Towards mathematically consistent air-sea coupling

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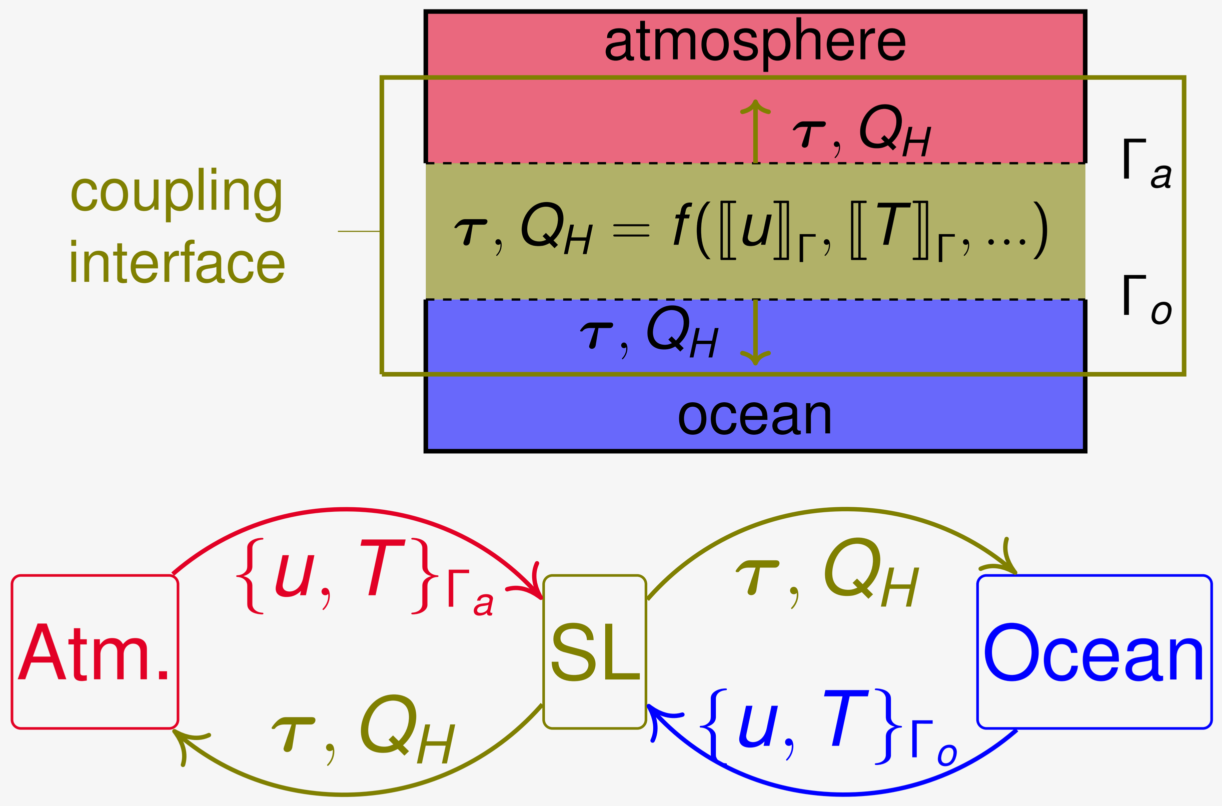
- Proper account of coupling mechanisms numerically improves both *long-term* and *extreme event* predictions.
- To our knowledge, theoretical studies of coupling embracing sub-grid eddy parameterizations have not been carried out yet.
- Our aim is to develop *Schwarz coupling algorithms* that would take into account these parameterizations, be non-intrusive, have low computational costs, and guarantee essential mathematical properties.
- Focus is on 1D air-sea columns in the Planetary Boudary Layer (PBL), where turbulent effects are due to the common interface. Both subdomains are separated by the Surface Layer (SL), where solutions are extended as log-profiles.

