Session 1: Propositional Logic and Basic Logical Connectives

- Propositions
- Basic Logical Connectives
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive OR
- Truth tables

What is Logic?

- Language of mathematics
  - Makes human language precise
    - Propositional logic exhibits problems we have in natural language with interpreting expressions such as “or”, “if … then”
  - Basis for mathematical proofs
  - Basis for automated reasoning
  - Omnipresent in computing

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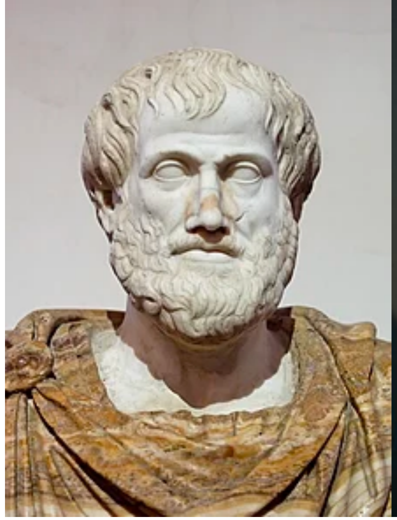
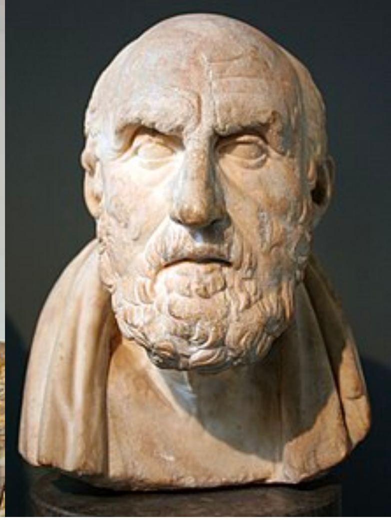
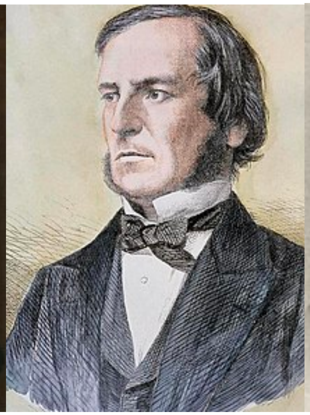
- **Propositional logic** is the most basic form of logic

Some History

Aristotle Chrysippus


Leibniz 1750 forgotten Boole 1860 De Morgan 1870

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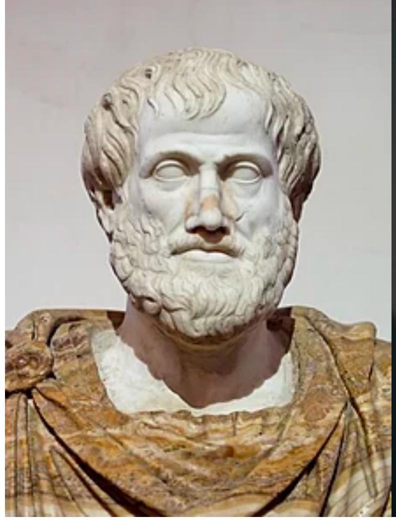
- Greek philosophers developed first logic formalisms, to formalize reasoning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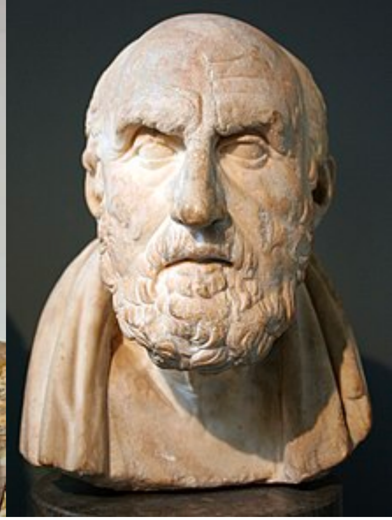
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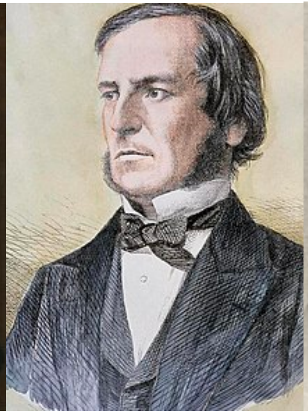
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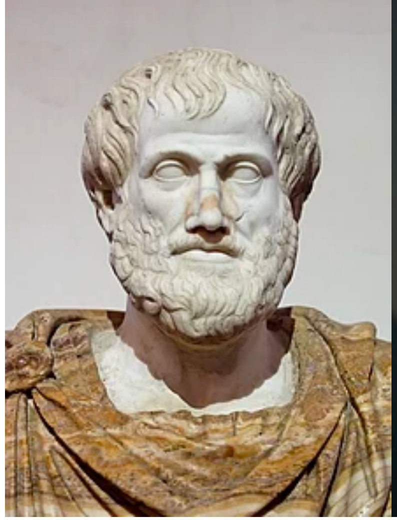

