chi}^2$$

$$\widetilde{u}_3^3 = -\frac{1}{\sqrt{3}}\left(\widetilde{\delta_1\widetilde{\psi}^2}^3 - \widetilde{\delta_2\widetilde{\psi}^1}^3\right) + \widetilde{\delta_3\chi}^3$$

$$\widetilde{\delta_1\alpha}^1 + \widetilde{\delta_2\alpha}^2 + \widetilde{\delta_3\alpha}^3 = 0$$

John Thuburn's (JCP, 2008) averaging guarantees for the linear dependency of gradient components.



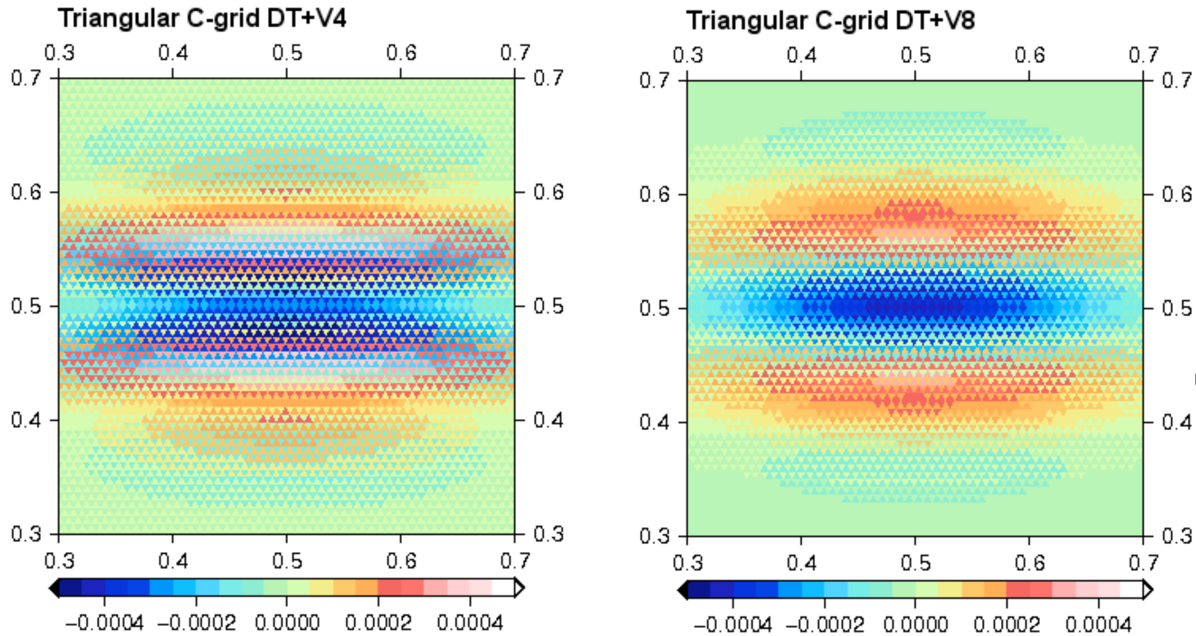


Observation:
To keep the linear dependency constraint working for the velocity components, the streamfunction has to be **placed in the center of a hexagon**!

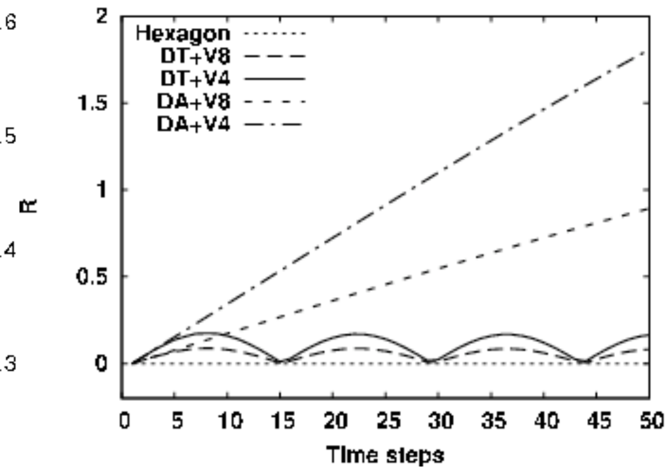
We would expect that the streamfunction is on the corner of a hexagon = center of a triangle!

The streamfunction always occurs averaged (nullspace is there).