1. Balanced fluxes



Balance fluxes $\leftrightarrow$ guarantee that the same turbulent stress and heat latent fluxes are used for:

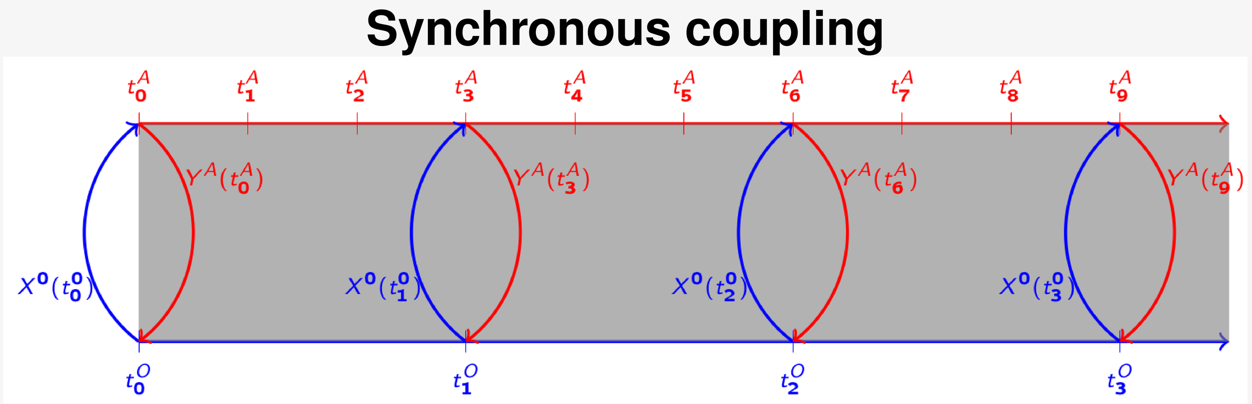
1. enforcing both submodels at the interface with the SL
2. computing those turbulent fluxes within the SL

Turbulent fluxes ought to be continuous through the interface.

2. Coupling non stationary problems

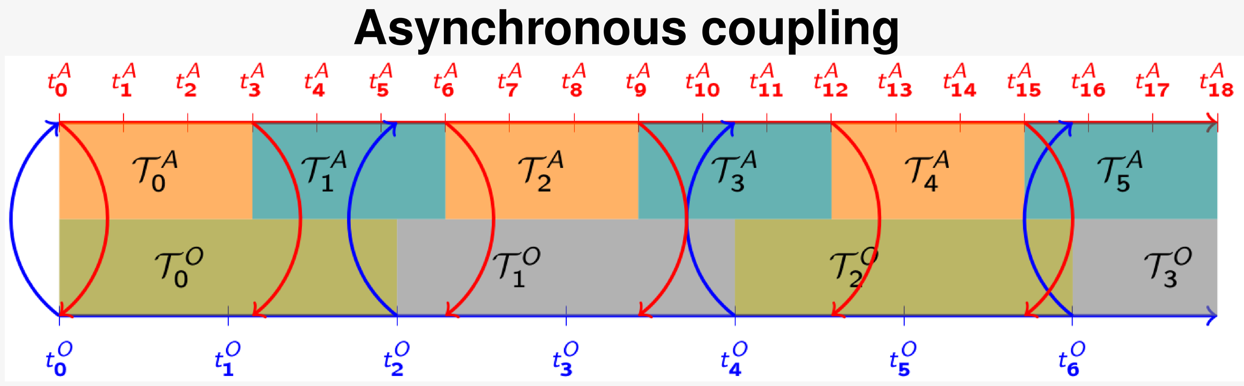
Without iterating, 2 ways of managing time inconsistency.

