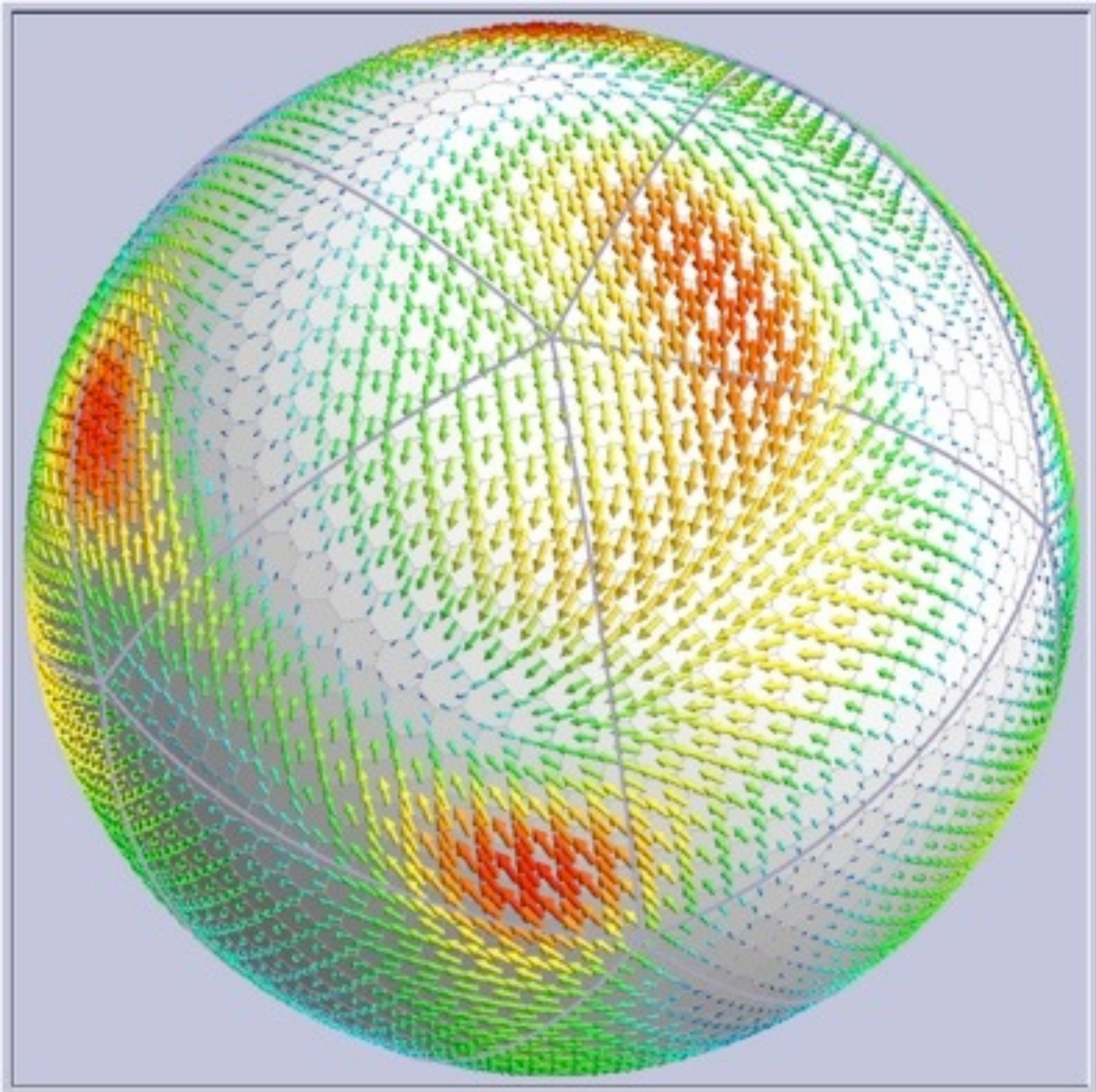
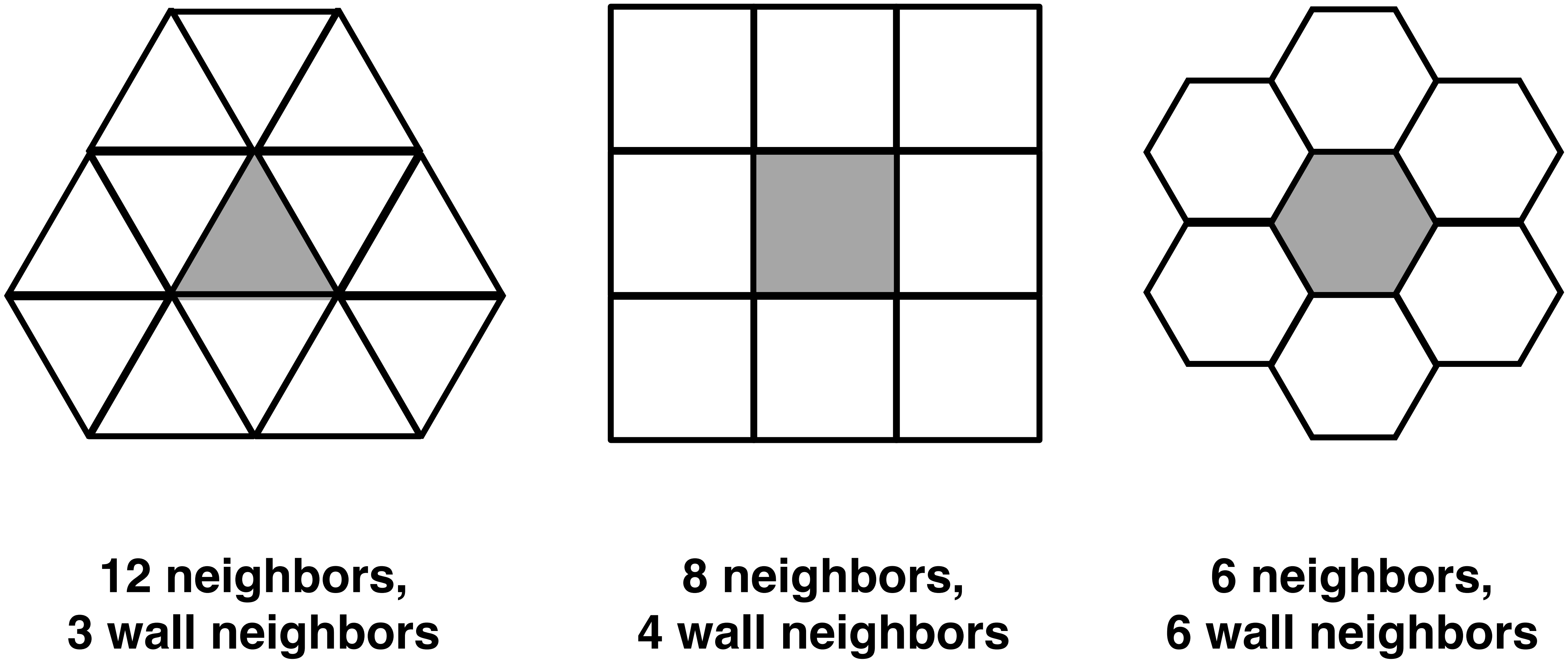
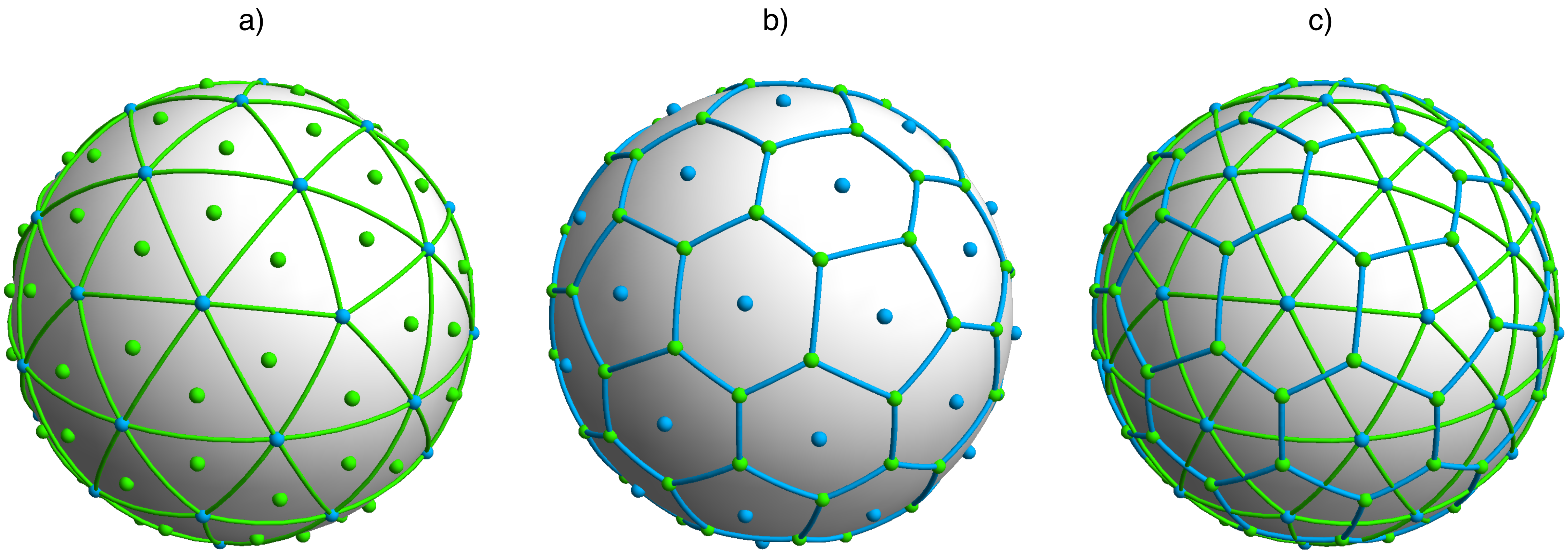
# Unified Zgrid Icosahedral Model



