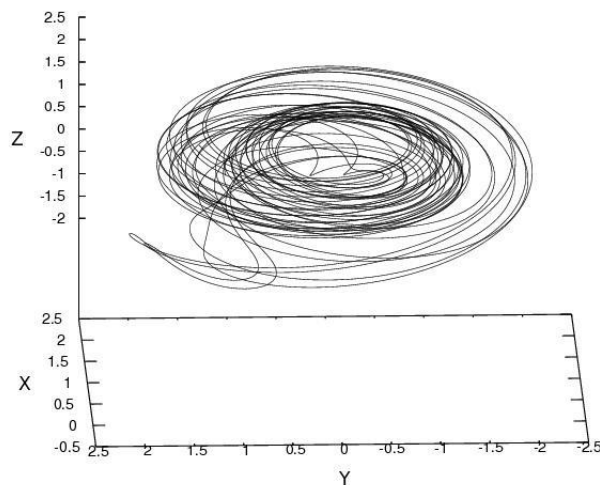
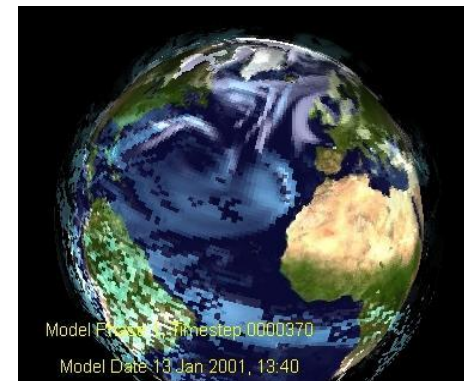


# Ensembles and Uncertainty I

Dave Stainforth

Acknowledgements to: Lenny Smith & Joe Daron

Centre for the Analysis of Timeseries and Grantham Research Institute on Climate Change and the Environment, **London School of Economics**.



**DCMIP Summer School**  
**Boulder**  
**30<sup>th</sup> July 2012**

*“Blizzards and what-not. Being fine today doesn’t Mean Anything. It has no sig – what’s that word? Well, it has none of that. It’s just a small piece of weather.”*

*Eeyore, Winnie-the-Pooh, A.A. Milne*



**LSE**

THE LONDON SCHOOL  
OF ECONOMICS AND  
POLITICAL SCIENCE ■

# Layout – 3 lectures

- Stimulate thought and discussion on the purpose of climate modelling.
  - What is climate prediction?
  - What is the role of ensembles? How can we interpret them?
- Take a dynamical systems perspective on climate models.
- Categorisation of uncertainties and ensembles:
  - Initial condition ensembles (ICEs)
  - Multi-Model Ensembles (MMEs)
  - Perturbed Physics Ensembles (PPEs)
- Discuss issues in the design of Perturbed Physics Ensembles.
- Consider issues of model weighting and model exclusion.

# Layout - Today

- The purpose of climate modelling
- Multidisciplinarity and climate modelling
- What do we mean by climate?
- A dynamical systems perspective and initial value sensitivity
- Lorenz 63; maps versus continuous systems
- L63 under changing forcing.

## Question: What's the purpose of climate modelling?

- Why are you are involved in climate modelling?
- What do you hope to achieve?