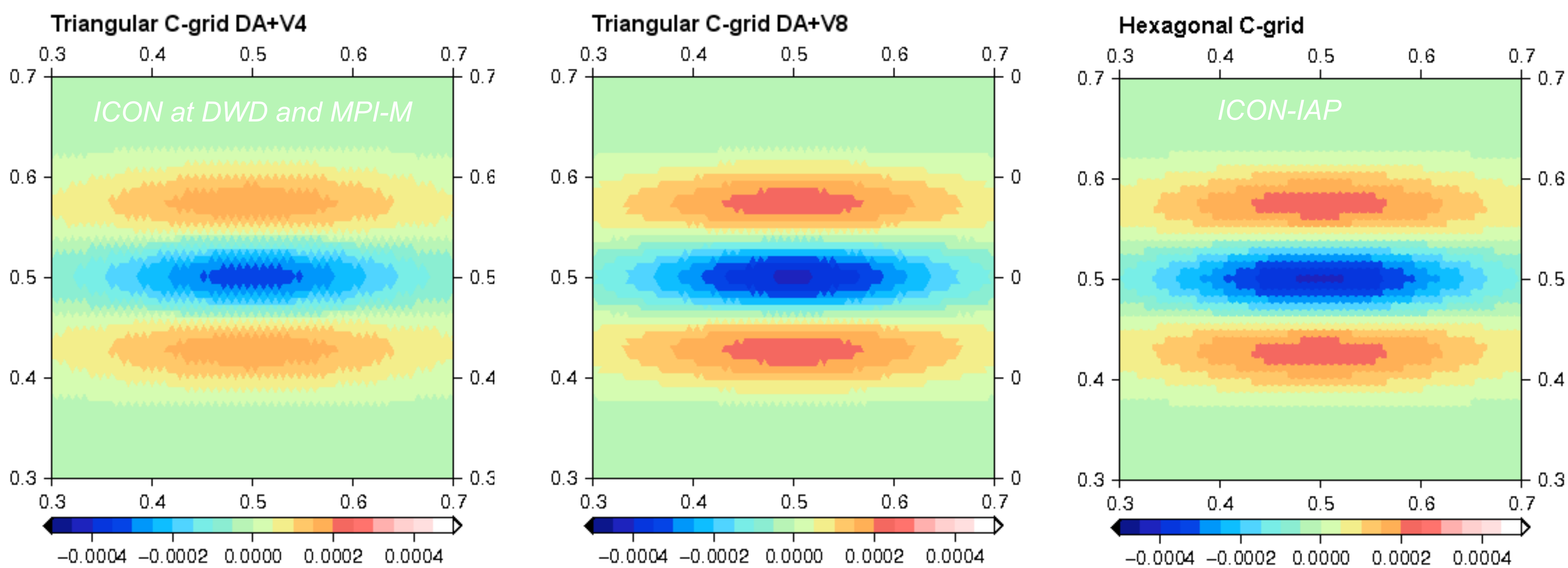
Checkerboard problem:
On can show that the non-fulfillment of the linear dependency constraint visualizes as a checkerboard pattern in that Stokes integral (circulation) which is defined on triangles.



Measurement of an occurring checkerboard



Divergence fields for a linear geostrophic adjustment problem with a badly resolved Rossby radius.



Consequences for the vorticity in the hexagonal C-grid....
Take the Laplacian...

$$\hat{\nabla}^2 u_1 = -\frac{1}{\sqrt{3}}\left(\delta_2 \hat{\nabla}^2 \widetilde{\psi}^3 - \delta_3 \hat{\nabla}^2 \widetilde{\psi}^2\right) + \delta_1 \hat{\nabla}^2 \chi$$

$$\hat{\nabla}^2 u_2 = -\frac{1}{\sqrt{3}}\left(\delta_3 \hat{\nabla}^2 \widetilde{\psi}^1 - \delta_1 \hat{\nabla}^2 \widetilde{\psi}^3\right) + \delta_2 \hat{\nabla}^2 \chi$$

