Model tuning II: Review of possible filtering operations and diffusive mechanisms in dynamical cores

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Overview of the talk

- Some general DCMIP announcements
- Reminder: Equations of motion in GCMs
  - How do diabatic and diffusive effects enter the equations?
  - Slight excursion: How to include moisture?
- Overview of diffusion, filters and fixers in GCMs with illustrative examples of selected processes:
  - Explicitly added and implicit (numerical) dissipation mechanisms
  - Spatial and temporal filters
  - *A posteriori* Fixers for mass, tracer mass and total energy
- Final Thoughts

General DCMIP announcements

- The recordings and slides of last week's lectures are available on the DCMIP web page.
- I would like to ask all modeling groups to prepare at least a few NetCDF data sets today that are compliant with the DCMIP standards. If you have not uploaded any data yet, please consult with Peter and me to conduct an additional check.
- We want to test whether the data can be uploaded correctly to the NOAA server, and other groups will get a chance to intercompare their own data.
- The workshop queue has a 6-hour wallclock limit. Is this sufficient?
- James Kent made NCL scripts available, see the information on the page http://earthsystemcog.org/projects/dcmip-2012/visualization
- Now is a good time to transition from data production to data analysis, and populate the DCMIP web page.

General DCMIP announcements

- There are many ways of writing the thermodynamic equation
- Here are three examples in advective form:

1) $$\frac{dT}{dt} + \frac{p}{c_v}\frac{d\alpha}{dt} = \frac{1}{c_v}Q$$

2) $$\frac{dT}{dt} - \frac{\alpha}{c_p}\frac{d\omega}{dt} = \frac{1}{c_p}Q$$

3) $$\frac{d\Theta}{dt} = \frac{1}{c_p}\left(\frac{p_0}{p}\right)^{\frac{R_d}{c_p}}Q$$

- If your thermodynamic equation belongs to category 1), the condensation and condensational heating in the DCMIP test cases 42, 43, 51 need to be modified (see next slide).

General DCMIP announcements

- If category 1 is used, c_p needs to be changed to $c_v = (c_p - R_d)$ in:

$$\frac{\partial T}{\partial t} = \frac{L}{c_v} C$$

$$C = \frac{1}{\Delta t}\left(\frac{q - q_{sat}(T)}{1 + \frac{L}{c_v}\frac{dq_{sat}(T)}{dt}}\right) = \frac{1}{\Delta t}\left(\frac{q - q_{sat}(T)}{1 + \frac{L}{c_v}\frac{\varepsilon L q_{sat}}{R_d T^2}}\right)$$

- The corresponding code changes in the simple-physics routine are marked in red:

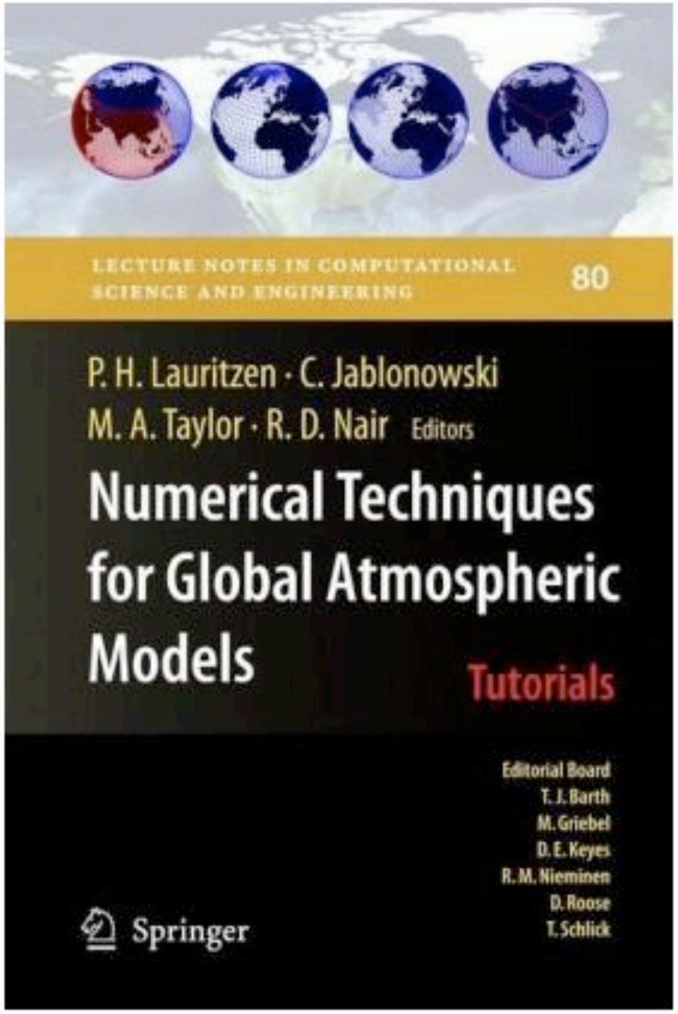
```
tmp  = 1._r8/dtime*(q(i,k)-qsat)/(1._r8+(latvap/(cpair-rair))*
          (epsilo*latvap*qsat/(rair*t(i,k)**2)))

dtdt(i,k) = dtdt(i,k)+latvap/(cpair-rair)*tmp
```

Lecture is documented in Springer Book (April 2011)

- Book based on the 2008 NCAR Dynamical Core Summer School
- 16 chapters with contributions from about 25 authors (> 550 pages)
- Chapter 13 by Christiane Jablonowski and David L. Williamson: **The Pros and Cons of Diffusion, Filters and Fixers in Atmospheric General Circulation Models,** pp. 381- 493

also available as an e-book



http://www.springer.com/mathematics/computational+science+%26+engineering/book/978-3-642-11639-1

Motivation for the Book Chapter and Lecture

- All dynamical cores need some form of dissipation, either explicitly added or implicitly included via the choice of the numerical scheme
- This is due to the truncation of the spatial scales:
  - Dissipation is needed to prevent an accumulation of energy at the smallest grid scales
  - This prevents numerical instabilities
- Dissipation mechanisms
  - are often hidden in the dynamical cores
  - are rarely fully documented in publications (maybe in technical reports), ask your mentor
  - and their coefficients are often empirically determined and resolution-dependent (‘tuning’ knobs in the dynamical cores), no physical basis

Key Words (in red briefly discussed here)

- Explicitly added dissipation mechanisms
  - Horizontal diffusion or hyper-diffusion
  - Divergence damping
  - Vorticity damping
  - External mode damping
  - Rayleigh friction and sponge layers near the model top
- Implicit numerical dissipation
  - Order of accuracy
  - Off-centering
  - Monotonicity constraints and flux limiters
  - Damping by interpolations in semi-Lagrangian schemes
- Filters:
  - Spectral Fast-Fourier-Transform (FFT) filters
  - Digital filters: e.g. Shapiro filters
  - Time filters: e.g. Asselin-filter
- *A posteriori* Fixers: Mass, tracer mass, total energy

Abstract View of a GCM

Time tendency from the dynamical core (adiabatic)

Time tendency from physical parameterizations (diabatic)

$$\frac{\partial \psi}{\partial t} = Dyn(\psi) + Phys(\psi) + \boxed{F_\psi}$$

Time tendency of the forecast variable ψ

Time tendency from dissipative mechanisms (mostly considered part of the dynamical core)

Dry 3D Primitive Equations:
What are the dissipative effects?

Horizontal momentum equation with $\vec{v}_h = (u, v)$

$$\frac{\partial \vec{v}_h}{\partial t} + \left(\vec{v}_h \vec{\nabla}_z\right)\vec{v}_h + w\frac{\partial \vec{v}_h}{\partial z} + f\vec{k} \times \vec{v}_h = -\frac{R_d T}{p_{dry}}\vec{\nabla}_z p_{dry} \boxed{+ \vec{F}_r}$$

temporal change | horizontal & vertical advection | Coriolis force | pressure gradient | Diffusion & mixing in dycore

Hydrostatic equation:

$$\frac{\partial p_{dry}}{\partial z} = -g\rho_{dry} = -\frac{g p_{dry}}{R_d T}$$

Equation of state:

$$p_{dry} = \rho_{dry} R_d T$$

Dry 3D Primitive Equations:
What are the diffusive effects?

Continuity equation

$$\frac{\partial \rho_{dry}}{\partial t} + \vec{\nabla}_z \bullet \left(\rho_{dry}\vec{v}\right) = 0$$

Thermodynamic equation:

$$\frac{D\Theta}{Dt} = \frac{\partial \Theta}{\partial t} + (\vec{v}\vec{\nabla})\Theta = \boxed{F_\Theta}$$

Dissipation

Tracer advection equation

$$\frac{\partial q}{\partial t} + \vec{v}_h \bullet \nabla_z q + w\frac{\partial q}{\partial z} = \boxed{F_q}$$

+ Tracer advection equations for other species

Moist (subscript m) 3D Primitive Equations: What are the diabatic and diffusive effects?

Horizontal momentum equation with $\vec{v}_h = (u,v)$

$$\frac{\partial \vec{v}_h}{\partial t} + \left(\vec{v}_h \vec{\nabla}_z\right)\vec{v}_h + w\frac{\partial \vec{v}_h}{\partial z} + f\vec{k} \times \vec{v}_h = -\frac{R_d T_v}{p_m}\vec{\nabla}_z p_m + \boxed{\vec{F}_r}$$

temporal change | horizontal & vertical advection | Coriolis force | pressure gradient

Dissipation (from physics and dycore)

Hydrostatic equation:

$$\frac{\partial p_m}{\partial z} = -g\rho_m = -\frac{g p_m}{R_d T_v}$$

Equation of state:

$$p_m = \rho_m R_d T_v$$

with virtual temperature:

$$T_v = T(1 + 0.608q)$$

Moist 3D Primitive Equations:
What are the diabatic and diffusive effects?

Continuity equation

$$\frac{\partial \rho_m}{\partial t} + \vec{\nabla}_z \bullet \left(\rho_m \vec{v}\right) = \boxed{S_{\text{moist air}}}$$

S: sources/sinks of water vapor

Thermodynamic equation:

$$\frac{D\Theta}{Dt} = \frac{\partial \Theta}{\partial t} + (\vec{v}\vec{\nabla})\Theta = \boxed{F_\Theta} + \boxed{\frac{1}{c_p}\left(\frac{p_0}{p}\right)^{R_d / c_p} Q}$$

Q: diabatic heating

Tracer advection equation (e.g. specific humidity)

