ANSI/ASQC B1-1996, ANSI/ASQC B2-1996, and ANSI/ASQC B3-1996

AMERICAN NATIONAL STANDARD

Guide for Quality Control Charts

Control Chart Method of Analyzing Data

Control Chart Method of Controlling Quality During Production

AMERICAN SOCIETY FOR QUALITY
611 EAST WISCONSIN AVENUE
MILWAUKEE, WISCONSIN 53202

AMERICAN NATIONAL STANDARD

Guide for Quality Control Charts
Control Chart Method of Analyzing Data
Control Chart Method of Controlling Quality During Production

[Reaffirmation of ANSI/ASQC B1-1985, ANSI/ASQC B2-1985, and ANSI/ASQC B3-1985]

Prepared by
American Society for Quality Standards Committee
for
American National Standards Committee Z-1 on Quality Assurance

An American National Standard Approved on February 26, 1996

American National Standard: An American National Standard implies a consensus of those substantially concerned with its scope and provisions. An American National Standard is intended as a guide to aid the manufacturer, the consumer, and the general public. The existence of an American National Standard does not in any respect preclude anyone, whether he or she has approved the standard or not, from manufacturing, marketing, purchasing, or using products, processes, or procedures not conforming to the standard. American National Standards are subject to periodic review, and users are cautioned to obtain the latest editions.

Caution Notice: This American National Standard may be revised or withdrawn at any time. The procedures of the American National Standards Institute require that action be taken to reaffirm, revise, or withdraw this standard no later than five years from the date of publication. Purchasers of American National Standards may receive current information on all standards by calling or writing the American National Standards Institute.

ASQ Mission: To facilitate continuous improvement and increase customer satisfaction by identifying, communicating, and promoting the use of quality principles, concepts, and technologies; and thereby be recognized throughout the world as the leading authority on, and champion for, quality.

10 9 8 7 6 5 4 3

Printed in the United States of America

Printed on acid-free paper

Published by:

American Society for Quality
ASQ™

Quality Press
611 East Wisconsin Avenue
Milwaukee, Wisconsin 53201-3005
800-248-1946
Web site http://www.asq.org

Contents

Foreword

(This Foreword is not a part of American National Standard ANSI/ASQC B1-1996, *Guide for Quality Control Charts,* nor of American National Standard ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data,* nor of ANSI/ASQC B3-1996, *Control Chart Method of Controlling Quality During Production.*)

Upon request by the War Department, the American Standards Association, in December, 1940, initiated a project on the application of statistical methods to the quality control of materials and manufactured products. Since, due to the national emergency, there was an urgent need for the prompt development of standards in this field, the ASA Defense Emergency Procedure (later called the War Emergency Procedure) was applied to this project, and the following Emergency Technical Committee (later, War Committee) was appointed to develop such standards:

H. F. Dodge, Bell Telephone Laboratories, Inc., Chairman
A. G. Ashcroft, Alexander Smith and Sons Carpet Company
W. Edwards Deming, Bureau of the Census
Leslie E. Simon, Ordnance Department, U.S. Army
R. E. Wareham, General Electric Company
John Gaillard, American Standards Association, Secretary

This committee developed these three standards. Drafts were submitted for criticism and comment to a number of key individuals in groups having a substantial interest in the subject of the standards. All of the comments were carefully reviewed by the committee and a number of changes were made in accordance with suggestions received. The revised drafts of Z1.1 and Z1.2 were unanimously approved by the Emergency Technical Committee and received ASA approval as American Defense Emergency Standards (later, War Standards) on May 27, 1941. Z1.3 was first approved in 1942.

In November, 1952, the ASA invited the American Society for Quality Control (ASQC) to accept the proprietary sponsorship for the three standards which had been developed by the War Committee. The invitation was accepted by ASQC in February, 1953, and the standards were turned over to the ASQC Standards Committee who assigned the designations ASQC B1, ASQC B2, and ASQC B3 to the standards which were later to become ANSI standards Z1.1, Z1.2 and Z1.3 respectively. The personnel of the committee at that time was as follows:

Irving W. Burr, Purdue University
W. Edwards Deming, Consultant in Statistical Surveys, New York University
Harold F. Dodge, Bell Telephone Laboratories, Inc. (retired) and Rutgers, The State University of New Jersey, Chairman
Eugene L. Grant, Stanford University
Ralph E. Wareham, Consultant in Quality Control

One of the duties of the ASQC as Proprietary Sponsor was the establishment of a national consensus on approval of the standards by industry. In August, 1956, a canvass of industry was instituted in which organizations believed to have a substantial interest in the subject of quality control were contacted. This canvass resulted in all but three of the organizations interested approving the standards as circulated by ASQC. After further review by the Standards Committee of ASQC, in the light of comments received in the course of the canvass, ASQC felt that the basic criticisms had been covered by making minor modifications and bringing the appendixes up to date, and accordingly submitted the standards to ASA for approval as American Standards. In the course of considering the submittal for a recommendation on approval, the Miscellaneous Standards Board, which had jurisdiction over this work, requested that those organizations which had objected be contacted again to ascertain their present feelings in the matter. This was done with the result that the organizations involved announced that they now approved the standards.

Accordingly, after receiving a favorable recommendation from the Miscellaneous Standards Board, the American Standards Association approved the standards as American Standards on November 21, 1958.

In 1981, the ANSI Z1 Statistical Methods Subcommittee recommended to the ASQC Standards Committee that the standards Z1.1, *Guide for Quality Control,* Z1.2, *Control Chart Method of Analyzing Data,* and Z1.3, *Control Chart Method of Controlling Quality During Production* can be updated to include more modern terminology and symbols in keeping with the American National Standard ANSI/ASQC A1-1978, *Definitions, Symbols, Formulas and Tables for Control Charts* and other publications such as the ASTM STP15D, *ASTM Manual on Presentation of Data and Control Chart Analysis.*

The task was assigned to the ASQC Statistics Division which formed a writing committee that completed the task of updating these standards. The writing committee combined the three standards, Z1.1, Z1.2, and Z1.3 under one cover as had been done previously with Z1.1 and Z1.2, because all three documents are concerned with the statistical quality control charts.

The significant changes are: (1) the redefinition of the sample standard deviation to be $s = \sqrt{\Sigma(X - \overline{X})^2 / (n - 1)}$; (2) the use of the words nonconforming and nonconformities to replace defectives and defects respectively; (3) the use of a subscript (0) to replace the prime symbol (') e.g. p_0 used in place of p', for designating a standard value, and (4) the amplification of the material on Warning Limits.

The redifinition of the sample standard deviation also required changing many factors used in control chart work (c_4 in place of c_2; A_3 in place of A_1; B_5 and B_6 in place of B_1 and B_2 respectively; E_3 in place of E_1). Table 6 *Factors for Computing Control Chart Lines* and the Table A2 in the Appendix now use the updated factors.

The above changes required many changes in these standards. This document is a reaffirmation of the 1985 version.

Scope

The scope of this document is intended to cover the Shewhart statistical quality control charts which are in general use in the United States manufacturing and service industries.

Writing Committee

The following individuals were members of the writing committee for the 1985 revision of ANSI Z1.1, Z1.2, and Z1.3 1958:

Sherman L. Babcock, Chairman
Hardy M. Cook, Jr.
Acheson J. Duncan
C. Allen Mannon
Harrison M. Wadsworth, Jr.
Oswald Willner

Suggestions for improvement of this standard will be welcome. They should be sent to the Standards Administrator, American Society for Quality Control, 611 East Wisconsin Avenue, Milwaukee, WI 53202.

ANSI/ASQC B1-1996
Guide for Quality Control Charts

1. SCOPE

1.1 This standard is intended as a guide for handling problems concerning the economic control of quality of materials, manufactured products, services, etc. It has particular reference to methods of collecting, arranging, and analyzing inspection and test records in a manner designed to detect lack of uniformity of quality.

For the sake of simplicity, the term *product* will be used hereafter, whether the object whose quality is being considered is a material, a semi-finished or finished product, or a service. Thus the words "manufacturer, purchaser, production process, etc." should be interpreted broadly to cover many fields of interest.

2. THE CONTROL CHART

2.1 It is recommended that the control chart be used for handling quality control problems, for example, for controlling quality during manufacture, for presenting the essential information of the quality records, as an aid in judging how well the quality is controlled, for continuing quality improvement, and as an indicator as to when action should be taken to prevent quality problems from occurring.

2.2 The control chart is a graphical record of quality. On it are placed a pair of *control limits.* These limits are of assistance in judging the significance of variations of product quality around the general level, particularly with a view to the more important function of exercising purposeful control over the quality. Moreover, they are placed such that a plotted point falling outside them during manufacture may be taken as an indication of a cause of variation that should be investigated.

3. SPECIFICATION LIMITS

3.1 Before the nature of control limits is explained, it may be well to visualize the nature of *specification limits.* These are the limits given in the product specification to define the extreme permissible values of a quality characteristic, to ensure correct performance of the *individual unit* of product. Specification limits are used by the manufacturer in his own plant, or by the purchaser when receiving a consignment, as a basis for checking the quality of each individual unit inspected.

A typical example of specification limits are the minimum and maximum limits for the diameter of a shaft.

4. CONTROL LIMITS

4.1 The control limits on a control chart are different from specification limits. They are used, not for checking the quality of each unit of the product, but as a basis for judging the significance of the quality variations from sample to sample, from lot to lot, or from time to time. They supply a criterion for deciding whether a production process is being disturbed by causes of variation that should be investigated.

Control limits apply to some measure of the *collective quality* of a group of units, this measure to be computed from observations made on the individual units in the group.

Quality measures commonly used are (1) the *average* of the observed values of the individual units under consideration, and (2) some measure of the dispersion of the observed values around their average, such as the *standard deviation* or the *range.* Where individual units are tested or examined primarily to determine whether they do or do not conform to a specified requirement or set of requirements, the quality measure *fraction nonconforming* is commonly used.

5. VARIATION OF QUALITY—ASSIGNABLE CAUSES

5.1 The quality of a product as measured from sample to sample, or from lot to lot, shows variations that are attributable to numerous causes. A variation can be classed in one of two ways: (1) as a variation that merits no investigation; (2) as a significant variation, indicative of an *assignable cause* (cause of trouble), which should be identified and corrected if practicable. The control limits effect this classification in an economic manner.

5.2 The control chart can only indicate when and where the trouble has occurred; the identification and elimination of the trouble is an engineering problem. An assignable cause of variation may be attributable to lack of uniformity in materials or in workmanship, or to irregular performance of manufacturing equipment or testing equipment. The removal of such causes decreases the variability of quality.

6. STATE OF CONTROL

6.1 When assignable causes have been eliminated from the production process to the extent that substantially all the points plotted on the control chart remain within the control limits, the process is said to be in a state of *statistical control* with respect to the measure of quality under consideration. When a state of statistical control is reached, no higher degree of uniformity in quality can be obtained with the production process in use. It may thus be concluded that greater uniformity can then be attained only through a basic change in the process itself.

For convenience, hereafter, a *state of statistical control* will usually be referred to simply as a *state of control.* Likewise, *statistical control* will usually be referred to as *control.*

6.2 When the quality is controlled and the level of control is found to be such that the product does not meet the specification limits in a satisfactory manner, either of two corrective measures may be taken:

1) Shift the level of control by making basic changes in the production process, or
2) Adjust the specification limits to encompass the existing level of control.

7. PRACTICAL ADVANTAGES OF A STATE OF CONTROL

7.1 To the manufacturer and the purchaser, the existence of a state of control presents the following practical advantages:

1) With control, the variation between individual units will be a minimum for the production process in use.
2) With control, data from samples of the product have the greatest possible reliability as a basis for judging product quality. Sampling and testing, and hence the cost of inspection, can be reduced to a minimum. As a result it often follows that sampling inspection is adequate, both for the manufacturer and the purchaser.

   The reliability of sampling results is particularly important when assurance regarding the quality of an entire quantity of similar articles or material must be based on the inspection of only a limited number of articles or test specimens. This will be the situation when 100-percent inspection is impracticable or impossible, as happens when the inspection test of an article is destructive or when tests are made to measure the physical or chemical properties of materials, bulk products, or the like.

   Even when 100-percent inspection is practicable, but where it is satisfactory to use sampling inspection in place of 100-percent inspection (as when 100-percent conformance to specification requirements is not essential), the possible savings in the cost of inspection are greatest when the quality is controlled and the level of control is a satisfactory one.
3) With control, the percentage of product whose quality will lie within any pair of limits may be predicted with the highest degree of assurance.
4) With control, there is a reliable basis for determining whether there would be a practical advantage in changing the specification limits.

   This question is important, for example, when the manufacturer has chosen between the use of ample specification limits combined with selective matching of components and the use of closer specification limits that permit interchangeability of components.
5) With control, acceptance of a product by the purchaser may often safely be based on the manufacturer's evidence of control, rather than on results of the purchaser's own sampling, shipment by shipment. This may be accomplished if continuing control chart records are by agreement made available by the manufacturer to the purchaser, who will then need to test only occasional samples as checks on the quality records furnished by the manufacturer.
6) With control, reduction in nonconformances can be achieved which results in lower production costs and better performance against schedules.
7) With control, the tendency to undercorrect or overcorrect is minimized.

8. ESTABLISHMENT OF CONTROL LIMITS

8.1 Control limits are established by computation, in which use is made of:

1) Data covering past and current production records for the particular product under consideration, and

2) Formulas that are derived from the statistical approach to the quality control problem, and whose dependability have been proved by practice.*

9. PRINCIPAL USES OF THE CONTROL CHART

9.1 For the purpose under consideration here, the control chart has two principal uses: assisting judgment with regard to whether a state of control exists (Section 10), and attaining and maintaining control of quality during production (Section 11).

The ultimate and most important purpose of the control chart is to provide a definite operational procedure for controlling quality in the manufacturing plant. However, before setting up a system of control of future operations, it is usually necessary to get a picture of the history of the quality of the product, and in consideration of this information, to decide what tentative control limits should initially be placed on the chart. Therefore, in practice, the first purpose for which the control chart is used is often that of analyzing the quality records of the past.

10. USE OF THE CONTROL CHART FOR JUDGING WHETHER CONTROL EXISTS†

10.1 To achieve this purpose the control chart is used for analyzing collectively an accumulation of quality data, comprising a number of "rational subgroups" of observed values. The control limits for this use of the control chart are computed entirely from the data under consideration.*

Subgrouping‡ on the basis of time is the most useful in practice. The control chart method is not restricted, however, to the basis of "rational subgrouping." The method applies equally to subgrouping based on technical factors that affect manufacture, such as the operator, the machine, or the lot of raw material.

10.2 Information regarding the state of control may be desired, for example, by a manufacturer who wishes to introduce the control chart method for attaining control of quality (see Section 11) and to find out what has been done, so far; or by a purchaser interested in having information regarding the uniformity of the quality of the product supplied to him by a manufacturer.

10.3 Usually, it may not safely be concluded that a state of control exists unless the plotted points for at least 25 successive subgroups fall within control limits constructed from the data in the manner described in ANSI/ASQC B2. On the other hand, lack of control may be evidenced by one or more failures to meet control limits with a much smaller number of subgroups.

Figure 1 indicates the features of the control chart as it is used for judging whether control exists.

11. USE OF THE CONTROL CHART FOR ATTAINING AND MAINTAINING CONTROL OF QUALITY

11.1 Here the control chart is used for keeping a continuing record of the quality of the product while production is going on, so as to detect any assignable cause of variation as soon as possible after it appears.

11.2 A control chart is set up and used as follows: A tentative standard level of control (central line or average line) for use in future operations is adopted usually on the basis of past experience (see Section 10) and control limits are set above and below the standard level at distances determined with a view to the number of observations to be taken each time the product is sampled. The values obtained from inspecting the samples are then plotted on the chart *as soon as they are obtained.* A plotted point falling outside the control limits is taken as an indication of the presence of a disturbing cause of variation in the production process. This cause of variation should be identified, and if practicable, eliminated. The control limits thus serve as a criterion for action, and for this reason are also referred to as *action limits. Warning limits* closer to the standard level are also often used in conjunction with *action limits*, to detect runs or patterns of points which may serve as an indication of potential out-of-control conditions. By repeating this procedure—the identification of causes of variation and their elimination—over and over again, and by taking steps to prevent recurrences, the quality is brought closer and closer to the state of control; and when this state is reached, maintained there or further improved. As time goes on, newly acquired data proved fresh information for computing revised control limits and, if desired, for adjusting the standard level as well.

Thus, the control chart method is a continuing, self-corrective one, aimed at the maintenance of statistical control.

---

* The question of the placing of the control limits is discussed in ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data* (Section 7) and in Appendix A.

† This particular use of the control chart is further explained in ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data.*

‡ For an explanation and illustration of rational subgroups, see Appendix B, reference 5.

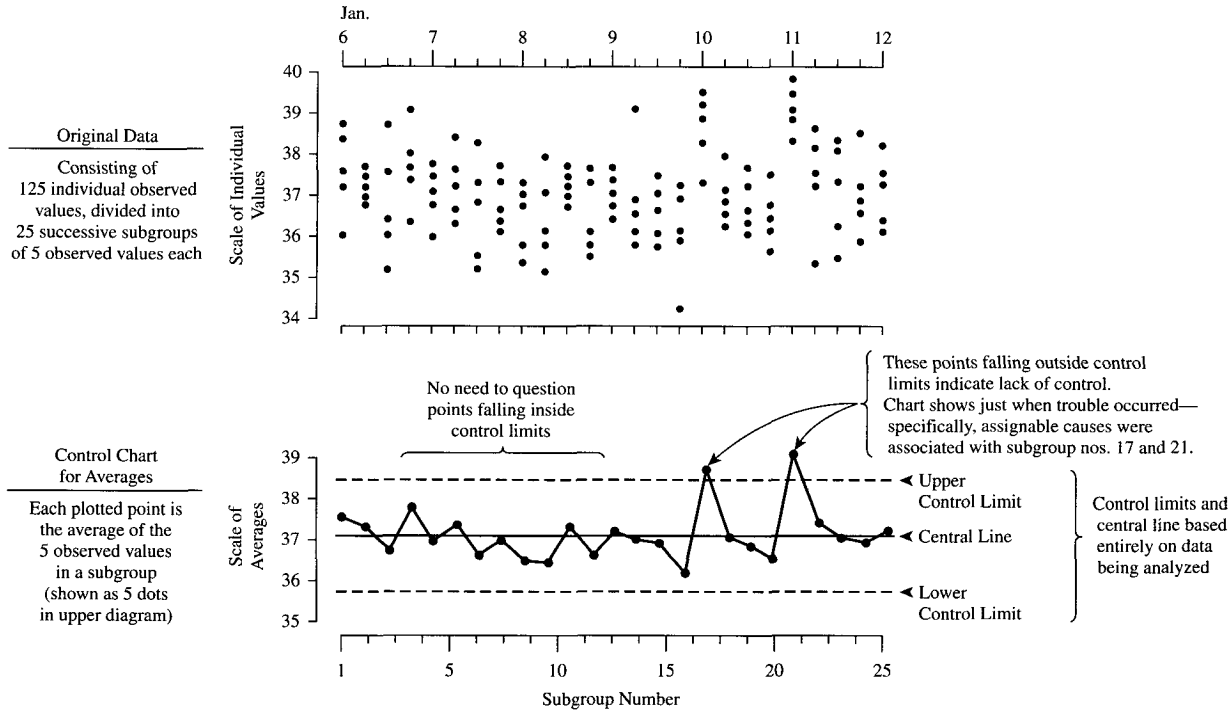


Figure 1 — Illustrating Features of the Control Chart as Used for *Analyzing* a Set of Data to Determine Whether There Has Been Lack of Control.

Every step in this direction (that is, the removal of any cause of trouble) is a step toward more economical production and often the reduction of product variability.

