Emergent Fractional Quasi-particles in Geometrically Frustrated Systems

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Abstract

Magnetic monopoles have been theoretically predicted to exist as elementary particles yet have not been observed in free space to this day. However, in a group of geometrically frustrated magnetic materials known as spin ice, emergent quasi-particles have been observed which behave as magnetic monopoles. Performing simulations on a kagome lattice, a corner-shared triangular lattice using a Monte-Carlo procedure, I was able to obtain evidence for these quasi-particles and their associated strings which were intrinsic properties of the lattice. Furthermore, it was found that these 'monopoles/anti-monopoles' interact via a magnetic analogy to the coulomb interaction with same charges repelling each other and opposite charges attracting one another and annihilating.

Contents

List of Figures

1 Introduction to the Material

Upon initial inspection of Maxwell's equations, in particular $\nabla \cdot B = 0$, it appears that the existence of a free magnetic monopole is prohibited due to the fact that all magnetic field lines must form closed loops. In other words a North or South pole cannot exist in the absence of the other, that they must exist as a pair.

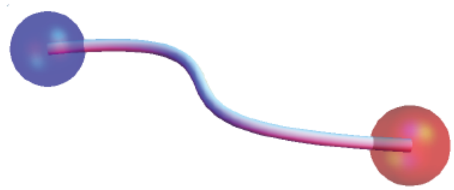


Figure 1: String connecting two monopoles

However, we can construct a system where two poles are tethered by an ideally elastic and flexible string-like tube which transports magnetic flux from the south to the north pole. These poles could then be moved arbitrarily far away from each other, and the magnetic field would exclusively emanate from or flow into the ends of the wire. These ends would still interact via Coulomb's law, yet the magnetic field lines still formed closed loops as required by Maxwell's equation $\nabla \cdot B = 0$.

The physicist Paul Dirac published a paper in 1931 predicting the existence of these exact magnetic monopoles.[1] Dirac showed that if any magnetic monopoles exist, then all electric charge in the universe must be quantized. The electric charge is in fact quantized, which is consistent with (but does not prove) the existence of magnetic monopoles.

Since Dirac's paper, several monopole searches have been undertaken, most notably the experiments in 1975[2] and 1982.[3] Both experiments produced events that were initially perceived as being monopoles but these experiments are now regarded as inconclusive. The question of whether a magnetic monopole exists in free space is still an open one.[4]

The history of physics shows that phenomena in high energy physics have counterparts in condensed matter physics which are described by similar mathematics and during the past few years, physicists have discovered that in synthesized exotic materials known collectively as spin ice, quasi-particles that behave like magnetic monopoles have been observed.

Spin-ice is a geometrically frustrated system of magnetic moments, similar to the frustration of hydrogen ions in water ice. In the spin-ice structure, these quasi-particles are present in a form of, borrowing the language of monopoles, 'monopole/anti monopole' pairs connected by a 'Dirac String' of overturned dipoles.

However, in studying these structures experimentally, it is extremely difficult to obtain a spacial image of the spin configurations without altering the state of the system. Instead, in order to observe these quasi-particles what I will be doing is simulating a 2 dimensional frustrated lattice with a kagome structure using Mathematica code provided by the UCD Theoretical Condensed Matter research group.

2 Geometrically Frustrated Systems

2.1 What is a Geometrically Frustrated System?

Geometrically frustrated systems are systems in which multiple configurations give the same total energy. A system exhibits this type of frustration when it is not possible to minimize the interaction energy between each of the elements of the system simultaneously. A degeneracy in the ground state is a characteristic of these frustrated systems. This degeneracy leads to an interesting effect, as the temperature of the system approaches zero the entropy reaches some finite value.

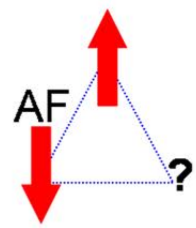


Figure 2: Frustration arising from anti-ferromagnetic interactions in a triangular spin arrangement

The above figure is an example of one of the most basic examples of this frustration, consider 3 spins which interact via anti-ferromagnetic interaction placed at the corners of an equilateral triangle. In this triangular lattice geometry, to align the third spin such that it minimises its total interaction energy gives the exact same energy whether the third spin is +1 or -1.