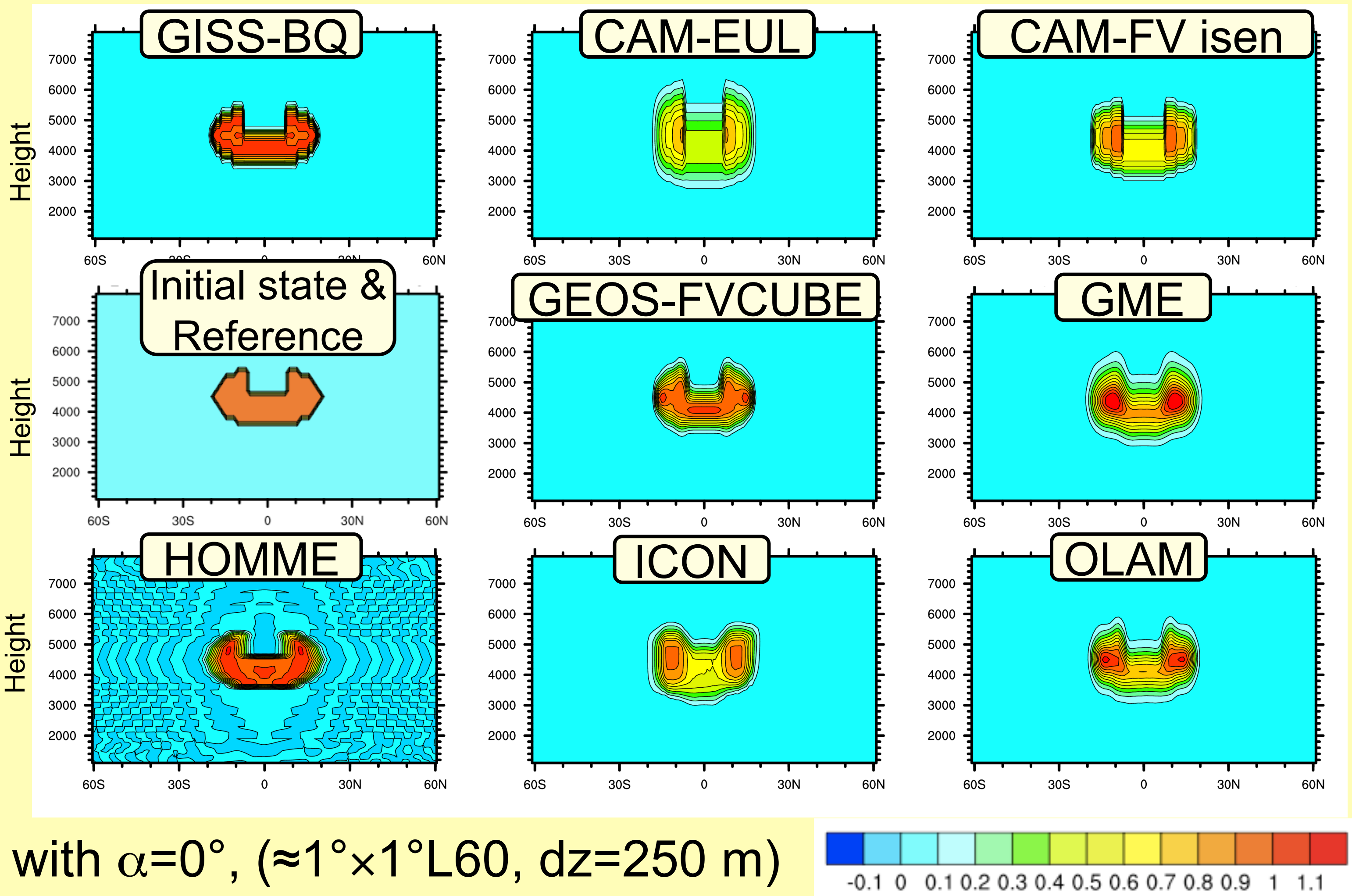
$$\frac{\partial q}{\partial t} + \vec{v}_h \bullet \nabla_z q + w \frac{\partial q}{\partial z} = \boxed{F_q} + \boxed{S_q}$$

S_q: sources/sinks

+ Tracer advection equations for other species

Examples:
Diffusive Signatures in Advection Schemes

Results from 8 dynamical cores during the 2008 NCAR Colloquium



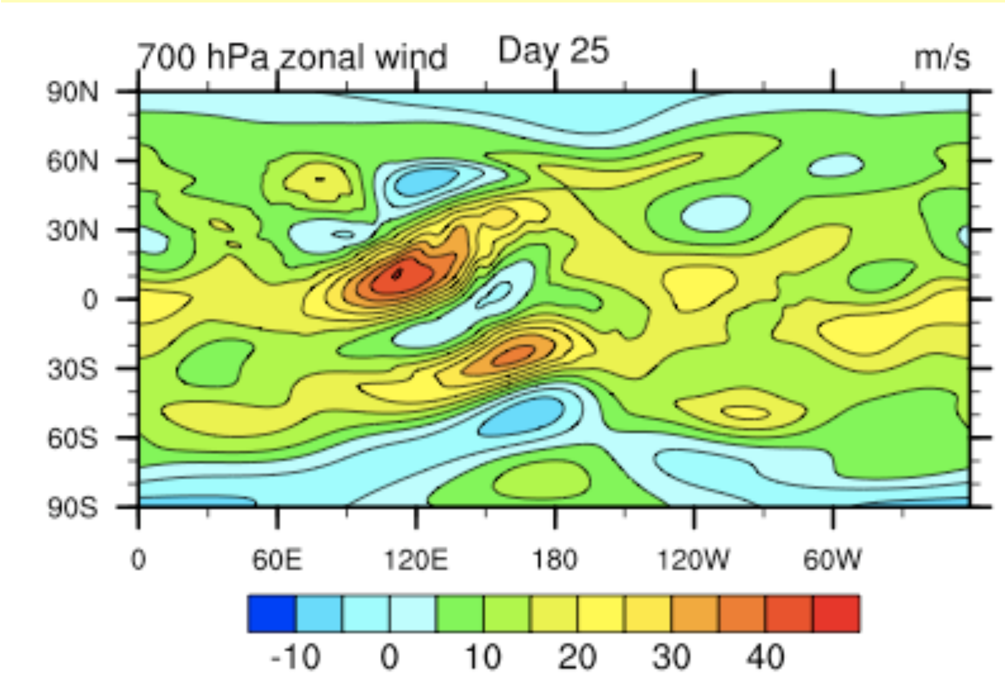
3D advection test case described in Jablonowski et al. (2008), similar to DCMIP test 11

with α=0°, (≈1°×1°L60, dz=250 m)

Different Dissipative Signatures in Dynamical Cores

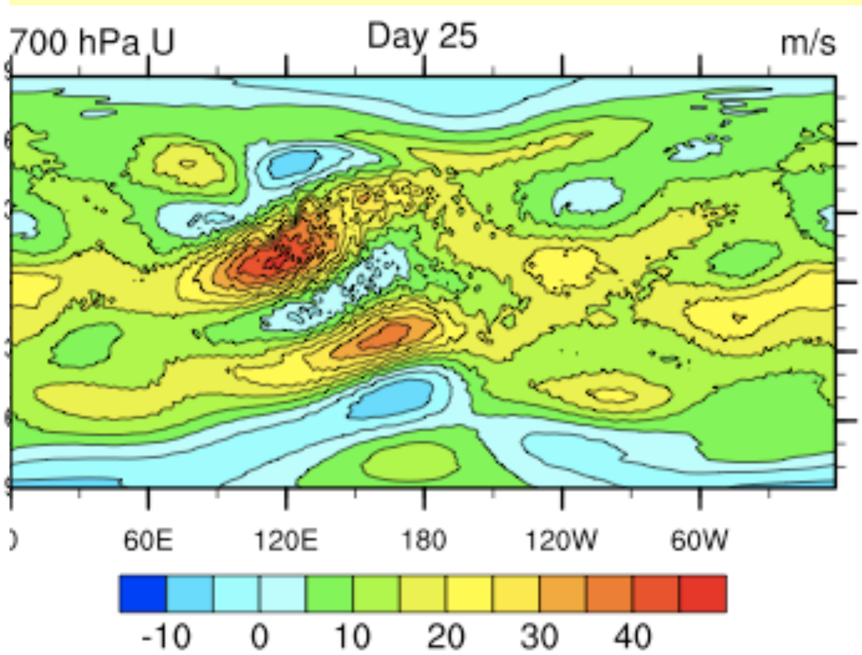
- Comparison of the 700 hPa zonal wind at day 25 in NCAR's CAM FV and CAM EUL with mountain-wave test

CAM FV 1ºx1ºL26



with monotonicity constraint, divergence damping

CAM EUL T106L26



With horizontal 4th-order diffusion with default coefficient

Explicitly Added Horizontal Dissipation

- Often: Diffusion applied to the prognostic variables
  - Regular diffusion ∇^2 - operator
  - Hyper-diffusion ∇^4, ∇^6, ∇^8 - operators: more scale-selective
  - Example: Temperature diffusion with order 2q

$$\frac{\partial T}{\partial t} = \cdots (-1)^{q+1} K_{2q} \nabla^{2q} T$$

  - K: diffusion coefficient (here constant), its e-folding time needs to depend on the horizontal resolution
  - Exact form depends on the choice of the prognostic variables, e.g. T could be replaced by Θ
- 2D or 3D Divergence damping
- Rayleigh friction and sponges

Horizontal Diffusion Coefficients

- Diffusion coefficients are scale-dependent
- Are guided by the so-called e-folding time: How quickly are the fastest waves damped so that their amplitude decrease by a factor of ‘e’?
- Typical 4th-order diffusion coefficients K_4 for CAM EUL

Eulerian spectral transform dynamical core(EUL)				
Spectral Resolution	# Grid points lat × lon	Grid distance at the equator	Time step Δt	Diffusion coefficient K_4 ($m^4\ s^{-1}$)
T21	32 × 64	625 km	2400 s	2.0×10^{16}
T42	64 × 128	313 km	1200 s	1.0×10^{16}
T85	128 × 256	156 km	600 s	1.0×10^{15}
T106	160 × 320	125 km	450 s	0.5×10^{15}
T170	256 × 512	78 km	300 s	1.5×10^{14}
T340	512 × 1024	39 km	150 s	1.5×10^{13}

Horizontal Diffusion Coefficients

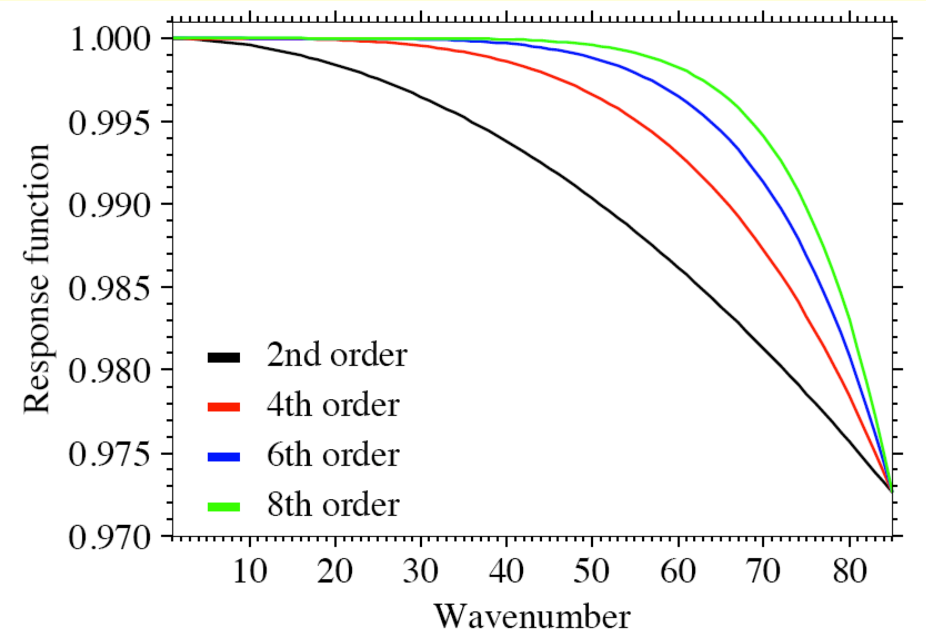
- Are guided by the so-called e-folding time τ: How quickly are the highest wavenumber n_0 damped so that the amplitude decrease by a factor of 'e'
- In spectral models:

$$K_{2q} = \frac{1}{\tau}\left(\frac{a^2}{n_0(n_0+1)}\right)^q$$

- In grid point models with spacing Δx:

$$K_{2q} = \frac{1}{2\tau}\left(\frac{\Delta x}{2}\right)^{2q}$$

Scale dependencies:

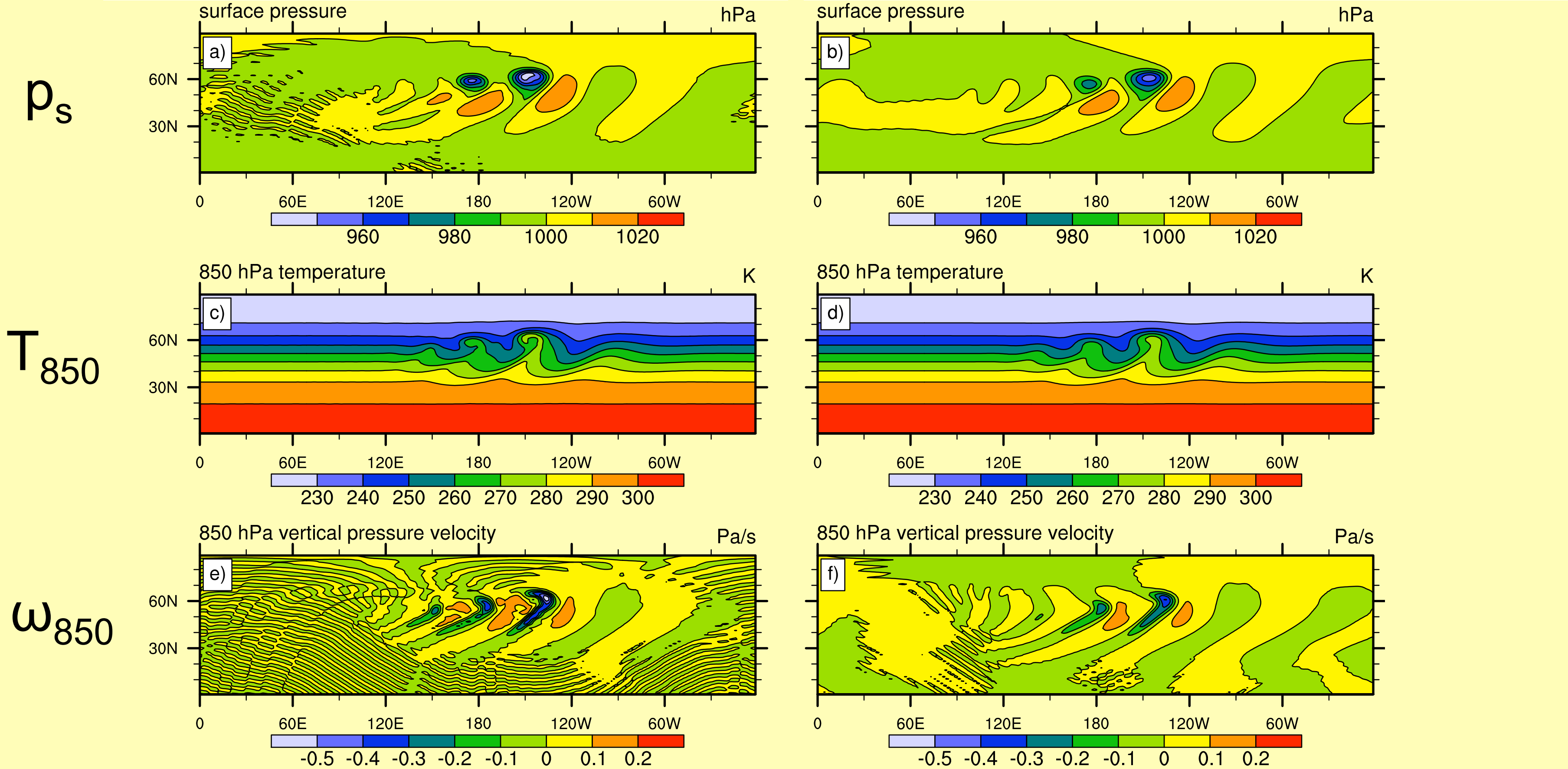


Higher order is more scale-selective, less damping at large scales (low wavenumbers)

Impact of Explicit Diffusion: Baroclinic Waves

- NCAR's CAM EUL T85L26 with two diffusion coefficients
- Let's evaluate the spectral noise (Gibb's oscillations)
- Right: Reduced noise, but **severe damping** of the circulation

Default $K_4 = 10^{15}\ m^4/s$ (8.6 h) | Increased $K_4 = 10^{16}\ m^4/s$ (0.86 h)



Divergence damping

- Divergence damping diffuses the divergent part of the flow

$$\mathbf{F_v} = (-1)^{q+1}\nabla\left(\nu_{2q}\nabla^{2q-1}\cdot\mathbf{v}\right)$$

- Example: 2nd-order (q=1) 2D divergence damping

$$\frac{\partial \vec{v}_h}{\partial t} = \cdots + \nabla(cD)$$

Divergence damping coefficient, units m^2/s

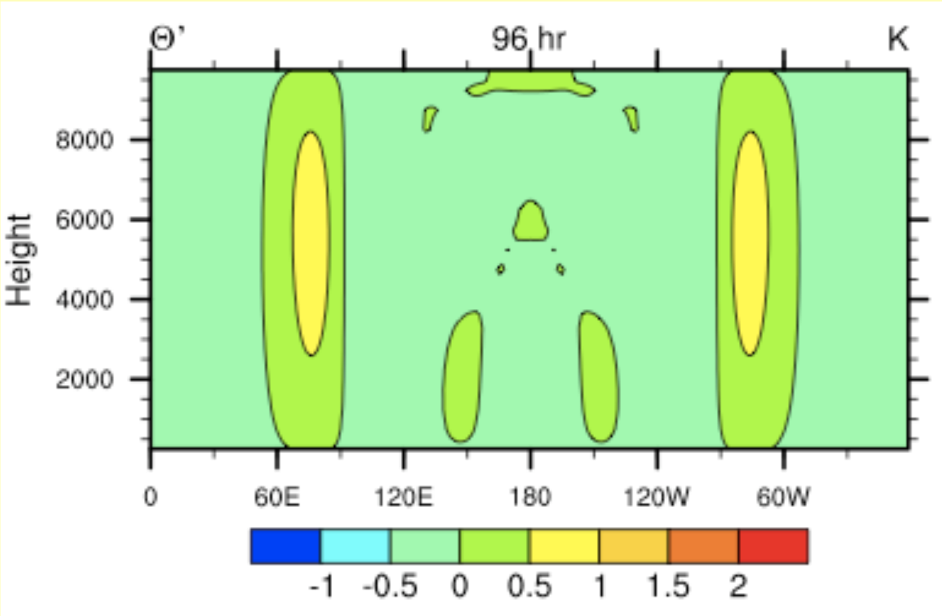
$$\Rightarrow \nabla \bullet \frac{\partial \vec{v}_h}{\partial t} = \cdots + \nabla \bullet \nabla(cD)$$

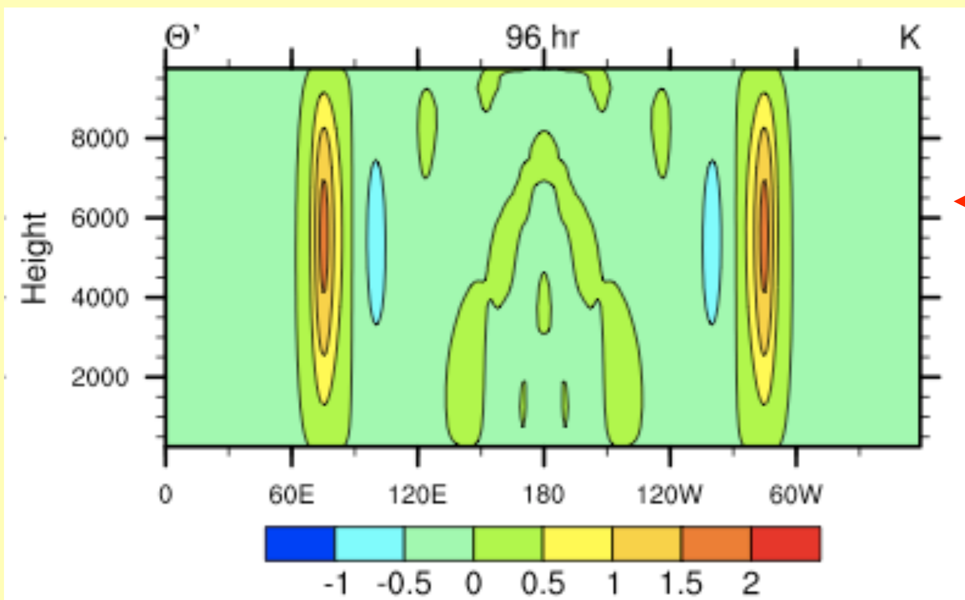
apply the div operator

$$\Leftrightarrow \frac{\partial D}{\partial t} = \cdots + \nabla^2(cD) \quad \text{with D : horizontal divergence}$$

- 2D divergence damping damps gravity waves
- 3D divergence damping damps sound waves
- Choice of the coefficient must satisfy stability constraints

2D Divergence damping: Effects

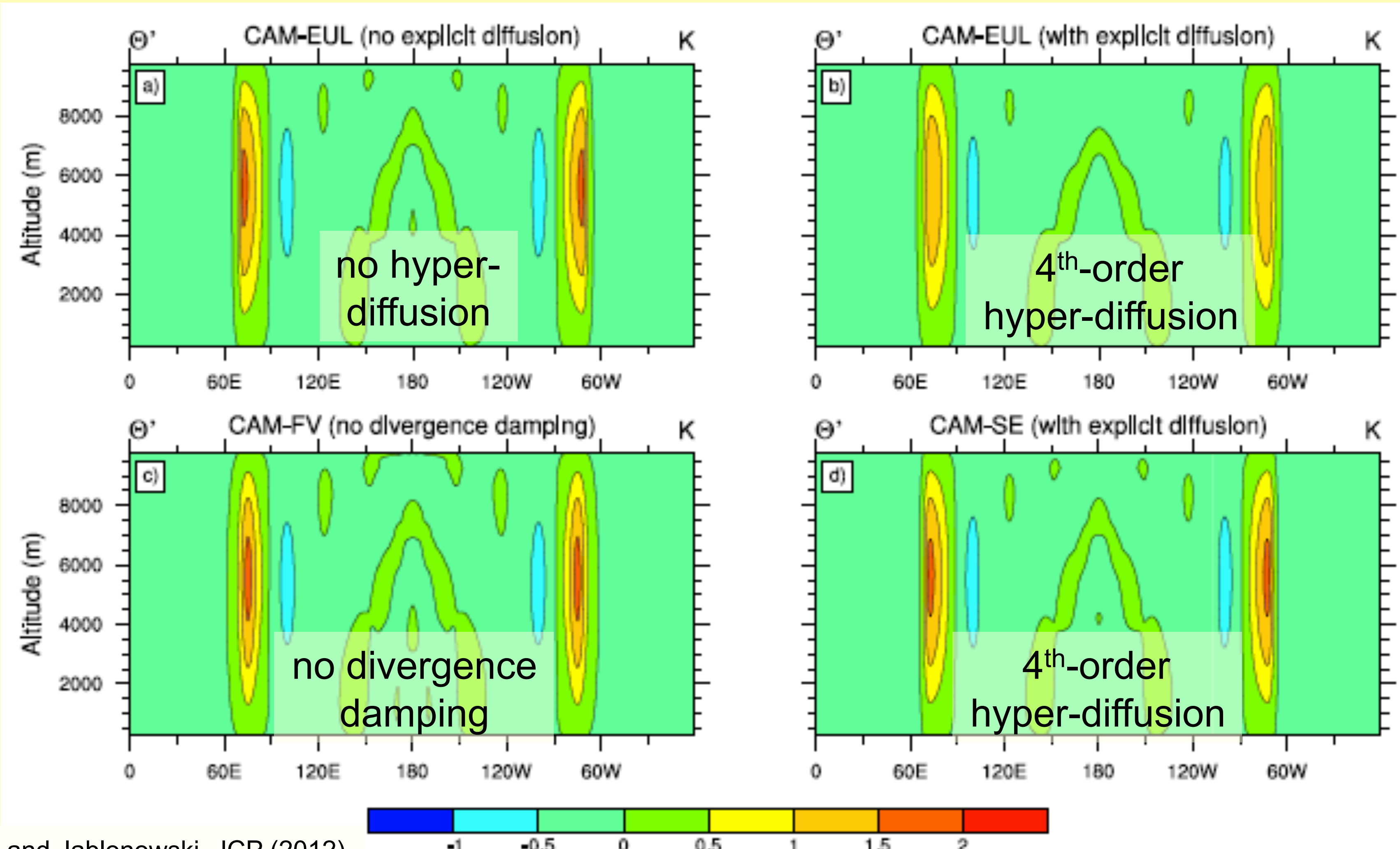




- Example: 3D gravity wave test, similar to DCMIP test 31
- Model CAM FV 1°x 1° L20 at day 4, cross section at equator
- with extreme 2nd-order divergence damping coefficient
- without divergence damping (bottom)
- Clear difference in the amplitudes of the gravity wave

