# Session 1: Propositional Logic and Basic Logical Connectives

- Propositions
- Basic Logical Connectives
  - Negation
  - Conjunction
  - Disjunction
  - Exclusive OR
- Truth tables

# What is Logic?

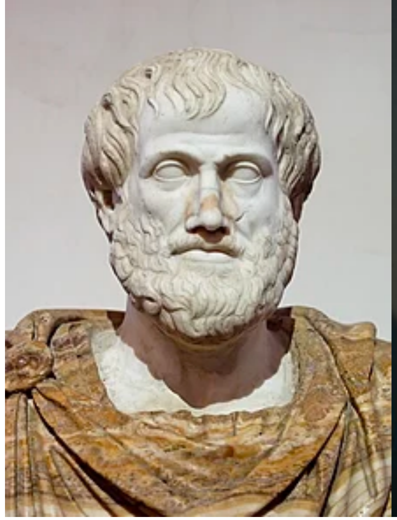
- Language of mathematics
  - Makes human language precise
    - Propositional logic exhibits problems we have in natural language with interpreting expressions such as “or”, “if … then”
  - Basis for mathematical proofs
  - Basis for automated reasoning
  - Omnipresent in computing

- **Logic is about statements that are either true or false**

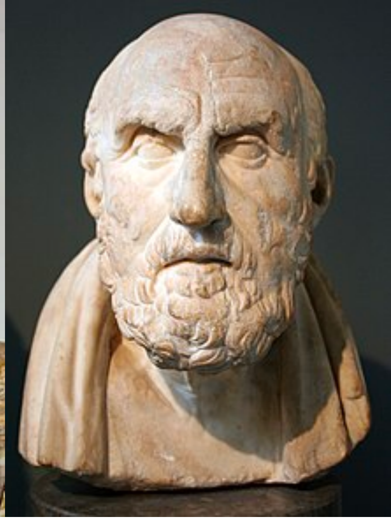
- **Propositional logic** is the most basic form of logic

# Some History

- Greek philosophers developed first logic formalisms, to formalize reasoning



Aristotle

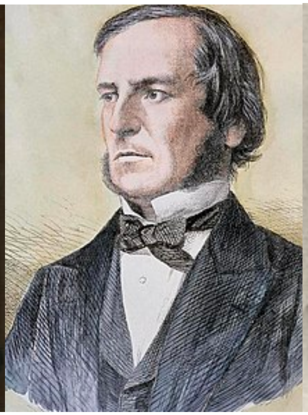


Chrysippus

- Modern mathematicians formulated propositional logic

- Propositional logic, though basic, introduces many fundamental concepts for mathematics
  - formal language, variables and operators, axioms, inference, proof, truth value

Leibniz 1750 forgotten



Boole 1860

De Morgan 1870

# Propositional Logic and Computing

- Propositional logic allows to
  - Formulate basic search queries (search engines)
  - Describe computer circuits
  - Specify properties of software systems
  - Formally describe games, like Sudoku

- Anything expressed in propositional logic can be **automatically decided** whether it is true or false
  - This is not the case for other logics!

# Propositions

- A **proposition** is a declarative sentence that is either **true** or **false**

- Examples of propositions
  - The Moon is made of green cheese.
  - The earth is round.
  - $1 + 0 = 1$
  - $0 + 0 = 2$

# Propositions

- A **proposition** is a declarative sentence that is either **true** or **false**

- Examples that are not propositions
  - Sit down!
  - What time is it?
  - $x + 1 = 2$
  - $x + y = z$

# Language of Propositional Logic: Atomic Propositions

**Atomic propositions:** Propositions that cannot be expressed in terms of simpler propositions

- Letters denote **Propositional Variables**: *p*, *q, r*, *s*, …

  **Example**: *p* denotes ”The Earth is round”

- The proposition that is always **true** is denoted by **T**
- The proposition that is always **false** is denoted by **F**

# Language of Propositional Logic: Compound Propositions

**Compound Propositions** are constructed from **logical connectives** and other propositions

- Negation $\neg$
- Conjunction $\wedge$
- Disjunction $\vee$
- Implication $\rightarrow$
- Biconditional $\leftrightarrow$

# Negation

Let $p$ be a proposition. The *negation of* $p$, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that $p$."

The proposition $\neg p$ is read "not $p$." The truth value of the negation of $p$, $\neg p$, is the opposite of the truth value of $p$.

**Example**

p := "The earth is round" (T)

q := "The moon is round" (T)

r := "The moon is made of green cheese" (F)

# Conjunction

Let $p$ and $q$ be propositions. The *conjunction* of $p$ and $q$, denoted by $p \wedge q$, is the proposition "$p$ and $q$." The conjunction $p \wedge q$ is true when both $p$ and $q$ are true and is false otherwise.

**Example**

p := "The earth is round" (T)

q := "The moon is round" (T)

r := "The moon is made of green cheese" (F)

# Disjunction

Let $p$ and $q$ be propositions. The *disjunction* of $p$ and $q$, denoted by $p \vee q$, is the proposition "$p$ or $q$." The disjunction $p \vee q$ is false when both $p$ and $q$ are false and is true otherwise.

**Example**

p := "The earth is round" (T)

q := "The moon is round" (T)

r := "The moon is made of green cheese" (F)

# Truth Tables

A **truth table** lists all possible truth values of the propositional variables occurring in a compound proposition, and the corresponding truth values of the compound proposition

Example: Truth table for disjunction

# Or in Natural Language

In natural language “or” has two distinct meanings

- Inclusive or
  “Candidates for this position should have a degree in mathematics or computer science”
  We assume that candidates need to have one of the degrees, but may have also both.

- Exclusive or
  “Soup or salad comes with this entrée”
  We do not expect to be able to get both soup and salad.

# Exclusive Or

- Disjunction: “Inclusive Or”.
  For p ∨ q to be true, either one or both of p and q must be true.

- “Exclusive Or” - called also Xor.
  For p ⊕ q to be true, one of p and q must be true, but not both.

# Summary

- Propositional Variable
- Atomic Proposition
- Compound Proposition
- Negation
- Conjunction
- Disjunction
- Exclusive OR
- Truth tables

Next: Implication and Biconditionals