11.3 The most effective use of quality control as an operation is to apply the control chart procedure at that point in the production process which is closest to the potential sources of trouble. The control limits then indicate the need for action with a minimum of delay. A less immediately effective yet highly valuable use can be made of the control chart by constructing limits in like manner around an already established level, and plotting the manufacturer's inspection data daily, weekly, or even monthly; or plotting the results of the purchaser's tests, shipment by shipment, or lot by lot.

Figure 2 indicates the features of the control chart as it is used in the operation of quality control.

11.4 The use of control limits as recommended in this guide is based on statistical methods whose practical value has been proved by extensive applications made during many years in actual manufacturing practice.

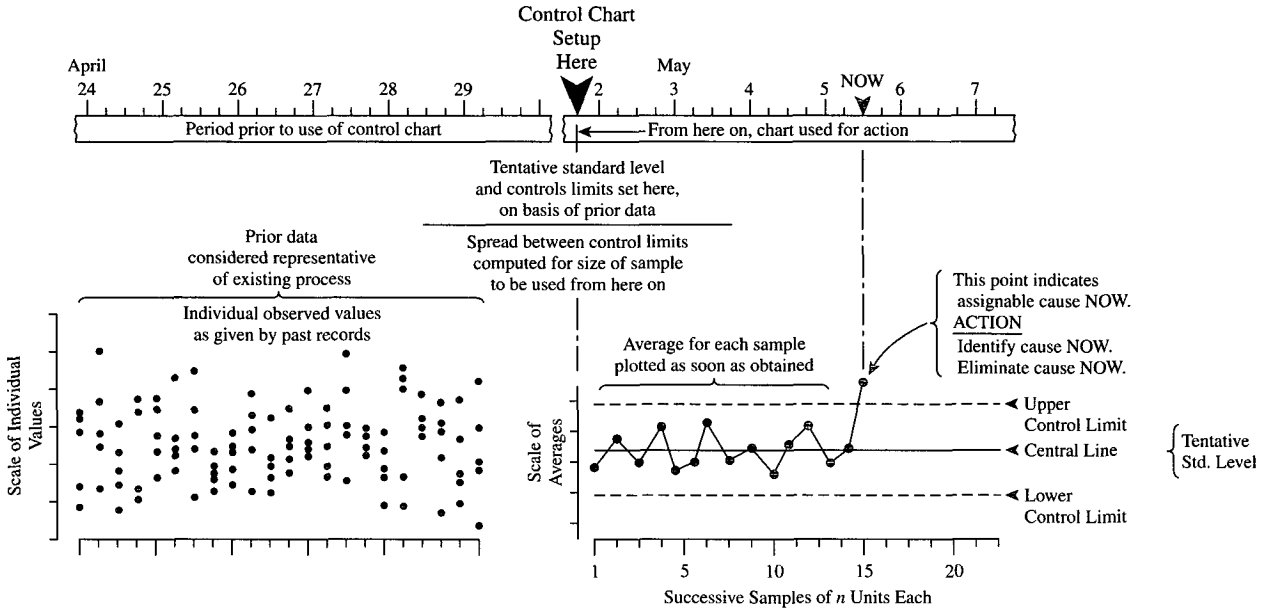


Figure 2 — Illustrating Features of the Control Chart as Used for *Controlling* Quality During Production. (This is a control chart for averages; each plotted point is the average of the n individual observed values for the n units in a sample.)

ANSI/ASQC B2-1996
Control Chart Method of Analyzing Data

1. SCOPE

1.1 This standard covers the control chart* method of analyzing a collection of data, with particular reference to quality data resulting from inspections and tests of materials and manufactured products.

1.2 The control chart method of analyzing data provides a graphical summary of the data that are being analyzed. This summary assists judgment in determining whether there is evidence of lack of control, and whether there is justification in using the summary as a basis for predicting the future performance of the processes that give rise to the data that are being analyzed—the performance, for example, of a production process.

2. NATURE OF THE DATA USED

2.1 What data should be collected to serve as a basis for judging the uniformity of the quality is a question of engineering, including statistical theory, and is to be decided in each individual situation. For a general discussion of this problem see for example the *ASTM Manual on Presentation of Data and Control Chart Analysis*, hereafter briefly called the ASTM Manual.†

2.2 Industrial experience indicates that where measurements are made on individual units of product or on test specimens (for instance, in determining weight in ounces, tensile strength in pounds per square inch, etc.) much of the useful information on the quality of a group or sample of n units is made available by presenting two simple statistical measures, their *average* ($\overline{X}$) and their *standard deviation(s),* along with the number n. If the number of units in a sample is very small (10 or fewer), the *range* (R) may be used in place of the standard deviation.

The *average* of a set of n observed values is the sum of the observed values divided by n. The sample *standard deviation*‡ of the set is the square root of the following: the sum of the squared deviations of the individual observations from their average ($\overline{X}$) divided by the quantity $(n - 1)$; i.e.:

$$s = \sqrt{\Sigma(X - \overline{X})^2 / (n - 1)}.$$

The *range* is the difference between the largest observed value and the smallest observed value.

There are two circumstances in which a lone statistical measure of quality, the *fraction nonconforming* (p), along with the number n, gives the essential information on the quality of a sample of n units. These circumstances exist (1) as a matter of choice when, although measurements are made on the individual units, interest centers almost wholly on whether the measured values fail to lie within specification limits; and (2) as a matter of necessity, when individual units are merely checked to determine whether they fail to meet a requirement (as when go/no-go gages are used in inspection, or when inspection consists of a visual examination for nonconformities with respect to workmanship, finish, appearance, and the like).

The *fraction nonconforming* is the ratio of the number of nonconforming units to the total number of units under consideration.

It should be mentioned that for some important measurable quality characteristics, there are circumstances in which there may be justification for using the fraction nonconforming (p), in addition to $\overline{X}$ and s, or in addition to $\overline{X}$ and R.

3. COMPUTATION OF THE STATISTICAL MEASURES

3.1 For methods of computing the average and the standard deviation, see Instruction Sheets 1 and 2; also the ASTM Manual or other references.

---

* An explanation of the fundamental concepts and terminology, and the two principal uses of the control chart, is given in ANSI/ASQC B1-1996, *Guide for Quality Control Charts.*

† Published by the American Society for Testing and Materials, Philadelphia, ASTM STP15D, 1990.

‡ Formerly the standard deviation of a sample was defined as the sample root-mean-square deviation and was represented by the symbol σ. Defining the sample standard deviation as equal to s is in line with modern usage. The symbol σ is reserved in this standard to represent the universe standard deviation and σ_0 is used to represent a standard value for σ. See ANSI/ASQC B3.

4. TYPES OF CONTROL CHARTS

4.1 When measured values on individual units are at hand, and not merely the fractions nonconforming (p), then control charts for $\overline{X}$ and s, one or usually both, are useful. Experience indicates that lack of control will most often be detectable by the control chart for $\overline{X}$ alone; hence, sometimes the second control chart (that for s) will not be needed. A control chart for R may be used as a substitute for the control chart for s with but a little loss in efficiency of detecting assignable causes, but only if the number of observed values in each subgroup is not more than 10. Since the use of R requires much less computation, it can be recommended wherever simplicity is of major importance, as in routine analysis in manufacturing plants.

4.2 Industrial experience indicates that assignable causes often come and go in an erratic fashion. For this reason the subgroups of observed values should be kept small, but usually the number of observed values in a subgroup should be not less than 4.

4.3 Each point on the control chart represents a set of observed values belonging to the same subgroup. In the control chart for $\overline{X}$, the plotted point for any subgroup is the average of the observed values for that subgroup. Likewise, in the control chart for s, the plotted point for any subgroup is the standard deviation of the observed values of that subgroup.

4.4 Where interest is centered primarily on the fraction nonconforming (p), a control chart would be constructed for p. Likewise, where interest is centered on the number of nonconforming units in successive subgroups, a control chart would be constructed for pn (fraction nonconforming times sample size). Similarly, a control chart for the number of nonconformities in a sample (c) such as the number of finish nonconformities in a sample area for plated metal sheets, the number of flaws in a sample area of cloth, the number of surface nonconformities in a sample length of insulated wire, etc.

5. CONSTRUCTION OF A CONTROL CHART

5.1 Instruction Sheet 1 shows the steps involved in constructing a pair of control charts for the average ($\overline{X}$) and the range (R), where the subgroups are composed of an equal number of observed values, this number being 10 or fewer. Instruction Sheet 2 gives similar information for a pair of control charts for the average ($\overline{X}$) and the standard deviation (s), where the subgroups are composed of an equal number of observed values, this number being 25 or fewer.

Instruction Sheet 3 shows the steps involved in constructing a control chart for the fraction nonconforming (p).

In the construction of other control charts, the same basic steps are to be followed, but the constants in the computation are different. When not all the subgroups are composed of the same number of observed values, control limits must be computed separately for each sample size. (See Appendix B, references 5 and 8.)

6. GENERAL INSTRUCTIONS FOR CONTROL CHART METHOD

6.1 General instructions for applying the control chart method of analysis and presentation of data, including certain formulas and tables applicable to various conditions met in practice, are given in Appendix B, reference 5.

7. CHOICE OF CONTROL LIMITS

7.1 Functions of Control Limits. Control limits perform two functions with regard to control: (1) in *assisting judgment* regarding whether a state of control exists; and (2) in providing *action limits* for attaining and maintaining control (see Section 9, 10, and 11 of ANSI/ASQC B1-1996, *Guide for Quality Control Charts.*)

7.2 Placing Control Limits. Although the nature of the control problem does not permit standardizing a precise and inflexible rule for computing control limits that will be found suited to all of the various conditions that may be encountered in practice, it is possible to make certain general recommendations here *on the basis of experience* that cover a wide range of industrial applications of the control chart.

It has been found satisfactory in dealing with most control problems to place the control limits above and below the grand average or other averages of the statistical measure ($\overline{\overline{X}}$, $\bar{s}$, $\overline{R}$, $\bar{p}$, etc.) that is being plotted, at distances of three times a computed value, commonly designated as the "sigma" of that statistical measure, for subgroups of the size under consideration. These are referred to as "3

sigma" limits.* In actual practice they are computed with the aid of tables (e.g., Table 6 of ANSI/ASQC B3).

7.2.1 Control Limits for the Judgment of Control. It is recommended that the "3 sigma" limits be used for control limits that are to serve as a basis for judging whether there is evidence of lack of control.

7.2.2 Control Limits as Action Limits for Attaining and Maintaining Control. In the choice of action limits for indicating when to look for assignable causes of variation, an attempt is made to strike an economic balance with respect to the net consequences of two kinds of "errors" that may occur in practice: namely, looking for trouble that does not exist, and not looking for trouble that does exist. Ordinarily, limits of the magnitude of "3 sigma" appear to approximate an economic balance between the costs resulting from the two kinds of errors, in a wide range of industrial applications. Therefore, it is recommended that the "3 sigma" limits be used, or limits having a spread differing but little from them, unless there is sufficient information concerning the relative costs just mentioned to warrant another choice.

7.2.3 Control Chart with Warning Limits. Comment: This type of chart is generally a variation of the Shewhart control chart in which two sets of limits are used instead of the single set of control limits. The usual control limits at ±3s are referred to as *action limits* and a second set, often at ±2s, are referred to as *warning limits*. The rules for using these limits differ according to practice. In general, the warning limits are used in the same manner as control limits. However, an important purpose of warning limits is to provide early warning or attention to potential shifts in level or out-of-control conditions, for example, runs of points lying between the warning and action limits may be used as an early warning signal for action as such points occur with low probability.

---

* It should be noted that this value "sigma" is not the computed standard deviation of the plotted points. In the case of the $\overline{X}$, s, and R charts, it is computed from the individual observed values within the subgroups and the size n of a subgroup.

A strict interpretation of the rule for using the "3 sigma" limits would imply that the control limits must be symmetrical about the central line, yet it will be noticed that on Instruction Sheets 1 and 2 the control limits for the range and standard deviation are unsymmetrical. The explanation is that since by definition neither R nor s can ever be negative, their lower control limits, when computed to be negative, are placed at 0 instead. The upper limits are nevertheless placed at the "3 sigma" distances above the central lines. As a result, the separation of the control limits will turn out to be unsymmetrical for R when the number n of items in each subgroup is 6 or fewer, and for s when the number n is 5 or fewer. The same is true for the statistical measures p and pn, when pn for any subgroup is less than about 9 (see Instruction Sheet 3).

† There are ways of placing control limits otherwise than just described. See for example the footnote to paragraph 3.2 of ANSI/ASQC B3, page 18. In the interests of brevity and clarity, subsequent procedures and examples will for the most part be based on use of the "3 sigma" limits.

INSTRUCTION SHEET 1

Procedure for Constructing Control Charts for *Average* ($\overline{X}$) and *Range* (R) — One of the Most Useful Pair of Charts Where the Number of Observed Values per Subgroup is 10 or Fewer.

Subgroup No.	1	2	3	4	5	6	7	8	9	10
Observed Values	38.4	37.4	39.5	37.4	38.0	36.1	38.7	39.5	38.1	39.0
of Quality	37.1	37.3	37.4	37.1	39.2	37.6	38.2	39.2	37.8	38.3
Measured	38.6	39.0	38.3	36.5	37.0	38.3	36.2	39.8	36.7	36.9
(Wt. in Grains)	38.5	37.7	37.7	36.3	38.2	39.2	38.8	40.8	38.3	38.8
Total	4|152.6	4|151.4	4|152.9	4|147.3	4|152.4	4|151.2	4|151.9	4|159.3	4|150.9	4|153.0
Average	38.15	37.85	38.23	36.83	38.10	37.80	37.98	39.83	37.73	38.25
Largest Value	38.6	39.0	39.5	37.4	39.2	39.2	38.8	40.8	38.3	39.0
Smallest Value	37.1	37.3	37.4	36.3	37.0	36.1	36.2	39.2	36.7	36.9
Difference = Range, R	1.5	1.7	2.1	1.1	2.2	3.1	2.6	1.6	1.6	2.1

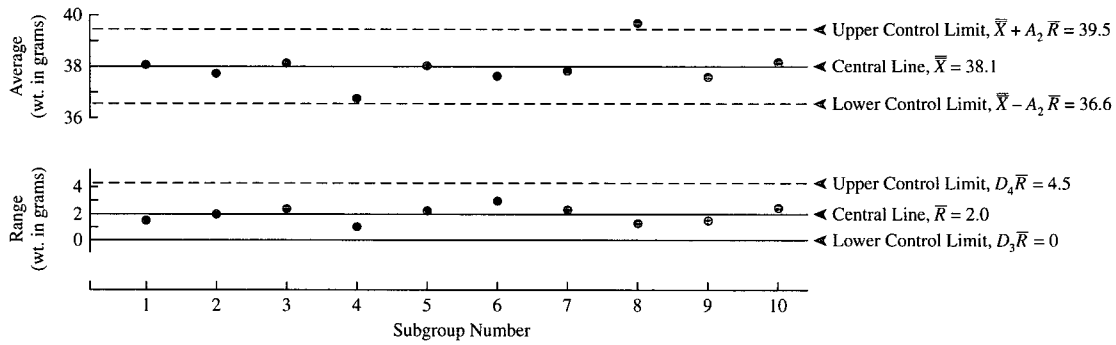
Grand Total . . . . . 40|1522.9

Grand Av, $\overline{\overline{X}} = 38.07$

Total of All Ranges . . . . . 10|19.6

Av Range, $\overline{R}$ 1.96

CONTROL CHARTS — APPLYING TO WEIGHT OF EXPLOSIVE CHARGE in Grains



Steps in Construction of Control Chart

1. *Develop Subgroups.*

Break up the total set of observed values into subgroups. The items of *any one subgroup* should have what is believed to be some important common factor; for example, units produced during the same short interval of time, or units coming from one of several distinct sources or locations. The *different* subgroups should represent possible or suspected differences in the processes that produced them, for example different intervals of time, or different sources or locations.

It is desirable that the subgroups be all of the same size. (In the example shown here, there are 10 subgroups of 4 observations each.) If they represent some more-or-less continuous series involving order in time or location, they should be small, but preferably not less than 4.

2. *Computations*

(a) *Average.* For each subgroup, add the observed values and find their Average (designated as $\overline{X}$).

(b) *Grand Average.* Compute the grand average of all observed values (designated as $\overline{\overline{X}}$).

(c) *Range.* For each subgroup, subtract the smallest value from the largest value. This difference is called the range (designated as R).

(d) *Average Range.* Compute the average of all the values of R. This is called the average range (designated as $\overline{R}$).

3. *Construction.*

(a) *Layout.* On a suitable form or cross-section paper, lay out a chart for $\overline{X}$ and a chart for R, with vertical scales at the left for $\overline{X}$ and for R, and with a horizontal scale for subgroup number. Plot the computed values for $\overline{X}$ on the chart for averages and plot the computed values for R on the chart for ranges.

(b) *Central Lines.* On these charts draw solid horizontal lines to represent $\overline{\overline{X}}$ and $\overline{R}$.

(c) *Placing Control Limits on Charts.* On the chart for $\overline{X}$, draw two horizontal dotted lines at $\overline{\overline{X}} \pm A_2\overline{R}$ and on the chart for R draw two horizontal dotted lines at $D_3\overline{R}$ and $D_4\overline{R}$, where A_2, D_3, and D_4 are based on n, the number of observations in a subgroup. (A_2, D_3, and D_4 are given in Table 6 of ANSI/ASQC B3-1996; for $n = 4$, $A_2 = 0.729$, $D_3 = 0$, and $D_4 = 2.282$. Note that when $n = 6$ or less, $D_3 = 0$, hence the lower control limit for R is taken as zero.)

4. *Interpretation of Chart.*

A value of $\overline{X}$ or R falling outside control limits is taken as an indication of lack of control.

INSTRUCTION SHEET 2
Procedure for Constructing Control Charts for *Average* ($\bar{X}$) and *Standard Deviation* (s).