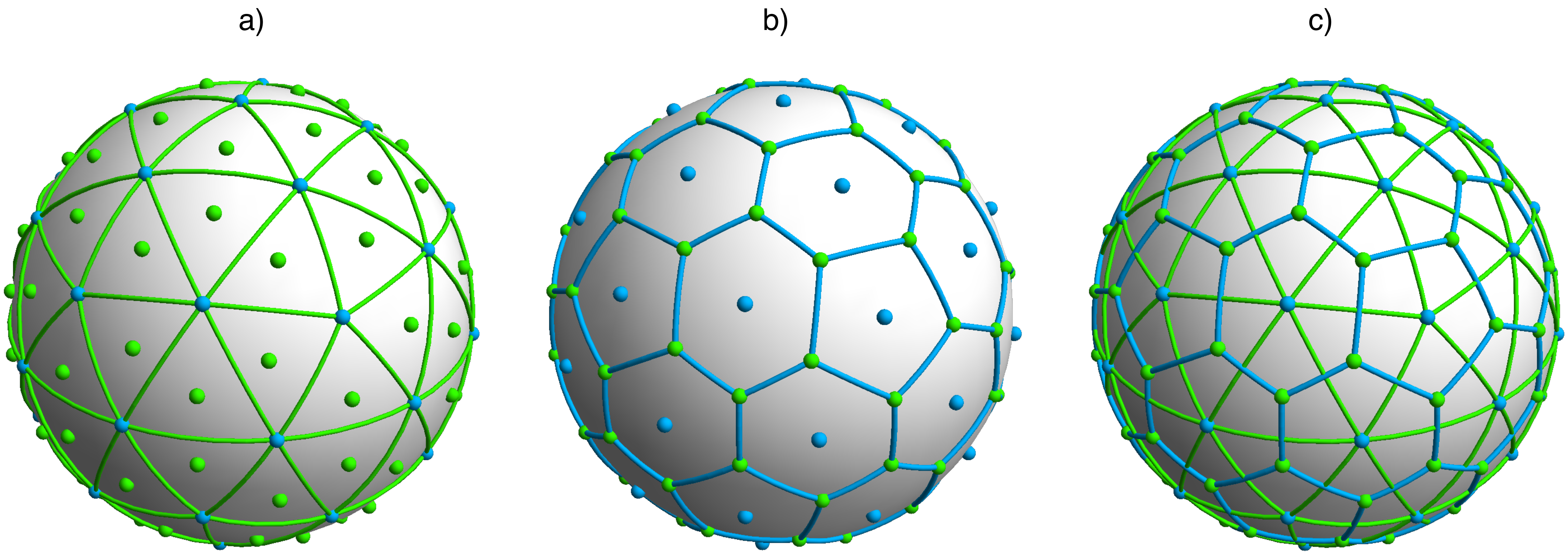



**We choose the hexagonal grid because of its high degree of symmetry.**

# Grid generation



# Grid generation

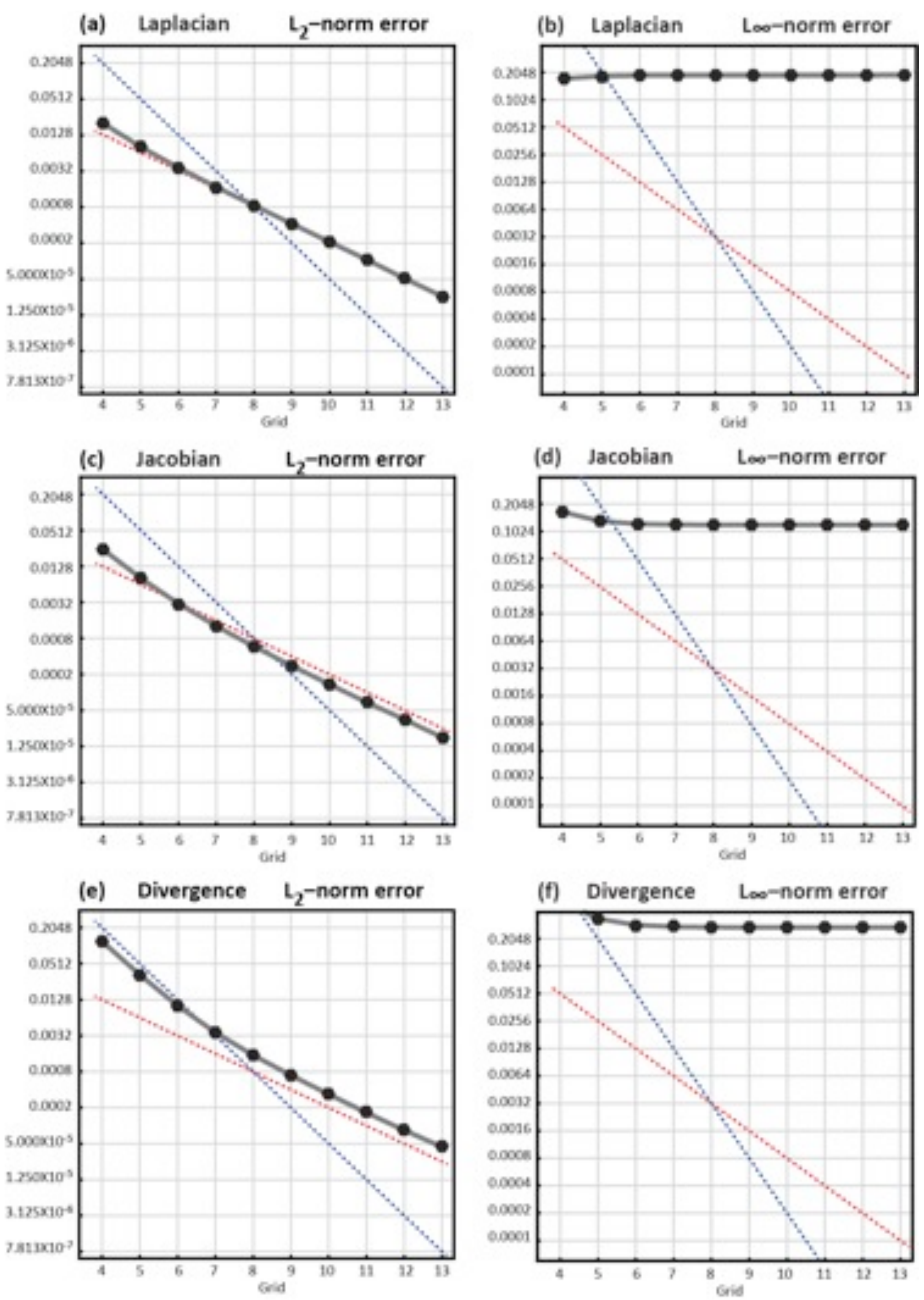



**The grid is optimized using a variational method. The goal of the optimization is to minimize the distance between the mid-point of the cell wall and the point where the grid segment intersects the cell wall, by displacing the cell centers relative to those of the raw grid.**

