INDR 220: Introduction to Computing for Operations Research
Homework 3: The Cell Tower Coverage Problem
Deadline: December 13, 2024, 11:59 PM

In this homework, you will implement a Python script that solves the cell tower coverage problem using CPLEX. A telecom company needs to build a set of cell towers to provide signal coverage for the inhabitants of a given city. $H \times W$ potential locations where the towers could be built have been identified. The towers have a fixed range (it covers its own region and its neighboring regions, namely, west, north, east, and south), and only a limited number of them can be built due to budget constraints. Given these restrictions, the company wishes to provide coverage to the largest percentage of the population possible. To simplify the problem, the company has split the area it wishes to cover into $H \times W$ regions, each of which has a known population. The goal is then to choose which of the potential locations the company should build cell towers on to provide coverage to as many people as possible.

The decision variables are

$$x_{ij} = \begin{cases} 1 & \text{if tower is built in region } (i,j), \\ 0 & \text{otherwise,} \end{cases}$$

$$y_{ij} = \begin{cases} 1 & \text{if region } (i,j) \text{ is covered by at least one tower,} \\ 0 & \text{otherwise,} \end{cases}$$

and the given information includes

$$\begin{aligned} H &= \text{height of city} \\ W &= \text{width of city} \\ T &= \text{maximum number of towers} \\ p_{ij} &= \text{population in region } (i,j). \end{aligned}$$

The integer linear programming formulation of this problem becomes

$$\begin{aligned} \text{maximize} \quad & z = \sum_{i=1}^{H} \sum_{j=1}^{W} p_{ij} y_{ij} \\ \text{subject to:} \quad & x_{ij} + \sum_{(k,l) \in \text{neighbors}(i,j)} x_{kl} \geq y_{ij} \qquad i = 1, 2, \ldots, H; \;\; j = 1, 2, \ldots, W \\ & \sum_{i=1}^{H} \sum_{j=1}^{W} x_{ij} \leq T \\ & x_{ij} \in \{0, 1\} \qquad i = 1, 2, \ldots, H; \;\; j = 1, 2, \ldots, W \\ & y_{ij} \in \{0, 1\} \qquad i = 1, 2, \ldots, H; \;\; j = 1, 2, \ldots, W. \end{aligned}$$

This problem will be represented using a `.txt` file, namely, `populations.txt`. This file contains the populations (i.e., p_{ij}) in H rows and W columns, and it is composed of the following lines for an example problem:

```
populations.txt
-----------------------
732 539 949 508 806 881
```

```
646 757 257 630 994 547
105 859 876 589 615 345
136 370 433 419 631 485
```

