Name: _____

Math 152 * Enrichment Session 3

1. [10=4+3+3 points] Let R be the region in the $xy-$ plane lying below the curve $y = \sqrt{x}\cos x$ and above that portion of the $x-$ axis with $0 \leq x \leq \frac{\pi}{2}$.

   (a) Set up, but do not yet evaluate, an integral in terms of a single variable that represents the volume of the solid obtained by rotating R about the x-axis. Justify your answer by drawing a picture, labeling a sample slice, and citing the method used.

   (b) Verify that the antiderivative of the function $f(x) = x\cos 2x$ is given by $F(x) = \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + c$

   (c)Evaluate the integral of part (a).

*Supported by NSF grant number 1317651

2. [10 points] A napkin ring is made by taking a solid wooden ball (sphere) of radius R and drilling a hole of radius a straight through the center. (The hole is cylindrical in shape with radius a, and the resulting solid has flat edges at its top and bottom.) Find the volume of the napkin ring, in terms of R and a.

3. [5 points] Set up, but do not evaluate an integral, in terms of a single variable, representing the volume of the solid obtained by rotating the region enclosed by the curves $y = \sqrt{x}$ and $y = x^{1/3}$ in the first quadrant around the line $x = 1$. Use the method of washers and draw a picture and label a sample slice.

4. [5 points] Set up, but do not evaluate **an** integral, in terms of a single variable, representing the volume of the solid obtained by rotating the region enclosed by the curves $y = 5 - x^2$ and $y = x^2 + 3x + 3$ about the x-axis. Draw a picture and label a sample slice.

5. [5 points] Set up, but do not evaluate an integral, in terms of a single variable, representing the volume of the solid obtained by rotating the region enclosed by the curves $y = \sqrt{x}$ and $y = x^3$ in the first quadrant around the line $x = 1$. Use the method of washers and draw a picture and label a sample slice.