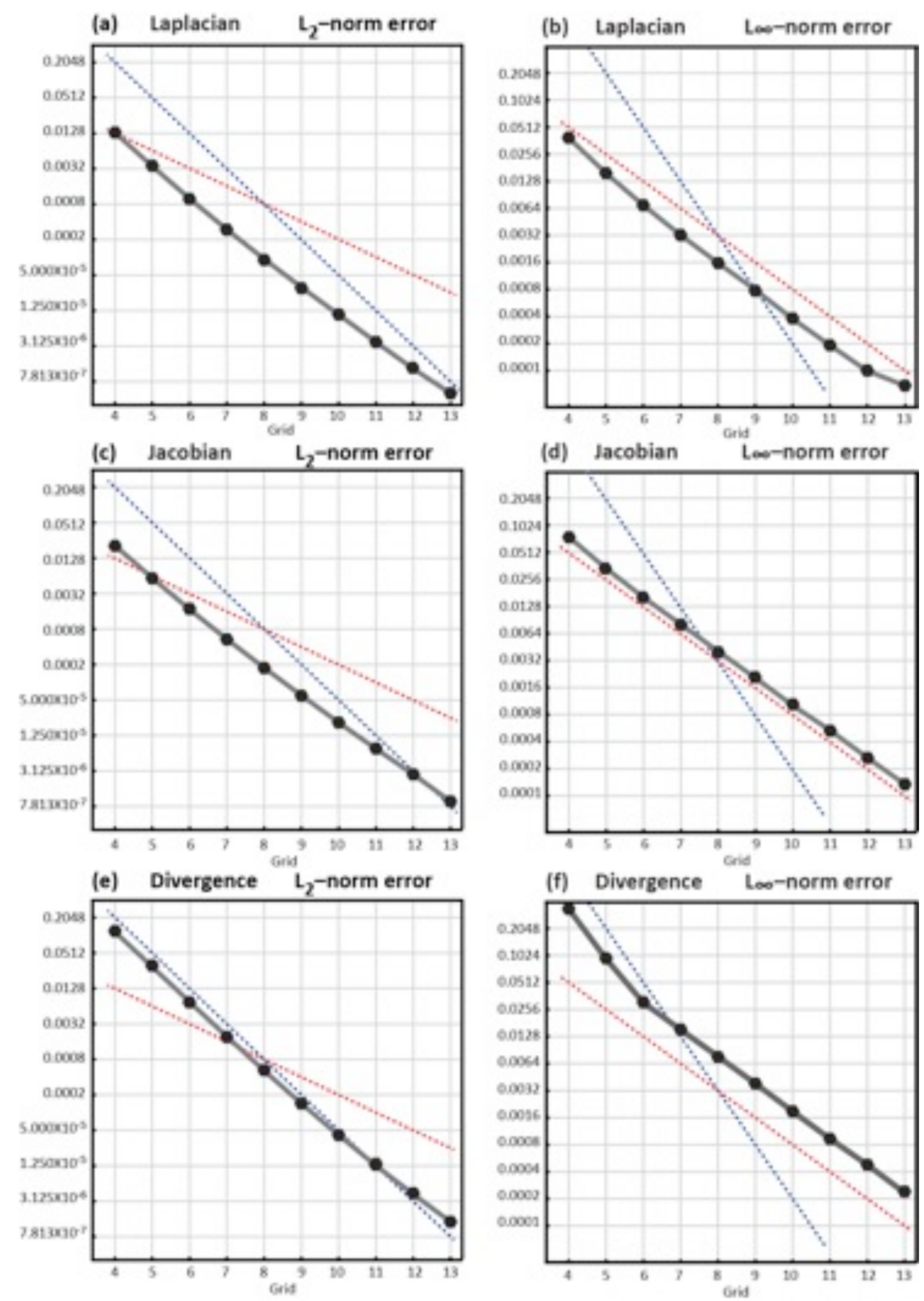
# Some grids of interest

| Level of recursion | Number of grid columns | Distance between grid columns, km |
|---|---|---|
| 9 | 2,621,442 | 15.64 |
| 10 | 10,485,762 | 7.819 |
| 11 | 41,943,042 | 3.909 |
| 12 | 167,772,162 | 1.955 |
| 13 | 671,088,642 | 0.977 |

# Optimization improves the accuracy.




**Raw grid**




**Optimized grid**

Z

$$\frac{\partial u}{\partial t} - fv + g\frac{\partial h}{\partial x} = 0$$

$$\frac{\partial v}{\partial t} + fu + g\frac{\partial h}{\partial y} = 0$$

$$\frac{\partial h}{\partial t} + H\delta = 0$$

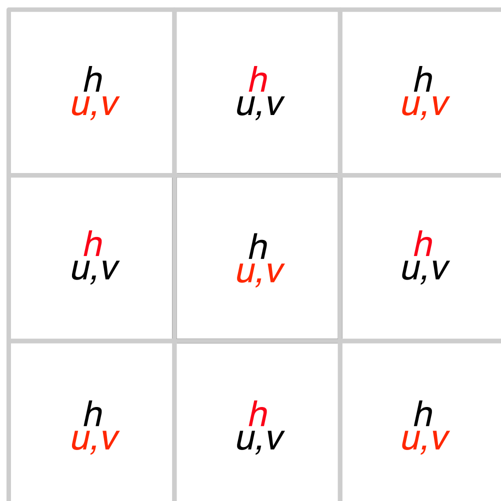
---

$$\frac{\partial \delta}{\partial t} - f\zeta + g\left(\frac{\partial^2}{\partial x^2}h + \frac{\partial^2}{\partial y^2}h\right) = 0$$

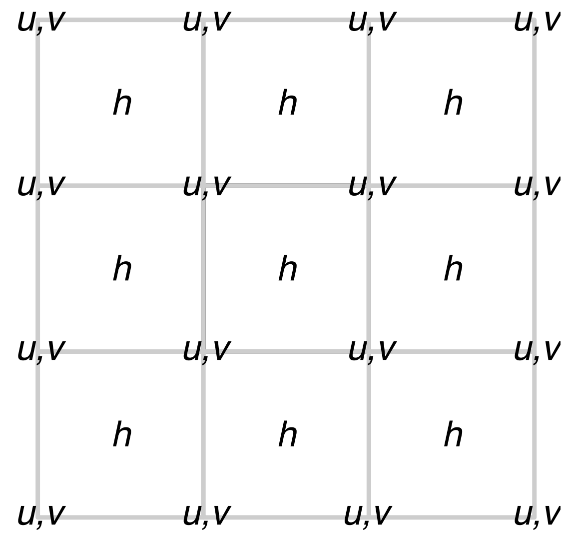
$$\frac{\partial \zeta}{\partial t} + f\delta = 0$$

$$\frac{\partial h}{\partial t} + H\delta = 0$$

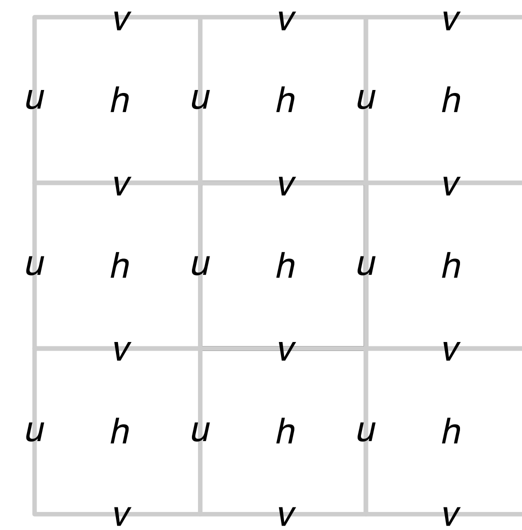
# Arranging the variables on square grids