Subgroup No.	1		2		3		4		5	
	X	X^2	X	X^2	X	X^2	X	X^2	X	X^2
(1) Observed Values* of Quality Measured (Wt. in Grains)	8.4	70.56	7.4	54.76	9.5	90.25	7.4	54.76	8.0	64.00
	7.1	50.41	7.3	53.29	7.4	54.76	7.1	50.41	9.2	84.64
	8.6	73.96	9.0	81.00	8.3	68.89	6.5	42.25	7.0	49.00
	8.5	72.25	7.7	59.29	7.7	59.29	6.3	39.69	8.2	67.24
(2) Total X	4|32.6		4|31.4		4|32.9		4|27.3		4|32.4	
Average $\bar{X}$	8.150		7.850		8.225		6.825		8.100	

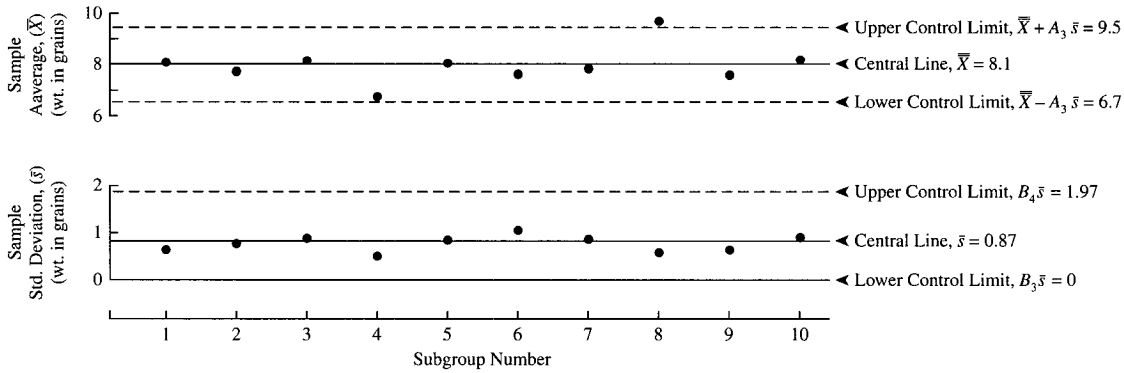
Subgroup No.	1	2	3	4	5
(4) ΣX^2	267.18	248.34	273.19	187.11	264.88
(5) $(\Sigma X)^2$	4|1062.76	4|985.96	4|1082.41	4|745.29	4|1049.76
(5) $(\Sigma X)^2 \div n$	265.69	246.49	270.60	186.32	262.76
(6) $\Sigma X^2 - ((\Sigma X)^2 \div n)$	3|1.49	3|1.85	3|2.59	3|.79	3|2.44
(7) Sample Variance (6) $\div (n-1)$	.497	.617	.863	.263	.813
(8) Square Root of Variance = Sample Standard Deviation, s	.705	.785	.929	.513	.902

Subgroup No.	6		7		8		9		10		
	X	X^2	X	X^2	X	X^2	X	X^2	X	X^2	
(1) Observed Values* of Quality Measured (Wt. in grains	6.1	37.21	8.7	75.69	9.5	90.25	8.1	65.61	9.0	81.00	
	7.6	57.76	8.2	67.24	9.2	84.64	7.8	60.84	8.3	68.89	
	8.3	68.89	6.2	38.44	9.8	96.04	6.7	44.89	6.9	47.61	
	9.2	84.64	8.8	77.44	10.8	116.64	8.3	68.89	8.8	77.44	Grand
(2) Total X	4|31.2		4|31.9		4|39.3		4|30.9		4|33.0		Total . . . . 40|322.9
Average $\bar{X}$	7.800		7.975		9.825		7.725		8.250		(3) Grand Av, $\bar{\bar{X}}$ = 8.07

Subgroup No.	6	7	8	9	10	
(4) ΣX^2	248.50	258.81	387.57	240.23	274.94	
(5) $(\Sigma X)^2$	4|973.44	4|1017.61	4|1544.49	4|954.81	4|1089.00	
(5) $(\Sigma X)^2 \div n$	243.36	254.40	386.12	238.70	272.25	
(6) $\Sigma X^2 - ((\Sigma X)^2 \div n)$	3|5.14	3|4.41	3|1.45	3|1.53	3|2.69	Total of
(7) Sample Variance (6) $\div (n-1)$	1.713	1.470	.483	.510	.897	All s's . . . . . 10|8.71
(8) Square Root of Variance = Sample Standard Deviation, s	1.31	1.21	.695	.714	.947	(a) Avg. $\bar{s}$ 871

* Same data as on Instruction Sheet 1; 30 subtracted from each observed value, giving smaller numbers to work with and thus simplifying computations.

CONTROL CHARTS — APPLYING TO WEIGHT OF EXPLOSIVE CHARGE in Grains



INSTRUCTION SHEET 2 (Continued)

Procedure for Constructing Control Charts for *Average* ($\overline{X}$) and *Standard Deviation* (s)

Steps in Construction of Control Chart

1. *Develop Subgroups.*

Break up the total set of observed values into subgroups. The items of *any one subgroup* should have what is believed to be some important common factor; for example, units produced during the same short interval of time, or units coming from one of several distinct sources or locations. The *different* subgroups should represent possible or suspected differences in the processes that produced them, for example, different intervals of time, or different sources or locations.

It is desirable that the subgroups be all of the same size. (In the example shown here, there are 10 subgroups of 4 observations each.) If they represent some more-or-less continuous series involving order in time or location, they should be small, but preferably not less than 4.

2. *Computations.*

(a) *Average.* For each subgroup, add the observed values and find their Average (designated as $\overline{X}$—pronounced X bar).

(b) *Grand Average.* Compute the grand average of all observed values (designated as $\overline{\overline{X}}$—pronounced X double bar).

(c) *Subgroup Variance.* For each subgroup, square each X value and find the sum of the squared values (Total X^2 *or* ΣX^2).

For each subgroup, sum the values of X and square the total ($(\Sigma X)^2$). Divide this number by the subgroup size (n).

For each subgroup, find the difference between the total of the X values squared and the sum of the X values squared divided by n.

For each subgroup, divide the difference found in (c) by one less than the subgroup size ($n - 1$). This yields the subgroup variance (designated as s^2) for each subgroup.

For each subgroup, take the square root of the subgroup variance. This yields the subgroup standard deviation (s for each subgroup*).

Compute the average of all the sample standard deviations (in this case there are 10 subgroups). This is designated $\bar{s}$ (pronounced s bar).

3. *Construction.*

(a) *Layout.* On a suitable form or cross section paper, lay out a chart for $\overline{X}$ and a chart for s, with vertical scales at the left for $\overline{X}$ and s and with a horizontal scale for subgroup number. Plot the computed values for $\overline{X}$ on the chart for averages and plot the computed values for s on the chart for standard deviations, by subgroup number.

(b) *Central Lines.* On these charts draw solid horizontal lines to represent $\overline{\overline{X}}$ and $\bar{s}$.

(c) *Placing Control Limits on Charts.* On the chart for $\overline{X}$, draw two horizontal dotted lines at $\overline{\overline{X}} \pm A_3\bar{s}$ and on the chart for s draw two dotted horizontal lines at $B_3\bar{s}$ and $B_4\bar{s}$ where A_3, B_3 and B_4 are based on n, the number of observations in the subgroup. (A_3, B_3 and B_4 are given in Table 6 in ANSI/ASQC B3 contained herein. For $n = 4$, $A_3 = 1.628$, $B_3 = 0$, and $B_4 = 2.266$. Note that when $n = 5$ or less, $B_3 = 0$, hence the lower control limit for s is taken as zero.)

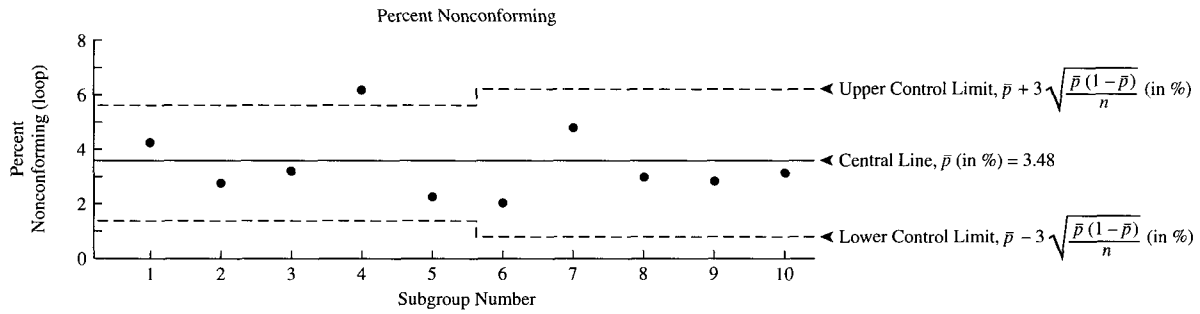
4. *Interpretation of Chart.*

A value of $\overline{X}$ or s falling outside control limits is taken as an indication of lack of control.

*Note—step (c) gives a computational form of the standard deviation(s) as defined on page 6.

INSTRUCTION SHEET 3
Procedure for Constructing Control Chart for *Percent Nonconforming* (*p*).

Subgroup No.	1	2	3	4	5	6	7	8	9	10	Total
No. of Units Inspected, n	600	600	600	600	600	400	400	400	400	400	5000
No. of Nonconforming Units	25	16	19	37	14	8	19	12	11	13	174
Fraction nonconforming, p	.042	.027	.032	.062	.023	.020	.048	.030	.028	.032	$p = \frac{174}{5000} = .0348 = 3.48\%$
Percent Nonconforming	4.2	2.7	3.2	6.2	2.3	2.0	4.8	3.0	2.8	3.2	$100\bar{p} = 3.48\%$
Control Limits:											
$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	0.0348 ± .0224 = .057 and .012 or 5.7% and 1.2%					0.0348 ± .0275 = .062 and .007 or 6.2% and 0.7%					



Steps in Construction of Control Chart

1. *Develop Subgroups*

Given a set of data comprising the number of units of product inspected and the number of units observed to be nonconforming. Break up the total set of data into subgroups. The units of product corresponding to *any one subgroup* should have what is believed to be some important common factors; for example, units produced during the same short interval of time, or units coming from one of several distinct sources or locations. The *different* subgroups should represent possible or suspected differences in the processes that produce them; for example, different intervals of time, or different sources or locations.

It is desirable that the subgroups be of the same size. (In the example shown, each of the first 5 subgroups represents 600 units of product and each of the last 5 subgroups, 400 units.) If the subgroups represent some more-or-less continuous series involving order in time or location, they should be reasonably small but preferably not so small that the number on nonconforming units per subgroup is less than 1.

2. *Computations*

(a) *Fraction Nonconforming.* For each subgroup, find the fraction nonconforming (designated as p) by dividing the number of nonconforming units by the total number of units in the subgroup (designated as n).

(b) *Average Fraction Nonconforming.* Compute the average fraction nonconforming (designated as $\bar{p}$) by dividing the total number of nonconforming units for all subgroups by the total number of units in all subgroups.

3. *Construction*

(a) *Layout.* On a suitable form or cross section paper, lay out a chart for $100p$ with a vertical scale at the left for $100p$ and with a horizontal scale for subgroup number. Plot the computed values of $100p$ on the chart.

(b) *Central Lines.* On this chart draw a solid horizontal line to represent $100\bar{p}$.

(c) *Placing Control Lines on Chart.* (a) If the subgroups are all of equal size, draw two horizontal dotted lines across the chart at $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$, where n is the number of units in each subgroup. (b) If the subgroups are not of the same size, that is, if n varies from subgroup to subgroup, draw horizontal dotted lines above and below each plotted point at $\bar{p} \pm 3\sqrt{\bar{p}(1-\bar{p})/n}$, using the appropriate value of n for each subgroup.

(In the above illustration two different pairs of control limits are shown for the two sizes of subgroups, $n = 600$ and $n = 400$. In general, if pn for any subgroup is less than about 9, the computed lower control limit will be negative and should, therefore, be plotted at $p = 0$.

(d) In presenting tabulations and charts, it is generally preferable to express results in terms of "percent nonconforming" rather than "fraction nonconforming." Numerically, percent nonconforming is $100p$.

4. *Interpretation of Chart*

A plotted point falling outside control limits is taken as an indication of lack of control. Also, see 7.2.3.

ANSI/ASQC B3-1996
Control Chart Method of Controlling Quality During Production

1. INTRODUCTION

1.1 Scope. This standard outlines the control chart method of controlling quality during production. The principles of procedure are given in general terms, and illustrations show how these principles might be applied in specific cases. A glossary of symbols and terms used in this standard is given on pages 37 and 38.

It is recommended that the manufacturer use this method for assistance in identifying and eliminating causes of trouble in repetitive processes, with the object of reducing variations in the quality of manufactured products and materials. The method is especially recommended where inspection can be performed by sampling only. Where inspection is 100 percent, the method is useful for controlling the percentage of rejections. The graphical display of continuing inspection results provides overall quality history at a glance.

The work involved in applying the method is largely engineering, based on judgment, knowledge of the processes, and technical skill in tracking down unwanted causes of variation to their source. The computations require only simple arithmetic.

Some advantages to be gained from a statistical program of quality control in the manufacturing plant are mentioned in ANSI/ASQC B1-1996, *Guide for Quality Control Charts.* In particular the following advantages are to be emphasized here:

1) Reduction in rejections

2) Prediction of impending trouble (anticipating trouble before rejections occur; speeding up production by avoiding rejections and interruptions)

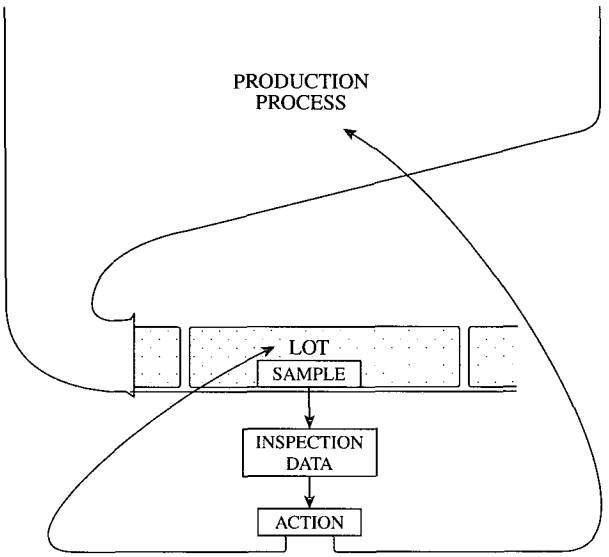
3) Reduction in cost of inspection

4) Narrowing of tolerances (thus making more efficient use of materials)

5) Better basis for establishing or altering specification requirements.

1.2 Two Purposes of Inspection. The manufacturer's inspection of their own product serves two purposes (Fig. 1):

Purpose A. *To provide a basis for action with regard to the product already at hand; as for instance, to decide whether the particular article or lot of product at hand should be allowed to go out, or some alternative disposition made (inspected further, sorted, repaired, reworked, scrapped, etc.)*



Purpose A, Action on the Lot	Purpose B, Action on the Process
(1) Release the lot for shipment, or (2) Hold the lot for disposition pending further action and decision, (inspect further, return to Operating Dept. for correction, release as of lower grade, scrap, etc.)	(1) Leave the process alone, or (2) Find and eliminate the disturbing cause.

Figure 1—Showing Two Purposes of Inspecting a Sample from a Current Lot of Product.

Purpose B. *To provide a basis for action with regard to the production process, with a view to future product; as for instance, to decide whether the process should be left alone, or action taken to find and eliminate disturbing causes.*

When a manufacturer has set up an inspection plan to serve primarily for Purpose A, whether by 100 percent inspection with criteria to provide a basis of disposing of individual articles one by one, or by sampling inspection with criteria to provide a basis for disposing of individual

lots of product one by one, the inspection data can *also* be used for Purpose B. Now if the manufacturers use their inspection data for Purpose B in accordance with the principles outlined herein and attain good control at a satisfactory quality level, then the amount of inspection needed for Purpose A is ordinarily decreased, often considerably (see 1.3). It is recommended, however, that *at the outset* for existing methods of checking the product (including sampling, inspection, and testing) be altered as little as possible.

1.3 Decrease in Lot-by-Lot Inspection Accruing from Statistical Control. When inspection data are used for Purpose B, each sample is considered, not by itself, but as one of a series of samples. The criterion for action is proved by the control limits on the control chart, which are used to indicate when something appears to be abnormal in the process. Action based on failures to meet the control criterion eliminates disturbing causes one by one, and leads to stabilization of the quality of the product.

The size of sample needed for Purpose A in any particular instance is usually dependent on the quality history of the product; for example, if the current lot is one of a series that in the past has exhibited statistical control at a satisfactory level, and if there is qualitative evidence that the conditions of production for this lot were essentially the same as for preceding lots, little additional quantitative information may be needed to provide a basis for letting the lot go out. All that is required is a sample sufficient to indicate whether control has been maintained. A relatively small sample is usually sufficient for this purpose, for it is not used as an isolated sample.

On the other hand, if no prior information is available (as in new production), or if prior information indicates unstable or doubtful quality, the sample from the lot must be large enough to provide quantitative information, adequate in itself to serve as a basis for action regarding the disposal of the lot. A relatively large sample may be needed for this purpose.

To sum up, small samples that will answer Purpose B will not necessarily at the same time fulfill Purpose A. Furthermore, the criteria for action on individual lots (Purpose A) as distinct from the criterion for action on the process (Purpose B) must be arrived at separately.

Although this standard restricts its attention to Purpose B, material is provided in Illustrative Example 1 (5.1.4.4 of pp. 26 and 27) to indicate the kind of relation that may exist between the criteria used for Purpose A and those used for Purpose B, where 100 percent inspection is not practicable. It is shown there that the control chart criterion used for action on the process may also serve as one of the criteria for taking additional samples from a current lot of product, but that the criteria to be used for releasing a lot or for otherwise disposing of it when lack of control is indicated must be based on other considerations. When a control procedure using frequent periodic samples is set up in the manner illustrated by Example 1, a single point falling outside the control limits may be considered grounds not only for further inspecting or otherwise checking or correcting the increment of product from which the sample was drawn, but also for taking like action with respect to a preceding portion, since the change indicated by the specific point out of control may have begun to operate to some greater or lesser extent prior to the indication of lack of control.

Note: When processes are better controlled (Purpose B), lots from such processes will also result in better quality on a lot-by-lot basis (Purpose A).

The reduction of inspection is only one of the advantages mentioned in 1.1 accruing from a state of control.

For control charts that have as a primary function the acceptance and rejection of lots, see discussions of acceptance control charts in reference 8 of Appendix B.

1.4 Quality Characteristics Requiring Sampling Inspection. In connection with inspection for Purpose A, it may be noted that for some quality characteristics, 100 percent inspection or testing is practicable, whereas for certain other quality characteristics, sampling alone is practicable. One hundred percent conformance to a specification for an individual article can be assured if it is possible to conduct an errorless inspection of every article and sort the bad from the good. However, experience has shown that 100 percent inspection is seldom perfect. When 100 percent conformance to specification is admittedly not essential, sampling may be used. When the inspection test is destructive, sampling must be used. It therefore follows that when the inspection test is destructive, and when therefore the marked articles are not themselves inspected (but are instead represented in the sample of articles that are inspected), it is particularly desirable that they be produced under conditions of statistical control so that the quality of the uninspected portions may be safely inferred from the results shown by samples.

Table 1 is an exhibit of some quality characteristics for which 100 percent inspection of the product is practicable and others for which, by necessity or economic reasons, sampling inspection alone is practicable.

TABLE 1

Two Groups of Quality Characteristics

Characteristics for Which 100 Percent Inspection is Practicable
DIMENSIONS subject to measurement go/no-go gaging
PERFORMANCE CHARACTERISTICS subject to nondestructive testing, such as the operating time of timers, the torque of motors
ELECTRICAL CHARACTERISTICS, such as the gain of transistors, the resistance of relays
CHARACTERISTICS OBSERVABLE BY VISUAL INSPECTION, such as assembly, finish, appearance, condition, or material defects; flaws in cloth or painted surface
Characteristics for Which 100 Percent Inspection Is Not Practicable
LIFE TESTS, measuring a maximum period of usefulness, such as the life of electric lamps and fan belts
PERFORMANCE CHARACTERISTICS subject to destructive testing, such as the firing of ammunition, blowing time of an electric fuse, the BTU content of gas, fuel oil, and coal
ULTIMATE PHYSICAL PROPERTIES, such as the tensile strength of steel, the bursting strength of boxboard, the torsional strength of steel rod, the tensile strength of welded joints of copper line wire; flash point and viscosity tests of oil

1.5 Need for Keeping Records of Quality. In whatever form the control chart method of controlling quality is introduced and developed in the manufacturing plant, it will require keeping and using records of inspection results. Where records have not been kept in the past, one of the first things to do is to start keeping records of the inspection results showing production sequence, and in a form that will permit grouping the data and examining them by the control chart method of analysis (see ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data*). Where records have already been kept in form, analysis of recent representative data for the purpose of starting the control chart can begin at once.

1.6 Different Types of Inspection Data. The data resulting from inspection of a quality characteristic may take one of the following forms:

1) A record of the actual measurements of the characteristic for the individual articles or specimens.

2) A record of the number of articles or specimens inspected and of the number found nonconforming (failing to conform to the specified requirement). An example is the go/no-go record for a number of articles.

   Hereinafter a nonconforming article (one containing one or more nonconformities) will be referred to as a nonconforming unit.

3) A record of the number of nonconformities that are observed in a sample, when the possible number of nonconformities per sample is very large compared with the average number of nonconformities per sample. The sample may be a single article or test specimen, a designated test length or area, a designated number of articles or specimens, etc. An example is a record of the number of pinholes in the paint film on a standard test panel.

For purposes of control, data of form (1) may be usefully summarized by taking two statistical measures, the average ($\bar{X}$) and the sample standard deviation (s), or the average ($\bar{X}$) and the range (R). Data of form (2) can be summarized in terms of fraction nonconforming (p), the ratio of the number of articles that are nonconforming to the total of articles inspected. When the number of articles (n) in the samples is nearly constant, it is usually more convenient to plot the number of nonconforming units (pn) directly instead of computing the fraction nonconforming (p).

