Week 6 Exercise Sheet

The following exercises have different levels of difficulty indicated by (*), (**), (***). An exercise with (*) is a simple exercise requiring less time or effort to solve compared to an exercise with (***), which is a more complex exercise.

1 Logistic Regression

1. (**) Derive π from $\log(\frac{\pi}{1-\pi} = \mathbf{w}^T\mathbf{x}$, i.e. derive the logistic sigmoid function from the logit function.

2. (*) In a binary (two-class) logistic regression model, the weight vector $\mathbf{w} = [4, -2, 5, -3, 11, 9]$. We apply it to some object that we'd like to classify; the vectorised feature representation of this object is $\mathbf{x} = [6, 8, 2, 7, -3, 5]$. What is the probability, according to the model, that this instance belongs to the positive class (i.e y=1)?

3. (***) Consider flipping a coin 20 times and recording each result $y_i = \{0, 1\}$. Using the log likelihood $l(\pi; \mathbf{y})$ derive the maximum likelihood estimation (MLE) for π by finding the derivative w.r.t π and setting this equal to zero ($\partial l(\pi; \mathbf{y})/\partial\pi = 0$). Is the result what you would have expected?

2 Automatic differentiation

Let $\mathbf{f}$ be a vector-valued function that maps from $\mathbb{R}^3$ to $\mathbb{R}^2$

$$y_1 = f_1(x_1, x_2, x_3) = x_1x_3 + \log(x_2 + x_1) \times \exp(-x_3), \tag{1}$$
$$y_2 = f_2(x_1, x_2, x_3) = \exp(-x_2) + \cos(x_1x_3). \tag{2}$$

1. (*) Compute the Jacobian using manual differentiation and evaluate the Jacobian at the point $(x_1 = 3, x_2 = 5, x_3 = 1)$

2. (*) Compute the Jacobian at the same point that in the previous point, but using finite difference approximation.

3. (*) Draw the computational graph.

4. (**) Compute the Jacobian using AD in forward mode. Write the expressions for all the intermediate variables $\dot{v}_i$ in the forward tangent trace.

5. (**) Compute the Jacobian using AD in reverse mode. Write the expressions for all the adjoints $\bar{v}_i$ in the reverse derivative trace.