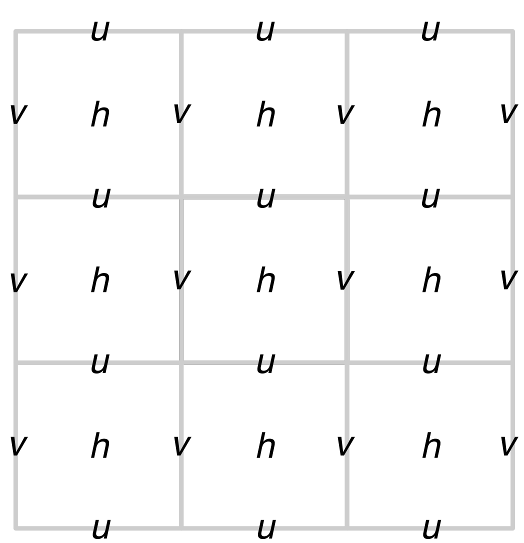

**A grid**



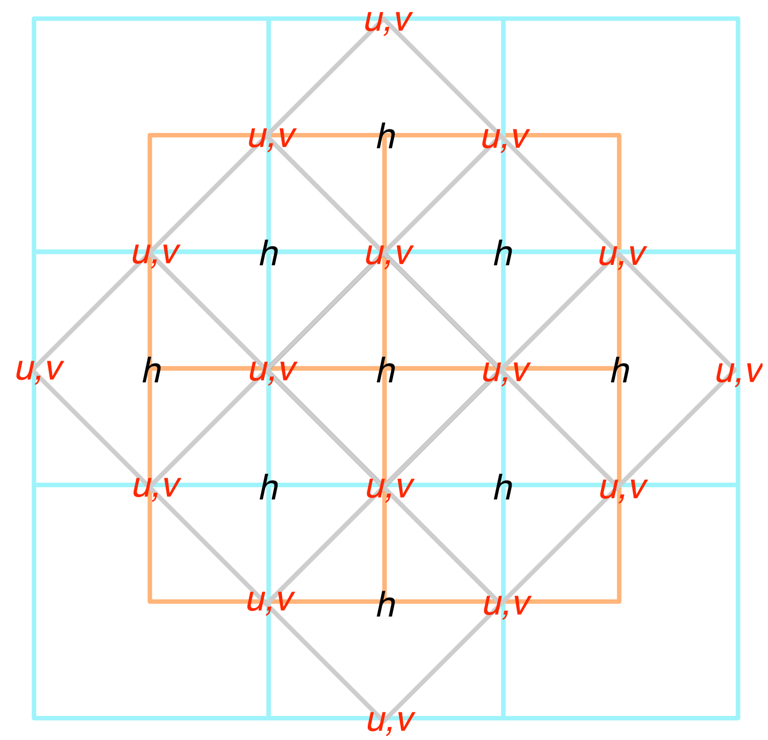

**B grid**



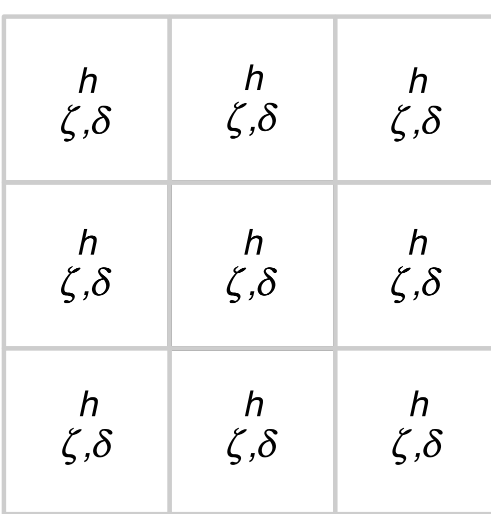

**C grid**




**D grid**

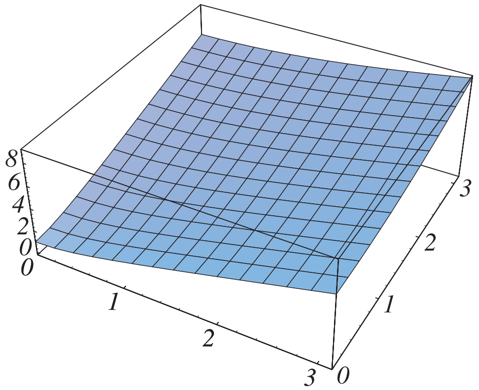



**E grid**



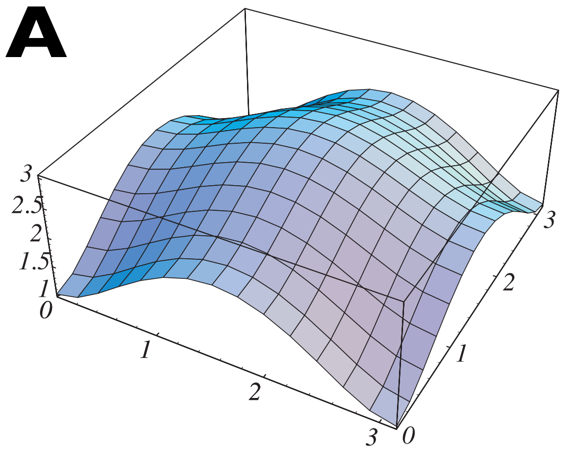

**Z grid**

**Right answer**

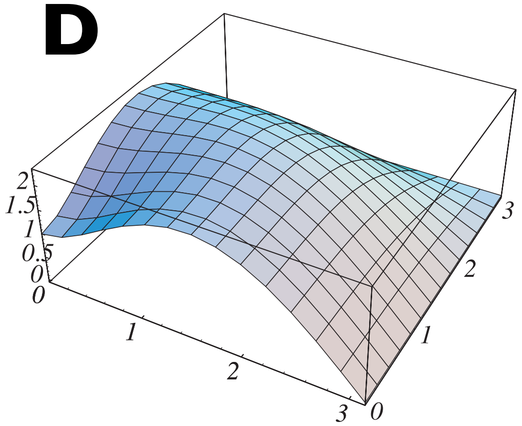


**These dispersion plots show how well (or how badly) geostrophic adjustment works on the various grids.**

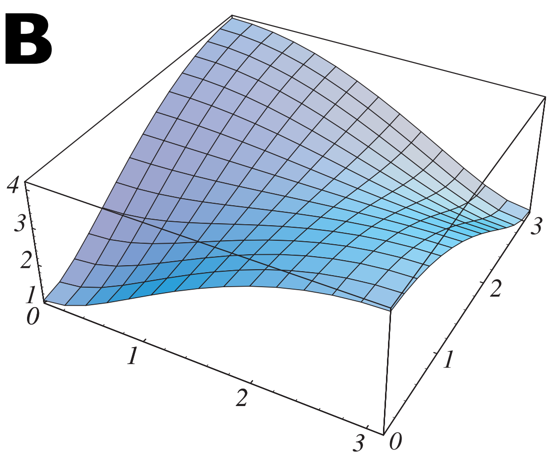
**A**



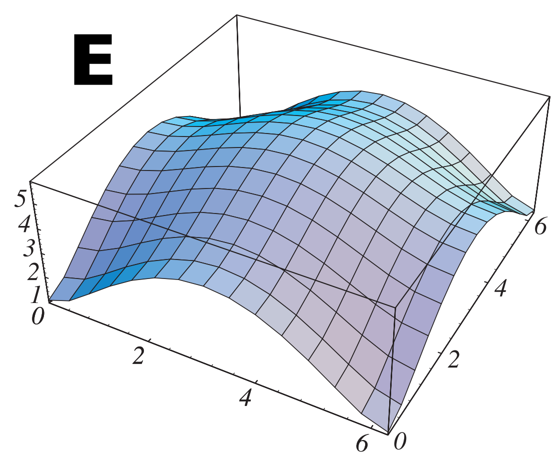
**D**



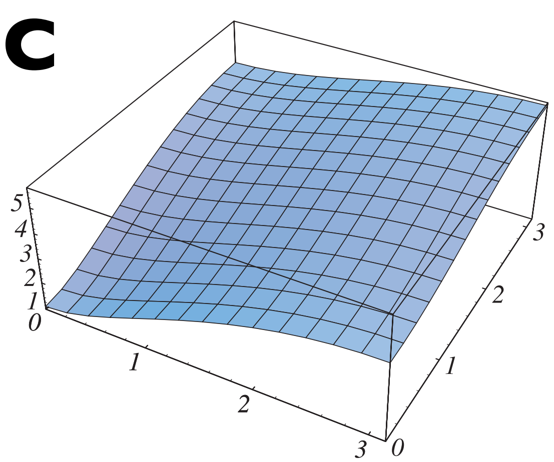
**B**



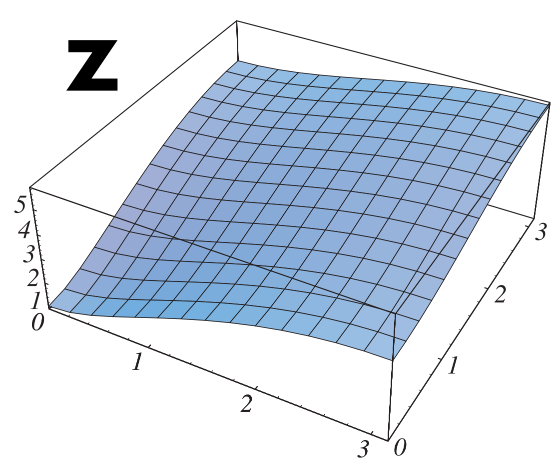
**E**



**C**



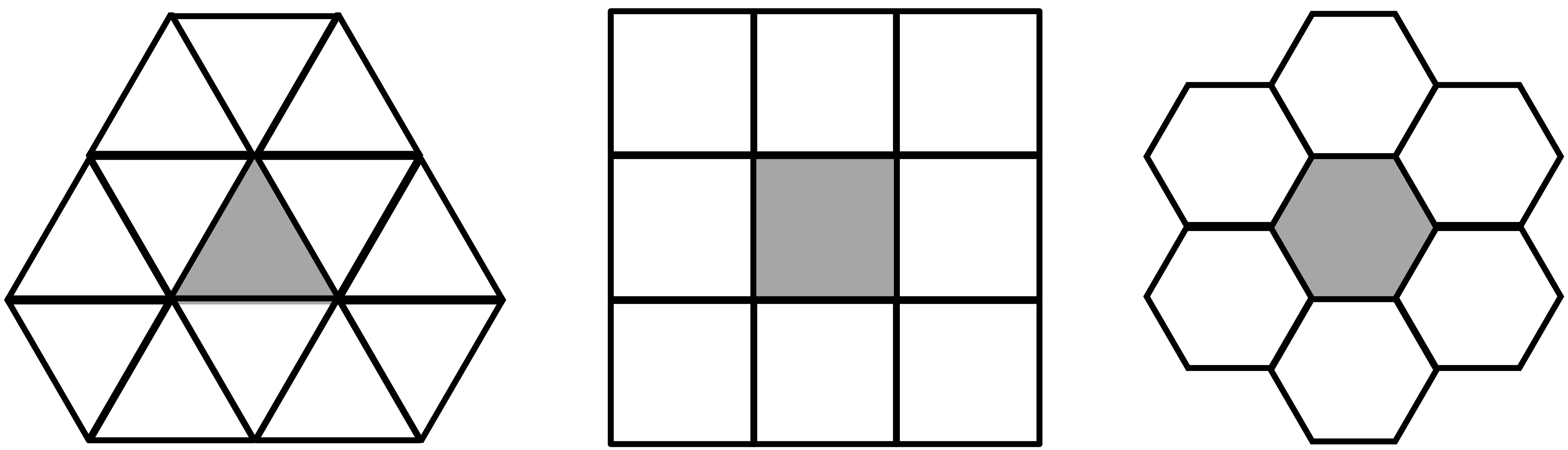
**Z**



**The two best choices are C and Z.**

# Generalization

**A "C" grid places the normal components of the winds on the walls of mass cells.**



**A C grid has computational modes on triangles and hexagons, but not on squares.**

# We choose the Z grid because:

$\zeta = \nabla^2 \psi$

- **It does a good job with geostrophic adjustment, and**
- **It has no computational modes.**