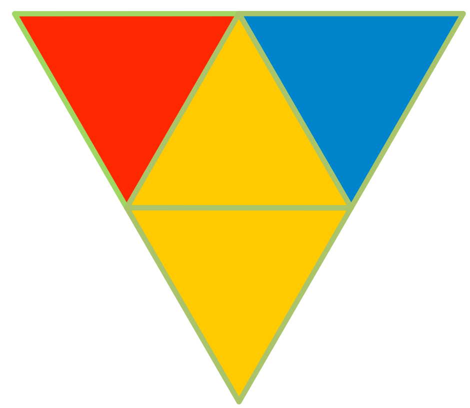
$$\hat{\nabla}^2 u_3 = -\frac{1}{\sqrt{3}}\left(\delta_1 \hat{\nabla}^2 \widetilde{\psi}^2 - \delta_2 \hat{\nabla}^2 \widetilde{\psi}^1\right) + \delta_3 \hat{\nabla}^2 \chi$$

There are 3 kinds of **vorticities** which are naturally located at edges.

$$\zeta_1 = \hat{\nabla}^2 \widetilde{\psi}^1 = 2(\delta_2 u_3 - \delta_3 u_2)/\sqrt{3}$$

$$\zeta_2 = \hat{\nabla}^2 \widetilde{\psi}^2 = 2(\delta_3 u_1 - \delta_1 u_3)/\sqrt{3}$$

$$\zeta_3 = \hat{\nabla}^2 \widetilde{\psi}^3 = 2(\delta_1 u_2 - \delta_2 u_1)/\sqrt{3},$$

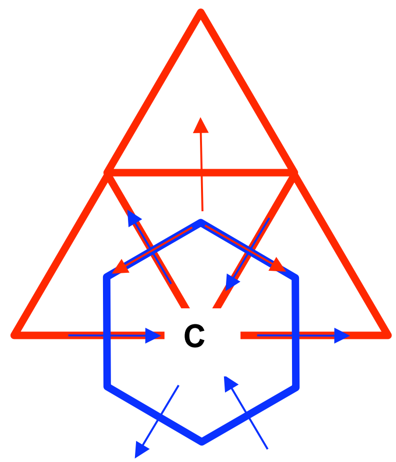


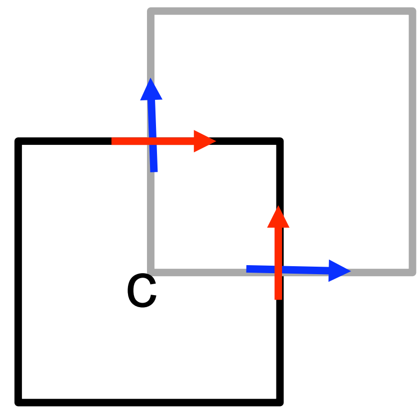
Which natural grid conforms with the red velocities?

hexagonal C-grid

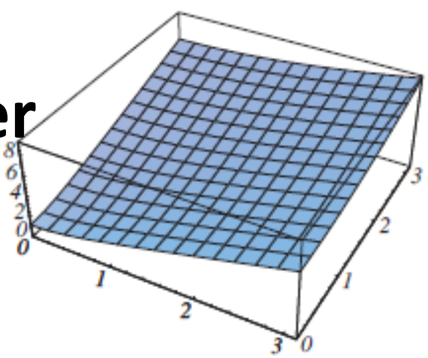
~~triangular C-grid~~

hexagonal D-grid

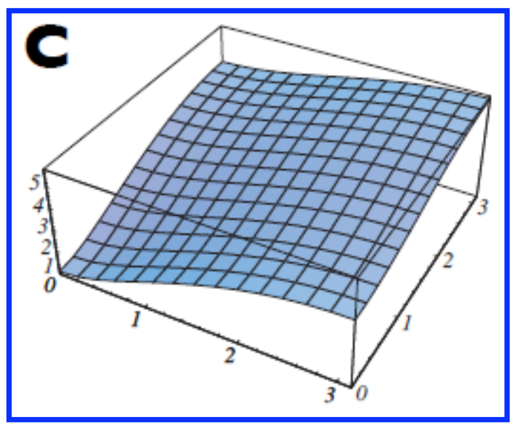


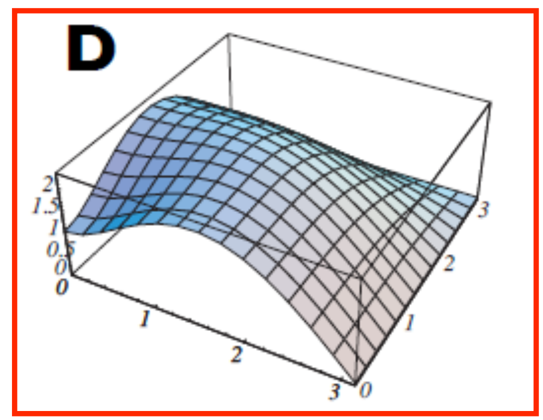


Right answer



from Dave Randall's slides last week





Conservation (conversion) issues

Lorenz (1967): The nature and theory of the general circulation of the atmosphere