Data of form (1) may be converted into terms of fraction nonconforming form (2) by noting, from the recorded measurements, how many of the individual articles fail to conform to specification. However, caution should be exercised in abandoning data of form (1) in favor of their converted form (2), since under some circumstances the use of p alone will be equivalent to throwing away useful information contained in the measurements.

2. PRELIMINARY STEPS IN SETTING UP A CONTROL PROCEDURE (PURPOSE B: PROCESS CONTROL)

2.1 Choice of Quality Characteristics. Select the quality characteristics for which a control program is desired.

Characteristics affecting the performance of the product should normally be the object of first attention. These may be features of the materials used, or of component parts of the product, as well as of the finished product delivered to the purchaser.

If the manufacturers formulate the specifications for the product, when they market a product of their own design, they are free to decide what quality characteristics should be controlled and at what levels. If they are working to a specification formulated by the purchaser, including limits of variation for a number of characteristics, they will probably have less freedom in determining the characteristics deemed essential, and in adjusting the levels about which the characteristics should be controlled.

2.2 Analysis of the Production Process. For any quality characteristic selected, study the production process in detail to determine the kind and location of causes that are likely to give rise to irregularities.

2.2.1 Review the requirements imposed by the specifications on the quality characteristic selected.

2.2.2 Study the relation between each production step and this particular characteristic, noting where and how quality irregularities may arise from causes associated with raw materials, component parts of the finished product, machine operations, human operations, etc.

2.2.3 Study the method of inspecting an individual article or specimen for the selected characteristic, noting particularly any factors that may give rise to errors of measurement or errors of observation. Irregularities evident in quality data may arise from errors in inspection as well as from faults in the production process. Errors in inspection may result from faulty gages, test apparatus, etc., a faulty procedure attributable to the inspector, or faulty instructions prepared for the inspector's use.

2.2.4 Decide whether the entire output of product should be considered as a single stream having a common system of causes, or as two or more distinct streams, each to be treated separately in the control program because they come from different cause systems—different conveyor lines, different machines or batteries of machines, different shifts of workers, etc.

2.2.5 Determine the earliest point in the production process at which inspection and testing can be carried out for the characteristic under consideration, to obtain information as promptly as possible regarding faults and irregularities that arise in the process.

2.3 Planning How to Collect and Subgroup Data. Decide, for the selected quality characteristic, what inspection data should be recorded, and how such data are to be divided into subgroups (see Glossary, p. 38).

2.3.1 Make sure that the instructions for inspecting an individual unit or specimen are clearly set forth and well understood.

2.3.2 Decide whether a measured value is to be recorded for each article or specimen, or whether merely a notation is to be made that the article or specimen does or does not conform to the requirement. This decision will govern what kind of quality control charts can be constructed ($\overline{X}$, *s*, *R*, *p*, etc.; see 2.4).

2.3.3 On the basis of the analysis of the process (2.2), decide how the observed results are to be grouped so that the articles in any subgroup are produced under the same essential conditions. Specifically, if like parts or raw materials in the articles belonging to any subgroup were produced under different conditions, important causes of variation might be obscured.

This part of the problem depends on technical knowledge of the production process and familiarity with the conditions under which the product is to be produced and the conditions under which inspection data are to be taken. The items of *any one subgroup* should have what is believed to be some important common factor; for example, units produced during the same short interval of time, or units coming from one of several distinct sources or locations. The *different subgroups* should represent possible or suspected differences in the processes that produced them, such as different intervals of time or different sources or locations.

The existing inspection procedure may provide a subgrouping of data that is directly suited for use in control analysis. Small subgroups in order of production are almost always satisfactory.

The application of the control chart procedure is reduced to its simplest form, with a minimum of computation effort, when samples, all containing the same number of articles or specimens, are selected on some planned basis, such as once during every half hour, hour, or day, or from every production lot, every batch, etc. To avoid bias, two precautions should be considered in the selection of consecutive samples. First, a periodic selection should not coincide with any relevant periodic factors in the process. Second, the selection of samples should not be made on a fixed time schedule, such as every hour on the hour, if prediction of the selection time would have an influence on the quality of the articles selected.

If single articles or specimens are selected and inspected at frequent time intervals, they may be grouped in sets of consecutive articles, perhaps 4, 5, or 10 to a set, and each set treated as a subgroup, the subgroups being arranged in order of their production.

Sometimes the existing inspection procedure consists of testing each article to get merely a go/no-go acceptance check. In such circumstances, the control procedure might involve taking a sample of 4 or 5 articles every half hour or every hour, obtaining a measured value of the quality characteristic for each article, recording the measured values, and plotting points on a control chart for each sample ($\overline{X}$ and *s* or $\overline{X}$ and *R*), for the purpose of detecting undesirable trends before they have gone so far as to cause rejections.

2.3.4 When the subgrouping is based on the order of production, use frequent small subgroups in preference to infrequent large subgroups.

Where inspection is necessarily confined to observations on relatively small portions of the whole, as in destructive testing, subgroups of 4 or 5 articles each are often found to be most efficient for detecting trouble. For

example, when constructing control charts for $\overline{X}$ and R, or $\overline{X}$ and s, 25 subgroups of 4 are to be preferred over 4 subgroups of 25. Individual subgroups of measured values should preferably comprise not less than 4 observed values. Where rational subgroups consist of 2 observations each, as when standard specifications call for "2 tension specimens per melt," a trial should show whether it is better, for purposes of the control chart program, to use the subgroups of 2, or to combine two successive subgroups into a composite subgroup of 4 observations.

Where inspection is necessarily confined to testing relatively small samples of the whole on a go/no-go basis, frequent small subgroups are likewise preferable to infrequent large subgroups, but the minimum size for efficient control operation is determined to some extent by the expected fraction nonconforming (p, ratio of the expected number of nonconforming articles to the total number of articles or specimens under consideration). When practicable, it is best to avoid using subgroups so small that one nonconforming unit must be taken as an indication of an assignable cause.

There are circumstances in which the fraction nonconforming (p) is so small, and the economically practicable sample size is also so small, that one nonconforming unit must be taken as an indication of an assignable cause. An alternative sometimes resorted to, although it incurs sacrifices including delayed detection of causes of trouble, is to consider the cumulative data for several samples as a subgroup, such as the total data for a day, a week, or even a longer period.

In more extreme cases, nonconformities may be so rare (such as one hazardous failure in 100,000 articles) that the control chart for p is of little value for controlling quality during production. The desired quality may then be better obtained by holding the process constant in other ways, such as by introducing tests of increased severity, or by controlling associated quality characteristics.

An example of a test of increased severity is a determination of the number of items failing under a load of 75 pounds, when the required minimum strength is 50 pounds, so that in effect one deals with a value of p (fraction failing at 75 pounds) that is much larger than the actual fraction nonconforming (fraction failing at 50 pounds). Although this does not exert a direct control on the fraction nonconforming, it may aid in detecting the presence of causes that tend to give an abnormal number of values close to the specified limit—often the forerunner of actual failures.

2.3.5 Keep records of the inspection data in such form and completeness that specific portions of the data may be identified with specific times or sources of production within the shop. This will facilitate quick association of any observed trouble with the location of the cause in the shop (See the form suggested in Fig. 5, Example 1). Since some causes may be transient, speed in locating trouble on the job is of first importance.

It is often helpful to label individual articles, tray loads, lots, or batches with identifying marks, numbers, or tickets, to show the time and specific source of its production (batch or raw material, machine, operator, etc.). This information may then be noted on the inspection data sheets for each subgroup.

2.4 Choice of Statistical Measures. Decide which measures of the quality characteristic selected in 2.1 should be chosen for the control chart or charts. In certain situations there will be no choice, as when a mere notation has been made to show whether each article passed or failed; the control chart for p or pn is then the only one possible (section 2.4.4).

2.4.1 Control Charts for $\overline{X}$ and s, or $\overline{X}$ and R. When measured values of the quality characteristics are at hand, charts for $\overline{X}$, s, and R are possible. Usually a pair of charts, one for $\overline{X}$ and the other plotted simultaneously for s or R, is used to advantage. The range (R) requires mere subtraction and is accordingly much easier to compute than s, which involves computing squares and square roots. When the number of observed values in each subgroup is 10 or less, and where simplicity is of importance, as in routine inspection work in the manufacturing plant, the use of R in place of s is particularly recommended.

2.4.2 Control Chart for $\overline{X}$ Alone. Use $\overline{X}$ alone where experience with control charts for $\overline{X}$ and s, or $\overline{X}$ and R has demonstrated the instances of lack of control are almost always associated with causes that affect $\overline{X}$ rather than s or R (see 5.2, Example 2).

When a control chart for $\overline{X}$ alone is used, values of s or R should be computed (though not plotted) to provide a basis for computing the control limits for $\overline{X}$.

2.4.3 Control Chart for R or s Alone. Use R or s alone where technical reasons render control of $\overline{X}$ unimportant, or where control $\overline{X}$ is known to be unjustifiably expensive.

2.4.4 Control Chart for p and pn. Use p and pn when the records of the inspection or testing show merely the number of articles inspected and the number found nonconforming.

Use p in addition to $\overline{X}$ and s, or in addition to $\overline{X}$ and R, when the observed values of the characteristic are obtained and recorded, and when interest centers on controlling the fraction nonconforming (p).

2.4.5 Control Chart for c. As indicated in 1.6(3), there are circumstances wherein the inspection consists in determining the number of nonconformities (c) in a sample. Such is the case, for example, in the examination of finishes, textiles, materials, wire, etc. The sample may then consist of a square yard, a foot of length, 100 feet of length, a sample of a 4 test specimens, or some other designated quantity. The nonconformities to be counted may be physical nonconformities, finish irregularities, pinholes, flaws, etc.

Use a control chart for c when quality is measured in the above terms.

2.4.6 Other Types of Control Charts — For other types of control charts such as U, Q, D, moving average, moving range, CUSUM, etc., see reference 10, Appendix B.

3. STARTING THE CONTROL CHART

3.1 Collection and Analysis of Preliminary Data. It is presumed that for the quality characteristic selected (2.1), the plan of subgrouping has been worked out (2.3) and the statistical measures to be used for the control charts have been chosen (2.4). In starting the control chart it remains to collect and analyze some inspection data for the purpose of providing preliminary control chart values that are needed for determining the central line and control limits to be drawn on each chart.

Start collecting data, sample by sample, in subgroups of a size and character decided upon in 2.3. When a sufficient number of such subgroups of data has been collected, analyze them by the control chart method (see Instruction Sheets 1 and 2, ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data*) with respect to the statistical measures ($\overline{X}$, *s*, *R*, *p*, *c*) that have been chosen. As recommended in ANSI/ASQC B2-1996 (p. 8), use 3-sigma control limits in this analysis.

It usually will be found advantageous to have at least 20 subgroups for this purpose. When speed is important, and it is desired to get the program going with a minimum of delay, as few as 10 successive subgroups may be used initially, pending the accumulation of more data.

Past data, representative of the current process, and in a form that permits subgrouping on the basis decided upon, may be used at once without waiting for the accumulation of new data.

3.2 Establishment of Control Limits—General—Standard Values. From the analysis of the preliminary data (3.1) and other pertinent information, establish control limits that are to be used as action limits for future inspection results as they are plotted subgroup by subgroup. Use the 3-sigma control limits unless there are reasons for using other limits.

The following material is quoted from Section 7.2.2 of ANSI/ASQC B2-1996: "In the choice of action limits for indicating when to look for assignable causes of variation, an attempt is made to strike an economic balance with respect to the net consequences of two kinds of 'errors' that may occur in practice: namely, looking for trouble that does not exist, and not looking for trouble that does exist. Ordinarily, limits of the magnitude of '3 sigma' appear to approximate an economic balance between the costs resulting from the two kinds of errors, in a wide range of industrial applications. Therefore, it is recommended that the '3-sigma' limits be used, or limits having a spread differing but little from them, unless there is sufficient information concerning the relative costs just mentioned to warrant another choice."

As indicated in Section 7 of B2-1996, there are ways of placing control limits other than by the 3-sigma method. For instance, control limits are sometimes placed so that they would include approximately some assigned percentage (99.8, 99, 95, etc.) of the plotted points *if* the process were controlled at the level used in computing the control limits, and *if* the distribution were of the form assumed in computing the limits. Such limits are referred to as "probability limits." Factors for computing probability limits for $\overline{X}$, *s*, and *R* for samples of 15 observations or less from a normal (Gaussian) distribution are shown in Appendix A. Further tables and charts are given in the books cited in references 2 and 5 of Appendix B. The 0.001 probability limits (99.8 percent within limits) have about the same spread as the 3-sigma limits, and in practice they yield essentially the same results.

As stated above, the purpose of control limits is to indicate when to take action on the process, as the points arising from future inspection results are plotted one by one. Whatever control limits are used, the purpose is *not* to predict what percentage of future product will fall beyond any designated limit or limits; this cannot be accomplished until a state of control has been attained. In practice, even though the quality is controlled, the actual percentages to be expected may differ substantially from the theoretical ones unless the actual control level is very closely determined—a condition that can be met only when data are available from an extremely large number of samples.

The establishment of the central line and the control limits involves two steps: first, the selection of standard values for the statistical measures that have been chosen; and second, the computation therefrom of the central line and control limits. These computations may be made by the use of simple formulas and tables (see Table 6).

The term *standard value* ($\overline{X}_0$, σ_0, p_0, etc.) designates a value of $\overline{X}$, *s*, *p*, etc., that is adopted at any time for computing the control limits to be used as criteria for action in the immediate future, or until additional evidence indicates need for revision. Standard values are thus always subject to change and may be different for different manufacturers using the same specification. In some circumstances, standard values may be desired values to be aimed at.

The selection of standard values (of $\overline{X}$, *s*, *p*, etc.) is perhaps the most basic problem in setting up a control procedure. The primary aim is not just to get control, but to get control at a satisfactory level. There may be a rather wide range in levels that would be satisfactory, or again the latitude may be small. What can be deemed satisfactory depends fundamentally on the needs of the consumer as defined by his specification: questions of cost of production and capability of the manufacturing process must, of course, also be taken into account in deciding on a level that will be economical from an overall point of view.

For processes that are set up to produce the quality desired, it will usually be found that the standard value can be based on data of the recent past (preliminary data). However, if these data do not show control, perhaps only a portion of them can be used for this purpose. Or, if the preliminary data indicate a level of quality that is unsatisfactory, then an arbitrary decision with respect to the immediate procedure to be followed in the control program may be necessary, as outlined more specifically in 3.3.

3.3 Establishment of Control Limits—Charts for $\overline{X}$ and *s*. In setting up a pair of control charts to be used for $\overline{X}$ and *s*, two selections are needed: a standard value of the average, designated as $\overline{X}_0$, and a standard value of the

standard deviation of individual values within subgroups, designated as σ_0.

The control chart analysis of the preliminary data (3.1) will have given the following two values, which appear as central lines on the charts for analysis of past data: $\overline{\overline{X}}$, the average of the subgroup values of $\overline{X}$; and $\bar{s}$, the average of the subgroup values of s. Whether these values will be satisfactory for setting the standard values, $\overline{X}_0$ and σ_0, depends on two considerations: first, whether all or only a part of the preliminary data can be taken as representative of the process; and second, whether an acceptable goal will be set if $\overline{X}_0$ is made equal to the value of $\overline{\overline{X}}$ obtained from such total or partial data, and σ_0 made equal to the value $\bar{s}/c_4$ *obtained from the same data.*

Usually, an acceptable goal in practice is one which, if attained in control, would show not more than an acceptable proportion of individual articles having measured values outside the specification limits. Consideration of an acceptable goal will take into account the requirements of the specification, the prospective use of the product, and the engineering problems that are met in the manufacturing.

In the relation $\sigma_0 = \bar{s}/c_4$, the factor c_4 is a contrast that depends on sample size. Values of c_4 are given in Table 6. If the sample size exceeds 25, take $c_4 = 1$. If the sample sizes for the preliminary data vary from sample to sample, then it is usually satisfactory as a simple working rule to use the average value of *observed* s/c_4 for the individual samples, in place of $\bar{s}/c_4$. These formulas have been selected with a view toward utmost simplicity in computation consistent with adequacy for control.

3.3.1 First Consideration: *Shall all or only part of the preliminary data be taken as representative of the process?* Whether all or only part of the preliminary data should be taken as representative of the process is determined by noting whether the analysis of the preliminary data gives any indications of the lack of control.

If the analysis of the preliminary data given no indications of lack of control, then the values $\overline{\overline{X}}$ and $\bar{s}$ computed from the preliminary data may be taken as representative of the process.

If the analysis of the preliminary data gives indications of lack of control, there may be engineering grounds for discarding some of the data. It is recommended, however, that portions of the preliminary data be discarded only if they are known for engineering reasons to be not representative of the normal process. In other words, the discarding of data should be based on knowledge of abnormal conditions prevailing during production rather than on the indications of statistical analysis.

If only a few plotted points for $\overline{X}$ and s are outside their control limits and are outside only marginally, then it will ordinarily be found satisfactory to retain all data and to proceed as though there had been no indications of lack of control. If, however, some of these few points are far outside their control limits, and if it is found on review that the wide deviations were attributable to some isolated irregularities in the production process or in the inspection, then the inspection data for these points should be discarded. If a substantial proportion of the plotted points for $\overline{X}$ and s are outside their control limits, then a careful review of conditions prevailing during production may be necessary to find whether there are grounds for discarding portions of the data. Particular attention should be given to the nature of any known disturbing causes and of any fundamental changes in the production or inspection practices, with a view to retaining only data that may be considered representative of the process from the present time on. If it is found, for example, that during the period covered by the preliminary data a fundamental change in the process was made—one which from an engineering standpoint would be expected to result in a substantial change in $\overline{X}$ or s—then data obtained prior to this change may have to be discarded.

If any data are so discarded, then revised values of $\overline{\overline{X}}$ and $\bar{s}$ should be computed for the data retained, and may be taken as representative of the process. On the other hand, if, with lack of control, no grounds have been found for discarding any of the data, then there is little alternative to taking the $\overline{X}$ and $\bar{s}$ of the entire preliminary data as representative of the process.

3.3.2. Second Consideration: *Will standard values based on the preliminary data provide an acceptable goal? Adoption of standard values,* $\overline{X}_0$ *and* σ_0. After representative values of $\overline{\overline{X}}$ and $\bar{s}$ are obtained from all or part of the preliminary data, the next step is to determine whether the level and spread based on $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 \simeq \bar{s}/c_4$ will be satisfactory in consideration of the specification limits for individuals.