# We choose the Z grid because:

$\zeta = \nabla^2 \psi$

- **It does a good job with geostrophic adjustment, and**

- **It has no computational modes.**

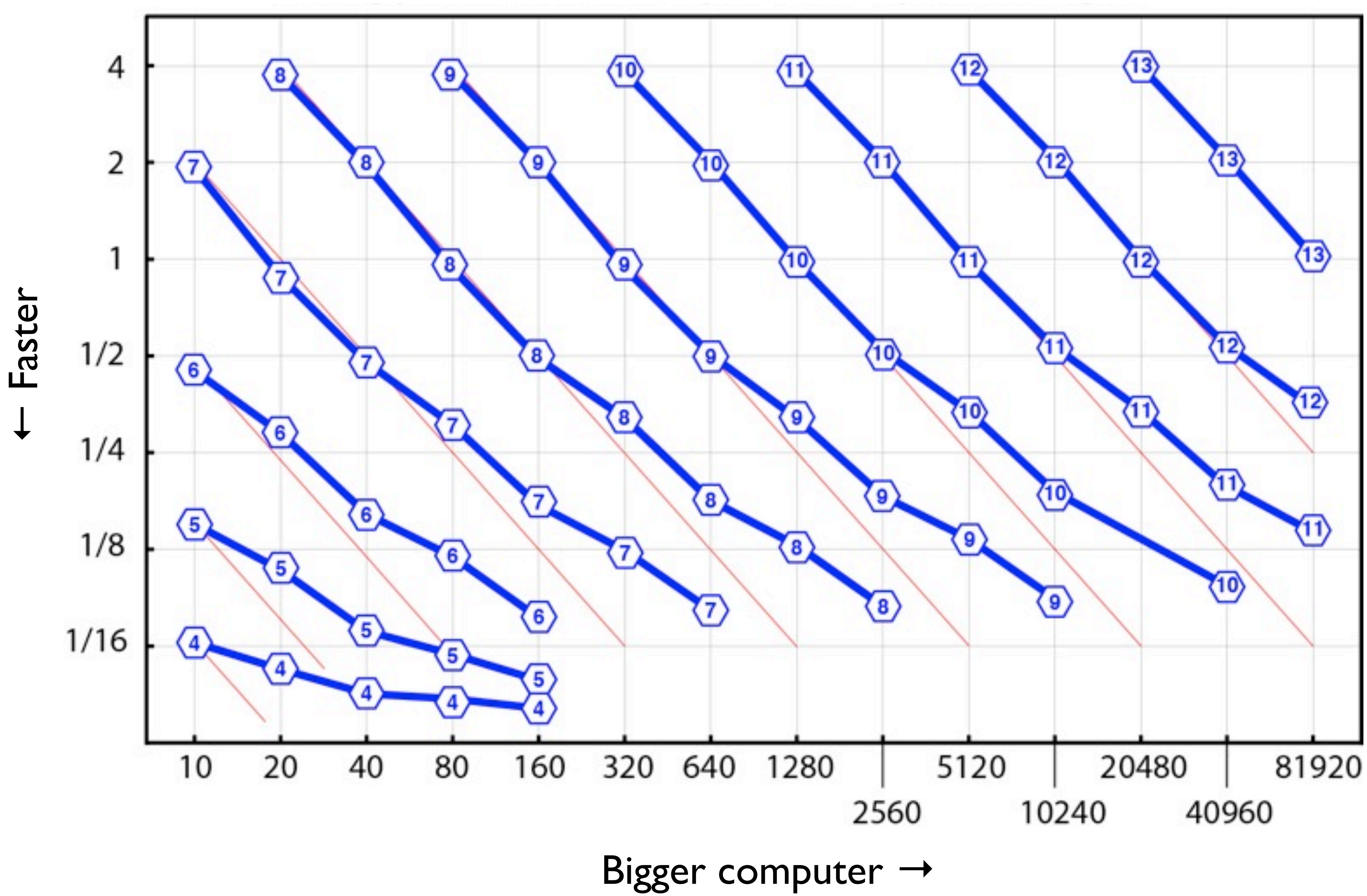
**The penalty: We have to solve a pair of elliptic equations at each level, on each dynamics time step.**

$$\zeta = \nabla^2 \psi \qquad\qquad \delta = \nabla^2 \chi$$

**This increases the run time of the dynamics by about 15%.**

**The elliptic solver is based on multigrid methods, which scale well to very high resolution.**

# Multigrid scaling




**The blue curves show how the speed increases *for a given grid*, as the computer gets bigger.**

There are two ways to filter sound waves.

**Quasi-hydrostatic system :**

$$\cancel{\frac{Dw}{Dt}} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -g$$

Vertical momentum equation becomes diagnostic.

*To satisfy this for all t, vertical velocity must be passive to other variables.*

**Anelastic system :**

$$\cancel{\frac{\partial \rho}{\partial t}} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Continuity equation becomes diagnostic.

*To satisfy this for all t, pressure gradient force must be passive to other forces.*

For cloud-resolving models, filtering must be this type.

## WHAT IS THE MINIMUM REQUIREMENT

## FOR FILTERING VERTICALLY PROPAGATING SOUND WAVES?

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho_{qs}}{\partial t} + \underline{\frac{\partial \delta\rho}{\partial t}}$$

$\rho_{qs}$ : "quasi-hydrostatic density" that satisfies $\partial p_{qs} / \partial z = -\rho_{qs} g$

$\delta\rho$ : "non-hydrostatic density" defined by $\delta\rho \equiv \rho - \rho_{qs}$

***Since vertically propagating sound waves are non-hydrostatic,***

***it is sufficient to drop only the*** $\partial \delta\rho / \partial t$ ***term for filtering those waves.***

## THE UNIFIED SYSTEM VS. OTHER SYSTEMS

### (a) Compressible non-hydrostatic

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

with no modification of the momentum equation

### (b) Quasi-hydrostatic

$$\frac{\partial \rho_{qs}}{\partial t} + \nabla \cdot (\rho_{qs} \mathbf{V}) = 0$$

with the hydrostatic equation

### (c) Anelastic non-hydrostatic

$$\nabla \cdot (\rho_0 \mathbf{V}) = 0$$

with an approximated vertical momentum equation

### (d) Unified

$$\frac{\partial \rho_{qs}}{\partial t} + \nabla \cdot (\rho_{qs} \mathbf{V}) = 0$$

with no modification of the momentum equation

## THE UNIFIED SYSTEM VS. OTHER SYSTEMS

**(a) Compressible non-hydrostatic**

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

with no modification of
the momentum equation

**(b) Quasi-hydrostatic**

$$\frac{\partial \rho_{qs}}{\partial t} + \nabla \cdot (\rho_{qs} \mathbf{V}) = 0$$

with
the hydrostatic equation

**(c) Anelastic non-hydrostatic**

$$\nabla \cdot (\rho_0 \mathbf{V}) = 0$$

with an approximated
vertical momentum equation

**(d) Unified**

$$\frac{\partial \rho_{qs}}{\partial t} + \nabla \cdot (\rho_{qs} \mathbf{V}) = 0$$

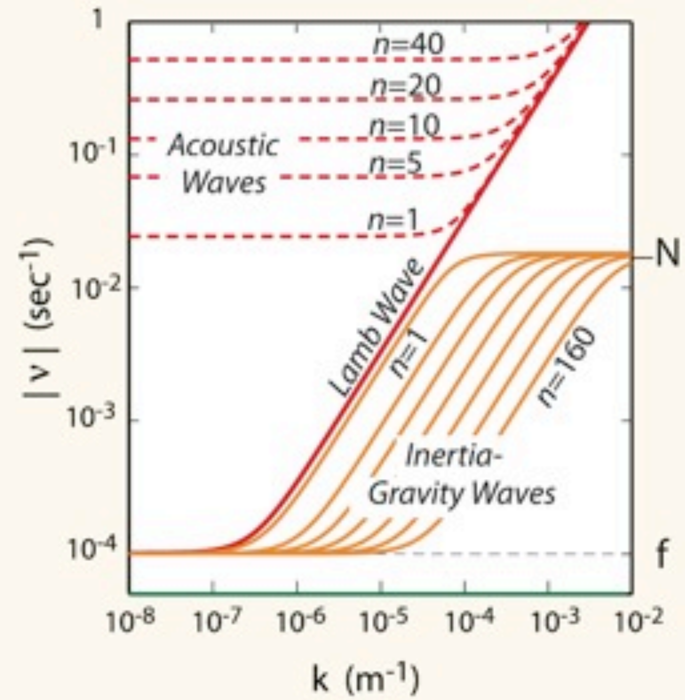
with no modification of
the momentum equation