# Climate Change

Complexity (and confusion) between many disciplines

*Hydrology*

*Economics*

*Numerical Analysis*

*Human Geography*

*Non-linear  
dynamical systems  
mathematics*

*Psychology*

*Computer  
Science*

*Climate Modelling*

*Development  
Policy*

*Impact Modelling*

*Climate Physics*

*Philosophy*

*Adaptation  
planning*

*Statistics*

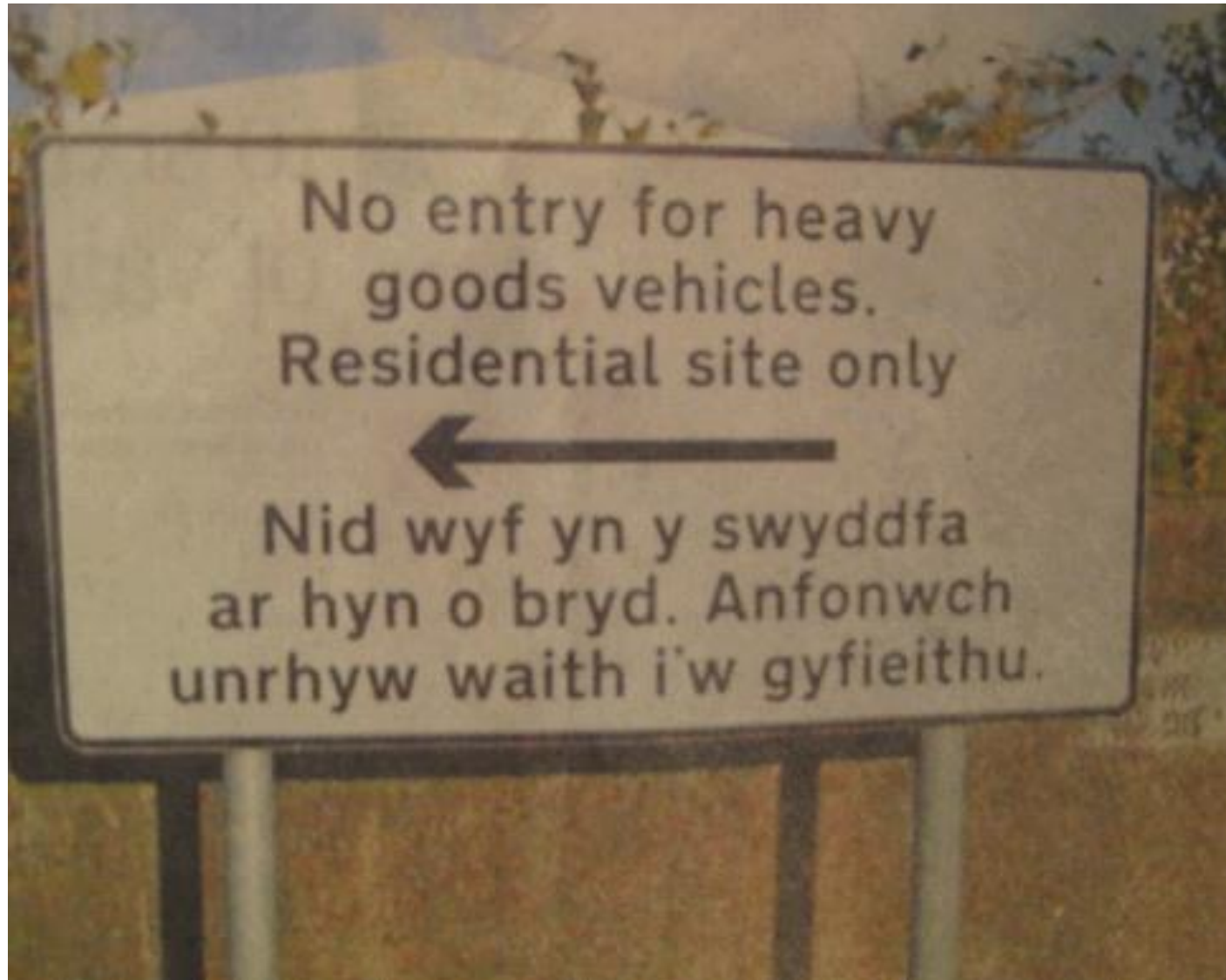
*Mitigation Policy*

*Climate chemistry*

*Agricultural science*

# Communication Can Be Difficult

There are disciplinary language problems



# Climate Change

Complexity (and confusion) between many disciplines

*Hydrology*

*Economics*

*Numerical Analysis*

*Human Geography*

*Non-linear  
dynamical systems*

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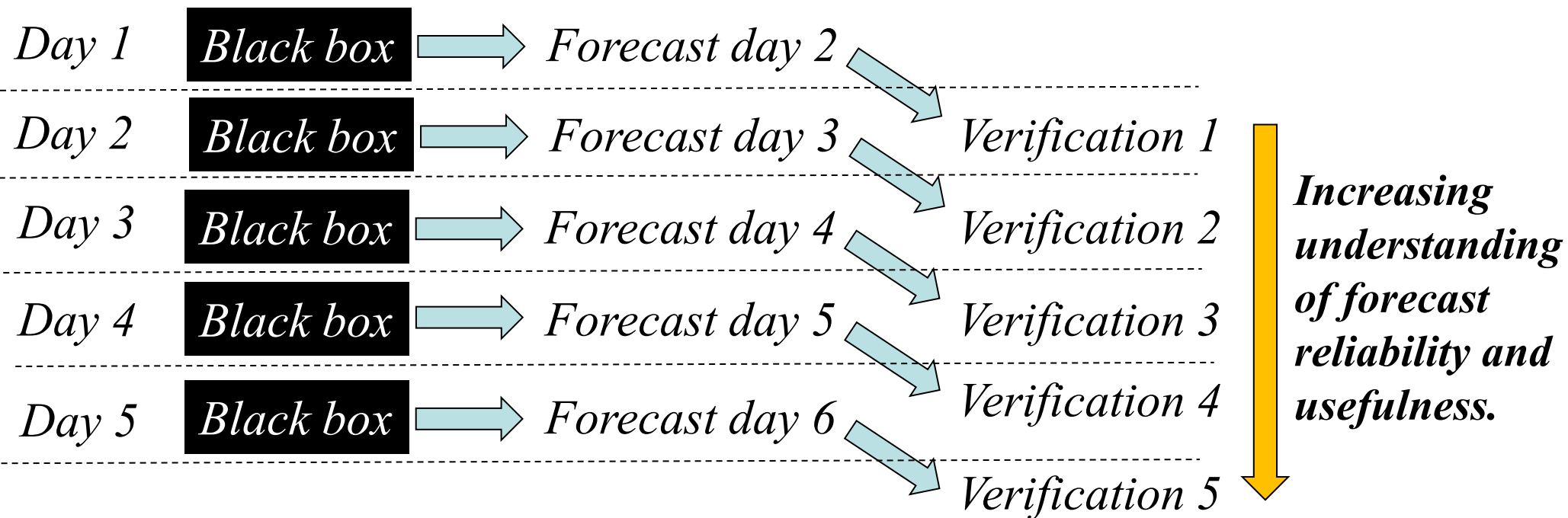
# Philosophy

- What is the relationship between a computer model and reality?
- What's the basis for making real world decisions on the results of computer models?
- What are the, often hidden, assumptions in model based results?
- Plus mathematical issues:  
A system with constant parameters may have an attractor. For a system with changing parameters the term “attractor” has no meaning so what is the object we are trying to study?