The following paragraphs give suggestions that may be helpful in deciding the above question. With attention restricted to types of symmetrical distributions that are usually met in practice, it may be said that for a controlled distribution having an average $\overline{X}_0$ and a standard deviation σ_0 practically all of the individual articles will fall within the band $\overline{X}_0 \pm 3\sigma_0$. If the specification prescribes minimum and maximum limits.* (X_{min} and X_{max}) applicable to the

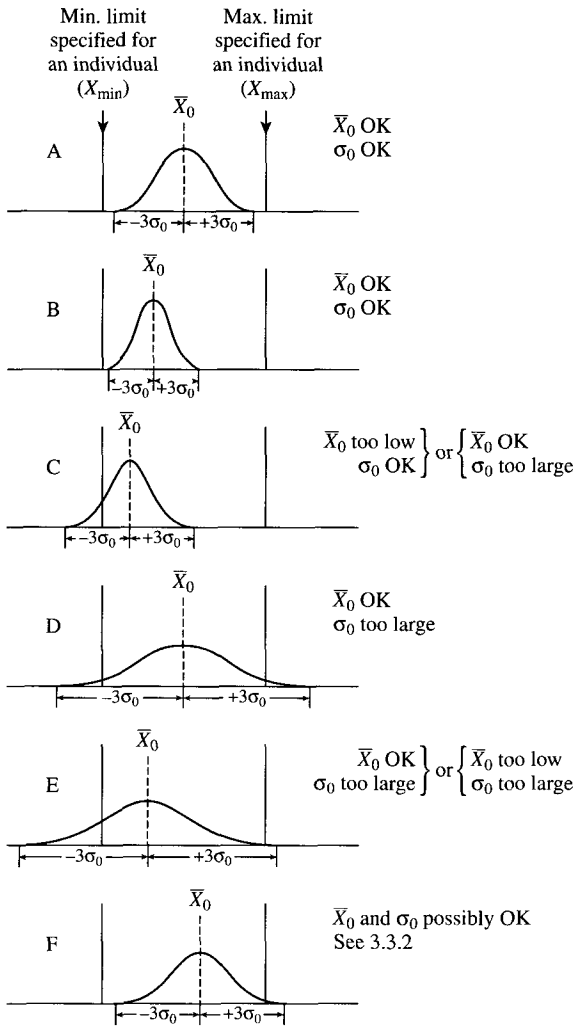
* The specifications may prescribe merely a minimum limit or a maximum limit, applicable to the individual article. The discussion applies equally well in these circumstances.

individual article, the criterion of satisfactoriness often used is that practically all of the distribution of quality for individual articles should fall within the specified limits. The adoption of this criterion requires that the band $\overline{X}_0 \pm 3\sigma_0$ fall within the band defined by the specified limits, X_{min} and X_{max}, which is to say that $\overline{X}_0$ should be located within the specified limits and at a distance of more than $3\sigma_0$ from each of them. This is illustrated in Fig. 2, which shows the effect of having too high or too low an $\overline{X}_0$ or too large a σ_0.

If, with consideration for its shape, the distribution defined by $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 = \bar{s}/c_4$ falls within the specification limits, as in diagrams A and B of Fig. 2, adopt $\overline{X}_0 = \overline{\overline{X}}$ and adopt $\sigma_0 = \bar{s}/c_4$. Or again, adopt these values if the distribution extends only a small amount outside X_{min} or X_{max} or both, as in diagram F of Fig. 2, and if this amount represents a percentage of individual articles falling outside specification limits that is judged to be acceptable and satisfactory.

The above use of the relation between the $\overline{X}_0 = \pm 3\sigma_0$ band for individuals and the specified limits X_{min} and X_{max} should be considered as a rough guide and not as a precise rule for determining the satisfactoriness of the values of $\overline{X}_0$ and σ_0 since the *shape* of the distribution of measured values for individual articles produced under controlled conditions determines what percentage of the articles may be expected to fall above or below the band defined by $\overline{X}_0 \pm 3\sigma_0$. How this percentage depends on the shape of the distribution is shown in Fig. 3, which provides a picture, drawn to scale, for three typical distributions. Exact information on the shape is rarely available in practice, but one may often know, from accumulated data, whether the distribution is nearly symmetrical, approximating that shown in diagram A of Fig. 3, or decidedly unsymmetrical, as in diagram C. If *for controlled quality,* the distribution is believed to be decidedly unsymmetrical, then it may be advisable to assume that the distribution of the quality of individual articles will extend in one direction as far as $4\sigma_0$ from $\overline{X}_0$, or even farther.

It should be held in mind that there does not exist a distribution that can be used for making predictions on a probability basis, unless quality is controlled.



Note: OK herein means "is at a satisfactory level."

Figure 2 — Showing Possible Relations Between the Distribution of Individuals and the Specified Minimum and Maximum Limits.

If the distribution defined by $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 = \bar{s}/c_4$ is unsatisfactory because it will allow too large a percentage of individual articles to fall outside the specified limits, the course to be followed must be based on an engineering decision. If, for example, the preliminary data comprise results from a substantial number of samples, say 25 or more, and if good control has been shown, but at the wrong level, or with too much spread, then one of two changes is needed: *a fundamental change in the production process* or *a change in specification limits*. Depending on circumstances, the change needed in the process is one that will result in shifting the average to a more favorable level, or reducing the standard deviation, or both. Failure to make such a change will result in too large a percentage of the product failing to meet the specification. It should be noted that arguments for changing specification limits are strongest when backed by evidence of a high degree of control, as when none out of 25 successive points on the control chart, no more than 1 out of 35, or no more than 2 out of 100, fall outside the 3-sigma control limits.

On the other hand, if the preliminary data are somewhat limited in quantity or if they do not indicate good control, then the arguments for making one of the above mentioned two changes may not be supported by sufficient evidence.

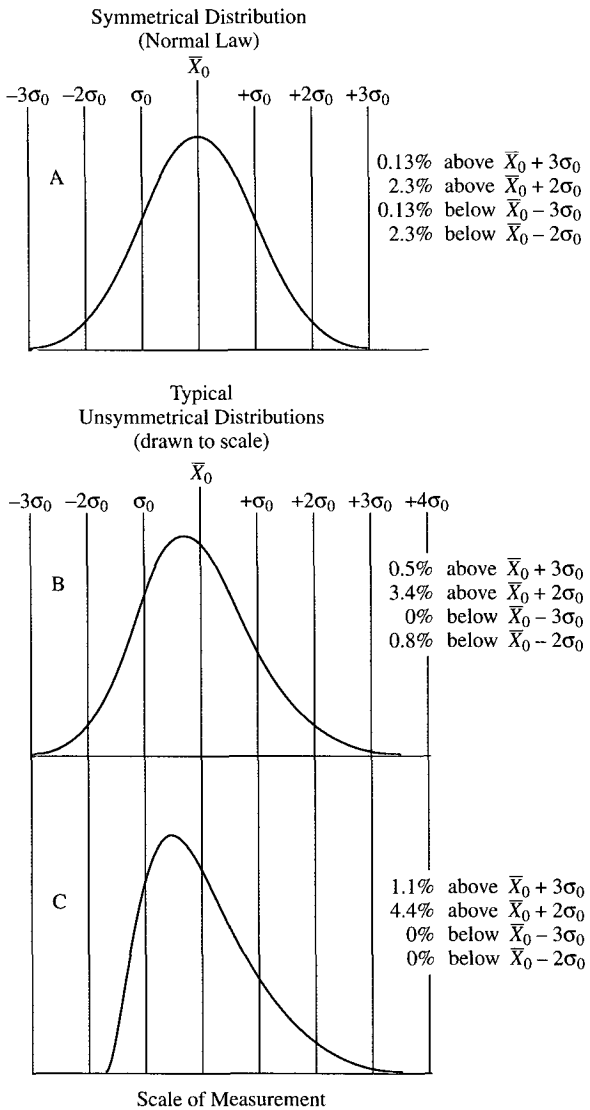


Figure 3 — Showing Percentages at the Far Ends of Distributions Having Different Shapes.

Either for this or other reasons, it may be judged inadvisable to make one of these two fundamental changes immediately, and the question arises as to what sort of control chart can be used in the interim. This is a purely practical problem not subject to the rule, but the following suggestions based on experience may be helpful. In some instances, it may be found advantageous to set up control charts at once at the level indicated by the preliminary data, even though this level is not satisfactory, in order to get a graphical impression of the behavior of the process, to see what the next step should be. Or again, it may be deemed advisable to make an arbitrary selection of standard values $\bar{X}_0$ and σ_0 that would appear reasonable and at the same time satisfactory, in order to provide a basis for portraying in a continuing manner the degree of concordance of current quality with such arbitrarily selected satisfactory standards. Often, for example, when the degree of unsatisfactoriness is only marginal, it may be found that there are indications of the lack of control for $\bar{X}$ but not for s, in which case it may be desirable for engineering reasons to adopt $\sigma_0 = \bar{s}/c_4$ and to adopt arbitrarily an acceptable value of $\bar{X}_0$ differing only moderately from $\bar{\bar{X}}$.

During the immediately succeeding period, there will remain a basic need for some change, either in the process or in the specification, and usually it will be found profitable to concentrate at once on the discovery of causes in the process that may be responsible for the unsatisfactory level. The aim should be to reach the point as quickly as possible where the quality levels of the past may be used to provide standard values for the immediate future.

If a fundamental change is made in the process, the procedures outlined above should be applied to the new data accumulated after the change is made, and new standards $\bar{X}_0$ and σ_0 adopted.

3.3.3 Computation of Control Limits. Using the standard values $\bar{X}_0$ and σ_0 that have been adopted (3.3.2), compute control limits for the sample size (n) that is to be used (formulas and tables for computing central lines and 3-sigma control limits are given in Table 6). If the sample size is to vary from sample to sample, then the control limits theoretically ought to be computed sample by sample as soon as the size is known. However, if the variations in sample size are not great, it may be satisfactory to use an average sample size.

While no general rule can be laid down, it may be noted that some have found it satisfactory in practice to use control limits based on the average sample size if the samples vary in size by a ratio of not greater than 2 to 1, and to determine the exact control limits only for questionable points—those falling fairly close to control limits, either inside or outside. To avoid the necessity of actually computing control limits on the job sample by sample, there should be provided in advance and made available to the person who is to plot the points, either a table showing control limits for different sample sizes, or a strip of cardboard with a scale marked along its edge giving for different sample sizes the relative positions of the central line and the control limits.

3.4 Establishment of Control Limits—Charts for $\bar{X}$ and R. In setting up a pair of control charts to be used for $\bar{X}$ and R, two selections are needed—the same as those needed for $\bar{X}$ and s: a standard value of the average, designated as $\bar{X}_0$;

and a standard value of the standard deviation of individual values within subgroups, designated as σ_0. The procedure is identical with that given in 3.3 except that here a value of σ_0 is derived from observed values of the range (R) instead of from observed values of the sample standard deviation (s).

The control chart analysis of the preliminary data will have given the following two values, which appear as central lines on the analysis charts: $\overline{\overline{X}}$, the average of the subgroup values of $\overline{X}$, and $\overline{R}$, the average of the subgroup values of R. Whether these values or some others should be used for establishing the standard values of $\overline{X}_0$ and σ_0 depends (as in 3.3) on two considerations: first, whether all or only part of the preliminary data can be used as representative of the process; and second, whether an acceptable goal will be set if $\overline{X}_0$ is made equal to the value $\overline{\overline{X}}$ obtained from the whole or the selected part of the preliminary data, and σ_0 made equal to the value $\overline{R}/d_2$, obtained from the same data.

These two considerations should be handled in the same manner as for charts for $\overline{X}$ and s, following the procedures of 3.3.1 and 3.3.2 but replacing the terms $\bar{s}$ and $\bar{s}/c_4$ where they appear in these sections by the terms $\overline{R}$ and $\overline{R}/d_2$.

In the relation $\sigma_0 = \overline{R}/d_2$, the factor d_2 is a constant that depends on sample size. Values of d_2 are given in Table 6. If the sample sizes for the preliminary data vary from sample to sample, then it is usually satisfactory as a simple working rule to use the average value of *observed* R/d_2 for the individual samples, in place of $\overline{R}/d_2$.

3.4.1 Choice of Values to Be Taken as Representative of the Process. Follow the procedure of 3.3.1 and decide upon values of $\overline{\overline{X}}$ and $\overline{R}$ that can be taken as representative of the process.

3.4.2 Adoption of Standard Values, $\overline{X}_0$ and σ_0. Using the values $\overline{\overline{X}}$ and $\overline{R}$ obtained from the preceding paragraph, follow the procedure of 3.3.2 and determine whether the adoption of $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 = \overline{R}/d_2$ would provide an acceptable goal. If so, adopt $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 = \overline{R}/d_2$. If not, adopt standard values $\overline{X}_0$ and σ_0 on the basis of the consideration outlined in 3.3.2.

3.4.3 Computation of Control Limits. Using the standard values of $\overline{X}_0$ and σ_0 that have been adopted (3.4.2), compute control limits for the sample size, n, that is to be used (Formulas and tables for central lines and 3-sigma control limits are given in Table 6). If the sample size is to vary from sample to sample, then the control limits theoretically ought to be computed sample by sample as soon as the size is known. (See 3.3.3 on the use of an average sample size).

3.5 Establishment of Control Limits—Charts for p or pn. In setting up a control chart for p or pn, only one selection is needed—a standard value of fraction nonconforming, designated as p_0. It is often convenient to use "number of nonconforming units (pn)" rather than fraction nonconforming (p) for measuring quality when all samples comprise the same number of articles (i.e., when n is constant).

The control chart analysis of the preliminary data will have given an average value of $\bar{p}$ of fraction nonconforming. The value $\bar{p}$ appears as the central line on the analysis chart when p (fraction nonconforming) is used, or is found by dividing by n the central line value $\bar{p}n$, when the chart for pn (number of nonconforming units) is used. Whether this or some other value should be adopted as the standard value p_0 depends (as in 3.3) on two considerations: first, whether all or only a part of the preliminary data can be taken as representative of the process; and second, whether a satisfactory goal will be set if p_0 is made equal to the value $\bar{p}$ obtained from the whole or the selected part of the preliminary data.

3.5.1 Choice of Values to Be Taken as Representative of the Process. If the analysis of the preliminary data gives no indication of lack of control, then value $\bar{p}$ computed from the whole of the preliminary data may be taken as representative of the process.

If the analysis of the preliminary data gives some indication of lack of control, follow the general procedure of 3.3.1 with respect to retaining and discarding data and decide upon a value of $\bar{p}$ that should be taken as representative of the process.

3.5.2 Adoption of Standard Value, p_0. Using the value of $\bar{p}$ obtained from 3.5.1, decide whether $p_0 = \bar{p}$ defines a quality of product that may be considered satisfactory. If satisfactory, adopt $p_0 = \bar{p}$. If not satisfactory, giving too large a percentage of nonconforming articles, decide as in 3.3.2 whether either of two changes should be made immediately—a fundamental change in the production process or a change in the specification requirements. If one of these two changes is not made at once, some temporary step is necessary as outlined in 3.3.2, such as adopting a standard value p_0 that is equal either to $\bar{p}$ given by the preliminary data or to some arbitrarily selected value.

3.5.3 Computation of Control Limits. Using the standard value p_0 that has been adopted (3.5.2), compute control limits for the sample size (n) that is to be used.

For p, the formula for the 3-sigma limits is $p_0 \pm 3\sqrt{p_0(1 - p_0)/n}$.

For pn, the formula for the 3-sigma limits is $p_0 n \pm 3\sqrt{p_0 n (1 - p_0)}$. If the fraction nonconforming (p_0) is small, say less than one-twentieth ($p_0 < 0.05$), the

simplified formula $p_0n \pm 3\sqrt{p_0n}$ may be used. The values given by the latter formula may be read directly from the solid curves of Fig. 4 substituting p_0n for c_0.

If the sample size is to vary from sample to sample, the control limits theoretically ought to be computed sample by sample. However, if the variations in sample size are not great, it may be satisfactory to use an average sample size (see 3.3.3 and the discussion in Illustrative Example 3, section 5.3.3). Another method for handling p charts with variable sample sizes is to use a stabilized p chart (see Appendix B, reference 8, page 404).

3.6 Establishment of Control Limits—Chart for *c*. In setting up a control chart to be used for c, the number of nonconformities in a subgroup or sample, only one selection is needed, a standard value of the number of nonconformities per sample, this number being designated as c_0. The sample may be a single article or specimen, a designated specimen length or area, a sample comprising a designated number of articles or specimens, etc. Consideration is given here only to subgroups or samples of *equal size*, so that the expected number of nonconformities is the same for each subgroup.

The control chart analysis of the preliminary data (3.1) will have given an average value of the number of nonconformities

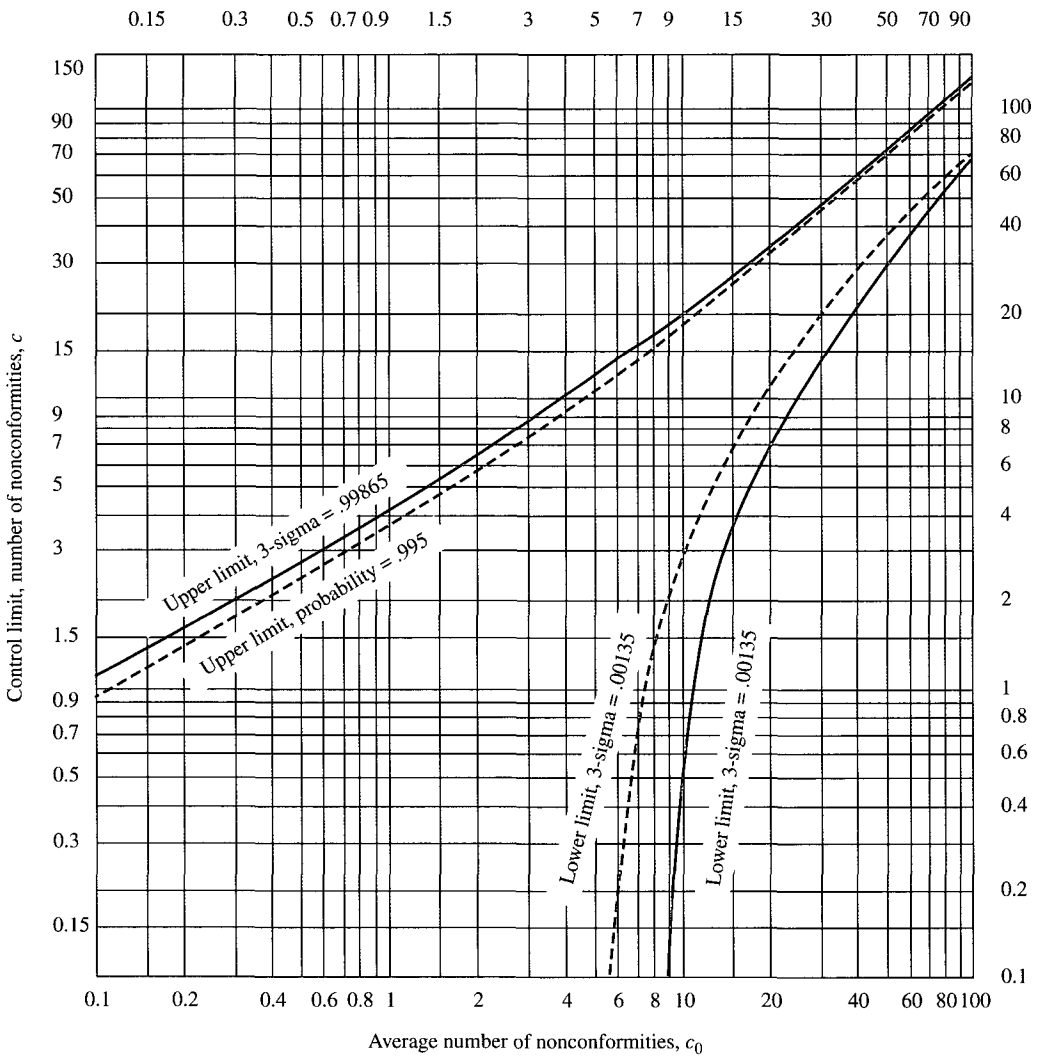


Figure 4 — Chart for Determining Control Limits for c, Number of Nonconformities in Sample.

in a subgroup, designed as $\bar{c}$, and this number appears as the central line on the analysis chart, and represents the average of the subgroup values of c. For example, if the preliminary data had covered the results of inspecting the paint on 30 test panels, each test panel being considered as a separate sample, and if on these 30 test panels a total of 18 pin holes was observed, then $\bar{c} = 18/30 = 0.60$, and this value will be used on the control chart as the central line. Whether this may be adopted as a standard value c_0 depends on the same two considerations as were outlined in the preceding sections: first, whether the analysis chart indicates that quality is controlled; and second, whether the value $c_0 = \bar{c}$ gives a quality of product that is satisfactory for adoption in the control program.

The general procedures outlined in 3.5 for selecting a standard value of p are applicable here for c.

The recommended 3-sigma control limits can be obtained from the solid curves of Fig. 4, or can be calculated as $c_0 \pm 3\sqrt{c_0}$.

The symmetric 3-sigma limits have been found in wide experience to approximate an economic balance between the net consequences of the two kinds of errors referred to in 3.2.