Synchronous coupling



- numerically unstable
- relies on instant fluxes
- conditional stability at best (see [Lemarié et al., 2015])

Asynchronous coupling



"Time windows" during which fluxes are considered constant

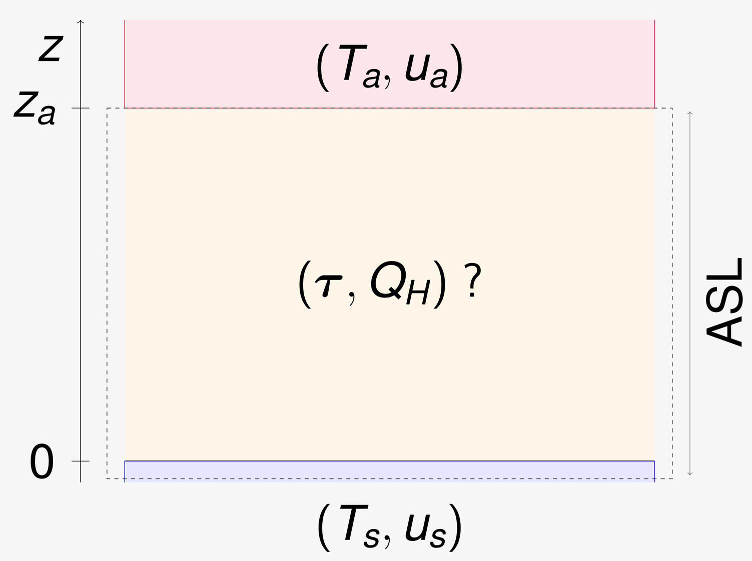
- more flexible & reliable
- still no numerical stability guarantees

Schwarz **iterative** algorithms are good candidates for coupling both subdomains in a stable way while being non-intrusive.

Need to study the BCs at the interface
How are turbulent fluxes computed?

3. *Bulk formulae*

Computing τ and Q_H ([Monin and Obukhov, 1954]):



Deemed constant in the "atmospherical surface layer" (ASL), through (u^*, θ^*) scales.

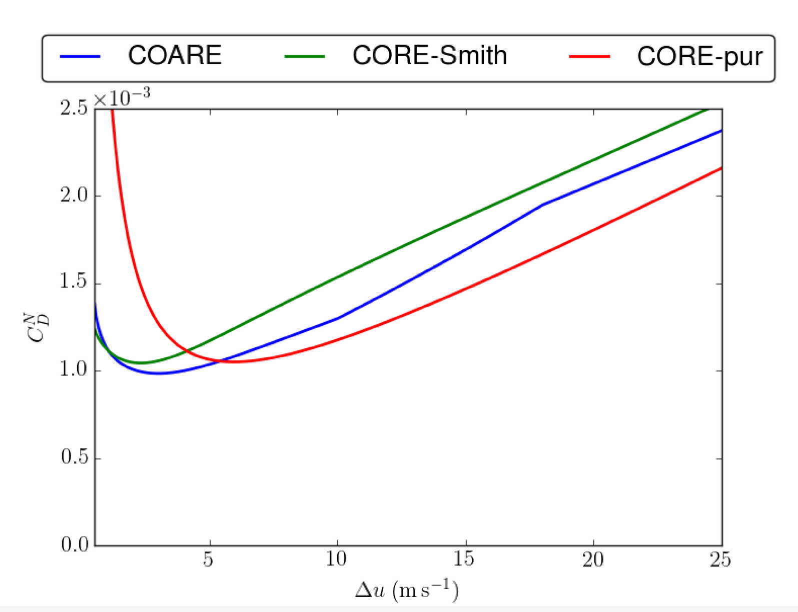
$$\tau = \rho\, u^{*2}\mathbf{u_h}/|\mathbf{u_h}|$$
$$Q_H = \rho c_p\, u^*\theta^*$$

$$u^{*2} = C_D(\delta u, \zeta) \times (\delta u)^2$$
$$u^*\theta^* = C_H(\delta u, \zeta) \times (\delta u)\,(\delta T) \qquad (*)$$

where $\delta x := x(z_a) - x_s$, $\zeta = \zeta(u^*, \theta^*)$: "stability parameter"