Why this occurs is because there is no way to orient the spin such that it minimizes it's interaction with the other two spins simultaneously resulting in a two-fold degenerate ground state. The existence of a degenerate ground state is a defining characteristic of a frustrated system. An important note to make here is that this kind of frustration is a result of the topology of a well-ordered system, rather than the result of a disorder in the system. Hence why this is referred to as geometric frustration.

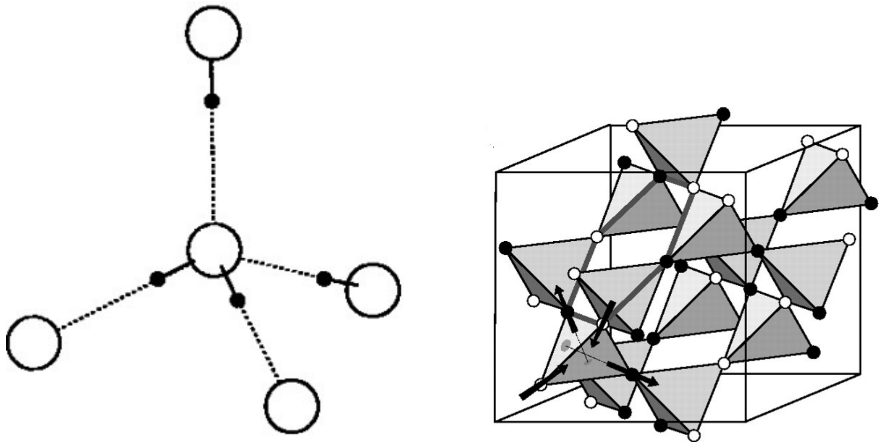
2.2 Geometrical Frustration in Water Ice

An example of frustration in a system can be seen by looking at ordinary water ice at low temperature. When water ice was investigated experimentally in 1933 by W.F. Giauque & M.F. Ashley, the entropy of ice at low temperatures did not agree with the predicted theoretical value. In fact, when the results were extrapolated toward absolute zero kelvin, it was found that the entropy of the system was non-zero.[5]

This result appeared to be in direct contradiction to the famous third law of thermodynamics which states that entropy as a measure of the disorder of a system should approach zero as temperature approaches 0. In 1935, Linus Pauling attributed this discrepancy to a frustration resulting from the geometric structure of the bonds between the oxygen and hydrogen atoms and thus the existence of a degenerate ground state.[6]

The frustrated structure of the water ice occurs in the I_h phase of water ice.[7] H^2O molecules are arrayed in such a way that the bond angle from the liquid phase is almost preserved. Each oxygen ion is bonded covalently to four hydrogen ions or protons.

However, the protons are not all positioned equidistant between the oxygen ions. Instead relative to each oxygen atom, the hydrogen atoms' positions are that two of the neighbouring hydrogen are relatively close and two relatively far away. This arrangement gives the lowest energy configuration in each tetrahedral unit.



(a) Locations of protons

(b) Tetrahedral water ice lattice

Figure 3: Structure of water ice.

Figure 3(a) shows one such arrangement with the large white circles and small black circles representing oxygen and hydrogen respectfully. Figure 3(b) is an example of how a segment of the lattice is constructed from the tetrahedral units.

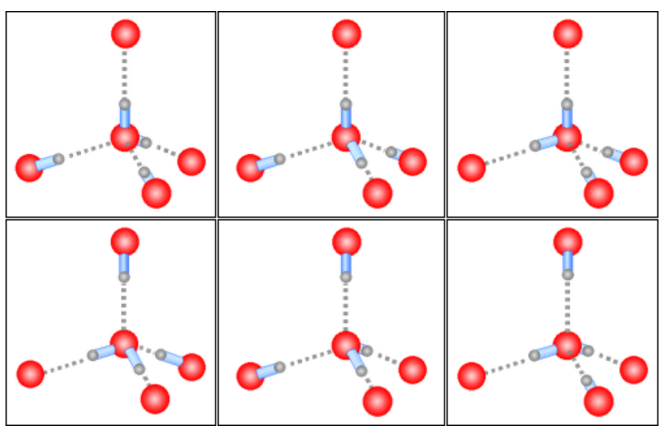


Figure 4: There are 6 different arrangements each tetrahedral unit can take on

This tendency toward a two-in, two-out configuration is known as the Bernal-Fowler ice rule. Pauling showed that for a macroscopic number of atoms the ice rules lead to a 6-fold degeneracy in the ground state, even at zero temperature. Thus the system is left with a residual entropy. Pauling calculated this residual entropy and it was in very good agreement with the extrapolated experimental value.