For increased attention to deviation in the direction of better quality (low value of c), in contrast with deviations in the direction of poorer quality (high values of c), and for other considerations, asymmetric limits may be preferred. One such choice is offered by the dashed curves in Fig. 4.*

3.7 Preparing the Control Chart for Use. On a suitable form, graph, or cross-section paper, lay out a chart with vertical scales at the left for the statistical measures chosen ($\overline{X}$, s, R, p, c), and with a horizontal scale for subgroup number (possibly designated by date and sample number; see Fig. 6).

It is usually satisfactory to use cross-section paper having 8 or 10 rulings to the inch (sometimes 20) so that the running history of control over an extended period of time can be kept on a single chart.

If charts for $\overline{X}$ and s, or for $\overline{X}$ and R, are being used, place the $\overline{X}$ chart at the top and the s or R chart below, using the same horizontal scale for both charts.

If the size of future samples is to remain constant, the central line and control limits for each chart (values computed in 3.3.3, 3.4.3, 3.5.3, 3.6) may be drawn at once on the chart. Draw the central line as a horizontal solid line and draw the control limits as horizontal dotted lines. Extend these lines into the future for a relatively small number of subgroups, possibly 10, to permit an early review and a decision regarding whether the standards ($\overline{X}_0$, σ_0, p_0, etc.) initially adopted should be retained for a longer period or revised in light of the results obtained for the next few samples.

4. USING THE CONTROL CHART DURING PRODUCTION

4.1 Plotting Points on the Control Chart; Taking Action. Decide on the general type of action that is to be taken if a point falls outside the control limits.

As noted in 1.2 and 1.3 and discussed in Illustrative Example 1, two kinds of action may be involved: action on the lot of product sampled (Purpose A), and action on the process (Purpose B). An example of action for Purpose A, not specifically covered by this standard, is the examination of additional samples from the lot sampled with a view to determining whether it should be released or held up. Examples of action for Purpose B, the particular objective of this standard, include the investigations of specific steps in the production process, of the suitability of raw materials, of the corrections of the testing procedure, etc., the identification of the cause, if possible, and corrective action to prevent its recurrence.

As soon as the results of inspection become available for a sample, compute the value of the statistical measure or measures ($\overline{X}$, s, R, p, c) and plot a point on the chart *at once*.

If a point falls outside control limits, take the action that is deemed appropriate.

Even though all the points fall within the control limits, trouble or change in the process is sometimes evidenced by unusual patterns or arrangements of the point, for example:

1) A series of points all falling close to one of the control limits
2) A long series predominantly above or below the central line
3) A series of points exhibiting a trend

Note: If "warning limits" are set at ±2 sigma, runs of points between the warning and action limits may also be taken as an indication of lack of control. For a discussion of pattern analysis, see reference 11.

4.2 When Control May Be Assumed to Exist. As stated above in 10.3 of ANSI/ASQC B1-1996, *Guide for Quality Control Charts,* it is usually not safe to conclude that a state of control exists unless the plotted points for at least 25 successive subgroups fall within the control limits. In addition, if not more than 1 out of 35 successive points, or not more than 2 out of 100, fall outside the control limits, a state of control may ordinarily be assumed to exist.

4.3 Review of Control Chart Standards. The standards ($\overline{X}_0$, σ_0, p_0, c_0) initially adopted for constructing the control

* Control limits read from the dashed curves of Fig. 4 are the so-called "probability limits" for the Poisson distribution having mean c_0. If the quality is statistically controlled at exactly the level c_0, then 0.5 percent of the points may be expected to fall above the upper control limit, and another 0.5 percent below the lower control limit. It is to be noted that the theoretical probability values associated with discrete numbers of nonconformities occur in jumps, and are not continuous, hence any probability values can only be approximated (cf., the reference to "probability limits" in the footnote to 3.2).

charts should usually be reviewed after 10 to 25 points have been plotted. If causes of trouble are located and eliminated, it may be found that the standards should be altered. As mentioned at the close of 3.3.2, after any major corrective steps have been taken, data accumulated following the change should be analyzed afresh by the control chart method (3.1; also ANSI/ASQC B2-1996) for determining whether the standards should be revised.

Even though a fairly high degree of control is attained, it is recommended that standards be subjected to periodic review, and revised as necessary in light of the results of analysis and other pertinent information regarding the process. It is recommended further that a definite schedule for such reviews be set up and followed as a routine matter; for example, every 50 points, every 100 points, every month, etc., depending on the frequency of sampling and the degree of control shown by the charted history.

4.4 Control Chart Records. It is recommended that the control charts be kept as a part of the permanent records of quality history. It is further recommended that brief notes be placed on the control chart record regarding causes of trouble that are found in the manufacturing process or inspection methods, and the corrective action that is taken. Fig. 6 gives a suggested form of control chart record. Such records may be used in producer and purchaser relations to provide graphical evidence of the level and degree of uniformity of quality, and, in favorable cases, to provide a basis for reducing the amount of acceptance inspection required by the purchaser.

5. ILLUSTRATIVE EXAMPLES

5.1 Example 1—Control Chart Using $\overline{X}$ and R—Control Procedure Illustrated Step by Step

5.1.1 Outline. This example illustrates a control procedure for a measurable quality characteristic of a manufactured article—destructive test; small equal periodic samples; control charts for the average ($\overline{X}$) and for the range (R) of observations in the samples.

The control procedure is outlined step by step. Depending on the local plan of departmental organization, setting up the sampling plan may rest with the quality control engineer, an inspection planning group, or others, subject to necessary approvals of the management. Responsibility for designing the data sheets for recording the original data, designing the control chart form, and placing the control limits on the chart is assumed to rest with the person who has charge of the control program. Responsibility for inspecting, recording data, computing the values of $\overline{X}$ and R, and plotting points on the control chart is assumed to rest with the person who conducts the test.

The characteristic illustrated is the "time of blow" of an electrical fuse whose specification states as one of its requirements that the fuse, when tested under specified circuit conditions, shall blow within a maximum time of 150 seconds. The sampling procedure illustrated is based on the testing of a sample of 5 fuses every hour. For simplification, and to permit illustrating several points of interest in the relatively short span of time covered by the chart of Fig. 6, conditions attending an actual case are modified and only a portion of the data is actual.

In carrying through the computations for this example, a number of control chart constants will be needed for a sample size of $n = 5$. These constants are given in Table 6, but are reproduced here in Table 2 for ready reference in following through the procedures of the subsequent sections.

TABLE 2
Factors for Computing Control Chart Lines

For 3-sigma control limits; sample size $n = 5$			
A	1.342	D_1	0
A_2	0.577	D_2	4.918
A_3	1.427	D_3	0
d_2	2.326	D_4	2.114

5.1.2 Starting the Control Chart

5.1.2.1 *Laying Out the Data Sheet.* Lay out a data sheet (see Fig. 5) with spaces for recording individual readings and for carrying out computations for $\overline{X}$ and R. On the form, provide spaces for recording identifying information regarding the individual samples, unit of measurement in which the data are to be recorded, and information regarding the product sampled, the production source, the inspector, etc.

5.1.2.2 *Sampling and Inspecting.* Select a sample of n articles from the production line (every half hour, every hour, every 1000 articles, etc., as provided by the sampling plan), and test each article in the sample to determine the observed value of the chosen quality characteristic for that article.

5.1.2.3 *Recording Data.* Record the n observed values in the order taken (see Fig. 5).

CONTROL CHART DATA SHEET

Product: Type XX fuse
Characteristic: Time of blow
Unit of Measurement: seconds
Specified Limits: – Min. / 150 sec. Max.
Specification No.: XXXX

Production Order No.: 00-000
Production Dept. No.: 000
Normal Daily Output: 0000
Sample: 5 per hour
Test Set No.: 00

Date: Nov. 3-4, 19
Inspector's Clock No.: 000

Signed: Jack Doe (Inspector)

Identification	Obs. Value	Ident.	Obs. Value	Ident.	Obs. Value	Ident.	Obs. Value	Ident.	Obs. Value	Ident.	Obs. Value
Sample No. 1	87	2	90	3	9	4	69	5	51	6	75
Time 8:35 A.M.	78	9:25	68	10:30	81	11:00	54	1:00 P.M.	57	1:50	51
Operator No. 00	42	00	42	00	81	00	36	00	59	00	132
Machine No. 000	65	000	45	000	80	000	77	000	78	000	74
---- No.	75		72		24		84		42		78
Total 5	347	5	317	5	275	5	320	5	287	5	410
Average, $\bar{X}$	69.4		63.4		55.0		64.0		57.4		82.0
Largest Value	87		90		81		84		78		132
Smallest Value	42		42		9		36		42		51
Difference: Range, R	45		48		72		48		36		81
Sample No. 7	60	8	20	9	62	10	118	11	109	12	136
Time 3:05	95	4:00	27	8:30 A.M.	39	9:25	113	10:30	91	11:35	109
Operator No. 00	60	00	18	00	15	00	69	00	93	00	78
Machine No. 000	138	000	60	000	84	000	153	000	112	000	94
---- No.	72		42		30		109		64		61
Total 5	425	5	167	5	230	5	562	5	469	5	478
Average, $\bar{X}$	85.0		33.4		46.0		112.4		93.8		95.6
Largest Value	138		60		84		153		112		136
Smallest Value	60		18		15		69		64		61
Difference: Range, R	78		42		69		84		48		75

NOTES: Sample No 10. 1 fuse exceeds max. limit. 35 add'l fuses tested. Data recorded on supplementary data sheet.

Figure 5 — Control Chart Data Sheet — Illustrative Example 1.

5.1.2.4 *Computing $\overline{X}$ and R.* For each sample:

1) Add the *n* recorded values, divide the sum by *n,* and record the result. This figure is the average and is designated as $\overline{X}$ (see Fig. 5).

2) Subtract the smallest recorded value from the largest recorded value. This figure is the range of variation in the sample and is designated as *R* (see Fig. 5).

5.1.2.5 *Laying Out the Form for the Control Chart Record.*

1) On a specially designed sheet or on standard cross-section paper, lay out a control chart form (see Fig. 6). On the form, provide spaces for recording essential information regarding the product inspected, such as:

   (a) Name of article

   (b) Quality characteristic

   (c) Specified maximum and minimum limits

   (d) Production order number

2) On the-cross section area of the form, mark a horizontal scale across the bottom (or top, if preferred) for the sample number. If several samples are taken in one day, add an auxiliary horizontal scale to show the dates; *e.g.,* Nov. 3, Nov. 4, etc. (shown at the top in Fig. 6). If the cross-section paper has 8 or 10 divisions to the inch, one division may be used to represent a sample.

3) Mark two vertical scales at the left-hand margin of the paper, one near the top for the purpose of plotting the averages ($\overline{X}$), and another below for plotting the ranges (*R*), as in Fig. 6.

It is usually convenient to choose vertical scales so that the spread between the control limits is from 5 to 15 percent of the horizontal length of the chart. It is desirable to have the spacing between the control limits on the *R* chart about the same as on the $\overline{X}$ chart.

4) If desired, mark a vertical scale at the left-hand margin for "number of nonconforming units" (number of failures to meet the specified minimum and maximum limits). This is not a necessary part of the control charts for $\overline{X}$ and *R,* but is advantageous where it is desired to have this information at a glance.

5.1.2.6 *Plotting Points on the Chart.* For each sample:

1) Plot a point to represent the observed average, $\overline{X}$, already computed according to 5.1.2.4(1). Plot the points in the order of the sample number.

2) Plot also a point to represent the range, *R,* already computed according to 5.1.2.5(2).

3) If a scale has been provided for "number of nonconforming units" [5.1.2.5(4)], plot also the number 0, or 1, or 2, etc., to show how many articles in each sample failed to meet the specification limits.

4) Optional step—Connect points with solid lines.

5.1.3 Analyzing the Preliminary Data

5.1.3.1 *Computing $\overline{\overline{X}}$.* After the data from preferably at least 25 (but not less than 10) samples have been plotted on the chart, compute the average of the observed averages. This figure is the grand average and is designated $\overline{\overline{X}}$. In Table 3, $\overline{\overline{X}} = 73.9$.

Over the period covered by the preliminary data, draw a solid horizontal line on the chart for averages, to represent $\overline{\overline{X}}$ (see Fig. 6).

5.1.3.2 *Computing $\overline{R}$.* From the same data, compute the average of the observed ranges (see Table 3). This figure is the average range and is designated $\overline{R}$. ($\overline{R}$ = 60.1 in Table 3.) Over the period covered by the preliminary data, draw a solid horizontal line on the chart for ranges to represent $\overline{R}$.

5.1.3.3 *Computing and Drawing Control Limits for the Preliminary Data.*

1) Compute control limits for the averages and draw them as two dotted lines on the control chart for $\overline{X}$ over the period covered by the preliminary data (the first 25 samples in Fig. 6).

2) Compute control limits for the ranges and draw them as two dotted lines on the control chart for *R* over the period covered by the preliminary data.

Procedure: The 3-sigma control limits for $\overline{X}$ are:

$$\overline{\overline{X}} \pm A_2\overline{R}$$
$$= 73.9 \pm 0.577 \times 60.1$$
$$= 108.6 \text{ and } 39.2$$

The 3-sigma control limits for R are:

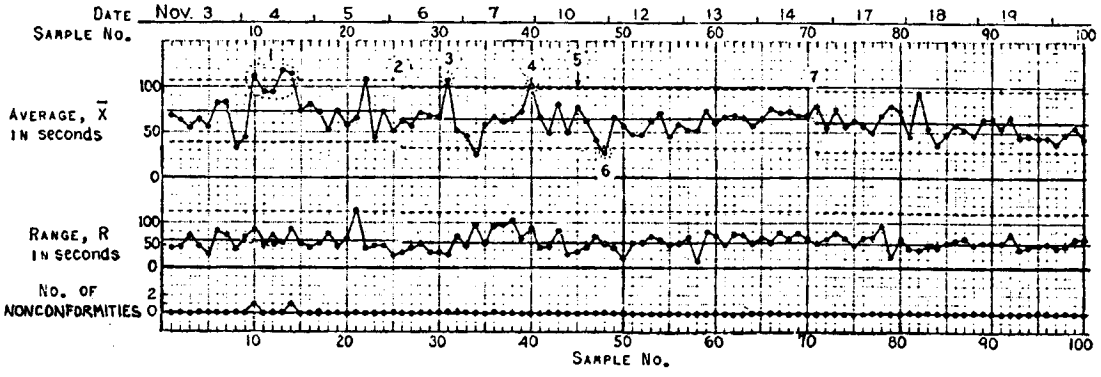
$$D_4\overline{R} \text{ and } D_3\overline{R}$$
$$2.114 \times 60.1 \text{ and } 0 \times 60.1$$
$$= 127.1 \text{ and } 0$$

$$\overline{X} = \frac{1848.0}{25} = 73.9$$

$$\overline{R} = \frac{1503}{25} = 60.1$$

CONTROL CHART RECORD

PRODUCT Type XX fuse
PRODUCTION ORDER NO. 00 - 000
DATE Nov. 3-4, 19
CHARACTERISTIC Time of blow
PRODUCTION DEPT. NO. 000
UNIT OF MEASUREMENT seconds
NORMAL DAILY OUTPUT 0000
SPECIFIED LIMITS – MIN. / 150 sec. MAX.
SAMPLE SIZE, GROUPS OF 5
SPECIFICATION NO. XXXX
POINTS BY Jack Doe INSPECTOR
LIMITS AND NOTES BY A.B. Smith Q.C. ENG'R.



NOTES:

(1) Faulty lot of raw material: rejected, replaced.
(2) End of preliminary data. Standard values adopted: $\bar{X}_o$ 65.7, σ_o=24.9.
(3) 35 add'l. fuses tested, one nonconformity found (data not plotted), Foreman notified, investigation started.
(4) 35 add'l. fuses tested, all O.K. (data not plotted), lot passed. Foreman notified, investigation continued.
(5) Completed investigation shows lack of uniformity of spring tension. Process changed, using weight instead of spring balance for adjusting spring tension.
(6) Foreman notified, cause not found.
(7) New standards adopted, based on last 25 samples: $\bar{X}_o$ = 60.5, σ_o = 24.8. (See 5.1.4.4 (4))

Figure 6 — Suggested Form for a Control Chart Record — Illustrative Example 1.

TABLE 3
Computation of $\overline{\overline{X}}$ and $\overline{R}$

The averages and ranges given here come from the control chart data sheet shown in Fig. 5.

Sample size, $n = 5$

Sample No.	Average $\overline{X}$	Range R
1	69.4	45
2	63.4	48
3	55.0	72
4	64.0	48
5	57.4	36
6	82.0	81
7	85.0	78
8	33.4	42
9	46.0	69
10	112.4	84
11	93.8	48
12	95.6	75
13	117.8	51
14	113.6	84
15	74.8	54
16	80.8	45
17	71.8	57
18	53.2	75
19	74.8	48
20	59.2	63
21	65.8	129
22	109.6	42
23	44.2	51
24	73.6	51
25	51.4	27
Total	1848.0	1503

5.1.4 Establishing Control Limits for $\overline{X}$ and R, for Actual Control Operation

5.1.4.1 *Adopting Standard Values $\overline{X}_0$ and σ_0.* If the analysis of the preliminary data (5.1.3.1 through 5.1.3.3) shows good control, take X and R for these data as representative of the process; otherwise determine what portion of the preliminary data should be used for obtaining values of $\overline{\overline{X}}$ and $\overline{R}$ that may be considered representative (3.4.1). Decide whether it will be satisfactory to adopt the standard values $\overline{X}_0 = \overline{\overline{X}}$ and $\sigma_0 = \overline{R}/d_2$ based on the values $\overline{\overline{X}}$ and $\overline{R}$ obtained from all or part of the preliminary data (3.4.2).

Procedure: The control charts for the preliminary data (Fig. 6) show a decided lack of control of $\overline{X}$, with extremely high values in the period for samples number 10–14, and scattered instances of lack of control of $\overline{X}$ or R for samples number 8, 21, and 22. The high level of $\overline{X}$ during the period for samples number 10–14 was definitely attributed to a faulty lot of raw material, which was thereupon withdrawn from the production line and replaced by a new lot. There were no known reasons for the lack of control indicated by samples number 8, 21, and 22. Samples number 10–14, only, were therefore disregarded, and new values of $\overline{\overline{X}}$ and $\overline{R}$ were computed for the remaining samples (see 3.3.1 and 3.4.1). With samples number 10–14 excluded, it was found that:

$$\overline{\overline{X}} = 65.7$$

$$\overline{R} = 58.0$$

Determine now whether it will be satisfactory to use these adjusted values of $\overline{\overline{X}}$ and $\overline{R}$ for establishing $\overline{X}_0$ and σ_0. Take $\overline{X}_0 = \overline{\overline{X}} = 65.7$, and compute $\sigma_0 = \overline{R}/d_2 = 58.0/2.326 = 24.9$, and see whether $\overline{X}_0$ turns out to be more than $3\sigma_0$ below the specified limit X_{max}. A simple procedure is to see whether $(X_{max} - \overline{X}_0)/\sigma_0$ exceeds 3; if so, $\overline{X}_0 = 65.7$ and $\sigma_0 = 24.9$ may be adopted. Here:

$$(X_{max} - \overline{X}_0)/\sigma_0 = (150 - 65.7)/24.9 = 3.4$$

Hence adopt $\overline{X}_0 = 65.7$ and $\sigma_0 = 24.9$ as standard values for use in the immediate future.

5.1.4.2 *Computing the Central Line and Control Limits for Actual Control Operation.* Compute the central lines and the 3-sigma control limits for $\overline{X}$ and R, for the sample size that is to be used in future samples (in this example $n = 5$).