# Statistics of forecast confirmation: weather

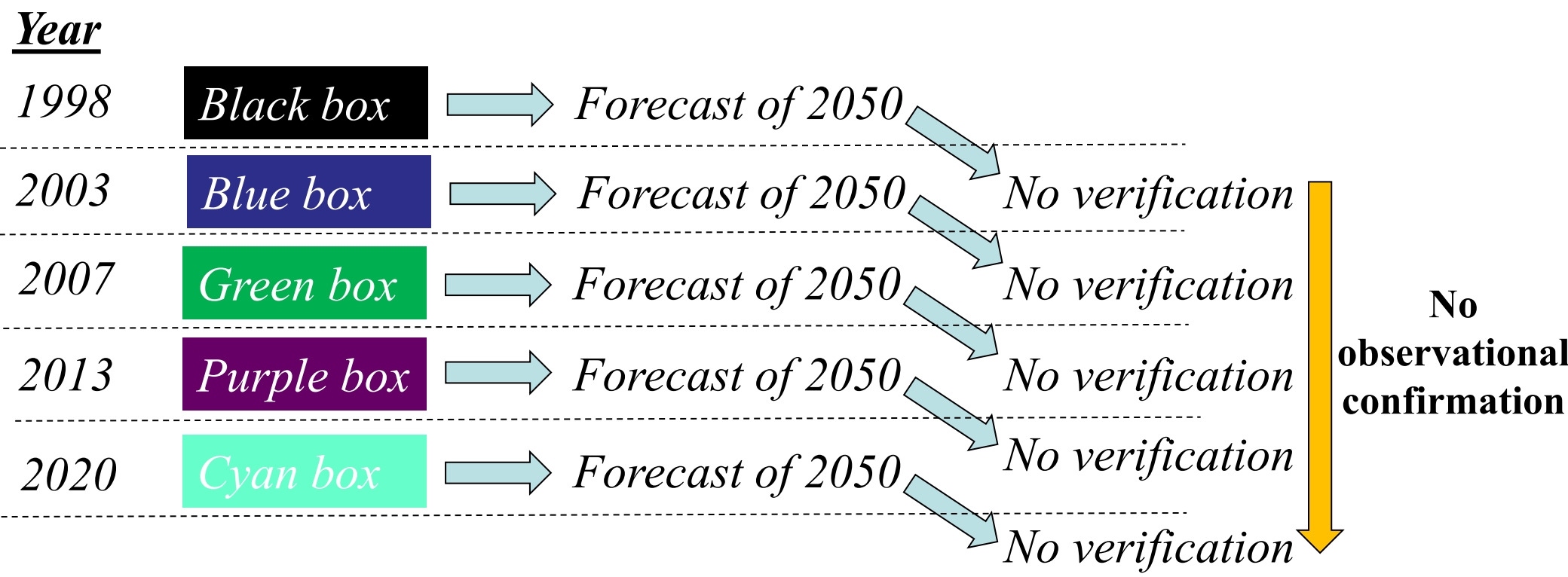
- Consider daily weather forecasts
- If there is a cycle of forecast and verification then we might choose to trust a forecast even if it comes from a black box. i.e. even if we know no details of its physical basis; maybe it has none.



- Many forecast-verification pairs enables the assessment of a forecast **in distribution**.  
i.e. to what extent are probability forecasts correct.

# Statistics of forecast confirmation climate

- In climate change prediction there is no cycle of forecast and verification
- So the physical basis of the model is the **fundamental basis** for any conclusions we draw about the real world.
- That's fine if we have a perfect model.
- It is arguably fine if we have empirically adequate models i.e. they are consistent with all past observations.
- If not ....



# Statistics of forecast confirmation climate

Model-based climate forecasts are **not** subject to the same out-of-sample confirmation as forecasts like: weather, stock markets, sports players/teams.

1. They are long term (distant in the future) so we have **no** observations, and
2. The models keep changing.  
(So even if we were only interested in 5 year forecasts we'd still only have a maximum of 1 verification point.)

Question: What is climate?