***The unified system is a generalization of both the quasi-hydrostatic and anelastic systems.***

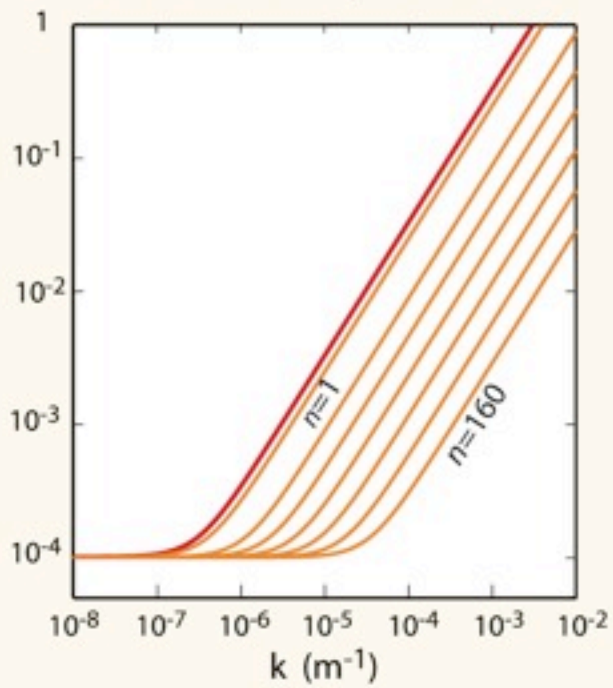
*(important in code development and evaluation)*

# DISPERSION RELATION FOR PERTURBATIONS ON A RESTING ISOTHERMAL ATMOSPHERE ON A f-PLANE (WITHOUT QUASI-GEOSTROPHIC APPROXIMATION)

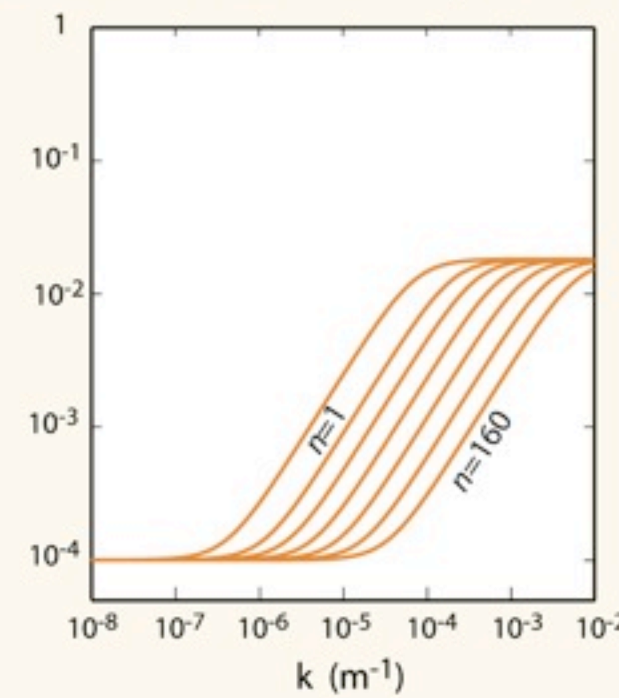
## (a) Compressible Non-Nydrostaic



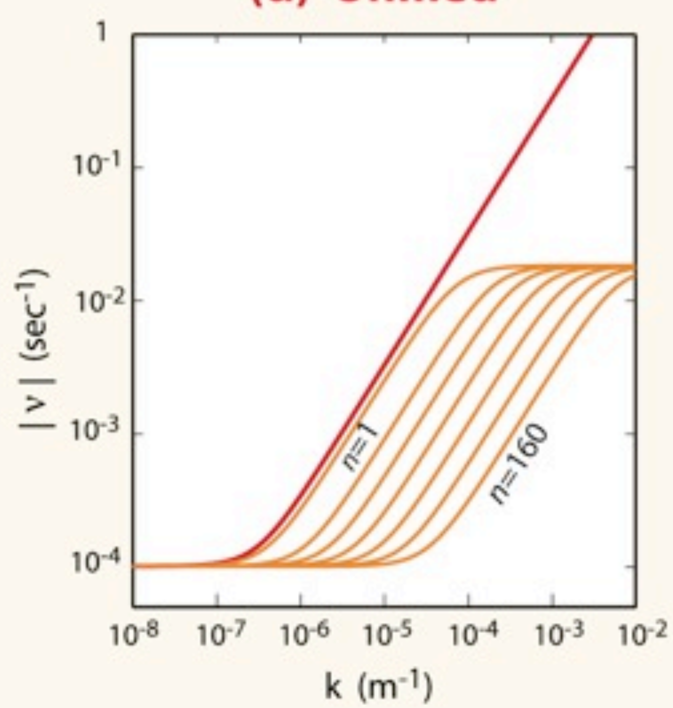

## (b) Quasi-Hydrostatic




## (c) Anelastic Non-Hydrostatic




## (d) Unified

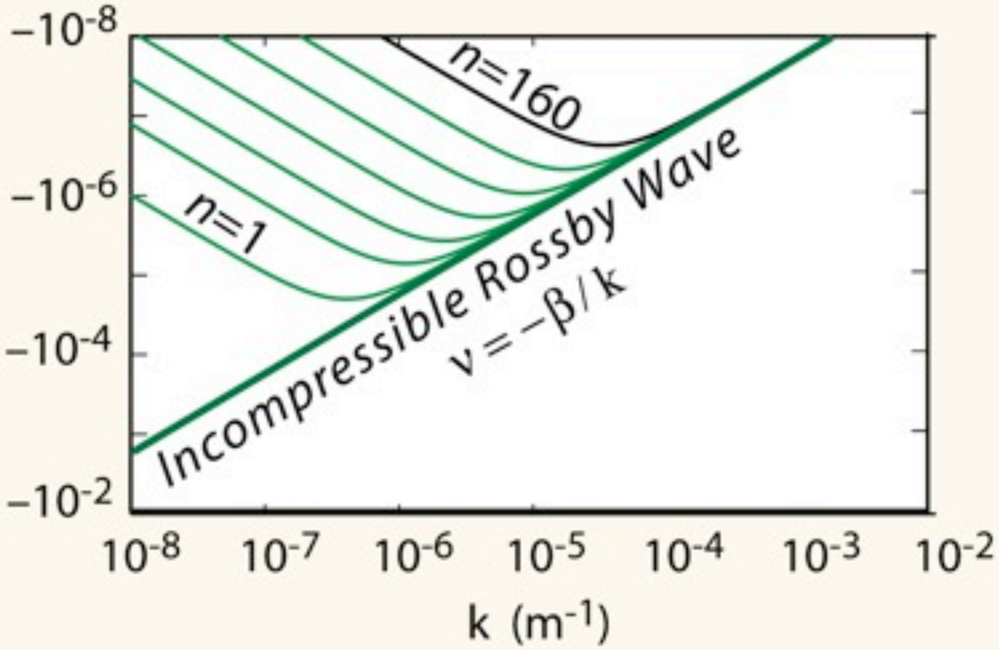



The real problem of the anelastic system is distortion of vertical structure, not in this dispersion relation.