$$\frac{\partial(\Phi + I)}{\partial t} = Q + D - C$$

$$\frac{\partial(K)}{\partial t} = -D + C + 0$$

ICON-IAP (dry):

$$\frac{\partial \mathcal{F}}{\partial t} = \boxed{\{\mathcal{F},\mathcal{H}\}} + \boxed{(\mathcal{F},\mathrm{f_r}) + (\mathcal{F},Q)}$$

$$\boxed{\{\mathcal{F},\mathcal{H}\}} = -\int_V \frac{\delta\mathcal{F}}{\delta\mathbf{v}}\cdot\left(\frac{\vec{\omega}_a}{\varrho}\times\frac{\delta\mathcal{H}}{\delta\mathbf{v}}\right)d\tau$$
$$-\int_V \left(\frac{\delta\mathcal{F}}{\delta\varrho}\nabla\cdot\frac{\delta\mathcal{H}}{\delta\mathbf{v}} - \frac{\delta\mathcal{H}}{\delta\varrho}\nabla\cdot\frac{\delta\mathcal{F}}{\delta\mathbf{v}}\right)d\tau$$
$$-\int_V \left(\frac{\delta\mathcal{F}}{\delta\tilde{\theta}}\nabla\cdot(\theta\frac{\delta\mathcal{H}}{\partial\mathbf{v}}) - \frac{\delta\mathcal{H}}{\delta\tilde{\theta}}\nabla\cdot(\theta\frac{\delta\mathcal{F}}{\delta\mathbf{v}})\right)d\tau,$$

$$(\mathcal{F},\mathrm{f_r}) = -\int_V \frac{\delta\mathcal{F}}{\delta\mathbf{v}}\frac{1}{\varrho}\nabla\cdot\overline{\varrho\mathbf{v}''\mathbf{v}''}d\tau$$

$$(\mathcal{F},Q) = \int_V \frac{\delta\mathcal{F}}{\delta\tilde{\theta}}\frac{\varepsilon}{c_p\pi}d\tau \qquad \varepsilon = -\overline{\varrho\mathbf{v}''\mathbf{v}''}\cdot\cdot\nabla\mathbf{v} > 0$$

<u>Motivation from Lorenz's words:</u>

„It also follows, since there is no long-term net heating by radiation and conduction, and since the remaining energy-conversion processes other than friction involve no heating at all, that the net total heating of the atmosphere-ocean-Earth system equals the net frictional heating."

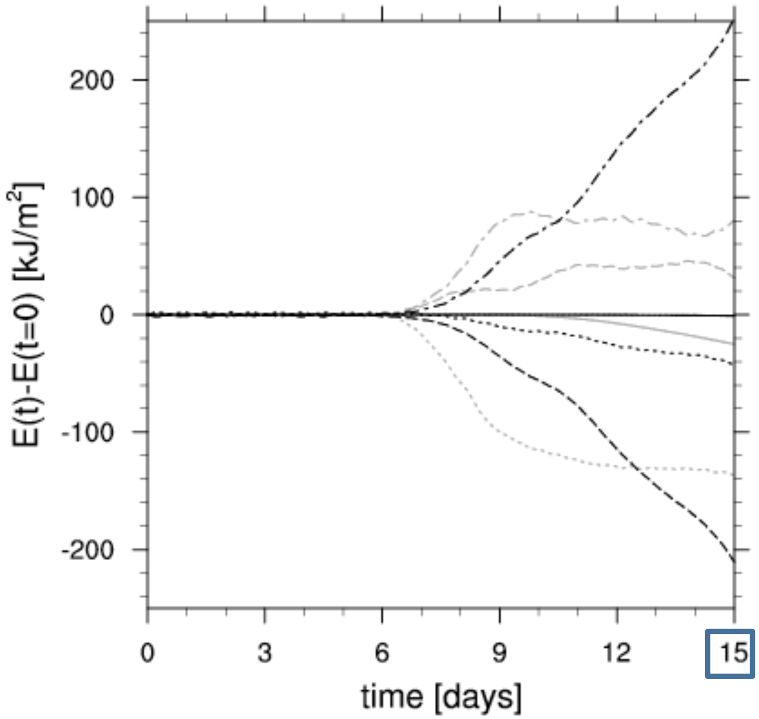
<u>Conclusions from Lorenz's words:</u>

In my opinion, any numerical model which is intended to be used in climate research has to mimick this physical law as closely as possible.