Impacts of both explicit and implicit diffusion

- Example: 3D gravity wave, cross section at the equator at day 4
- Shape of Θ perturbation differs, check sharpness of leading edge

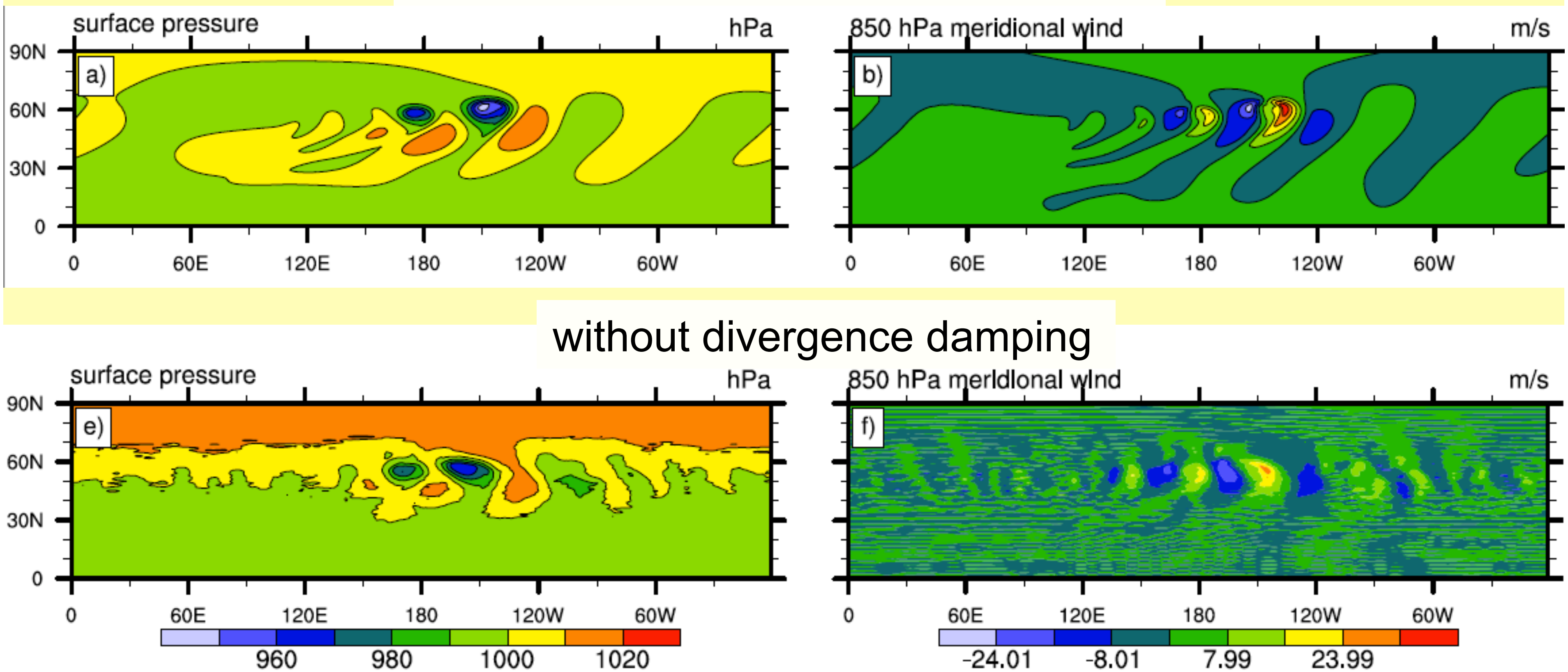


Ullrich and Jablonowski, JCP (2012)

2D Divergence damping: Needed for stability?

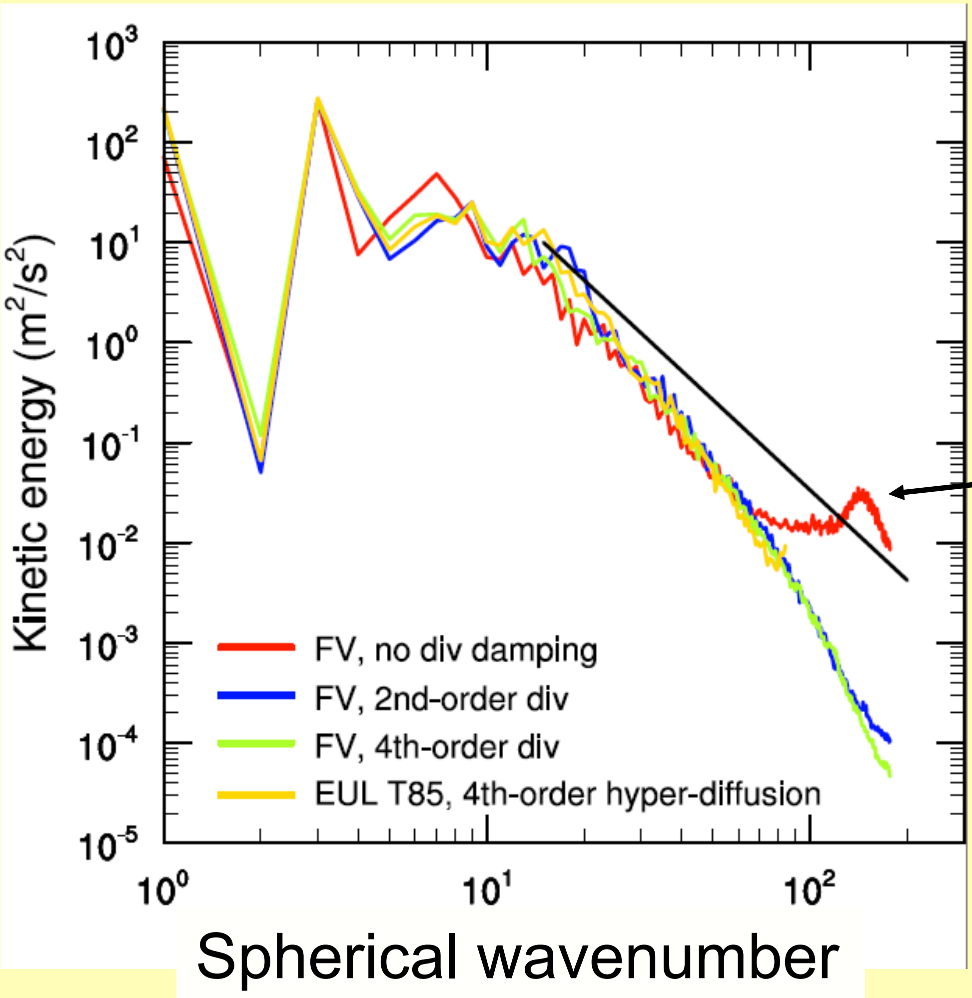
- Model NCAR CAM FV 1°x 1° L26, baroclinic wave at day 9
- Numerical stability of CAM FV depends on div damping

with 2nd-order 2D divergence damping



2D Divergence Damping

- Effects of the different divergence damping mechanisms on the kinetic energy spectrum

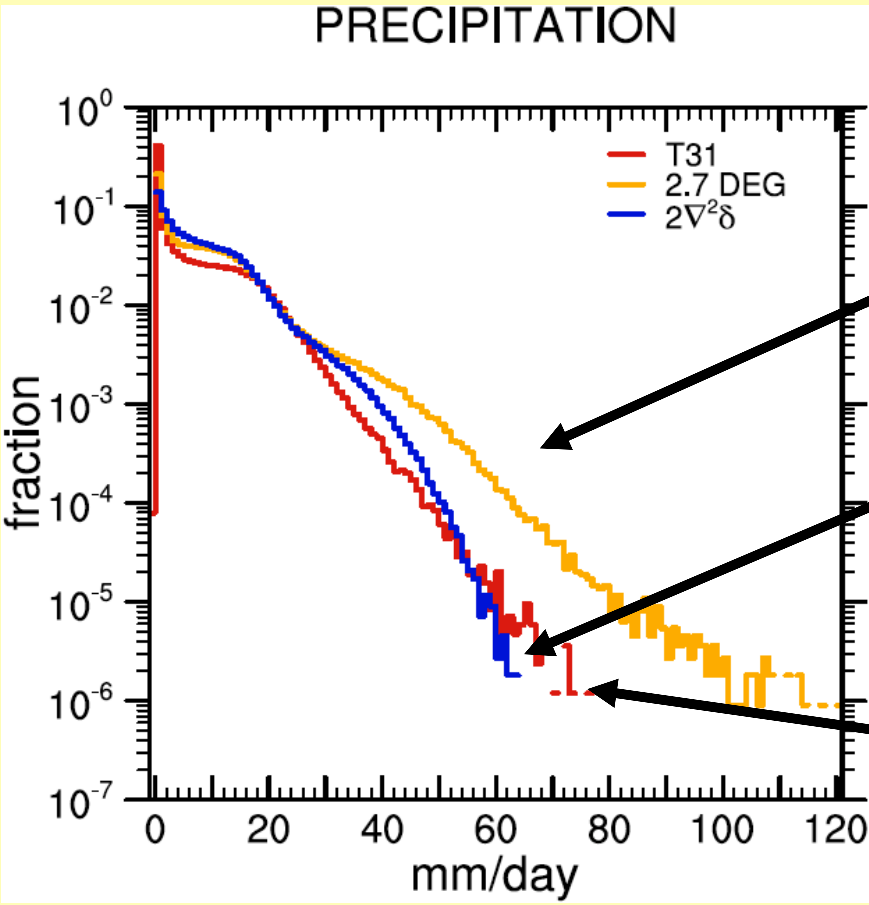


Very harmful: Accumulation of energy at small scales without divergence damping

700 hPa KE spectra from CAM-FV and CAM-EUL baroclinic wave simulations at 1°x1° and T85

2D Divergence Damping

- Divergence damping influences the likelihood of heavy precipitation in the tropics, at least at low resolutions



Default in CAM FV

Doubled coefficient in CAM FV

Comparison to CAM EUL T31

Figure: courtesy of P. Lauritzen

Sponge layers and Rayleigh friction

- Often desired: a wave-absorbing layer near the top of a GCM
- Prevents wave reflections of upward traveling waves that would normally leave the domain
- Some upper boundary conditions, e.g. that the model top is placed at a fixed height and w=0 m/s, are perfect reflectors, undesirable
- Practical approaches: Sponge layer near the model top, needs to be deep (at least one scale height)
- Examples are enhanced 2nd-order diffusion or Rayleigh friction, e.g. of the types:

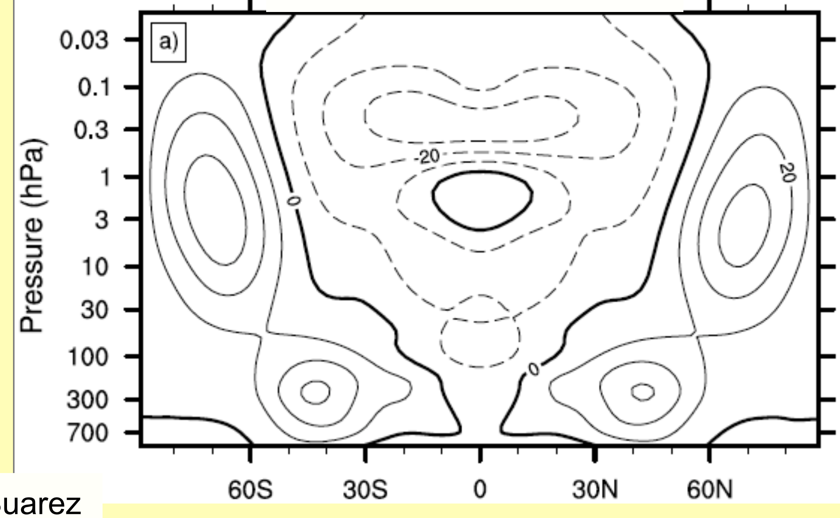
$$\frac{\partial u}{\partial t} = \ldots + K\nabla^2 \mathrm{u} \qquad \text{or} \qquad \frac{\partial u}{\partial t} = \ldots - \tau\left(u - \overline{u}\right)$$

Remember: no physical justification

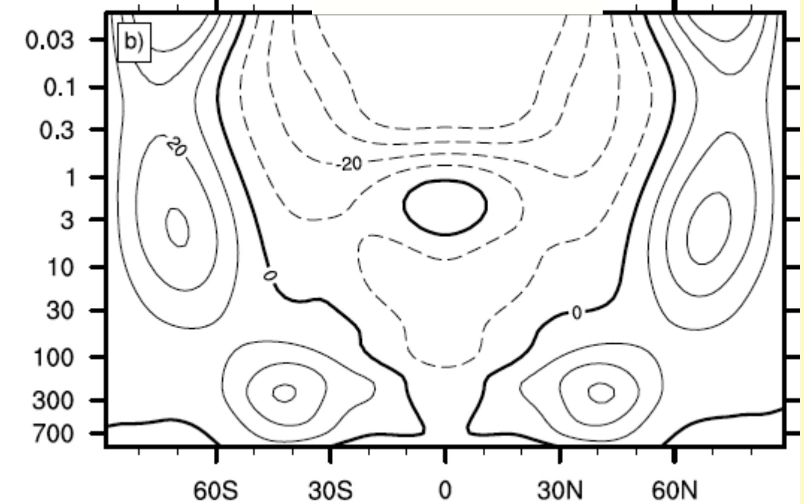
Examples of sponge layer effects

- Model CAM SLD T63L26: time-mean zonal-mean u wind

2nd order diffusion

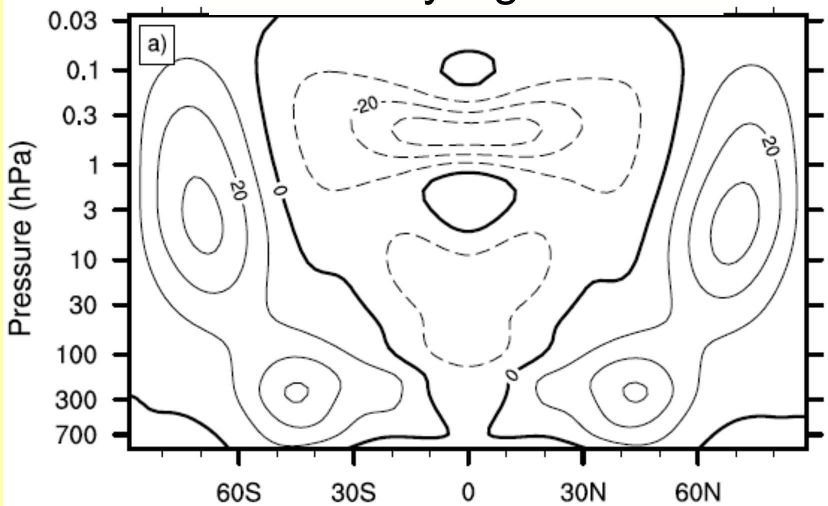


No diffusion

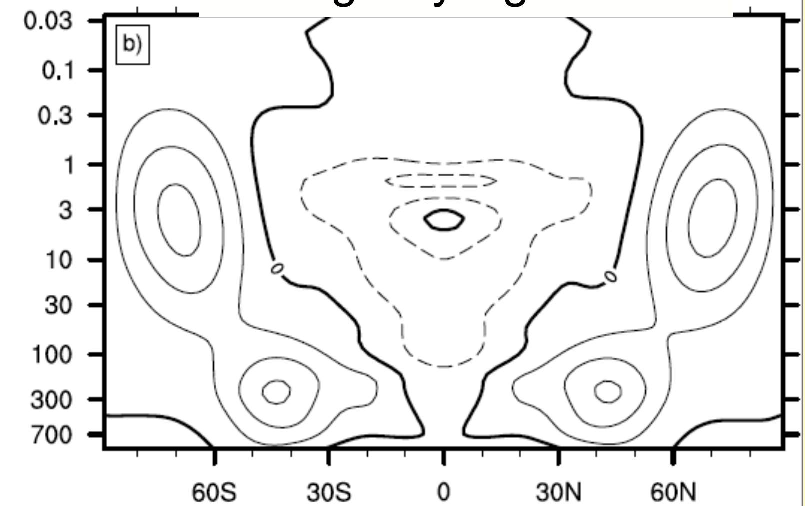


Held-Suarez experiment

Weak Rayleigh friction



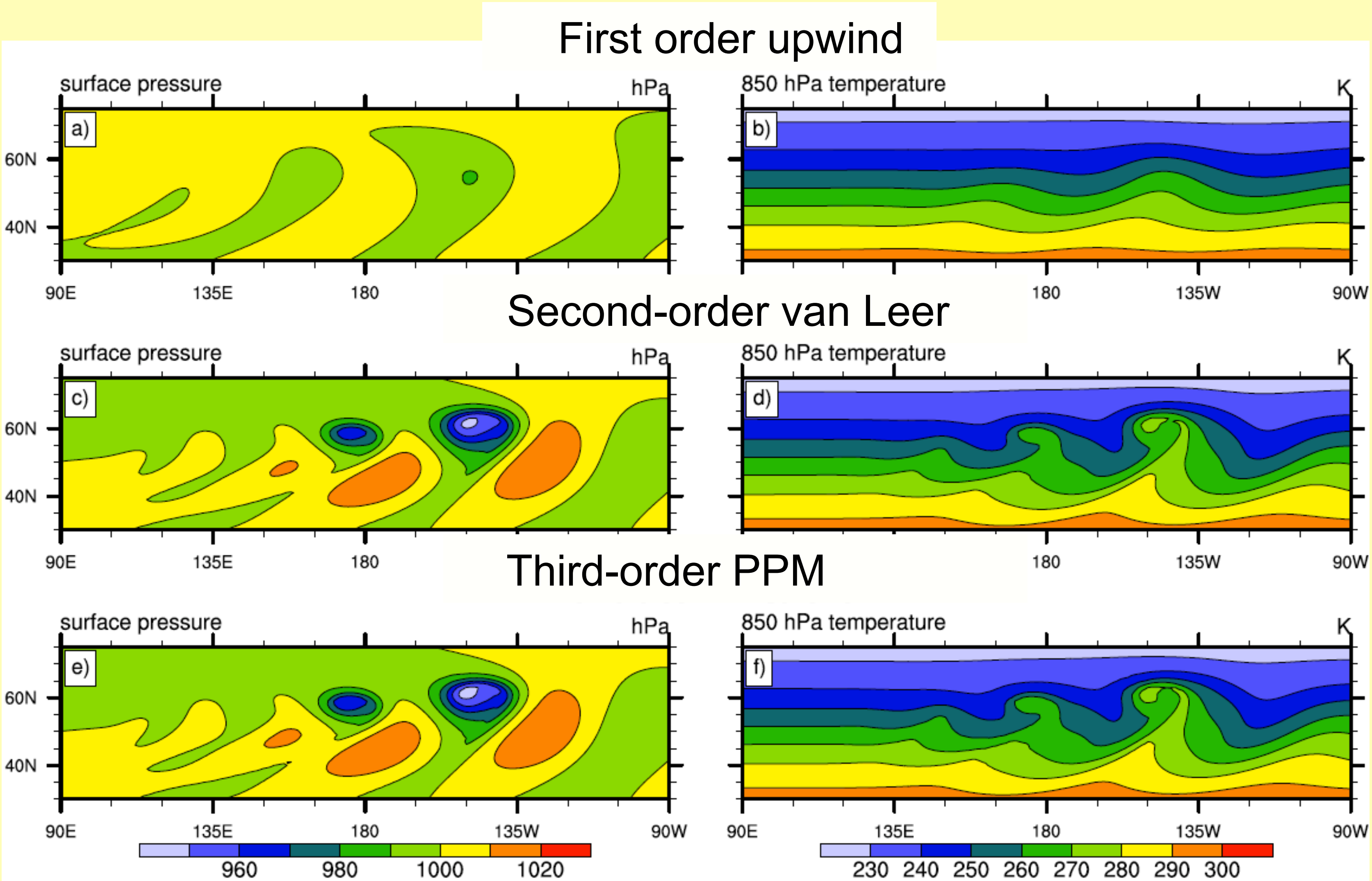
Strong Rayleigh friction



Implicit / Numerical Diffusion

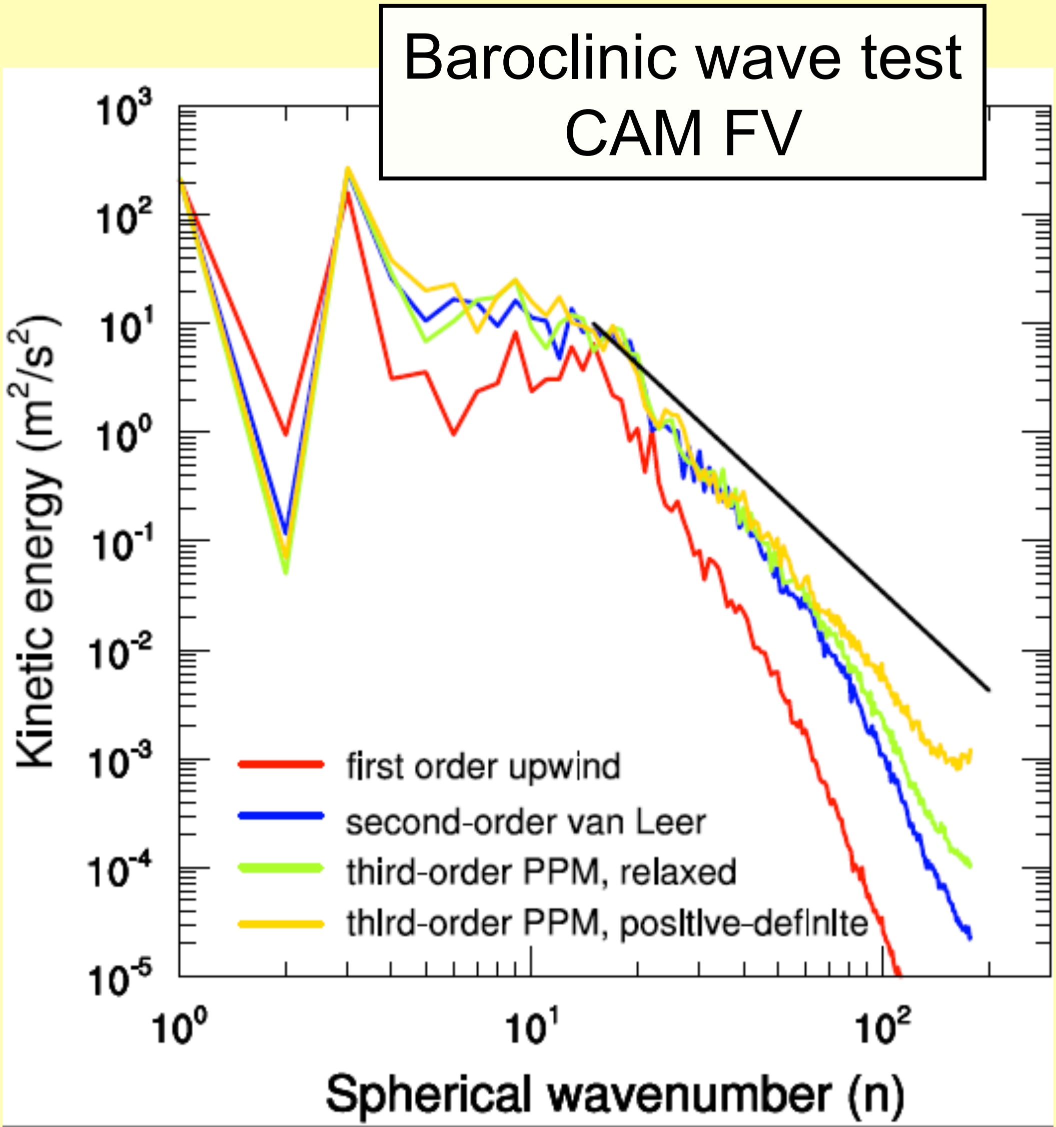
- Implicit diffusion: diffusion that is inherent in the numerical scheme
- Sources of implicit / numerical diffusion:
  - Order of accuracy: 1st order, 2nd order, 3rd order, …, higher order schemes
  - The higher the order, the less diffusive
  - Monotonicity constraints
  - Off-centering parameters in semi-implicit time-stepping schemes, or off-centered trapezoidal time-stepping schemes