$\mathbf{C_D}$, $\mathbf{C_H}$: **" transfer coefficients "**

$(*)$ is a difficult fixed-point problem, depending on which "bulk" is chosen:

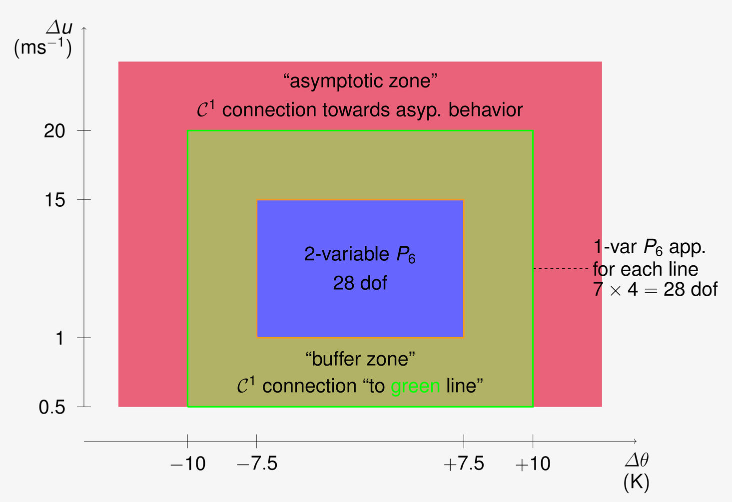


To our eyes, a "good bulk" would be:

- explicit
- cheap
- modular

Fig: bulks taken from [Large, 2006] and [Fairall et al., 2002].

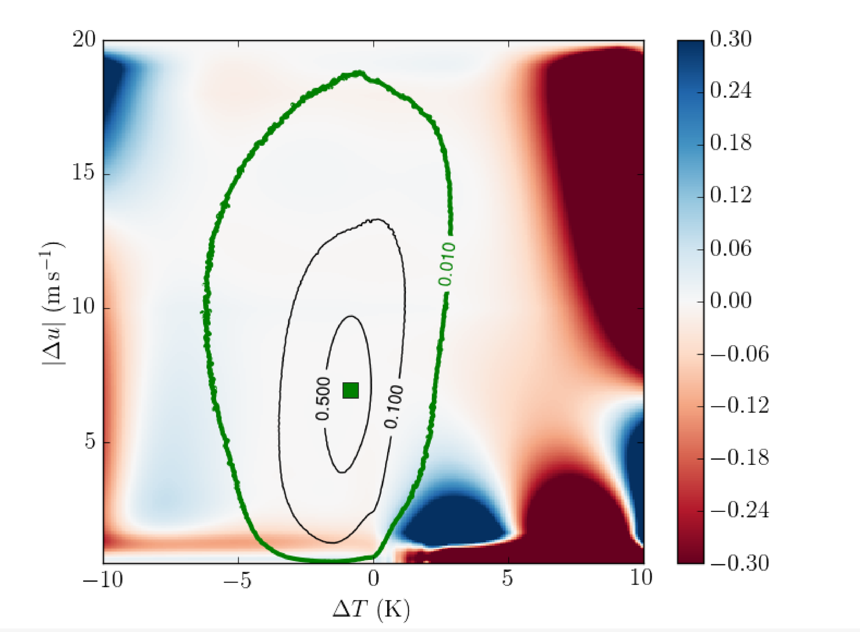
4. Polynomial fit



"Blue zone" ($\sim$ most frequent phys. config.):