# What is climate?

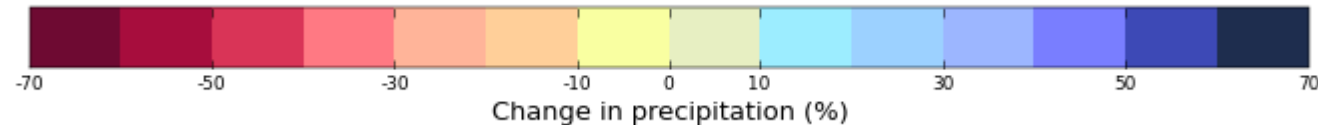
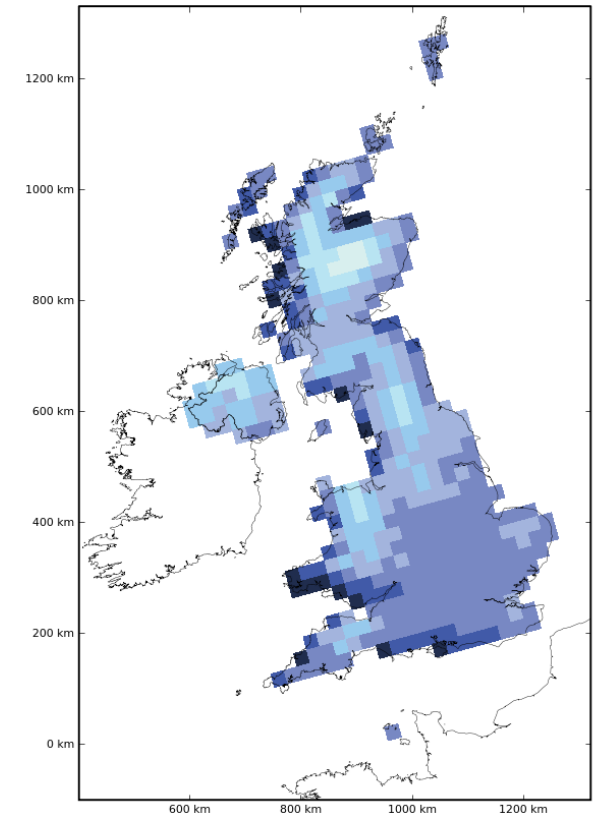
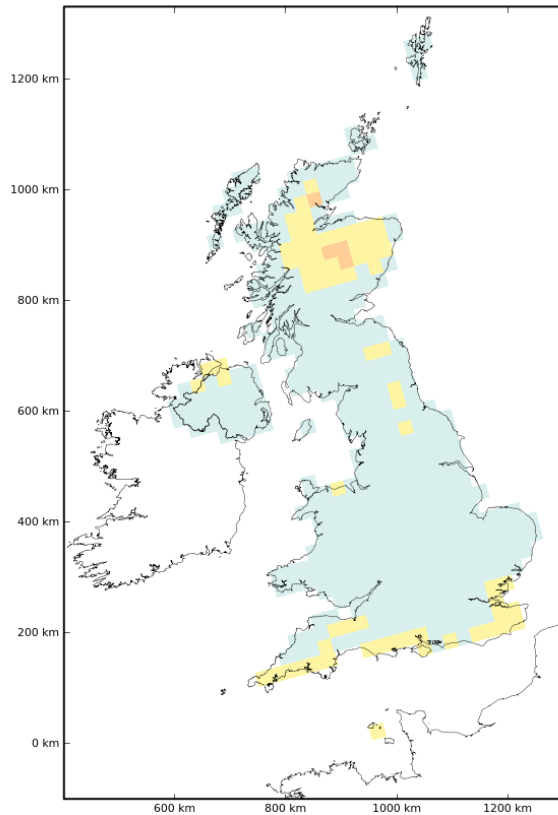
## UK Climate Projections 2009 (UKCP09): Mean Winter Precipitation

December / January /  
February average.

Taken over 30 years.

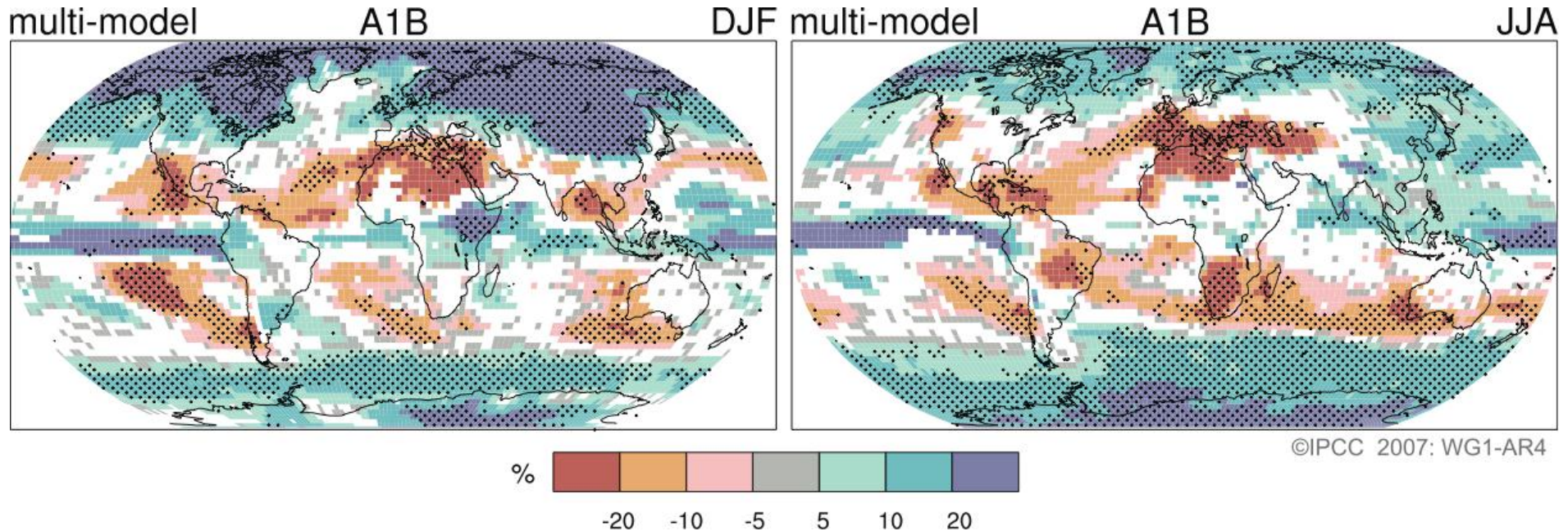
2080s : 10% probability level:  
very unlikely to be less than

2080s: 90% probability level:  
very unlikely to be greater than



*Source: UKCP09 website*

# Many Means



**Source: IPCC AR4 WG1 Summary for Policy Makers**

**Figure SPM.7.** Relative changes in precipitation (in percent) for the period 2090–2099, relative to 1980–1999. Values are multi-model averages based on the SRES A1B scenario for December to February (left) and June to August (right). White areas are where less than 66% of the models agree in the sign of the change and stippled areas are where more than 90% of the models agree in the sign of the change. {Figure 10.9}

*Seasonal means, multi-year means, multi-model means.*

# Climate definitions

American Meteorological Society – 2002:

***Climate*** - The **slowly varying aspects** of the atmosphere–hydrosphere–land surface system. It is typically characterized in terms of suitable **averages of the climate system** over periods of a month or more, taking into consideration the variability in time of these averaged quantities. Climatic classifications include the spatial variation of these **time-averaged variables**.