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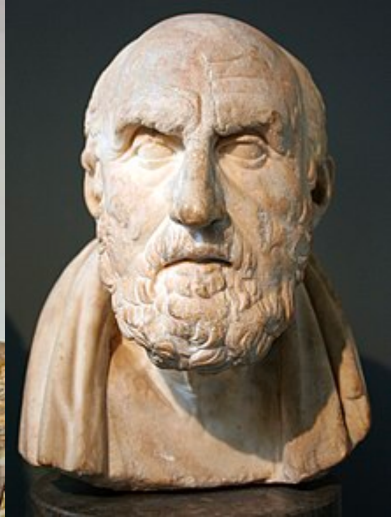

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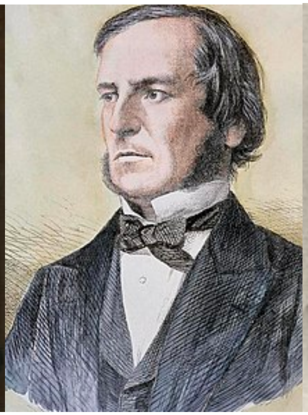

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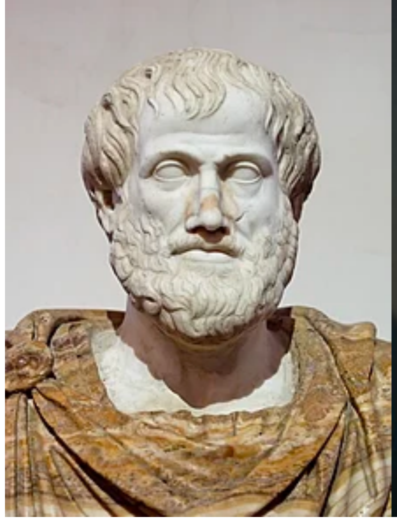

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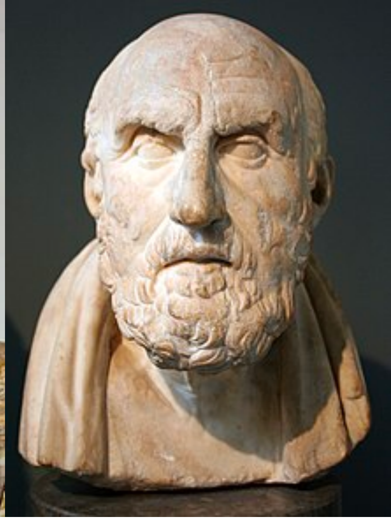

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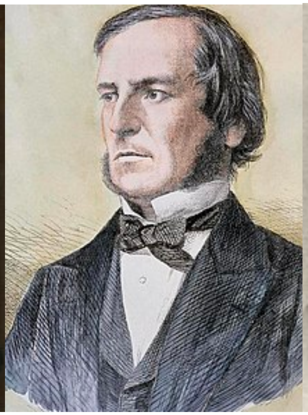