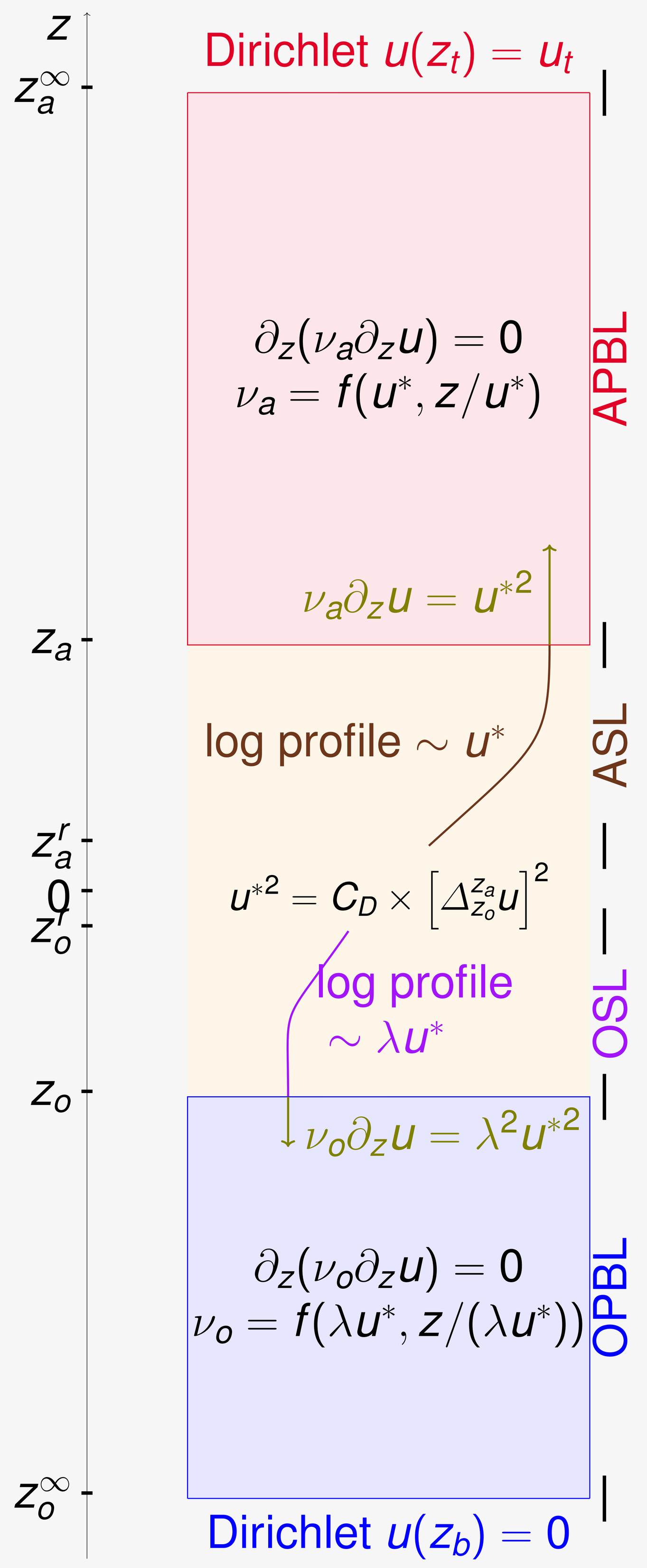
$$C_X = \sum c_{k,l}(\delta u)^k(\delta T)^l$$
$$X \in \{D, H\}$$

Error: $O(1\%)$ for $> 99\%$ of physical settings
$\rightarrow$ fast, **explicit**, modular bulk formula



Green level lines: physical configuration statistical quantile regions.

5. Ekman layer fully-coupled problem



Hyper idealized 1D case, including both sides of the PBL with 2 Dirichlet BCs at their respective ends, separated by the SL.

Physics is reduced to a stationary homogeneous heat equation:

$$\partial_z(\nu_\alpha\partial_z u) = 0, \ \alpha \in \{O, A\}$$

Aim: use the same u^* turbulent scale for the whole PBL, in both subdomains **and** for through the SL *via* the bulk formula.

Continuity of fluxes: $u_o^* = \lambda u^*$ where $\lambda := \sqrt{\rho_a/\rho_o}$.