Implicit diffusion: Order of accuracy



Baroclinic wave test: CAM FV 1°x 1° L26 $T_{850\ hPa}$ at day 9

Implicit diffusion: Order of accuracy



- 700 hPa kinetic energy spectrum (day 30) at 1° horizontal resolution
- Third-order (PPM)
- Second-order (van Leer scheme)
- Tail of 2nd-order scheme drops faster

Implicit diffusion: Off-centering

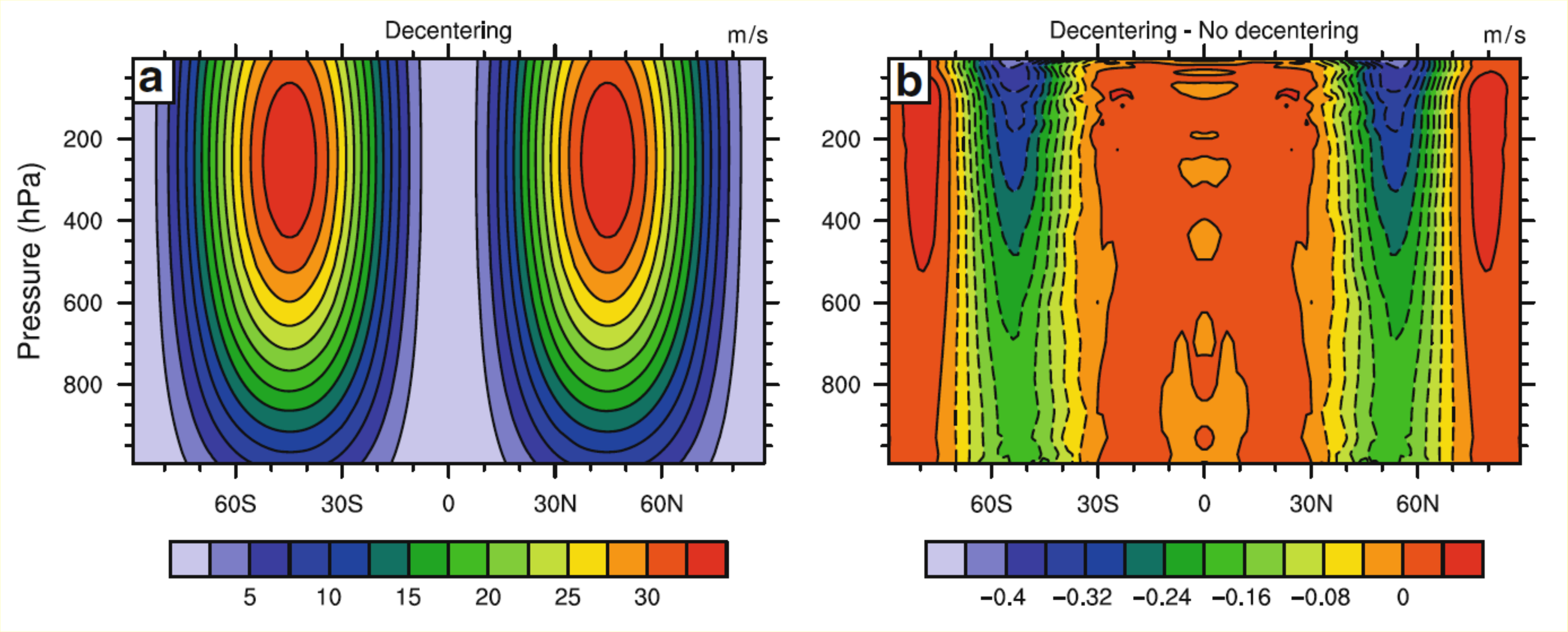
- Often used in semi-Lagrangian (SL) models like the U.K. Met Office Models
- Principle: introduce a weighting between the future and current time step
- The off-centering parameter ε is typically small (smaller than 0.2)
- Off-centering introduces implicit diffusion, needed to suppress orographic resonance waves in SL models, degrades formal order of accuracy

$$\frac{D\psi}{Dt} = S$$

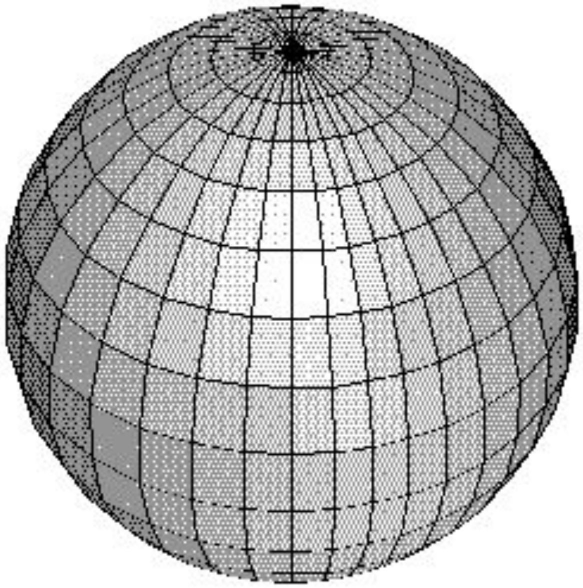
$$\psi^{j+1} - \psi_d^j = \Delta t\left(\frac{1+\epsilon}{2} S^{j+1} + \frac{1-\epsilon}{2} S_d^j\right)$$

Implicit diffusion: Off-centering

- Example: Zonal-mean zonal wind at day 30 in CAM-SLD (semi-Lagrangian spectral transform model)
- Difference between a steady-state experiment with off-centering (default) and no off-centering, wind speed decreases in an off-centered model

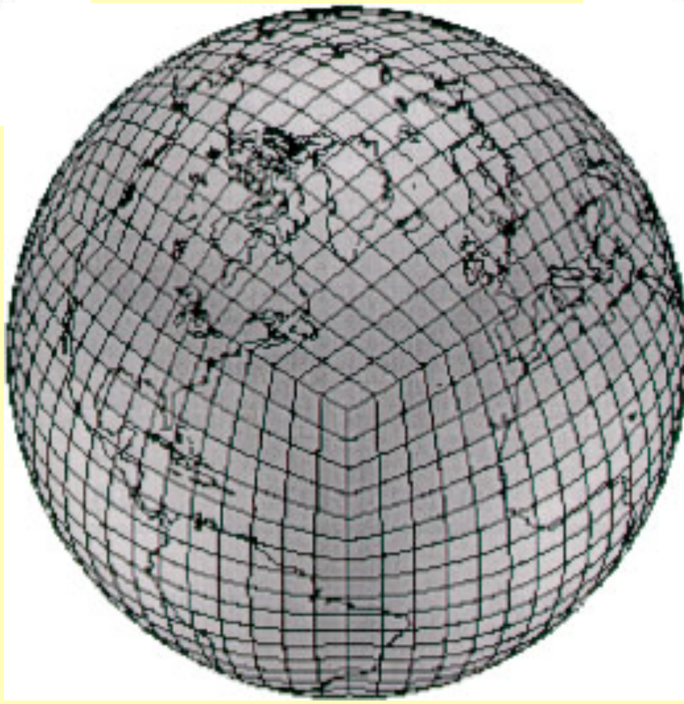


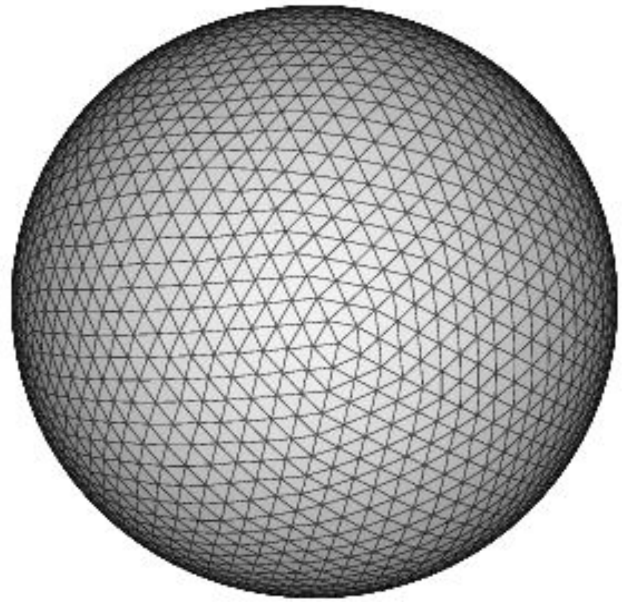
Computational grids (horizontal)



Latitude-longitude grid: needs polar filtering due to convergence of meridians

Cubed sphere





Icosahedral grid

No polar filter required

Spatial filters

- Most popular and most effective polar filter: 1D Fourier filter (spectral filter), used in the zonal (x) direction
- **Basic idea:**
  - Transform the grid point data into spectral space via Fourier transformations
  - Eliminate or damp high wave numbers (noise) by either setting the spectral coefficients to 0 or multiplying them with a damping coefficient $\in [0,1]$
  - Transform the field back from spectral space into grid point space: result is a filtered data set
- Filter strength is determined by the spectral damping coefficients, can be made very scale-selective and dependent on the latitude (e.g. less strong towards equator)
- Drawback: needs all data along latitude ring (poor scaling)

Polar Fourier Filters

- A Fourier filter application for all zonal wavenumbers k can be written as: $$\hat{\phi}(k)_{filtered} = a(k)\hat{\phi}(k)$$

  where $\hat{\phi}(k)$ are the Fourier coefficients and *a(k)* are the filter coefficients
- The filter coefficients depend on latitude *ψ*, they are e.g. defined by (with n: # grid points in the zonal direction)

$$a(k) = \min\left[1., \max\left(0., \left(\frac{\cos\psi}{\cos\psi_o}\right)^2 \frac{1}{\sin^2(\pi k/n)}\right)\right]$$

- Coefficients become small (or zero) at high latitudes and for high wave numbers. Filter becomes inactive (*a(k)*=1) at latitude ψ_0 (often chosen to be between 30-45 degrees N/S).