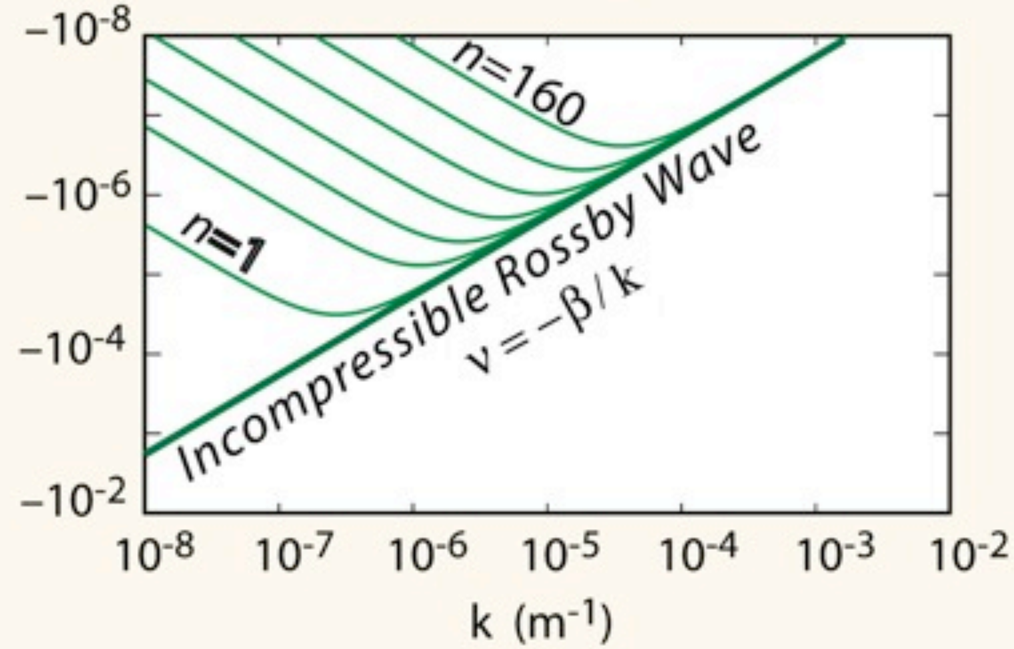
## DISPERSION RELATION FOR PERTURBATIONS ON A RESTING ISOTHERMAL ATMOSPHERE ON A $\beta$-PLANE

(WITH QUASI-GEOSTROPHIC APPROXIMATION)