It follows, that we have to look on our so-called smoothing algorithms more from a physical perspective. How much do numerical noise, smoothers, fixers, and dampers affect the physical credibility of our models?

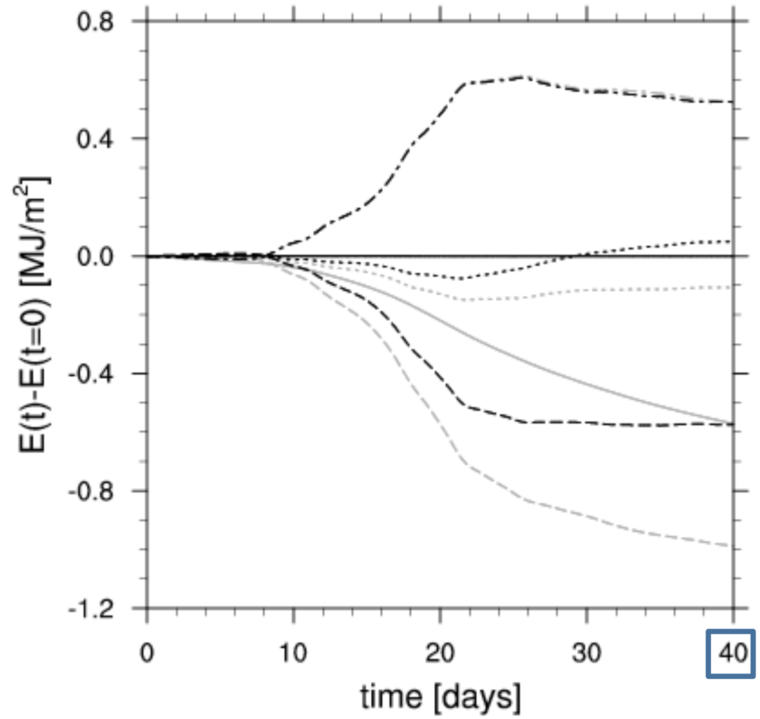
Momentum diffusion and dissipative heating

Energetics of a baroclinc wave:



without momentum diffusion

gray: Hollingsworth instability



with momentum diffusion

gray: without frictional heating

kinetic energy

potential energy

internal energy

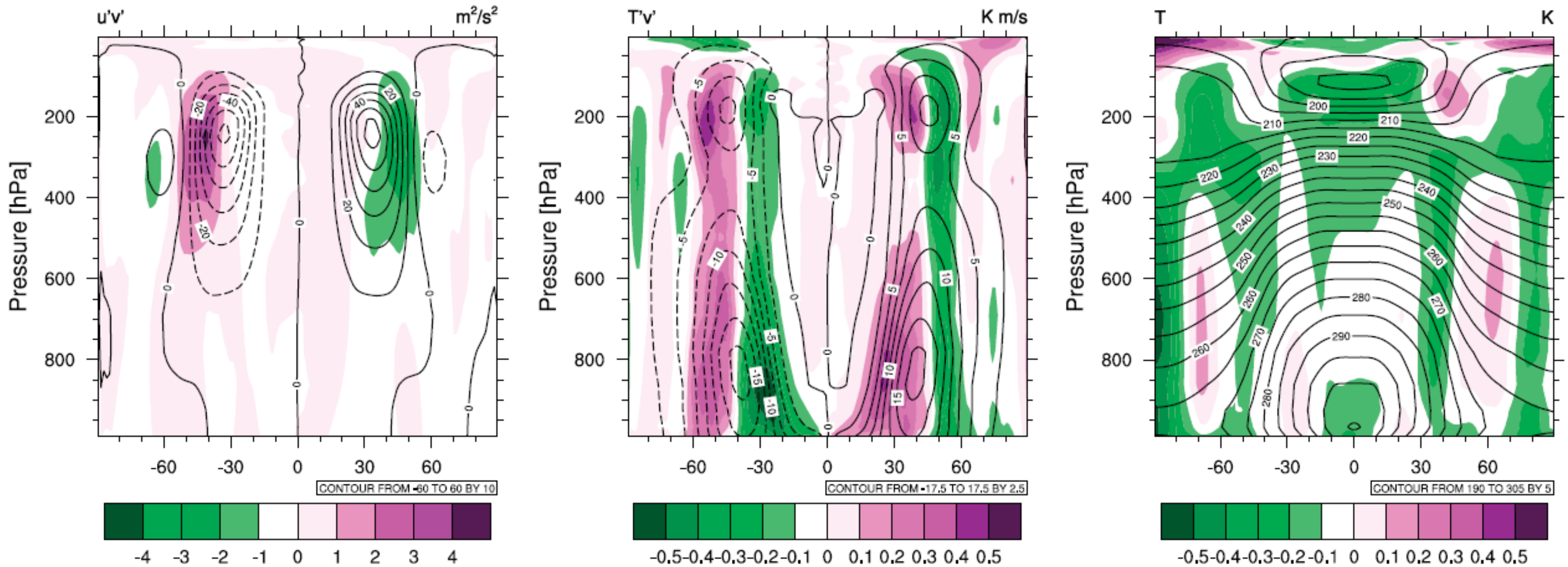
total energy

Further reading: Gassmann (2012,QJRMS)

Momentum diffusion and dissipative heating

Held-Suarez test

Temperature relaxation towards an equilibrium temperature.



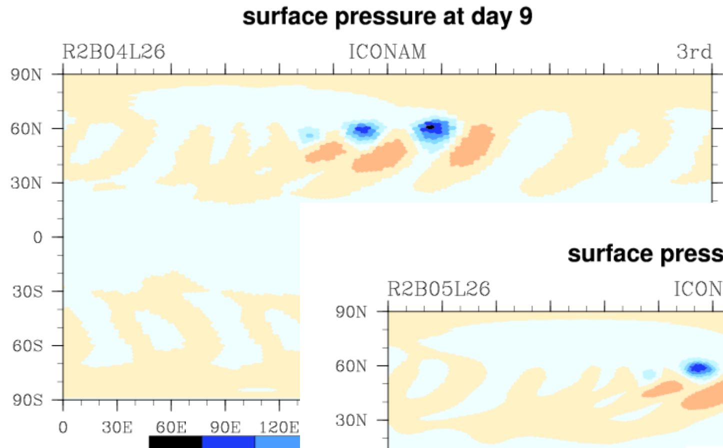
Contours: run with dissipative heating

Colours: Differences from run with dissipative heating to run without dissipative heating

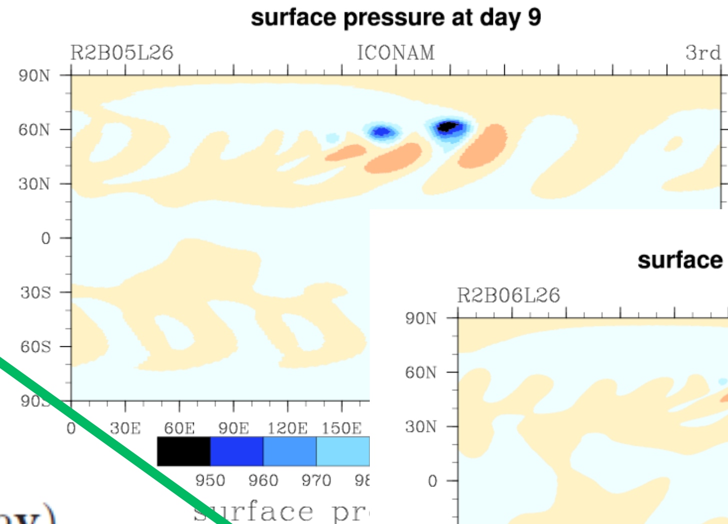
Results average last 1000 days of a 1250 days run.