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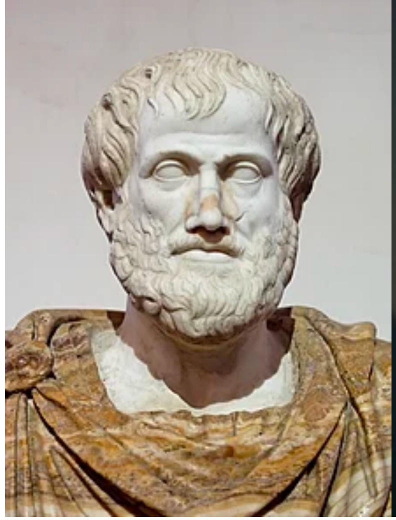
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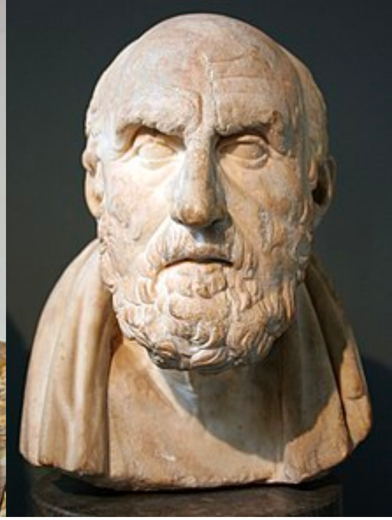
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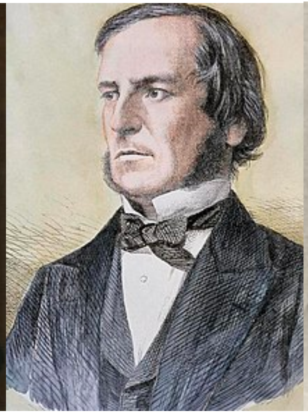
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- Modern mathematicians formulated propositional logic
- Propositional logic, though basic, introduces many fundamental concepts for mathematics
  - formal language, variables and operators, axioms, inference, proof, truth value

Propositional Logic and Computing

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  - Formulate basic search queries (search engines)
  - Describe computer circuits
  - Specify properties of software systems
  - Formally describe games, like Sudoku

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- Anything expressed in propositional logic can be **automatically decided** whether it is true or false
  - This is not the case for other logics!

Propositions

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  - $x + 1 = 2$
  - $x + y = z$

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- The proposition that is always **true** is denoted by **T**
- The proposition that is always **false** is denoted by **F**

T, F, p, q, r, ...

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Compound Propositions are constructed from **logical connectives** and other propositions

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- Negation $\neg$
- Conjunction $\wedge$
- Disjunction $\vee$
- Implication $\rightarrow$
- Biconditional $\leftrightarrow$

Negation

Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

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Example

p := "The earth is round" (T) $\neg p$ = "The earth is not round" (F)

q := "The moon is round" (T)

r := "The moon is made of green cheese" (F) $\neg r$ (T)

Conjunction

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

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Example

p := "The earth is round" (T) $p \vee q$ (T)

q := "The moon is round" (T) $p \vee r$ (T)

r := "The moon is made of green cheese" (F) $r \vee r$ (F)

Truth Tables

A **truth table** lists all possible truth values of the propositional variables occurring in a compound proposition, and the corresponding truth values of the compound proposition

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$	$p \vee q$
T	T	T	T
F	T	F	T
T	F	F	T
F	F	F	F

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- Exclusive or
  “Soup or salad comes with this entrée”
  We do not expect to be able to get both soup and salad.

Exclusive Or

- Disjunction: “Inclusive Or”.
  For $p \vee q$ to be true, either one or both of p and q must be true.

- “Exclusive Or” - called also Xor.
  For $p \oplus q$ to be true, one of p and q must be true, but not both.

p	q	$p \oplus q$	$p \vee q$
T	T	F	T
F	T	T	T
T	F	T	T
F	F	F	F

Summary

- Propositional Variable
- Atomic Proposition
- Compound Proposition
- Negation
- Conjunction
- Disjunction
- Exclusive OR
- Truth tables

Next: Implication and Biconditionals