INDR 220: Introduction to Computing for Operations Research
Homework 2: The Coin Distribution Problem
Deadline: December 5, 2024, 11:59 PM

In this homework, you will implement a Python script that solves the coin distribution problem using CPLEX. A parent is trying to evenly distribute N coins to his/her M children. The monetary values of these N coins are represented by $c_1, c_2, \ldots, c_N$. Assuming that these coins can be divided into M equally valued partitions (i.e., each child gets the same amount), the decision variables are

$$x_{ij} = \begin{cases} 1 & \text{if coin } i \text{ is assigned to child } j, \\ 0 & \text{otherwise.} \end{cases}$$

The integer linear programming formulation of this problem becomes

$$\begin{aligned} \text{maximize} \quad & z = \sum_{i=1}^{N} \sum_{j=1}^{M} x_{ij} \\ \text{subject to:} \quad & \sum_{i=1}^{N} c_i x_{ij} = \frac{1}{M} \sum_{i=1}^{N} c_i \quad j = 1, 2, \ldots, M \\ & \sum_{j=1}^{M} x_{ij} = 1 \quad i = 1, 2, \ldots, N \\ & x_{ij} \in \{0, 1\} \quad i = 1, 2, \ldots, N; \;\; j = 1, 2, \ldots, M. \end{aligned}$$

This problem will be represented using a `.txt` file, namely, `coins.txt`. This file contains the monetary values (i.e., c_i) in a single row, and it is composed of the following line for an example problem:

```
coins.txt
----------
10 5 25 10
```

The example problem with four coins and two children can be formulated as

$$\begin{aligned} \text{maximize} \quad & z = x_{11} + x_{12} + x_{21} + x_{22} + x_{31} + x_{32} + x_{41} + x_{42} \\ \text{subject to:} \quad & 10x_{11} + 5x_{21} + 25x_{31} + 10x_{41} = 25 \\ & 10x_{12} + 5x_{22} + 25x_{32} + 10x_{42} = 25 \\ & x_{11} + x_{12} = 1 \\ & x_{21} + x_{22} = 1 \\ & x_{31} + x_{32} = 1 \\ & x_{41} + x_{42} = 1 \\ & x_{11} \in \{0, 1\} \quad x_{12} \in \{0, 1\} \\ & x_{21} \in \{0, 1\} \quad x_{22} \in \{0, 1\} \\ & x_{31} \in \{0, 1\} \quad x_{32} \in \{0, 1\} \\ & x_{41} \in \{0, 1\} \quad x_{42} \in \{0, 1\}. \end{aligned}$$

An optimum solution of the example problem is as follows:

$$x^\star_{11} = 1 \qquad x^\star_{12} = 0$$
$$x^\star_{21} = 1 \qquad x^\star_{22} = 0$$
$$x^\star_{31} = 0 \qquad x^\star_{32} = 1$$
$$x^\star_{41} = 1 \qquad x^\star_{42} = 0.$$

Implement your algorithm to solve the coin distribution problem in a single interactive Python notebook using Azure Lab Services. Your notebook should include at least the following function definition that takes the file path of the input file and the number of children as parameters and returns the solution found.

```
def coin_distribution_problem(coins_file, M):
    # your implementation starts below

    # your implementation ends above
    return(X_star)
```

What to submit: You are provided with a template file named as `0099999_hw2.ipynb`, where `99999` should be replaced with your 5-digit student number. You are allowed to change the template file between the following lines.

```
    # your implementation starts below

    # your implementation ends above
```

You need to submit your source code in a single file (`0099999_hw2.py` file that you will download from Azure Lab Services by following "File" / "Save and Export Notebook As..." / "Executable Script" menu items).

How to submit: Submit the file you edited to LearnHub by following the exact style mentioned. Submissions that do not follow these guidelines will not be graded.

Late submission policy: Late submissions will not be graded.

Cheating policy: Very similar submissions will not be graded.