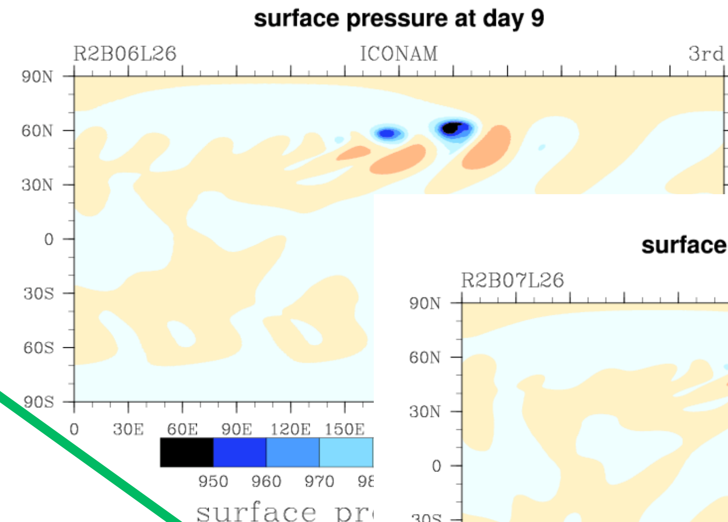
Further reading: Gassmann (2012,QJRMS)

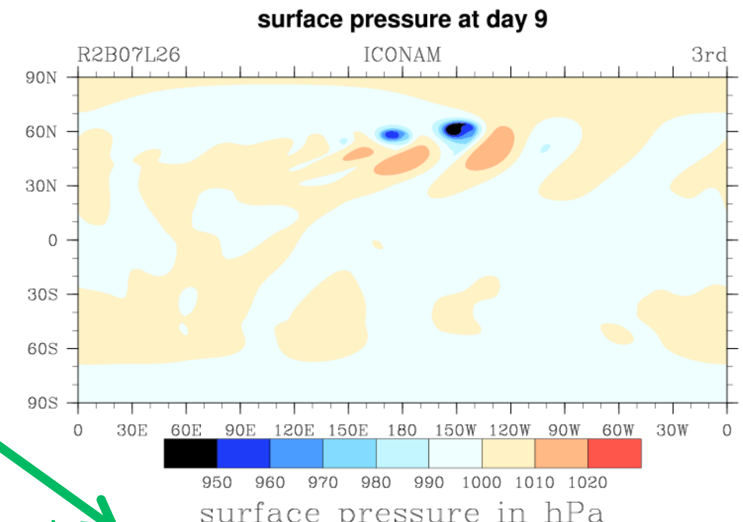
Advection scheme for scalars



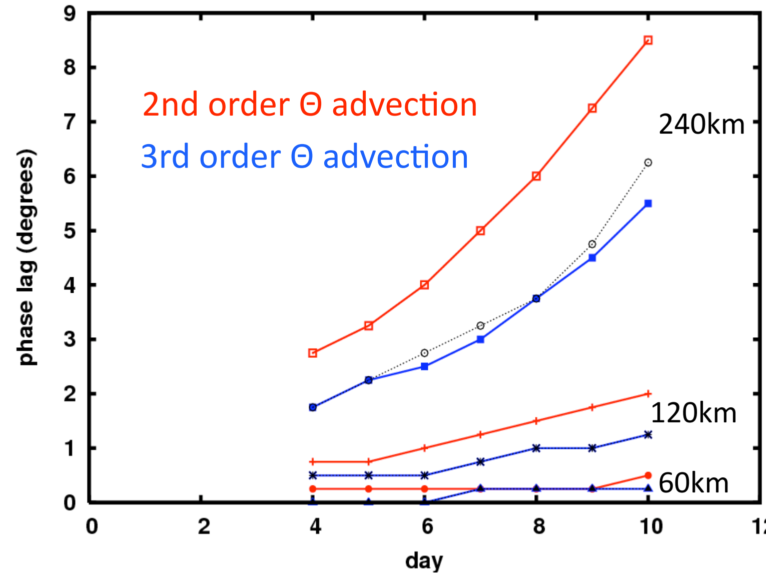
coarse (240km)







fine (30km)



$$\frac{\partial \tilde{\theta}}{\partial t} = -\nabla \cdot (\theta \varrho \mathbf{v})$$

$$\theta_{e,k} = \overline{\theta}^{e}_{e,k} - \Delta x^2 \frac{1}{6} \nabla_x^2 \theta|_{\text{upstream}}$$

Higher order transport of potential temperature proves to be advantageous.

Further reading: Skamarock & Gassmann (2011, MWR)

7. Time stepping (The first con? Or is it a pro?)

As our motivation is from the ***conservation of mass, energy***... side, we have to consider the time stepping scheme under the same perspective. This leads to the conclusion:

All terms in the equations have to be handled with the same time step!
No time splitting is allowed!

Well, we can still use the *HE-VI = horizontal explicit, vertical implicit* philosophy.