Digital filters

- Digital or algebraic filters are local grid-point filters that only take neighboring grid points into account
- Examples are the Shapiro filters (Shapiro, 1975)
- 4th order (n=2) Shapiro filter (i is the grid point index):

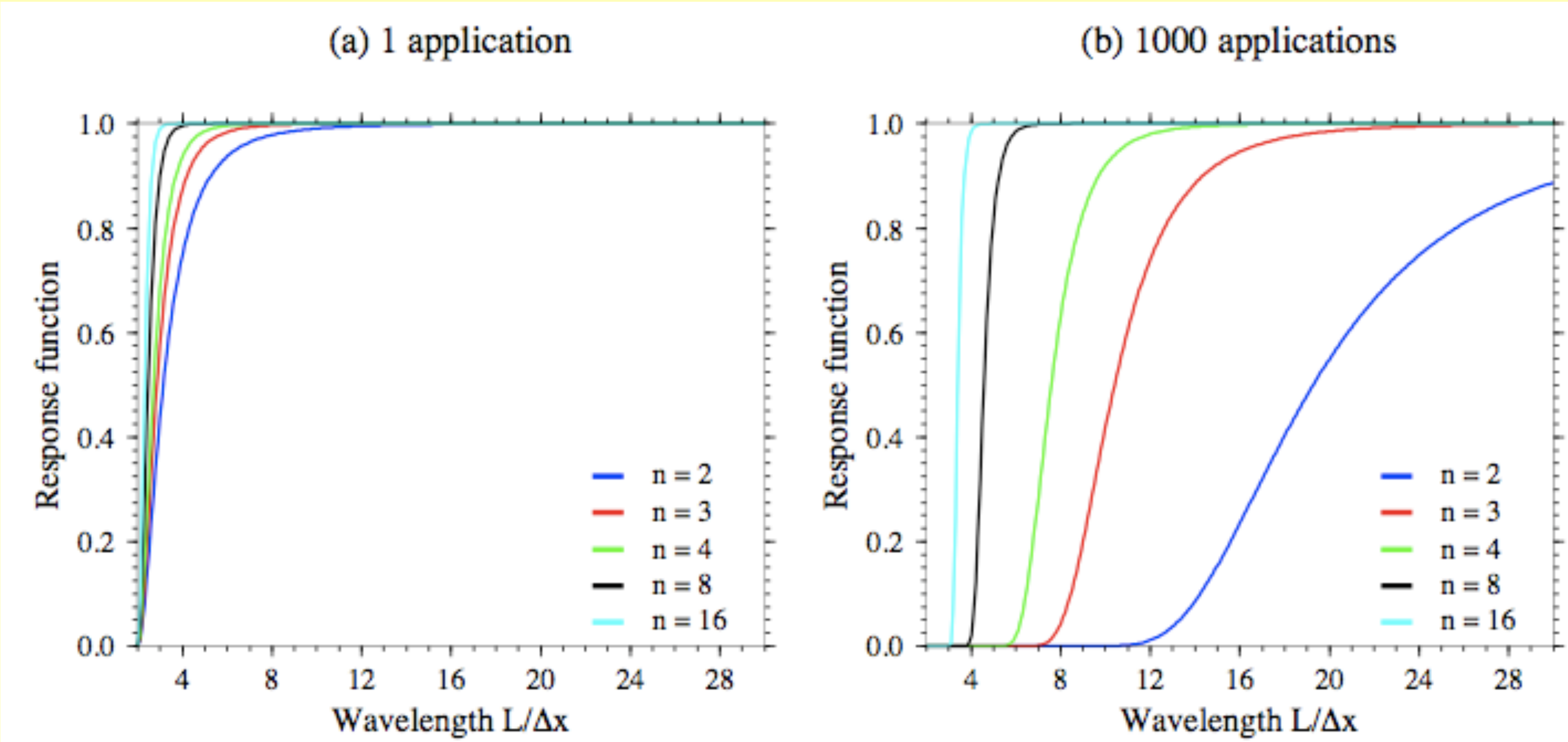
$$\bar{f}_i = \frac{1}{16}\left(- f_{i-2} + 4f_{i-1} + 10f_i + 4f_{i+1} - f_{i+2}\right)$$

- The filter response/damping function is (Shapiro, 1971)

$$\begin{aligned} \rho_n(k) &= 1 - sin^{2n}\left(k\frac{\Delta x}{2}\right) \\ &= 1 - sin^{2n}\left(\pi\frac{\Delta x}{L}\right) \end{aligned}$$

2n: order

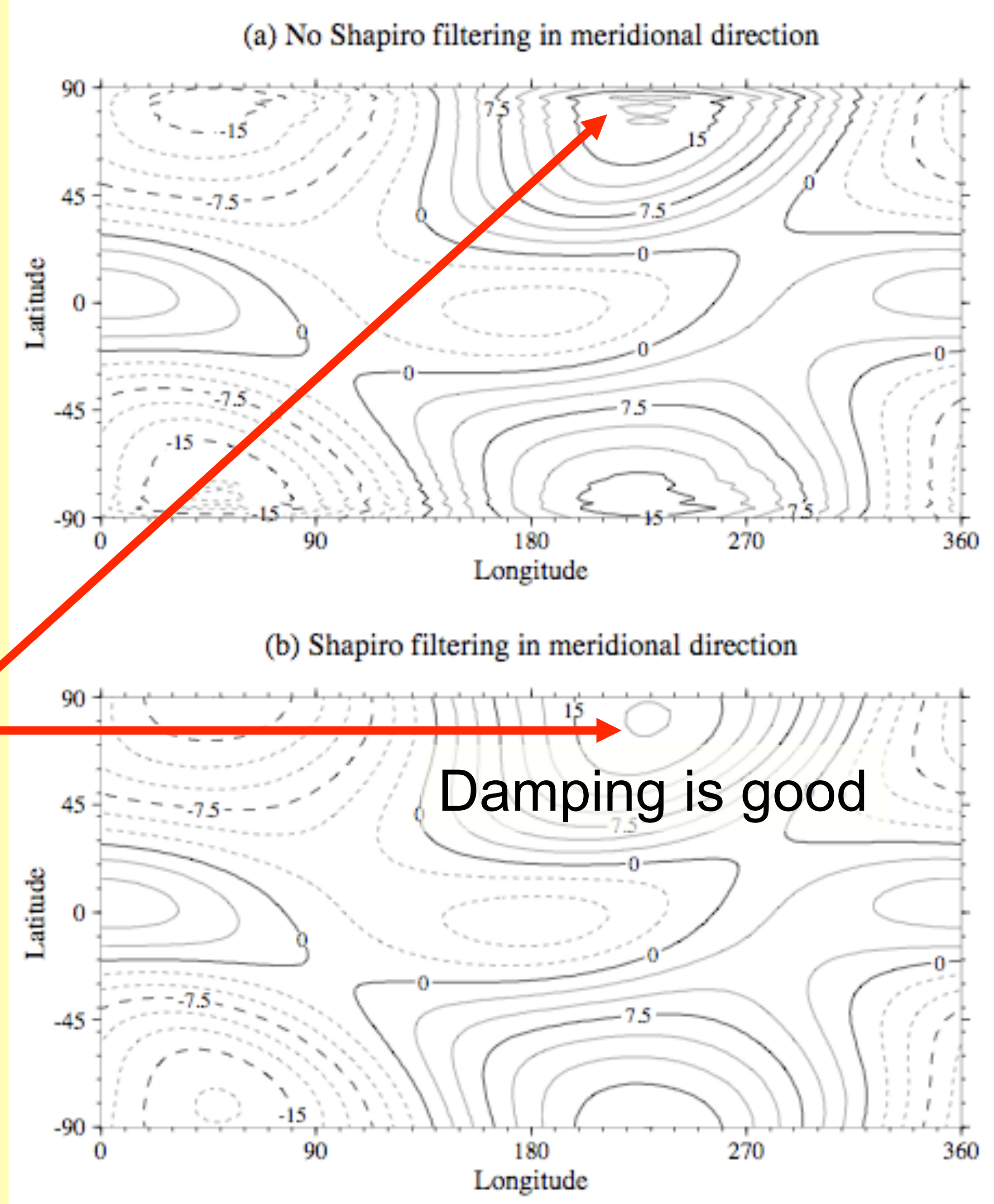
Digital filters: Response function



- Response function of different Shapiro filters after (a) 1 application and (b) 1000 applications. $2n$ indicates the order of the Shapiro filter. Higher orders need more data points.

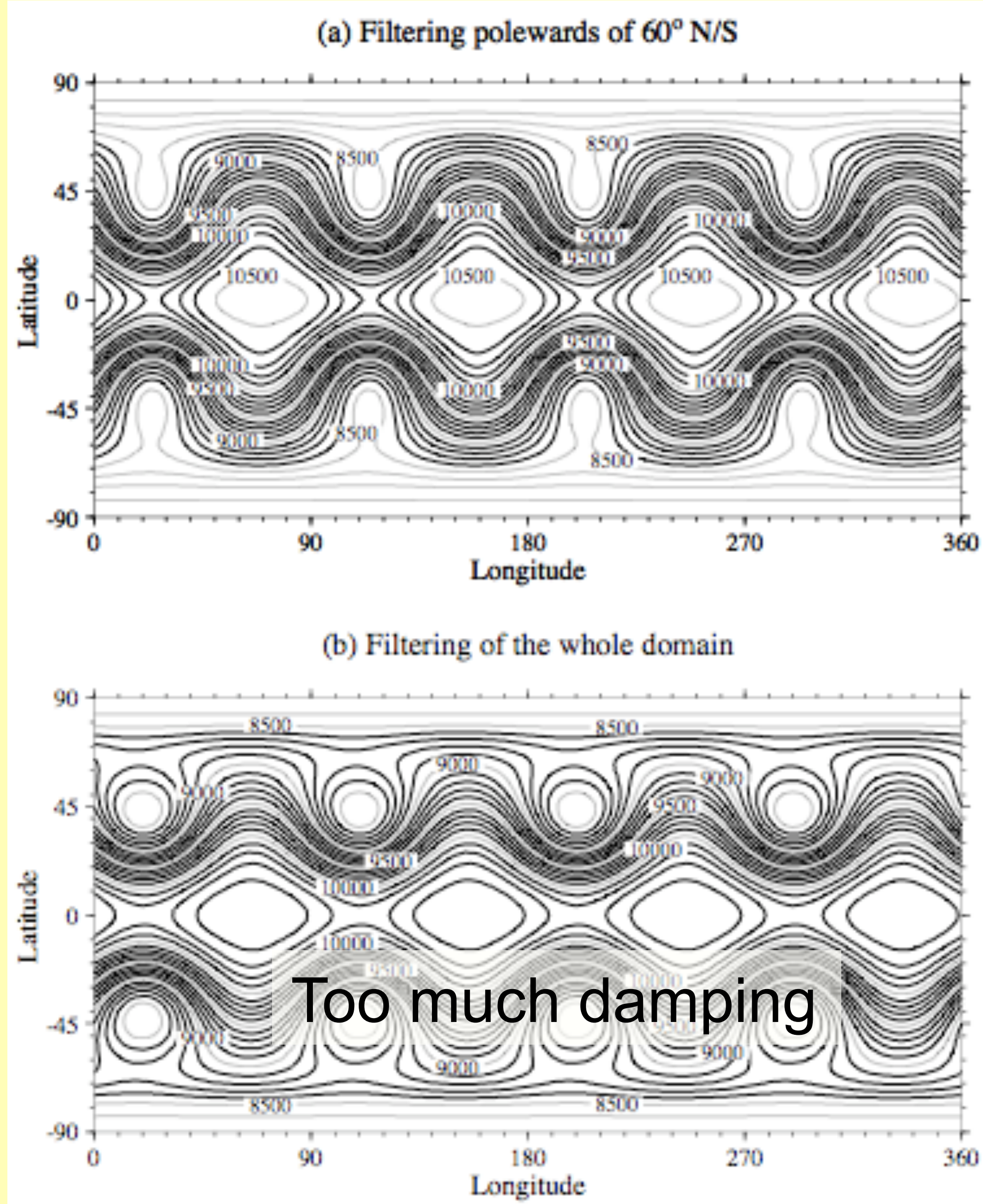
Digital filters

- Can provide a strong damping effect
- Use very selectively
- Example: SW simulation, digital filtering in y-direction applied near the pole points