The example problem with a 4×6 region and maximum 3 towers can be formulated as

$$\begin{aligned}
\text{maximize} \quad & z = 732y_{11} + 539y_{12} + 949y_{13} + 508y_{14} + 806y_{15} + 881y_{16} + 646y_{21} + 757y_{22} + \\
& \quad 257y_{23} + 630y_{24} + 994y_{25} + 547y_{26} + 105y_{31} + 859y_{32} + 876y_{33} + 589y_{34} + \\
& \quad 615y_{35} + 345y_{36} + 136y_{41} + 370y_{42} + 433y_{43} + 419y_{44} + 631y_{45} + 485y_{46} \\
\text{subject to:} \quad & x_{11} + x_{12} + x_{21} - y_{11} \geq 0 \\
& x_{11} + x_{12} + x_{13} + x_{22} - y_{12} \geq 0 \\
& x_{12} + x_{13} + x_{14} + x_{23} - y_{13} \geq 0 \\
& x_{13} + x_{14} + x_{15} + x_{24} - y_{14} \geq 0 \\
& x_{14} + x_{15} + x_{16} + x_{25} - y_{15} \geq 0 \\
& x_{15} + x_{16} + x_{26} - y_{16} \geq 0 \\
& x_{11} + x_{21} + x_{22} + x_{31} - y_{21} \geq 0 \\
& x_{12} + x_{21} + x_{22} + x_{23} + x_{32} - y_{22} \geq 0 \\
& x_{13} + x_{22} + x_{23} + x_{24} + x_{33} - y_{23} \geq 0 \\
& x_{14} + x_{23} + x_{24} + x_{25} + x_{34} - y_{24} \geq 0 \\
& x_{15} + x_{24} + x_{25} + x_{26} + x_{35} - y_{25} \geq 0 \\
& x_{16} + x_{25} + x_{26} + x_{36} - y_{26} \geq 0 \\
& x_{21} + x_{31} + x_{32} + x_{41} - y_{31} \geq 0 \\
& x_{22} + x_{31} + x_{32} + x_{33} + x_{42} - y_{32} \geq 0 \\
& x_{23} + x_{32} + x_{33} + x_{34} + x_{43} - y_{33} \geq 0 \\
& x_{24} + x_{33} + x_{34} + x_{35} + x_{44} - y_{34} \geq 0 \\
& x_{25} + x_{34} + x_{35} + x_{36} + x_{45} - y_{35} \geq 0 \\
& x_{26} + x_{35} + x_{36} + x_{46} - y_{36} \geq 0 \\
& x_{31} + x_{41} + x_{42} - y_{41} \geq 0 \\
& x_{32} + x_{41} + x_{42} + x_{43} - y_{42} \geq 0 \\
& x_{33} + x_{42} + x_{43} + x_{44} - y_{43} \geq 0 \\
& x_{34} + x_{43} + x_{44} + x_{45} - y_{44} \geq 0 \\
& x_{35} + x_{44} + x_{45} + x_{46} - y_{45} \geq 0 \\
& x_{36} + x_{45} + x_{46} - y_{46} \geq 0 \\
& x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} + x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} + \\
& \quad x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} + x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} \leq 3 \\
& x_{11} \in \{0,1\} \;\; x_{12} \in \{0,1\} \;\; x_{13} \in \{0,1\} \;\; x_{14} \in \{0,1\} \;\; x_{15} \in \{0,1\} \;\; x_{16} \in \{0,1\} \\
& x_{21} \in \{0,1\} \;\; x_{22} \in \{0,1\} \;\; x_{23} \in \{0,1\} \;\; x_{24} \in \{0,1\} \;\; x_{25} \in \{0,1\} \;\; x_{26} \in \{0,1\} \\
& x_{31} \in \{0,1\} \;\; x_{32} \in \{0,1\} \;\; x_{33} \in \{0,1\} \;\; x_{34} \in \{0,1\} \;\; x_{35} \in \{0,1\} \;\; x_{36} \in \{0,1\} \\
& x_{41} \in \{0,1\} \;\; x_{42} \in \{0,1\} \;\; x_{43} \in \{0,1\} \;\; x_{44} \in \{0,1\} \;\; x_{45} \in \{0,1\} \;\; x_{46} \in \{0,1\} \\
& y_{11} \in \{0,1\} \;\; y_{12} \in \{0,1\} \;\; y_{13} \in \{0,1\} \;\; y_{14} \in \{0,1\} \;\; y_{15} \in \{0,1\} \;\; y_{16} \in \{0,1\} \\
& y_{21} \in \{0,1\} \;\; y_{22} \in \{0,1\} \;\; y_{23} \in \{0,1\} \;\; y_{24} \in \{0,1\} \;\; y_{25} \in \{0,1\} \;\; y_{26} \in \{0,1\} \\
& y_{31} \in \{0,1\} \;\; y_{32} \in \{0,1\} \;\; y_{33} \in \{0,1\} \;\; y_{34} \in \{0,1\} \;\; y_{35} \in \{0,1\} \;\; y_{36} \in \{0,1\} \\
& y_{41} \in \{0,1\} \;\; y_{42} \in \{0,1\} \;\; y_{43} \in \{0,1\} \;\; y_{44} \in \{0,1\} \;\; y_{45} \in \{0,1\} \;\; y_{46} \in \{0,1\}.
\end{aligned}$$

The optimum solution of the example problem is as follows:

$$\begin{array}{llllll}
x_{11}^{\star}=0 & x_{12}^{\star}=1 & x_{13}^{\star}=0 & x_{14}^{\star}=0 & x_{15}^{\star}=0 & x_{16}^{\star}=0 \\
x_{21}^{\star}=0 & x_{22}^{\star}=0 & x_{23}^{\star}=0 & x_{24}^{\star}=0 & x_{25}^{\star}=1 & x_{26}^{\star}=0 \\
x_{31}^{\star}=0 & x_{32}^{\star}=0 & x_{33}^{\star}=1 & x_{34}^{\star}=0 & x_{35}^{\star}=0 & x_{36}^{\star}=0 \\
x_{41}^{\star}=0 & x_{42}^{\star}=0 & x_{43}^{\star}=0 & x_{44}^{\star}=0 & x_{45}^{\star}=0 & x_{46}^{\star}=0
\end{array}$$

Implement your algorithm to solve the cell tower coverage problem in a single interactive Python notebook using Azure Lab Services. Your notebook should include at least the following function definition that takes the file path of the input file and the maximum number of towers as parameters and returns the solution found.

```
def cell_tower_coverage_problem(populations_file, T):
    # your implementation starts below

    # your implementation ends above
    return(X_star)
```

What to submit: You are provided with a template file named as `0099999_hw3.ipynb`, where `99999` should be replaced with your 5-digit student number. You are allowed to change the template file between the following lines.

```
    # your implementation starts below

    # your implementation ends above
```

You need to submit your source code in a single file (`0099999_hw3.py` file that you will download from Azure Lab Services by following "File" / "Save and Export Notebook As..." / "Executable Script" menu items).

How to submit: Submit the file you edited to LearnHub by following the exact style mentioned. Submissions that do not follow these guidelines will not be graded.

Late submission policy: Late submissions will not be graded.

Cheating policy: Very similar submissions will not be graded.