But we cannot compute wave terms more often and advection terms less frequently. Our time step is restricted by the CFL number for horizontal propagation of sound waves. ***This is a con!***

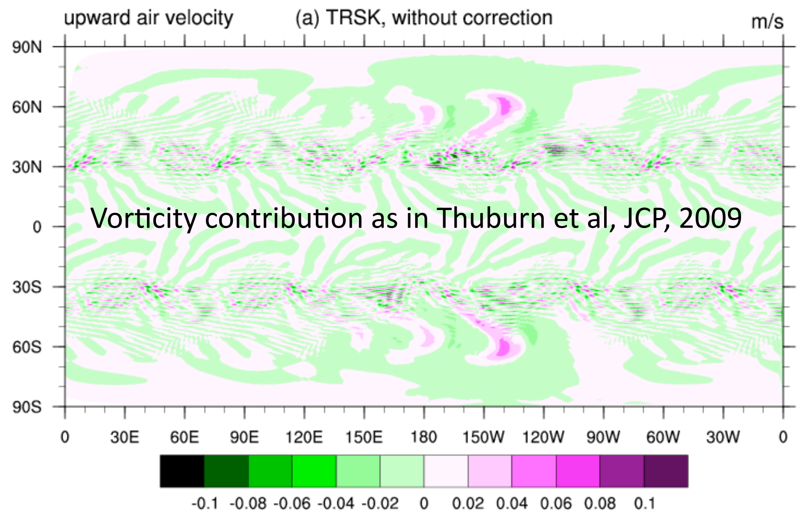
However, the consideration of energy conservation lead us to the derivation of

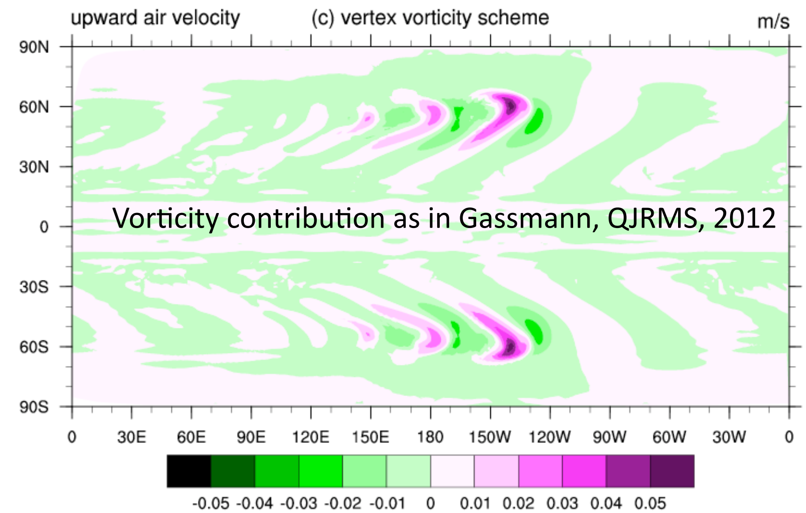
- physically based implicit weights in the VI-part
- physically based extrapolation weights in the HE-part

for the pressure gradient term. From the perspective of scientific insight ***this is a pro!***

A future version of ICON-IAP could focus on possible compromises.

Hollingsworth instability





Often the vorticity equation is considered crucial, but here the problem occurs in the ***divergence*** equation.

$$\frac{dD}{dt} + D^2 - 2J_p(u,v) + \nabla_p\omega \cdot \frac{\partial \mathbf{v}_h}{\partial p} + \beta u - f\zeta = -\nabla_p^2 \Phi$$

$$\nabla \cdot (\mathbf{v}_h \cdot \nabla \mathbf{v}_h) = \nabla \cdot (\mathbf{k}\zeta \times \mathbf{v}_h + \nabla K_h)$$ Does not hold in the discretisation.

Which effect is introduced by this error?

$$\partial_y(-u\partial_y u + \partial_y(u^2/2)) = \epsilon$$

quasi-geostrophic viewpoint

$$\left(\partial_{yy} + \frac{f^2}{\sigma}\partial_{pp}\right)\partial_t \Phi = \partial_t \epsilon$$

$$\left(\partial_{yy} + \frac{f^2}{\sigma}\partial_{pp}\right)\omega = -\frac{1}{\sigma}\partial_{tp}\epsilon$$

Lessons learned:

Classical linear wave analysis!

Shallow-water equations do not tell the whole story!

Take physical laws into consideration right from the beginning!