Procedure: On the basis of the standard values just adopted, the

Central line for $\overline{X}$ is $\overline{X}_0 = \overline{\overline{X}} = 65.7$

Control limits for $\overline{X}$ are $\overline{X}_0 \pm A\sigma_0 = 65.7 \pm 1.342 \times 24.9 = 99.1$ and 32.3

Central line for R is $d_2\sigma_0 = 2.326 \times 24.9 = 58.0$

Control limits for R are $D_2\sigma_0$ and $D_1\sigma_0 = 4.918 \times 24.9$ and $0 \times 24.9 = 122.5$ and 0

Note that if in 4.1.4.1 all of the preliminary data had been retained in establishing the standard values $\overline{X}_0$ and σ_0 for control purposes, and further, if the same sample size were to be retained, no new computations would be needed, since the control limits already obtained in 5.1.3.3 from the analysis of the preliminary data would merely be extended into the future beyond the preliminary data.

5.1.4.3 *Drawing the Control Chart Lines.* Draw the central lines and control limits on the charts for $\overline{X}$ and R to cover a suitable future period beyond the preliminary data.

5.1.4.4 *Using the Control Chart During Production.* To make the most effective use of the control chart during production, the following recommendations may be carried out, with suitable variations:

1) Post the chart in a conspicuous place where it may be readily viewed by those interested, such as the chief inspector, quality control engineer, operating foreman, manager.

2) Plot the points for the individual samples at once.

3) When any point ($\overline{X}$ or R) falls outside its control limits, take this as an indication of an assignable cause. When this happens, two problems immediately arise as described under Purpose A and Purpose B of 1.2:

   (a) What action should be taken with regard to the lot of product from which this sample was drawn?

   (b) What action should be taken with regard to the process?

With respect to Purpose B, investigate the process with a view to finding the cause and eliminating it if it is undesirable and correctable.

The scope of this standard embraces only Purpose B, but a possible way of handling some of the problems associated

with Purpose A is illustrated here with the object of pointing out that the two purposes must often be met simultaneously, and that the procedures and criteria used to achieve them, though different, may be related and may react on each other.

Procedure: Indication of lack of control was shown at sample number 31, the average $\overline{X}$ falling above the upper control limit (see Fig. 6). No nonconforming unit was observed in the sample.

With respect to Purpose B, the quality control engineer notified the foreman, who started an investigation of the process to determine the cause of lack of control.

With respect to Purpose A, the output of fuses is considered as a succession of lots, a lot being defined as the quantity of fuses turned out in an hour's time. Often, as in this case, where inspection can be performed on a sampling basis only, acceptance of a lot is based in part on the number of nonconforming units in a sample of prescribed size. Until some acceptable evidence of control is at hand, the inspection required for the disposition of the product, lot by lot (Purpose A), may amount to as much as 8 samples of 5 fuses during the hour, whereas, *once control has been established*, one hourly sample of 5 suffices for application of the following rules.

If *both*

1) $\overline{X}$ and R for the current hourly sample are below their upper control limits, *and*
2) no nonconforming units are observed in the current sample, release the lot for shipment.

   Lack of control in the direction of better quality (low $\overline{X}$ or low R) will not require additional inspection of the current lot, but lack of control in the direction of poorer quality will. Therefore, if *either*

3) $\overline{X}$ or R for the current hourly sample falls above it upper control limit,* *or*
4) one or more nonconforming units are found in the current hourly sample,

test 35 additional fuses from the same lot. Select these by taking 7 additional samples of 5 fuses each, spread over the output for the same one hour period of production.† If in the total of 40 fuses (the original 5 plus the additional 35) not more than one nonconforming unit is found, release the lot for shipment. If, however, 2 or more nonconforming units are found in the total of 40 fuses, classify the lot as nonconforming and refer the disposition of the lost to a suitable authority who has been charged with this responsibility, such as the quality control engineer. Depending on the character and extent of the nonconforming units, the designated authority shall then decide on the action to be taken—pass the lot as nonconforming, return it to the operating department for detailed examination and correction, scrap it, or make some other disposition of it.

Actually, in the present instance (sample number 31) the required additional samples totaling 35 fuses were selected and tested, and one nonconforming unit (failure to meet the specified maximum limit of 150 seconds) was observed. There being only one nonconforming unit in the total of 40 fuses (the nonconforming unit being in the additional 35 fuses), the lot was released for shipment.

The average ($\overline{X}$) for the regular hourly sample number 34 fell below the lower control limit, and the average ($\overline{X}$) for the hourly sample number 40 fell above the upper control limit. These two points outside control limits indicated that trouble still existed.

At about this time, the investigation mentioned above in connection with sample number 31 disclosed lack of uniformity in the tension of a spring that is used as a component part of the fuse. Starting at sample number 45, the method of adjusting the spring tension was changed, a weight being used thereafter instead of a spring balance [continued in paragraph (4) below].

NOTE: In the above discussion, only certain features of an overall inspection procedure for control and disposition of lots have been touched upon. The general plan of operation might be based on a set of procedures somewhat as follows:

(i) Initially inspect k samples of 5 per hourly lot, k having a value of possibly 4 or 6. These samples will serve the normal acceptance purposes, prior to the establishment of control, and will also provide preliminary data for starting the control program (5.1.3.3 and 5.1.4.1).

(ii) After control has been established, if $\overline{X}$ and R for m successive samples of 5 fall within their control limits (m having a prescribed value such as 10 or 20), *initial qualification* for reduced inspection is attained. This reduced inspection might be but one sample of 5 per hourly lot, and this one sample then suffices both for control purposes and for lot-by-lot disposition, with rules similar to items (1) and (2) under Procedure above. However,

(iii) If the $\overline{X}$ and R for any hourly sample of 5 falls *outside* its control limits, revert to procedure (i) for subsequent product until *requalification* is achieved. Requalification for reduced inspection may correctly be less stringent than initial qualification per procedure (ii), requiring fewer successive samples within control limits.

(iv) If the current hourly sample of 5 indicates the need for additional inspection according to criteria such as items (3) and (4) under Procedure above, test additional samples from the lot to determine what disposition should be made of it, as is illustrated in the paragraph following item (4) under Procedure above.

(v) If in the preceding M samples of 5 there is not at least one run of m or more successive samples all within control limits, this constitutes *disqualification* for reduced inspection, and initial qualification again becomes necessary in accordance with procedures (i) and (ii) (M may have any suitable prescribed value, such as 4 or 5 times m).

Appropriate criteria for initial qualification for reduced inspection, disqualification, requalification, reinspection of the current lot, etc., must be worked out to fit individual cases.

4) After a major cause is eliminated, and also periodically thereafter, review the standards and revise them if necessary.

Procedure: Following the change in the process at sample number 45 (see the above paragraphs), the subsequent 25 samples indicate relatively good control, with only one $\overline{X}$ outside the limits (sample number 48). The cause of the low $\overline{X}$ at sample No. 48 was not found. New standard values of $\overline{X}_0$ and σ_0 were adopted, using data for samples number 46–70; the new control limits were established at sample number 71, and extended into the future.

---

* In some circumstances where only a maximum limit is specified and where the standard level $\overline{X}_0$ is located at a very safe distance from the maximum limit, it may be desirable to use some arbitrarily chosen upper limiting values of $\overline{X}$ and R, rather than their upper control limits, as criteria in determining whether additional inspection of the lot should be made. In other circumstances, it may be desirable to use both the upper and lower control limits (or other selected values) as criteria for further inspection of the lot, especially if both a maximum and a minimum limit are specified.

† In any specific problem, the size of the second sample depends upon what has been adopted as a suitable criterion for releasing a lot when lack of control is indicated. This criterion might be 0 nonconforming units in a sample of 15, 1 nonconforming unit in a sample of 25, 1 nonconforming unit in a sample of 40, or the like. The second sample should almost always be at least twice as large as the initial sample, and where a nonconforming unit is relatively serious from a service standpoint, much larger still.

Computations:

$$\bar{X}_0 = \bar{\bar{X}} = 60.5; \bar{R} = 57.5$$

$$\sigma_0 = 57.7/d_2 = 57.7/2.326 = 24.8$$

The 3-sigma control limits for $\bar{X}$ are

$$\bar{X}_0 \pm A\sigma_0 = 60.5 \pm 1.342 \times 24.8$$

$$= 93.8 \text{ and } 27.2$$

The 3-sigma control limits for R are

$$D_2\sigma_0 \text{ and } D_2\sigma = 122 \text{ and } 0$$

(For the constants, see Table 6.)

Following a period of fairly satisfactory control, a routine schedule of reviewing standards once a month was tentatively adopted.

5) Place the completed control charts in the permanent file, as a record of the quality of the product.

5.2 Example 2—Control Chart Using $\bar{X}$ Only—Process Control Problem

5.2.1 Outline. This example is an illustration of the control chart method applied to an internal process control problem. It differs from Example 1 in two respects. First, the only statistical measure of the quality characteristic plotted (see Fig. 7) is the average, $\bar{X}$, since prior experience indicated that lack of control was almost always associated with causes that affected the average level $\bar{X}$ of the measured quality characteristic rather than the dispersion or scattering of individual values, as measured by s or R (see 2.4.2).

Second, lack of control of the quality characteristic measured (percent fat content of a certain type of woolen yarn), or failure of the manufacturer to maintain it rigorously within the self-imposed limits does not result in specific failures in performance or immediately discernible reduction in serviceability. Through experience, however, the manufacturer knows that maintenance at the adopted standard level will result in the dyeing of clearer colors at lower cost and better serviceability even though the effect of this control is not noticeable to the untrained observer or to the average consumer.

5.2.2 Construction of the Control Chart. The standard level of $\bar{X}$ used in the control chart of Fig. 7 was determined from accumulated data for a prior period. The spread between the 3-sigma control limits was determined from the average range ($\bar{R}$) for sample groups of 5 observed values each, obtained during the prior period.

5.2.3 Comments. Fig. 7 shows an actual series of events for which the control chart record was not only a history but was a major impetus toward the correction of conditions that caused lack of control. The normal condition of control was thrown out of gear by isolated causes (see items 3, 4, and 7 in Fig. 7). Then followed a change in conditions (item 8 in Fig. 7) involving changes in wool types, increased oil application in processing the fibers, and changes in scouring detergents. A long period of experimentation followed, and finally, changes in oil application and in the equipment and process of scouring brought about a level and degree of control consistent with that shown at the beginning of the period covered by the figure.

5.3 Example 3—Control Chart Using Fraction Nonconforming p—Control of Spoilage, 100-Percent Inspection of a Product Consisting of Individual Units

5.3.1 Outline. This example illustrates an application of the control chart for fraction nonconforming p, for controlling the percentage of units rejected due to spoilage during manufacture. Control of spoilage is often a factor of broad interest, as in times of emergency when conservation of materials, as well as speed of production, is important.

The data reflected by Fig. 8 represent the results of 100 percent inspection, and each point on the chart indicates the percentage of units (parts) found nonconforming and rejected in a single day's output as a result of this inspection. A portion of the original data is shown in Table 4.

TABLE 4
Portion of Data Used in the Control Chart of Fig. 8

Date	Number of Parts Produced	Number of Parts Nonconforming	Percent Nonconforming
June			
2	1717	37	2.2
3	2160	50	2.3
4	2019	42	2.1
5	1236	21	1.7
6	1013	52	5.1
9	1710	77	4.5
10	2064	48	2.3
11	2225	45	2.0
12	1239	19	1.5
13	1772	42	2.4
16	1403	21	1.5
17	1943	55	2.8
18	1785	52	2.9
19	1716	50	2.9
20	1442	37	2.6

The product covered by this illustration is a component part of a certain type of electrical apparatus. These parts are made by several operators on automatic winding machines. They are inspected 100 percent for a number of characteristics, and the data show the percentage of parts that failed to conform to four specified electrical characteristics measurable on a test set. The production rate varied from time to time, depending on schedules, and averaged about 1500 per day in June, about 725 in July, about 900 in

CONTROL CHART RECORD

PRODUCT 2-ply Woolen Yarn

DEPT. Dyehouse

CHARACTERISTIC Cleanliness after Scouring

NORMAL DAILY OUTPUT 50,000 lb.

UNIT OF MEASUREMENT Percent Fat by Ether Extraction

SAMPLE SIZE, GROUPS OF 5

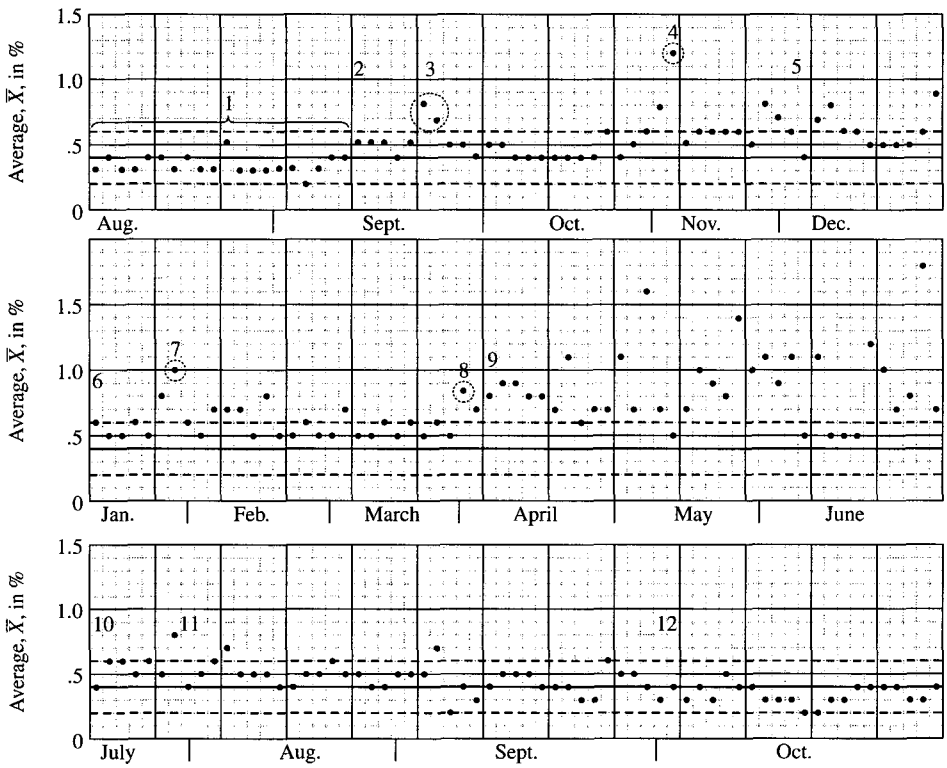
SPECIFIED LIMITS 0.1% Min. 1.0% Max.

POINTS BY Chem. Lab.

CONTROL LIMITS Based on Range

LIMITS BY Product Engineer

CHART FOR AVERAGES ($\bar{X}$)



1. Period of normal control.
2. Change in scouring — started use of new soap.
3. Operator failed to hold specified alkali reading.
4. Faulty electrode on alkali motor — replaced.
5. Speed of yarn through bowls reduced.
6. Change in scouring — started use of another soap.
7. Causes not determined.
8. Excessive oiling of yarn due to change in method. Not accepted as cause at first by production unit. Corrected finally in August.
9. Period of experimental changes to bring about control.
10. Started change in scouring epuipment — counterflow.
11. Further changes and adjustments in equipment.
12. Back in control — due to normal oiling and changes in methods.

Figure 7 — Control Chart Record of Quality of a Material — Illustrative Example 2.

August, and about 1200 in September and October. The number of parts submitted to the inspector as a lot as any one time ranged from 30 to 50; 20 to 40 lots were produced each day. Several different kinds of parts of the same general type are included. All were made under the same essential process conditions and, though the specified electrical limits applying to one kind of part differ from those specified for another, the inspection procedures were the same for all parts. Thus, while the total product covered by Fig. 8 does not comprise identical parts, a single overall system of causes, encompassing all production and inspection operations, is covered by this illustration.

Note that when 100 percent inspection is performed, the usual statistical procedure can be followed by considering each day's *output* as a sample of the production process. The number of units in a day's output is then treated as the sample size *(n)* for the day. The running record of the values of the fraction nonconforming observed day after day is taken to reflect the behavior of the production process.

A control chart for daily values of p will aid in disclosing either of two conditions:

1) The occasional presence of assignable causes in the manufacturing process, or possibly in the inspection
2) The inherent inability of the existing manufacturing process to turn out the quality of work demanded by the specification tolerances

An example of Condition (1) would be daily values of p that from time to time fall outside the control limits. The elimination of causes associated with the abnormally high values of p will result in reducing the average value of p (spoilage) and may thus serve not only to conserve materials but also to speed up production and to reduce the production cost per accepted unit. Condition (2) would be indicated if the daily values of p run consistently high, with practically all plotted points falling within control limits around an average value of p that is higher than can be tolerated by the user. This is a condition under which a 100 percent screening inspection is necessary. Furthermore, a quality improvement effort to remove the causes of nonconformity should be undertaken.

5.3.2 Construction of the Control Chart. The data charted in Fig. 8 show the results, plotted in percent nonconforming ($100p$), for the latter part of the control program.

A few months before the beginning of the chart, the percent nonconforming was greater than 4. From data available just prior to the first of June, a standard value of 2.65 percent ($P_0 = 0.0265$) was adopted for the purpose of starting the control chart of Fig. 8. This value of 2.65 percent was retained through June, July, and August, and the 3-sigma control limits for each month were computed from the formula

$$p_0 \pm 3\sqrt{p_0\left(1-p_0\right)/n}$$

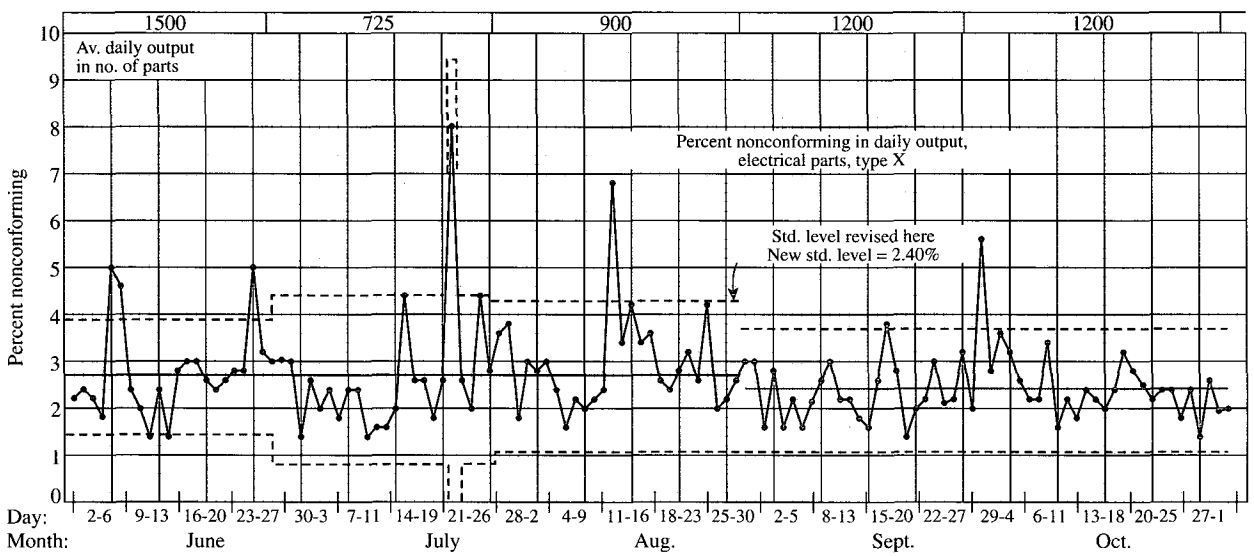


Figure 8 — Control of Spoilage for a Product Consisting of Individual Units — Illustrative Example 3.

p_0 being 0.0265, and n the estimated average daily output for that month, as provided by advance production estimates. These values are shown in the table below.

3-Sigma Control Limits

Month	Average Daily Output	Control Limits
June	1500	1.41 and 3.89%
July	725	0.86 and 4.44%
August	900	1.04 and 4.26%

Plotting control limits in advance on the basis of an estimated average daily variation for this job had not been excessive. However, see the comment below concerning the point on July 26.