Intergovernmental Panel on Climate Change – 2007:

***Climate*** - Climate in a narrow sense is usually defined as the ‘**average weather**’, or more rigorously, as the **statistical description in terms of the mean and variability** of relevant quantities over a period of time ranging from months to thousands or millions of years. [...] Climate in a wider sense is the state, **including a statistical description**, of the climate system. The classical period of time is 30 years, as defined by the World Meteorological Organization.

# Climate has changed

## It was different when my parents were young

American Meteorological Society – 1959:

***Climate*** - “The synthesis of weather” (C.S. Durst); the long term manifestations of weather, however they may be expressed. More rigorously, the climate of a specified area is represented by the statistical collective of its weather conditions during a specified interval of time (usually several decades).

Climatology, A. Austin Miller, Sixth edition. Methuen & Co London. 1931:

Weather types are the integrals which go into the make up of the climate whole and there is a danger of losing their individuality unless climate is carefully examined, as it were microscopically, to appreciate its texture. Average figures create an illusion of steadiness and uniformity which is seldom justified by the facts; the study of weather types provides the corrective.



# Climate as a Distribution

- Dice

# Climate under climate change – a changing distribution

- More dice

Climate is not an average  
Neither is it an average, plus skew, plus kurtosis.

- The distributions are multi-variate and often far from Gaussian.

# Climate as a Distribution

## Climate Change as a Changing Distribution

- Scientifically this is nice.
- But there will only ever be one 21<sup>st</sup> century.

# Statistics of forecast confirmation climate

Model-based climate forecasts aren't subject to the same out-of-sample confirmation as forecasts like: weather, stock markets, sports players/teams.

1. They are long term (distant in the future) so we have **no** observations, and
2. The models keep changing.  
(So even if we were only interested in 5 year forecasts we'd still only have a maximum of 1 verification point.)
3. **Under climate change in the 21<sup>st</sup> century there will only ever be one verification point.**  
**If the blue dice are climate in 2050 – we'll only ever have one throw.**  
**Statistical forecast confirmation is not on the cards..**

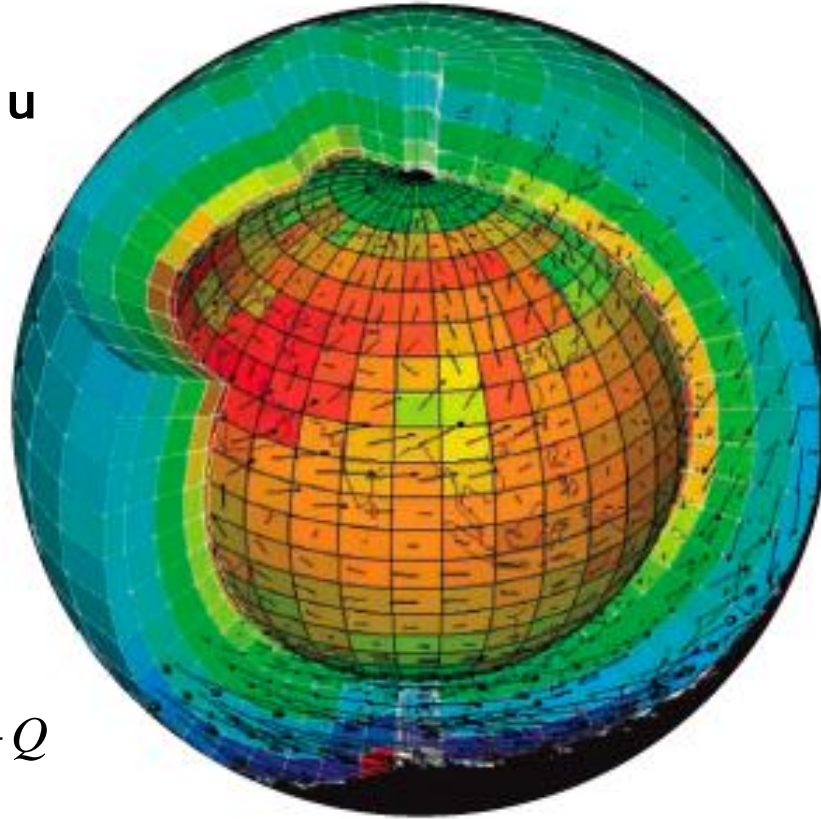
# Equations of Motion

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}$$

$$\frac{D\mathbf{u}}{Dt} = \frac{-\nabla p}{\rho} - 2\boldsymbol{\Omega} \times \mathbf{u} - \mathbf{g} - \frac{\eta}{\rho} \nabla^2 \mathbf{u}$$

