

1 Deterministic existance of edges

Given a set of vertices V which have two attributes, A and B , we will have an edge between vertices $u, v \in V$ having opposing advantages in A and B . That is $uv \in E$ the set of edges $\iff u_A > v_A$ and $u_B < v_B$ or $u_A < v_A$ and $u_B > v_B$. Suppose the attribute values are independently selected from a distribution of the integers from 1 to k inclusive. Then the probability of an edge forming is $2 * (\frac{1 - \Pr[u_A = v_A]}{2} * \frac{1 - \Pr[u_B = v_B]}{2}) = \frac{(1 - \frac{1}{k})^2}{2}$. In the case where $k = 10$ this probability is 0.405.

This gives rise to an Erdos-Renyi graph with edge probability $\frac{(1 - \frac{1}{k})^2}{2}$. The sharp boundary for connectivity in an Erdos-Renyi is an edge probability of $\frac{\ln n}{n}$ where $n = |V|$. Thus for a given k we must select an n s.t. $\frac{(1 - \frac{1}{k})^2}{2} > \frac{\ln n}{n}$. Because $\frac{\ln n}{n}$ is monotonically decreasing for all $n > 1$ and $n = 1, 2$ fulfill this criteria all values of n will meet the Erdos-Renyi threshold for a connected graph. However because there is a $\frac{2}{k^2}$ probability of a vertex having either both the highest or the lowest attribute values in expectation there will be $\frac{2n}{k^2}$ vertices that have no neighbors and so depending on the value of n the graph may have a high probability of being disconnected.

When instead of k options we select a real number from 0 to 1 we instead have an edge probability of $\frac{1}{2}$. Similarly any value of n will give a connected component. Here the probability that a vertex has the max or min attribute values is $2 * (\frac{1}{2})^{n-1}$

By simulation this tends to give shorter average path lengths than a Kleinberg grid which is expected given the known expected path lengths in Erdos-Renyi and Kleinberg grid graphs.

2 Probabilistic existance of edges

Consider a formulation where the attributes determine the probability of an edge existing. We will introduce a new parameter C with defines the initial edge probability. Thus we start with an Erdos-Renyi graph having edge probability C .

Next as before we discard all edges where the incident vertices do not have attribute differentials. Then we keep edges with probability $-1 * (u_A - v_A) * (u_B - v_B)$.