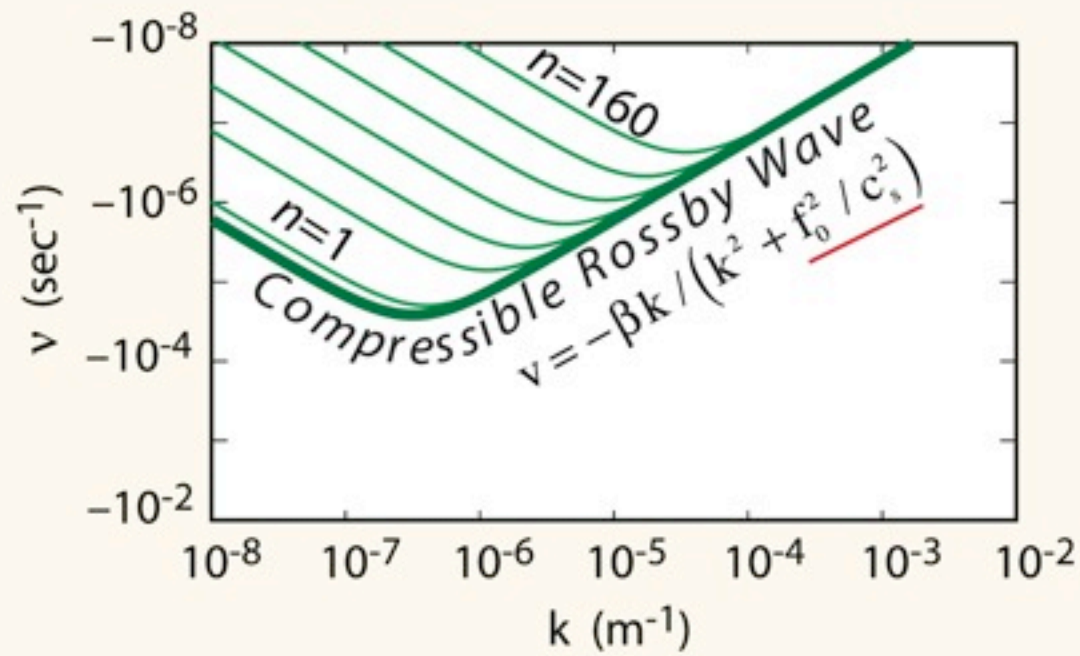
### Anelastic



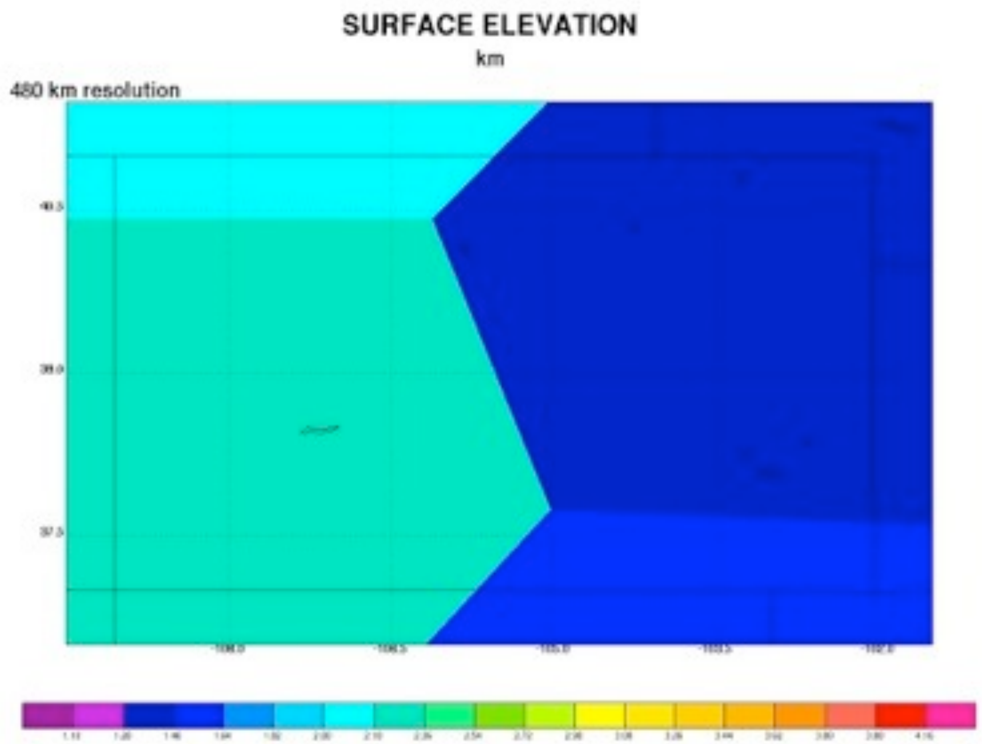

### Pseudo-Incompressible

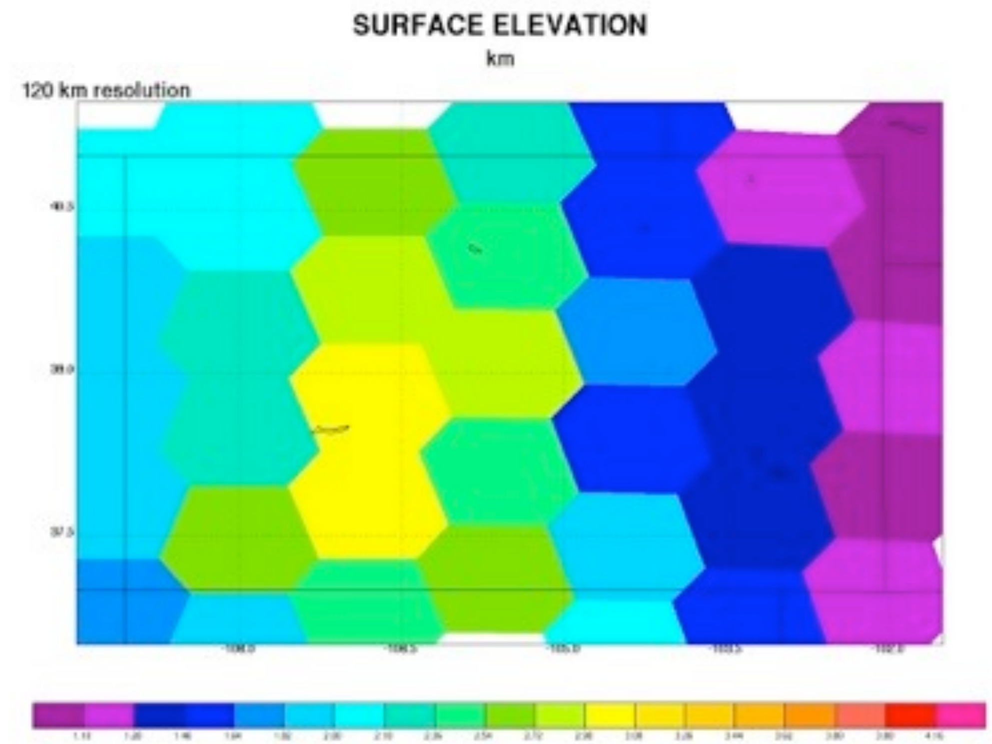


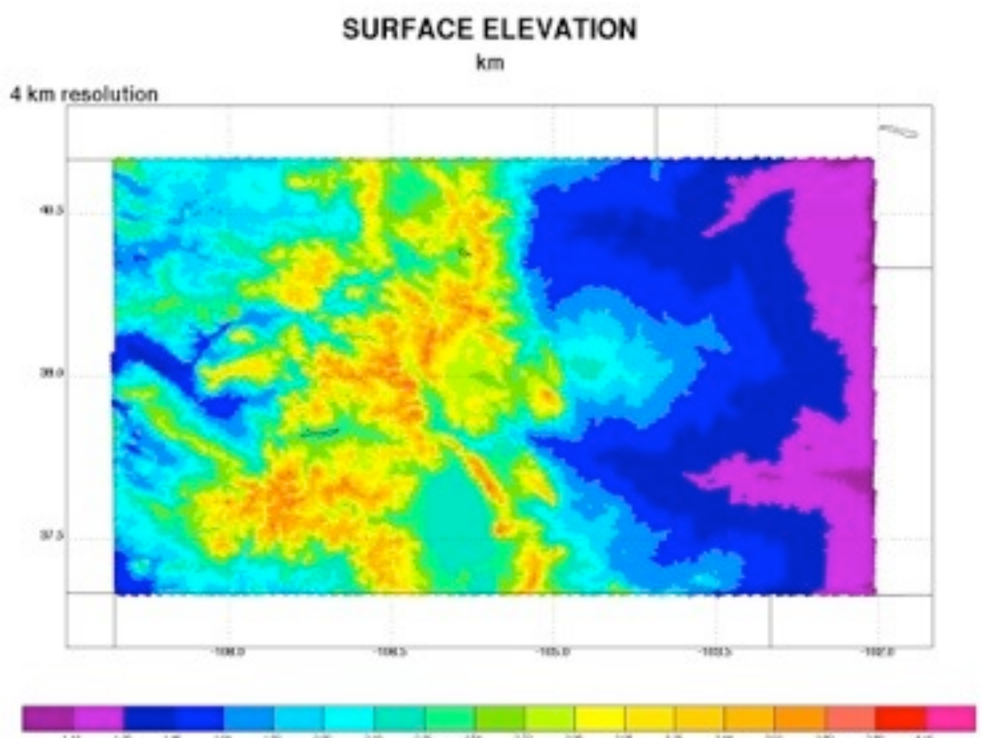

### Compressible Non-Hydrostatic, Unified & Quasi-Hydrostatic

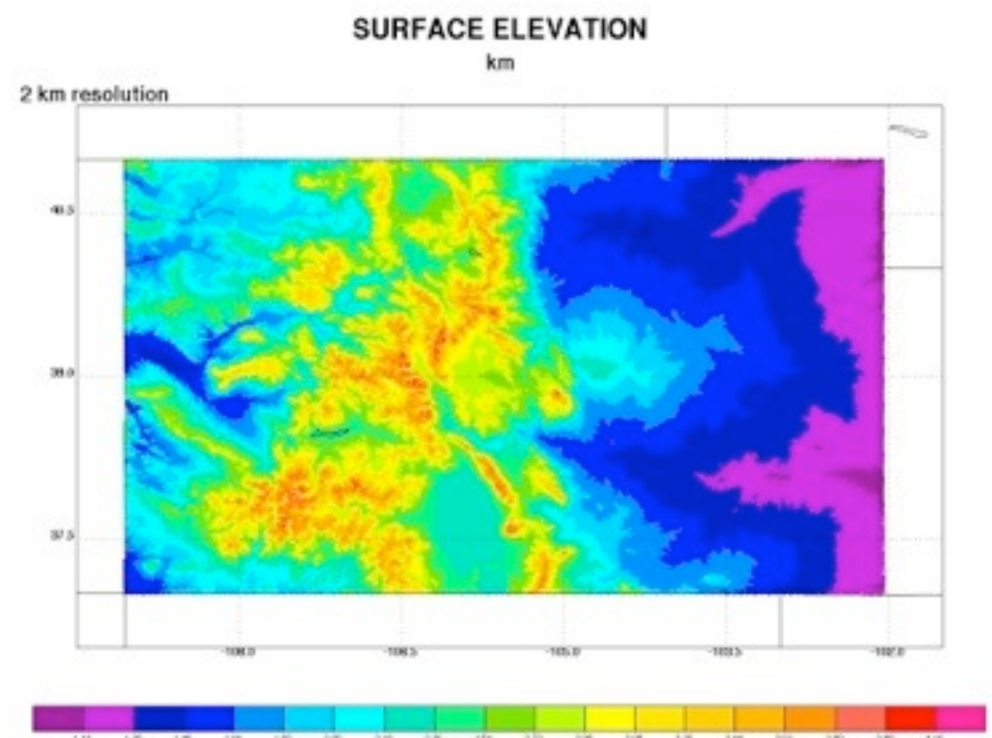


# Colorado topography



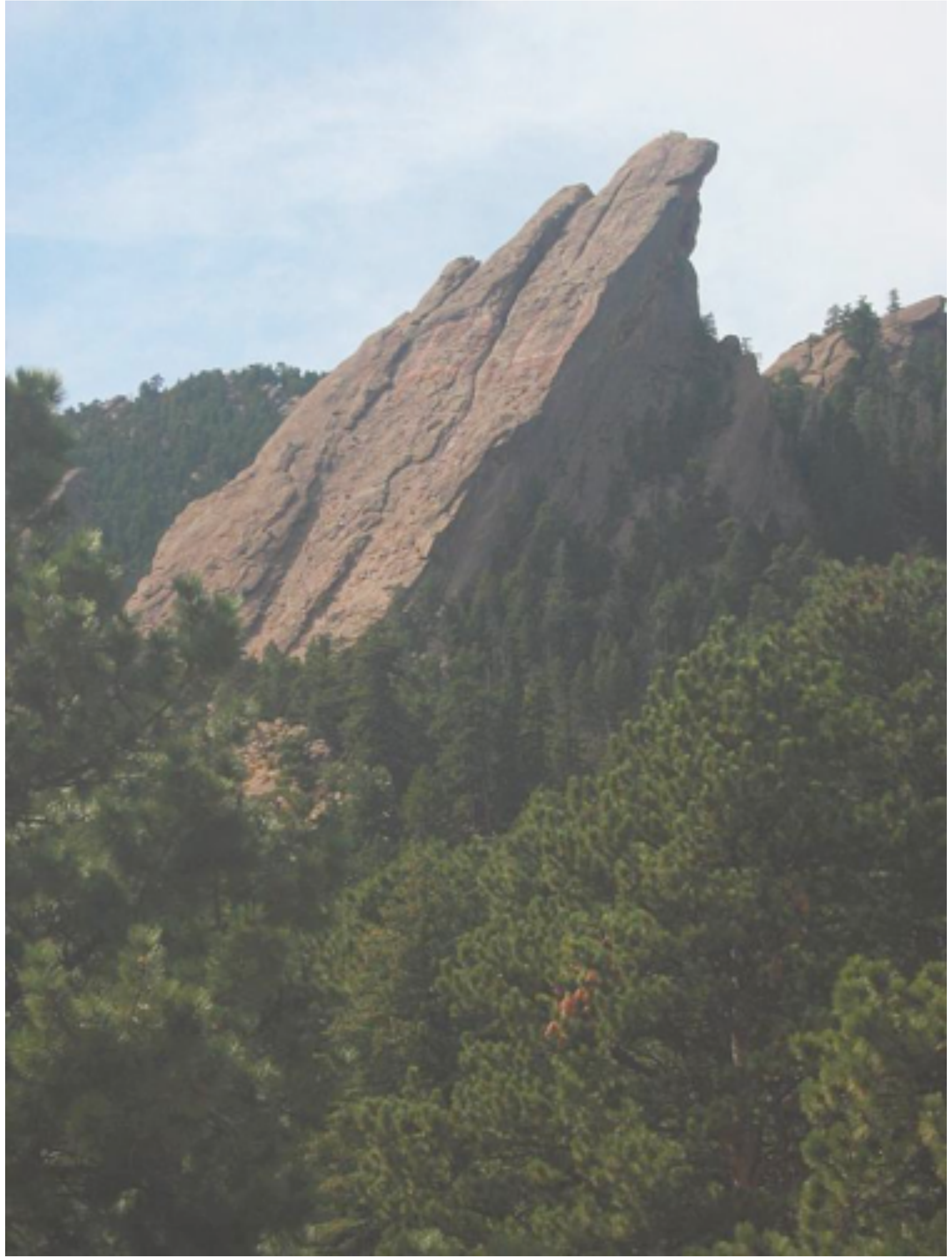



# Topography

- With high resolution, terrain slopes become very large.
- Terrain-following coordinates do not work well under those conditions.
- We will begin by implementing mountains with the height coordinate.
- In order to do this, we have to modify our elliptic solver.
- In the future we may move to a hybrid z-theta coordinate.



# Other aspects

- Scalar advection is third-order in 3 dimensions, with a sign-preserving FCT
- Physics
  - High res option based on SAM (Marat Khairoutdinov's cloud-resolving model).
  - Plan to implement CAM physics too, for low resolution.

# Summary

- **We use a hexagonal-pentagonal geodesic grid because of its good symmetry properties.**
- **We use the Z grid because it simulates geostrophic adjustment well and it has no computational modes.**
- **Our multigrid solver scales well to high resolution.**
- **We use the Unified System to filter vertically propagating sound waves.**
- **Topography is a work in progress, but will be based on the height coordinate.**

**U** *nified* **Z** *grid* **I** *cosahedral* **M** *odel*