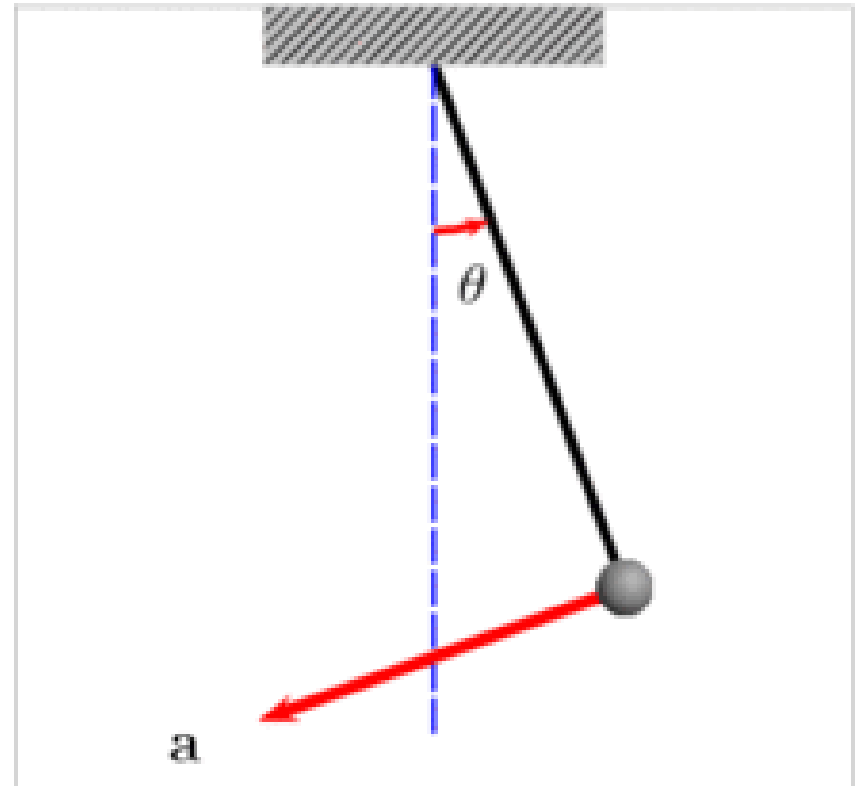
$$c_p \frac{DT}{Dt} = \frac{1}{\rho} \frac{Dp}{Dt} + Q$$

$$p = \rho RT$$



Some simple equations are simple to solve

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$



*Image source: wikipedia*

Coupling simple equations can lead to complex behaviour





Coupling simple equations can lead to complex behaviour



# Climate as a Nonlinear Dynamical System

- Simple equations when coupled can lead to complex, indeed chaotic, behaviour.

# What do we expect from massively coupled equations?

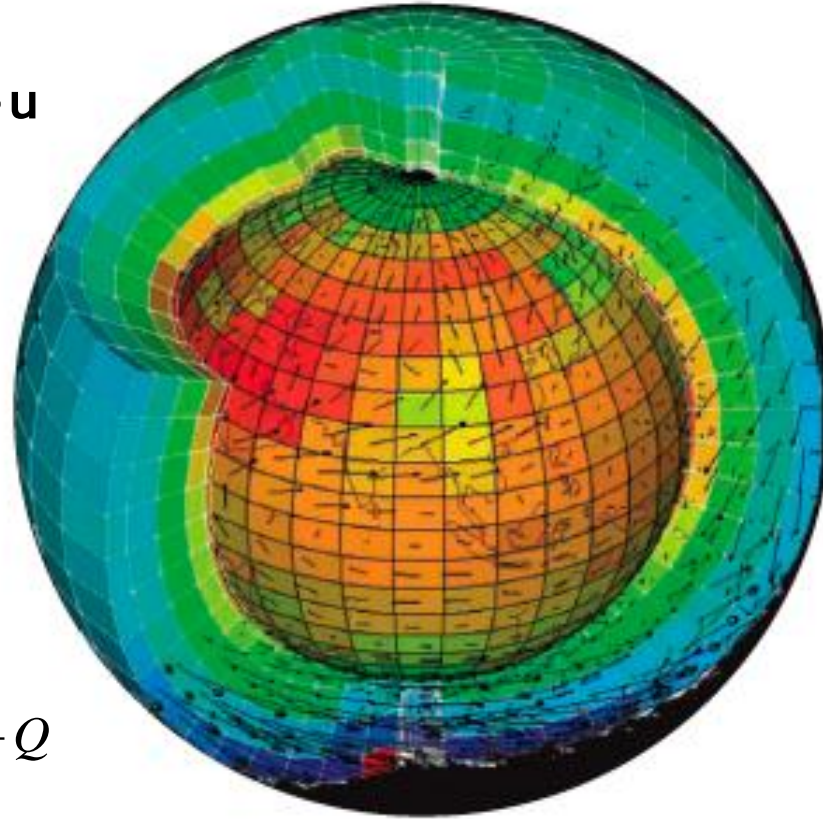
## Complex Climate Models / Global Circulation Models (GCMs)

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u}$$

$$\frac{D\mathbf{u}}{Dt} = \frac{-\nabla p}{\rho} - 2\boldsymbol{\Omega} \times \mathbf{u} - \mathbf{g} - \frac{\eta}{\rho} \nabla^2 \mathbf{u}$$

