Regular expressions: derivations

1. Which of the following statements is true? If it is true, give a derivation; if not, explain.
   (a) $a \in \mathsf{L}(a+b)$
   (b) $ab \in \mathsf{L}((a+b))$
   (c) $ab \in \mathsf{L}((a+b)(a+b))$
   (d) $aa \in \mathsf{L}(a+a)$
   (e) $\varepsilon \in \mathsf{L}(b*)$
   (f) $b \in \mathsf{L}(b*)$
   (g) $bb \in \mathsf{L}(b*)$

Regular expressions: properties

Two regular expressions r and r' are equivalent if forall xs, $xs \in \mathsf{L}(r)$ if and only if $xs \in \mathsf{L}(r')$.

Prove the following regular expressions are equivalent, for all regular expression a, b, c.

Clearly state how the proof is constructed, either by using rule induction or applying rules. When using rule induction, state the cases and hypotheses available at every step:

(a) a and $a + 0$
(b) $a + a$ and a
(c) $a + b$ and $b + a$
(d) $a + (b + c)$ and $(a + b) + c$
(e) $1a$ and a
(f) $(a^*)^*$ and a^*

Evaluation of lambda terms

Given the following definitions:

$$\begin{aligned} I &= \lambda x.x \\ K &= \lambda xy.x \\ S &= \lambda xyz.(xz)(yz) \end{aligned}$$

Given a derivation of following terms to a normal form, using the rules presented in class:

1. Ia
2. $KIab$
3. $(IK)(II)$
4. $S(K(Ka))(Kb)c$

The typed lambda calculus

Let Γ be an environment including:

- one : N
- isEven : $N \to B$
- not : $B \to B$
- add : $N \to N \to N$

Give typing derivations for the following terms using the rules presented in class:

1. isEven one
2. add one one
3. $\lambda x : B.\, \text{not}(\text{not } x)$
4. $\lambda x : N.\, \text{one}$
5. $\lambda x : N . \lambda y : N . \text{isEven } x$
6. $\lambda x : (N \to N).\text{not}$