Consistency equation:

$$\frac{u^{*2}}{C_D} = \left[\delta\overset{\infty}{u} - u^{*2}\left(\int_{z_a}^{z_t}\frac{\mathrm{d}z}{\nu_a\left(u^*, \frac{fz}{cu^*}\right)} + \lambda^2\int_{z_b}^{z_o}\frac{\mathrm{d}z}{\nu_o\left(\lambda u^*, \frac{-fz}{c\lambda u^*}\right)}\right)\right]^2 \qquad (\mathcal{E})$$

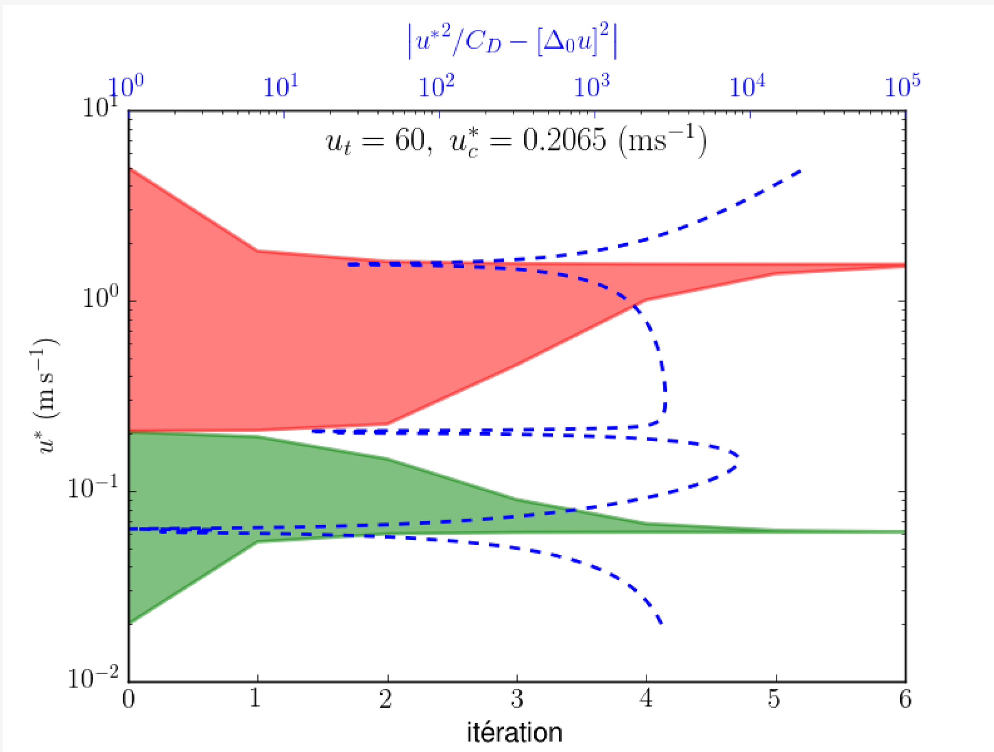
Adjusted KPP-profiles for ν (see [McWilliams and Huckle, 2006])

Solving $(\mathcal{E})$ for u^* requires "inverting" $u^* \mapsto \int \nu^{-1}(u^*)$, which is prohibitive.

6. An iterative algorithm

Distinguish **two** u^* scales:

- u_ν^*: used as an input for computing turbulent viscosities
- u_{bc}^*: used for enforcing turbulent BCs on both subdomains



1. define $u_{\nu,0}^*$
2. compute $\int \nu^{-1}$ with that $u_{\nu,0}^*$
3. compute $u_{bc,0}^*$ so that $(\mathcal{E})$ is satisfied (polynomial equation)
4. define $u_{\nu,1}^* := u_{bc,0}^*$
5. iterate

Result: fast convergence ($\sim$ 5 iterations) towards 2 possible u^* values, depending on $u_{\nu,0}^*$. Both satisfy $(\mathcal{E})$.

Red value $u^* \sim 1.5\ \mathrm{m\,s^{-1}}$ is non-physical: almost all u variation would be concentrated within the SL.

References

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