$$c_p \frac{DT}{Dt} = \frac{1}{\rho} \frac{Dp}{Dt} + Q$$

$$p = \rho RT$$

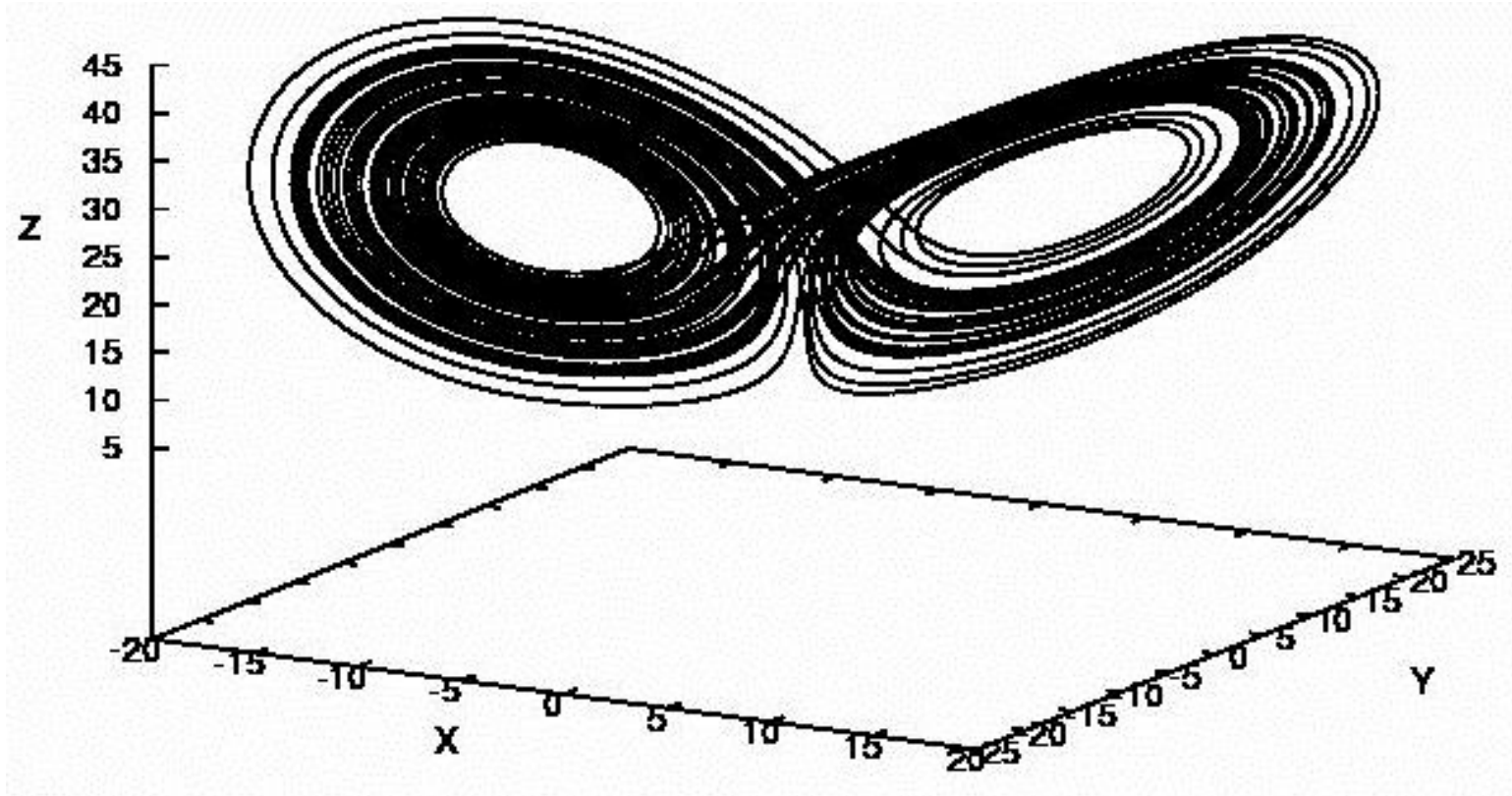


# Equations on Computers

- Computers don't solve continuous equations.
- Under what circumstances do we expect the numerical solution to approximate the continuous equations?

# What can we learn from simpler models?

## State Space and the Attractor of Lorenz '63



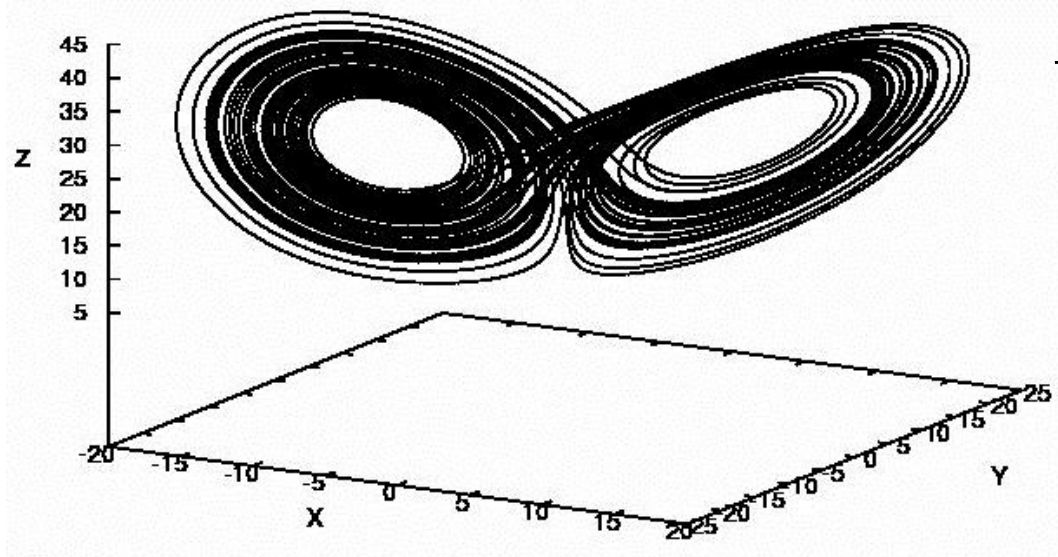
$$\frac{dx}{dt} = -\sigma x + \sigma y$$