Based on the data for June, July, and August, but with the omission of points that were outside control limits during this period for known reasons, a new standard level of 2.40 percent was adopted at the beginning of September. The control limits were based on an average daily output value of 1200 parts (actual average output: September, 1208; October, 1207).

5.3.3 Comments. The control chart history shown in Fig. 8 shows a reduction in the frequency of indications of lack of control as time goes on. Some were the result of faulty workmanship, as when a new operator limited experience was added to the working force.

Note that the control limits for July 26 have been specially constructed. For this particular day the output was only 50 parts. The plotted value of percent nonconforming does not fall outside the control limits computed for an output of 50 parts. This is an instance where a sample size differs so greatly from the average sample size that the control limits must be specially constructed for this day to avoid a false indication of lack of control.

For the two months of September and October, the average percent nonconforming was about 2.4. This reflects a reduction of more than 40 percent in the spoilage relative to that experienced at the outset of the control program (over 4 percent). When a high degree of control is exhibited so that practically no points fall outside of limits, one may be assured that the process is stabilized. Then the only way of reducing the spoilage is to make a fundamental change in the production process or, if acceptable to the interested parties, a change in the specification requirements for the features being inspected (see 3.5.2). This state of affairs has almost but not quite been reached during the last two months shown in Fig. 8, September and October. A control chart analysis of the data for this period, made in accordance with ANSI/ASQC B2-1996, showed that only 2 out of the 52 points for this period fall outside of the control limits.

5.4 Example 4—Control Chart Using $\bar{X}$ and R—Control of Spoilage for a Bulk Product

5.4.1 Outline. This example is an extension of the principles of Example 3, and is intended to illustrate how the control chart technique may be applied to control the percent nonconforming (spoiled, out-of-specification, substandard, etc.) for a bulk product, or material that does not consist of individual units that can be used for counting the proportion of good and bad. For example, products whose output can be expressed only in terms of length, area, volume, weight, etc. Examples are textiles in yards or pounds, coal in tons, steel in tons, paint in gallons, etc., for which, as a result of continuing mass production in a steady flow or in large batches, individual natural articles or unit quantities cannot be separately classified as conforming or nonconforming to specified requirements. Essentially the problem is one of classification.

In Example 3 the fraction nonconforming (spoilage) was measured by the ratio of the number of nonconforming units to the total number n of units in a subgroup, and there was a sample size n that could be used in the formula

$$p_0 \pm 3\sqrt{p_0\left(1-p_0\right)/n}$$

for the control limits (above). But in the present example, it is assumed that there are no natural units for counting the sample size, hence this formula does not apply. While the method here outlined could be used for Example 3, it would be relatively much less efficient for detecting assignable causes and is not to be recommended as a substitute for the method of Example 3, even for bulk products, when a suitable basis of counting (and sorting good units from bad) is available.

The procedure here given is suggested where successive increments of product (yards per day, tons per shift, etc.) are reasonably alike in quantity or when it is possible to break down the output into increments that do not vary too greatly in quantity. Variations through a range of 2 to 1 or even 3 to 1 are usually not too large in practice.

The data represented by the control chart of Fig. 9 indicate for a certain type of pile floor covering the percentage of total daily output in yards that is segregated out because it is of a grade that is poorer than standard. Due to nonconformities in materials, workmanship, finish appearance, etc., some products may be reprocessed, or classified as of lower grade, rather than scrapped. The floor coverings under consideration here are not of such a character that each foot or yard may be considered as a separate unit of product subject to independent acceptance and rejection. Hence, the data consists of two simple quantities—total

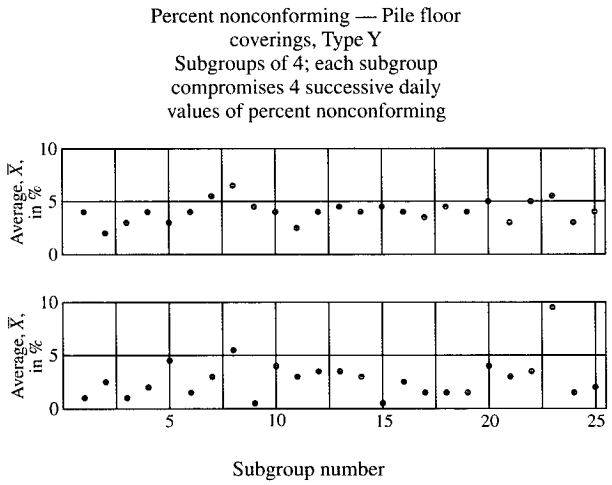


Figure 9 — Control of Spoilage for a Bulk Product — Illustrative Example 4.

daily production in number of yards and number of nonconforming yards, as is illustrated in Table 5.

5.4.2 Procedure. Treat the value of "percent nonconforming" *(X)* as the measured characteristic to be controlled, and take four successive values of percent nonconforming (values of X for four successive days) as a subgroup. Treat each subgroup of four in the manner outlined in Example 1, finding the average value $\bar{X}$ and the range R of the four values. Construct control charts for $\bar{X}$ and R.

Subgroups of five or six, representing, say, the working days of a complete week, could equally well be used, but groups of four are recommended in order to get indications of lack of control with a minimum of delay. In this case, with the relatively large quantities of product, had the data of Table 5 been made available for shorter periods, such as a two-hour period, a half day, etc., the efficiency of catching trouble might have been improved by shortening the time interval for successive subgroups. In individual instances engineering judgment, possibly supplemented by trial, should be exercised in the choice of the most advantageous quantity of output for which a value of percent nonconforming should be determined.

5.4.3 Comments. The control chart of Fig. 9 shows the actual data for a series of 100 successive production days (25 subgroups of four days each). Previous history had let to the adoption of tentative standard values of $\bar{X}_0$ = 3.98 percent and σ_0 = 1.33 percent, whence $\bar{R} = d_2\sigma_0$ = 2.75 percent as shown on the chart. The chart shows a period during which the overall manufacturing process had become pretty well controlled, following action during prior periods in which a number of causes of abnormally high values of percent of nonconforming product were eliminated.

Two instances of lack of control are indicated: the first shown by a large $\bar{X}$ for sample number 8; and the second shown by a large R for sample number 23. The high $\bar{X}$ for sample number 8 was attributable to shutting down looms because of shortage of raw materials, which caused an abnormal proportion of "run-out" nonconformities. The high value of R for sample number 23 indicates a wide variation in the percent nonconforming for the four days covered in this subgroup. A wide range (high R) indicates some abnormality of procedure, such as accumulating the nonconformities for repair, and then sending them through all at once.

Thus the control chart record provides information whereby the control engineer can appraise the fluctuation in percent nonconforming as time goes on, and enables the engineer to decide at any time whether there are causes present that can be found, or whether there is such uniformity that the only way to reduce the percent nonconforming is to make a fundamental change in the overall manufacturing process.

TABLE 5
Portion of Data Used in the Control Chart of Fig. 9
Pile Floor Coverings, Type Y; Nonconforming Product

Subgroup Number	Date	Production in Yards	Non-conforming Yards	Percent Non-conforming
1	Jan. 2	25,366	798	3.1
	3	28,176	1184	4.2
	6	33,335	1099	3.3
	7	23,904	985	4.1
2	8	33,343	734	2.2
	9	42,154	1342	3.2
	10	42,459	511	1.2
	13	31,849	623	2.0
3	14	33,888	767	2.3
	15	28,196	890	3.2
	16	38,377	1281	3.3
	17	36,505	923	2.5
4	20	38,252	2123	5.6
	21	44,469	1658	3.7
	22	35,863	1568	4.4
	23	34,830	1373	3.9

TABLE 6
Factors for Computing Control Chart Lines

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Standard Deviations: Factors for Central Line c_4	$1/c_4$	Factors for Control Limits B_3	B_4	B_5	B_6	Chart for Ranges: Factors for Central Line d_2	$1/d_2$	Factors for Control Limits d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8862	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5908	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.079	0	2.004
7	1.134	0.419	1.182	0.9594	1.0424	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.205	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.307	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.686	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0253	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0230	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.923	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.740	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.609
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.733	1.489	5.889	0.404	1.596
20	0.671	0.180	0.680	0.9869	1.0132	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.606	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0120	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.660	5.979	0.435	1.565
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.711	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.032	0.452	1.548
25	0.600	0.135	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.805	6.056	0.459	1.541

The above table is a copy of Table 27 (pages 134–135) from ASTM STP15D and is reprinted with permission, from the ASTM Manual on Presentation of Data and Control Chart Analysis (ASTM STP15D), MNL 7 copyright 1990, American Society for Testing and Materials, Philadelphia, Pennsylvania.

NOTES:

For $n > 25$

$$A = \frac{3}{\sqrt{n}}, \quad A_3 = \frac{3}{c_4\sqrt{n}}, \quad c_4 \cong \frac{4(n-1)}{4n-3}, \quad B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}}, \quad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}}, \quad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$

Formulas

Purpose of Chart	*Chart for*	*Central Line*	*3-Sigma Control Limits*
For analyzing past inspection data for control ($\overline{X}$, $\bar{s}$, $\overline{R}$ are average values for the data being analyzed)	Averages	$\overline{\overline{X}}$	$\overline{\overline{X}} \pm A_3\bar{s}$ or $\overline{\overline{X}} \pm A_2\overline{R}$
	Standard deviations	$\bar{s}$	$B_3\bar{s}$ and $B_4\bar{s}$
	Ranges	$\overline{R}$	$D_3\overline{R}$ and $D_4\overline{R}$
For controlling quality during production ($\overline{X}_0$, σ_0, R_0, are selected standard values; $R_0 = d_2\sigma_0$ for samples of size n)	Averages	$\overline{X}_0$	$\overline{X}_0 \pm A\sigma_0$ or $\overline{X}_0 \pm A_2R_0$
	Standard deviations	s_0 or $c_4\sigma_0$	$B_5\sigma_0$ and $B_6\sigma_0$
	Ranges	R_0 or $d_2\sigma_0$	$D_1\sigma_0$ and $D_2\sigma_0$

6. Glossary of Symbols and Terms*

X— The observed value of a quality characteristic; specific observed values are designated X_1, X_2, X_3, etc.

$\overline{X}$ *(X bar)*— The average, or arithmetic mean; the average of a set of n observed values is the sum of the observed values divided by n.

$$\overline{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

n— The number of observed values; the sample size, or number of units (articles, parts, specimens, etc.) in the sample.

σ— The *universe standard deviation,* which is defined as the root mean square (rms) deviation of the universe values from their mean.

s— The *sample standard deviation,* which is an estimate of the universe standard deviation, σ.

$$s = \sqrt{\frac{\Sigma\left(X_1 - \overline{X}\right)^2}{n-1}}$$

or expressed in a form convenient for computation purposes

$$s = \sqrt{\frac{\Sigma X_2\left[\left(\Sigma X\right)^2 / n\right]}{n-1}}$$

where Σ in both formulas means the sum of all values from $i = 1$ to $i = n$.

R— The *range*, the difference between the largest observed value and the smallest observed value.

p— The *fraction nonconforming,* the ratio of the number of nonconforming units (articles, parts, specimens, etc.) to the total number of units under consideration; sometimes referred to as the "proportion nonconforming." The use of *percent nonconforming* ($100\,p$) is often preferred.

pn— The *number of nonconformities* (nonconforming units) in a sample of n units.

c— The *number of nonconformities in a sample;* almost invariably the number of nonconformities in a sample of constant size (for example, one square foot, 100 feet). A nonconforming unit may have more than one nonconformity.

$\overline{\overline{X}}$, $\bar{s}$, $\overline{R}$, $\bar{p}$, $\bar{p}n$, $\bar{c}$— The average of the set of sample values of $\overline{X}$, s, R, p, pn, c, under consideration.

$\overline{X}_0$, s_0, p_0, p_0n, c_0— The *standard value* of $\overline{X}$, s, p, pn, c; in quality control work, a value adopted at any time for computing the control limits to be used as criteria for action in the immediate future, or until additional evidence indicates need for revision.

c_4— A factor, varying with sample size n, that is equal to the ratio of the expected value of s for samples of size n to the standard deviation (σ) of the universe sampled; c_4 = expected value of s/σ. (Values of c_4 are given in Table 6, based on a normal distribution.)

---

* See ANSI/ISO/ASQC A3534-2-1993, *Statistics—Vocabulary and Symbols—Statistical Quality Control* for a complete listing.

d_2— A factor, varying with sample size n, that is equal to the ratio of the expected value of R for samples of size n to the σ_0 of the universe sampled; d_2 = expected value of $(R)/\sigma$. (Values of d_2 are given in Table 6, based on a normal distribution).

$A, A_2, A_2, D_1, D_2, D_3, D_4$— Factors for computing control limits (see Table 6).

Unit— One of a number of similar articles, parts, specimens, etc.

Sample— A group of units, or a portion of material, taken from a larger collection of units, or quantity of material, which serves to provide information that can be used as a basis for judging the quality of a large quantity, or as a basis for action on the larger quantity or on the production process. Also used in the sense of a "sample of observations."

Lot, Batch:— A specific quantity of similar material or collection of similar units from a common source. In inspection work, the quantity offered for inspection and acceptance at any one time. It may be a collection of raw material, parts, or subassemblies, or a consignment of finished product to be sent out for service.

Nonconformity— A failure to meet a requirement imposed on a unit with respect to a single quality characteristic; also an irregularity in material, surface, finish, etc. Requirements often include so-called "accepted standards of good workmanship" as well as specifically stated limitations.

Nonconforming Unit— A unit containing one or more nonconformities with respect to the quality characteristic(s) under consideration.

Assignable Cause— A factor contributing to the variation in quality that it is economically feasible to identify. Assignable causes must be identified and removed to attain statistical control.

Subgroup— One of a series of groups of observation obtained by subdividing a large group of observations. Alternatively, the data obtained from one of a series of samples taken from one or more universes. One of the essential features of the control chart method is to break up the inspection data into *rational subgroups*, that is, to classify the observed values into subgroups, *within* which variations may for engineering reasons be considered to be due to nonassignable chance causes only, but *between* which there may be differences due to assignable causes whose presence is considered possible.

Appendixes

These Appendices are not a part of American National Standard ANSI/ASQC B1-1996, *Guide for Quality Control Charts,* nor of American National Standard ANSI/ASQC B2-1996, *Control Chart Method of Analyzing Data,* nor of ANSI/ASQC B3-1996, *Control Chart Method of Controlling Quality During Production,* but are included for information purposes only.

APPENDIX A

Factors for Computing Probability Control Limits

(For the significance of probability limits, see footnote to 3.2.)

Table A1
Factors to Be Used with the Standard Value, σ_0

Number of Observations in Sample	Chart for Averages		Chart for Standard Deviations					Number of Observations in Sample
	A		Factor for Central Line	B_L		B_U		
	Probability Level			Probability Level		Probability Level		
n	0.001	0.005	c_2	0.001	0.005	0.995	0.999	n
2	2.185	1.821	0.5642	0.001	0.004	1.985	2.327	2
3	1.784	1.487	0.7236	0.026	0.058	1.879	2.146	3
4	1.545	1.288	0.7979	0.078	0.134	1.792	2.017	4
5	1.382	1.152	0.8407	0.135	0.203	1.724	1.922	5
6	1.262	1.052	0.8686	0.187	0.262	1.671	1.849	6
7	1.168	0.974	0.8882	0.233	0.311	1.628	1.791	7
8	1.092	0.911	0.9027	0.274	0.352	1.592	1.744	8
9	1.030	0.859	0.9139	0.309	0.386	1.562	1.704	9
10	0.977	0.815	0.9227	0.339	0.417	1.536	1.670	10
11	0.932	0.777	0.9300	0.367	0.443	1.513	1.640	11
12	0.892	0.744	0.9359	0.391	0.466	1.493	1.614	12
13	0.857	0.714	0.9410	0.413	0.486	1.475	1.591	13
14	0.826	0.688	0.9453	0.432	0.505	1.459	1.570	14
15	0.798	0.665	0.9490	0.450	0.521	1.445	1.552	15

Table A2
Factors to Be Used with the Standard Value R_0

Number of Observations in Sample	Chart for Averages		Chart for Ranges			Number of Observations in Sample
	A_2			D_L	D_U	
	Probability Level		d_2	Probability Level	Probability Level	
n	0.001	0.005		0.005	0.995	n
2	1.937	1.614	1.128	0.009	3.518	2
3	1.054	0.879	1.693	0.100	2.576	3
4	0.751	0.626	2.059	0.185	2.259	4
5	0.594	0.495	2.326	0.254	2.085	5
6	0.498	0.415	2.534	0.308	1.973	6
7	0.432	0.360	2.704	0.351	1.897	7
8	0.384	0.320	2.847	0.386	1.837	8
9	0.347	0.289	2.970	0.415	1.791	9
10	0.318	0.265	3.078	0.441	1.755	10
11	0.294	0.245	3.173	0.463	1.724	11
12	0.274	0.228	3.258	0.482	1.698	12
13	0.257	0.214	3.336	0.498	1.676	13
14	0.242	0.202	3.407	0.511	1.656	14
15	0.230	0.192	3.472	0.524	1.639	15

Formulas — Charts for Controlling Quality During Production

Chart for	Central Line	Control Limits
Averages, using σ_0	$\overline{X}_0$	$\overline{X}_0 \pm A\sigma_0$
Averages, using R_0	$\overline{X}_0$	$\overline{X}_0 \pm A_2R_0$
Standard deviations	$c_4\sigma_0$	$B_3\sigma_0$ and $B_6\sigma_0$
Ranges	R_0	D_1R_0 and D_2R_0

Explanations

$\overline{X}_0$ is the selected standard value for the mean.
σ_0 is the selected standard value for the standard deviation.
R_0 is the selected standard value for the range of a sample size n.
$R_0 = d_2\sigma_0$
B_5 and D_1 are factors for the lower limits.
B_6 and D_2 are factors for the upper limits.

APPENDIX B

References

1) SHEWHART, WALTER A. *Economic Control of Quality of Manufactured Product.* D. Van Nostrand Company., Inc. New York, 1931, available through American Society for Quality Control, Milwaukee, Wisconsin

2) SHEWHART, WALTER A. *Statistical Method from the Viewpoint of Quality Control.* The Graduate School, U.S. Department of Agriculture, Washington, D.C. 1939

3) SIMON, LESLIE E. *An Engineer's Manual of Statistical Methods.* John Wiley and Sons, Inc., New York, 1941

4) DUDDING, B.P. AND JENNETT, W.J. *Quality Control Charts.* British Standard 600R, British Standards Institution, London, 1942

5) *ASTM Manual on Presentation of Data and Control Chart Analysis.* The American Society for Testing and Materials, Philadelphia, STP15D, 1990

6) GRANT, EUGENE I. and RICHARD S. LEAVENWORTH *Statistical Quality Control,* Fifth edition. McGraw-Hill Book Company, Inc. New York, 1980

7) BURR, IRVING W. *Statistical Quality Control Methods*, Marcel Dekker, Inc., New York, New York, 1976

8) DUNCAN, ACHESON J. *Quality Control and Industrial Statistics*, Fourth edition. Richard D. Irwin, Inc. Chicago, 1974

9) OTT, ELLIS R. and E. G. SCHILLING. *Process Quality Control Troubleshooting and Interpretation of Data*, Second Edition. McGraw-Hill Book Company, Inc., New York, 1975

10) American National Standard, ANSI/ASQC A1-1978, *Definitions, Symbols, Formulas and Tables for Control Charts,* American Society for Quality Control, Milwaukee, Wisconsin

11) Statistical Quality Control Handbook, Western Electric Co. Second Edition, Sixth Printing 1982 by The Mack Printing Co. Easton, Pa.

12) *Glossary and Tables for Statistical Quality Control,* Third Edition, 1996, ASQC, Milwaukee, Wisconsin

13) DEMING, W. EDWARDS *Quality, Productivity, and Competitive Position*, Massachusetts Institute of Technology, 1982

American Society for Quality

Quality Press
611 East Wisconsin Avenue
Milwaukee, Wisconsin 53201-3005
800-248-1946
Web site http://www.asq.org

T59
Printed in the United States of America