Spatial Filters

- Can provide a strong damping effect
- Example: Rossby-Haurwitz wave in SW FV model, height at day 14
- (a) Fourier (90º-75º N/S) and digital Shapiro filtering (75º-60º N/S)
- (b) Digital Shapiro filter also applied between 60ºN - 60ºS, very diffusive, not suitable



Time filters

- Used in models with 3-time level schemes (e.g. Leapfrog)
- Most often used: Robert-Asselin filter (Asselin, 1972)
- Avoids that the even and odd time steps separate
- Basic idea: Second-order diffusion in time
- Example with time levels n-1, n, n+1:

$$\overline{\psi}^{n} = \psi^{n} + \alpha\left(\overline{\psi}^{n-1} - 2\psi^{n} + \psi^{n+1}\right)$$

- Filter strength is determined by the coefficient α
- Often used $\alpha \approx 0.05$

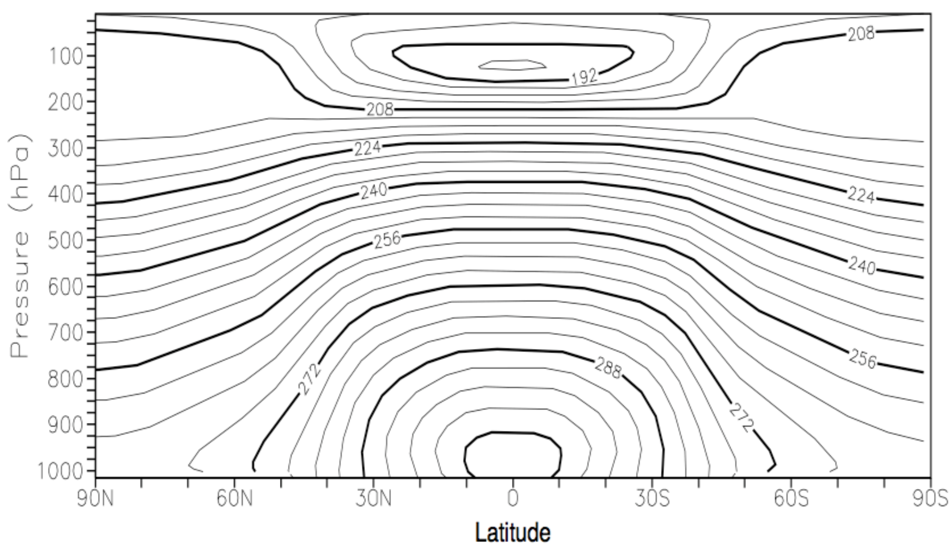
Conservation of Mass: Mass fixers

- Some dynamical cores are not mass-conserving by design
- But: Conservation of mass is needed in long-term climate simulations, less important in short weather prediction runs
- These models might apply an *a posteriori* mass fixer
- Basic idea behind the mass fixer: adjust the mean value of p_s after each time step, adjustment modifies all grid points at the surface
- This technique does not (!) alter the pressure gradients which are the driving force in the momentum equations
- Sounds okay? Let’s see (next slide):

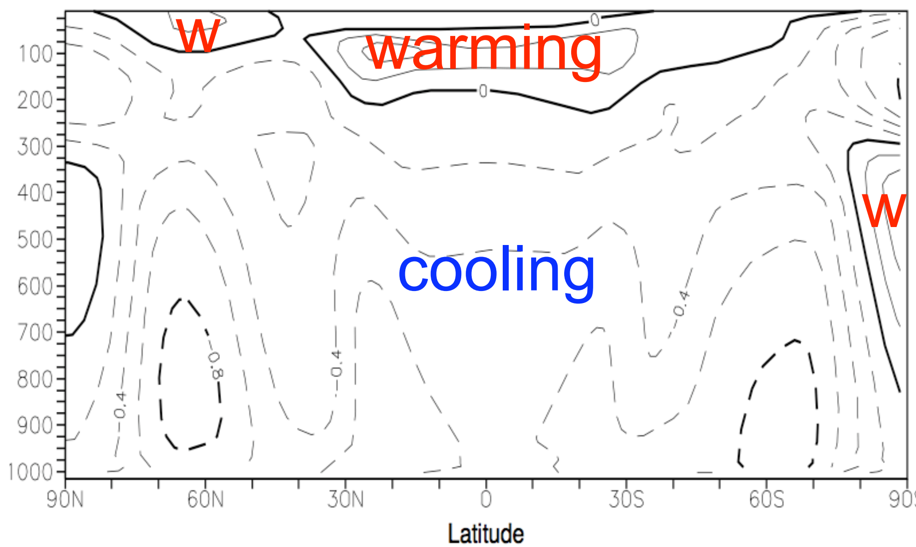
Conservation of Mass: The potential impact of mass fixers

- Weather forecast model IFS run with the Held-Suarez test
- Compare the time-mean zonal-mean temperature of a run with and without mass fixer

Temperature
(without mass fixer)



Temperature difference
(with mass fixer - without)



Conservation of Total Energy (TE)

- The question is whether TE is a conserved quantity in a dynamical core with numerical discretizations.
- Should we care?
  – in Weather Prediction Models
    - The answer is ‘not necessarily’
  – in Climate Models
    - The answer is ‘yes’
- When running for long times the violation of the total energy conservation leads to artificial drifts in the climate system (e.g. ocean heat fluxes)

Total Energy Fixer

- In nature:
  - conservation of total energy
  - energy lost by molecular diffusion provides heat
- In atmospheric models:
  - Energy is lost due to explicit or implicit (numerical) diffusion processes
  - Molecular diffusion is not represented on the model grid (spatial scale in models in way too big)
  - Numerical scheme might also lead to increase in total energy
- Therefore: some models provide an *a posteriori* energy fixer that restores the conservation of total energy by modifying the temperature

A posteriori Total Energy Fixer

- Goal: Total energy at each time step should be constant
- Compute the residual: $RES = \hat{E}^{+} - E^{-}$
- Compute the total energy before (-) and after (+) each time step

$$\hat{E}^{+} = \int_A \left\{ \left[\sum_{k=1}^{K} \left(\frac{(\hat{\mathbf{v}}_k^{+})^2}{2} + c_p \hat{T}_k^{+} \right) (p_0 \, \Delta A_k + \hat{p}_s^{+} \, \Delta B_k) \right] + \Phi_s \hat{p}_s^{+} \right\} dA$$

$$E^{-} = \int_A \left\{ \left[\sum_{k=1}^{K} \left(\frac{(\mathbf{v}_k^{-})^2}{2} + c_p T_k^{-} \right) (p_0 \, \Delta A_k + p_s^{-} \, \Delta B_k) \right] + \Phi_s p_s^{-} \right\} dA$$

A posteriori Total Energy Fixer

- **Idea:** Correct the temperature field to achieve the conservation of total energy (mimics heating by molecular diffusion)
- Option: **Fixer 1**, correction proportional to the magnitude of the local change in T at that time step

$$T^{+}(\lambda,\varphi,\eta) = \hat{T}^{+}(\lambda,\varphi,\eta) + \beta_1 |\hat{T}^{+}(\lambda,\varphi,\eta) - T^{-}(\lambda,\varphi,\eta)|$$

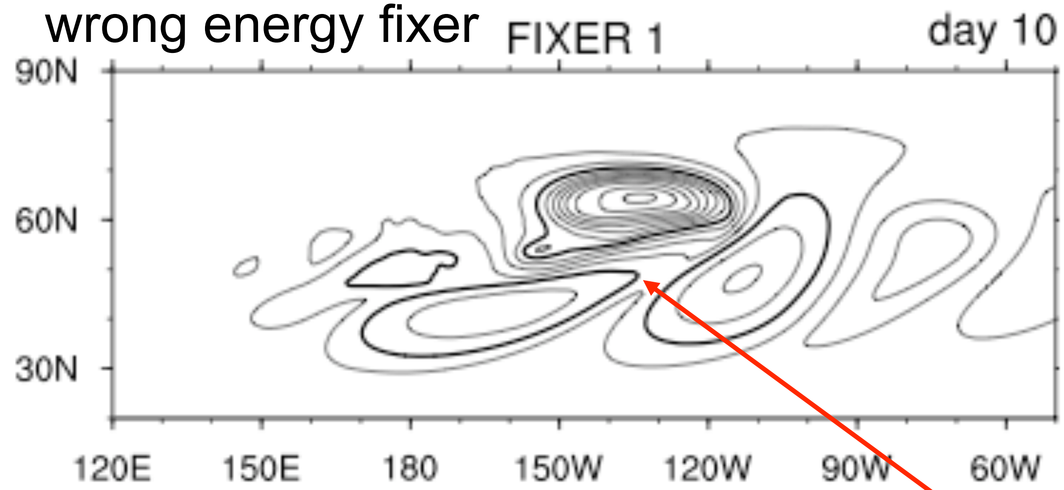
- Option: **Fixer 2**, correction is constant everywhere

$$T^{+}(\lambda,\varphi,\eta) = \hat{T}^{+}(\lambda,\varphi,\eta) + \beta_2$$

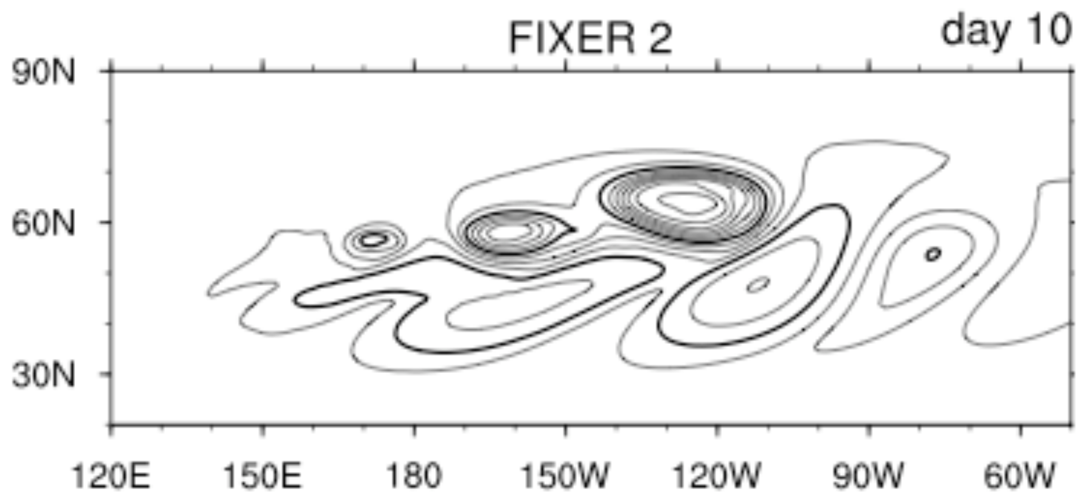
- Fixer 1 looks physical, but leads to wrong results

Energy Fixer: Surprising Consequences

wrong energy fixer



corrected energy fixer



- Baroclinic wave, p_s at day 10
- CAM SLD with an 'inadequate' and 'corrected' choice of an energy fixer
- Inadequate choice leads to wrong circulation pattern

Williamson, Olson & Jablonowski, (MWR, 2009)

Final Thoughts

- There are many design decisions in GCMs. They should be based on physical principles.
- Diffusion and filters help maintain the numerical stability.
- Some **diffusion** (either explicit or implicit) is **always needed** to prevent an accumulation of energy at the smallest scale (due to truncated energy cascade).
- But: Use the techniques selectively and know their consequences. Ask your mentor about your model!
- Test and intercompare as much as possible.
- Word of caution: It is very easy to compute nice-looking smooth, highly diffusive, but very inaccurate solutions to the equations of motion.

References

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