$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

# What can we learn from simpler models?

## State Space and the Attractor of Lorenz '63



$$\frac{dx}{dt} = -\sigma x + \sigma y$$

$$x_{n+1} = x_n + \tau(-\sigma x_n + \sigma y_n)$$

$$\frac{dy}{dt} = xz + rx - y$$

$$y_{n+1} = y_n + \tau(-x_n z_n + r x_n - y_n)$$

$$\frac{dz}{dt} = xy - bz$$

$$z_{n+1} = z_n + \tau(x_n y_n - b z_n)$$

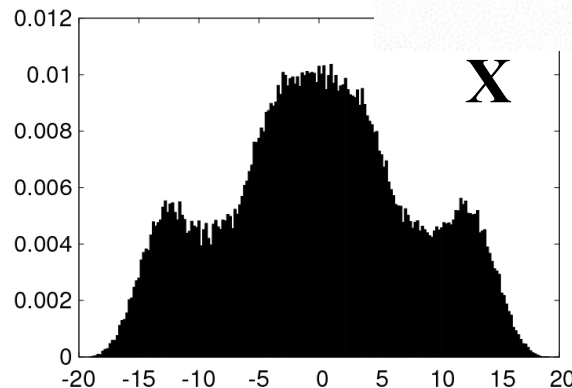
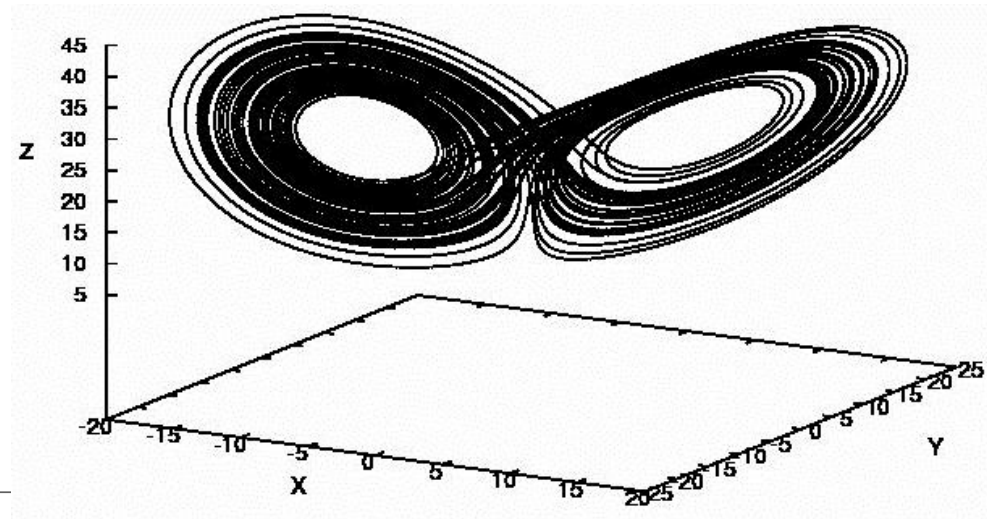
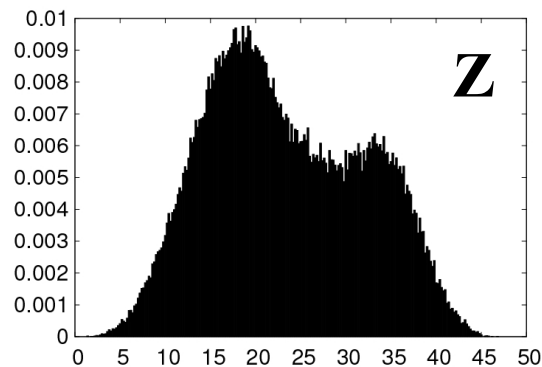
*Simple euler method used  
for illustration)*

# The State Space of a Climate Model

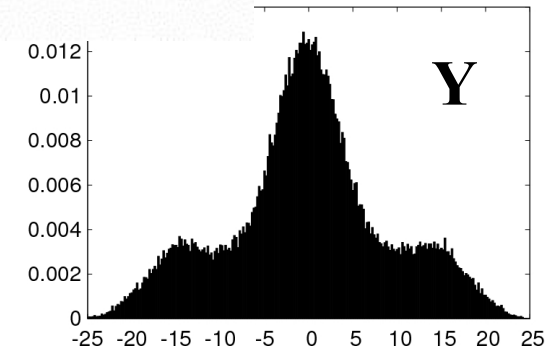
- $\sim 10^5$  atmospheric grid points
  - $\sim 10^5$  ocean grid points
  - $> 6$  physical variables at each grid point
  - ➔  $\sim 10^6$  dimensional state space.
- 
- We also have diurnal and seasonal cycles:
  - Number of timesteps in a year  $\sim 10^4$
  - ➔  $\sim 10^{10}$  dimensional state space
- 
- Our climate model attractor lives in a state space with dimensionality of this order

# Climates of a Mathematical Model

- The states representing the natural invariant measure .
  - For some systems this may be an attractor.
  - For others there may be no attractor but some constraining manifold.



**Results from Joe  
Daron's thesis.  
Papers in preparation**





# What is climate?

– Smith and Stainforth, paper in preparation.

- Climate is the collection (distribution) of all states of the Earth System consistent with properties of a state of the system on a given date. Given the divergence of physically possible trajectories, this distribution is expected to grow richer as the reference date recedes into the past.

“If anyone knows anything about anything it is Owl who knows something about something..”

*Winnie the Pooh, The House at Pooh Corner, A.A. Milne*