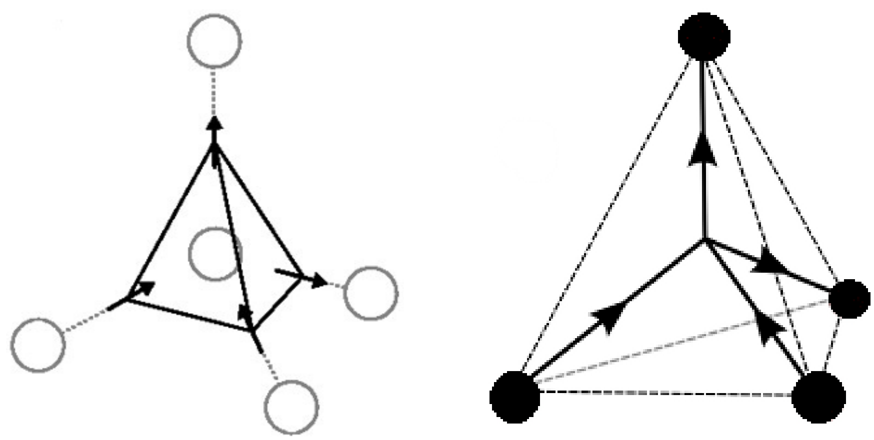
Unfortunately, studying the properties of water ice specifically to see 'monopole/anti-monopole' pairs is inherently difficult because of the need for immaculately pure crystals of water ice. To put this into perspective, the water ice crystals needed would require purities rivalling that of modern silicon crystals used for semiconductors but with the added challenge of keeping the crystals at a structure preserving temperature and ensuring the temperature is uniform to a sufficient degree throughout the crystal.

2.3 Spin Ice Structures

There is however another material that has the same tetrahedral structure as water ice. Philip Anderson realised this when he noticed there was a straightforward mapping between Pauling's water ice model and an anti-ferromagnetic two state Ising model on a pyrochloric lattice.[10]

The problem with such a two state model is not that it would require all of the spins to point either parallel or anti-parallel to a single global z-axis but given there is no reason for the system to prefer this particular orientation over another in a magnetic material exhibiting global cubic symmetry makes this a physically unrealistic model.

In a paper published in 1997, it was discovered that there was a mapping between the water ice model and a ferromagnetic, rather than anti-ferromagnetic, two state Ising model, also on a pyrochloric lattice. However, there is a difference from the previous mapping in that the spins must be constrained to point either toward or away from the centre of the tetrahedron.



(a) Collinear Spins (b) Non-Collinear Spins

Figure 5: Tetrahedral unit spin configurations

In the above Figure 5. the tetrahedrals outlined in both figures correspond directly with each other but Figure (b) has striped the neighbouring oxygen atoms to make it easier to visualize the orientation of the hydrogen spins. It is clear from this picture that the spin axis are collinear in (a) yet in (b) they are non-collinear. Figure (b) shows a strong magnetic anisotropy or preference for the spins to point along the axis which connects a spin to the center of it's tetrahedron. In crystallography this is known as the <111> axis.

In this model, the degenerate ground states of a single tetrahedron obeys the ice rule, with two spins pointing in and two pointing out. M.J. Harris and his team referred to the model as the 'spin ice model' and experimentally observed it to be approximated by the magnetic pyrochlore oxide $Ho_2Ti_2O_7$.[11].

This frustration arising from ferromagnetic interactions is quite unexpected, yet the experimental value for the residual entropy of the spin ice $Dy_2Ti_2O_7$ investigated by A.P. Ramirez et al. in 1999 was been found to agree reasonably well with Pauling's value for the residual entropy of water ice.[12] This is direct experimental evidence for the existence of a macroscopically degenerate ground state and an ice-rule obeying spin ice ground state.

3 Experimental Constraints

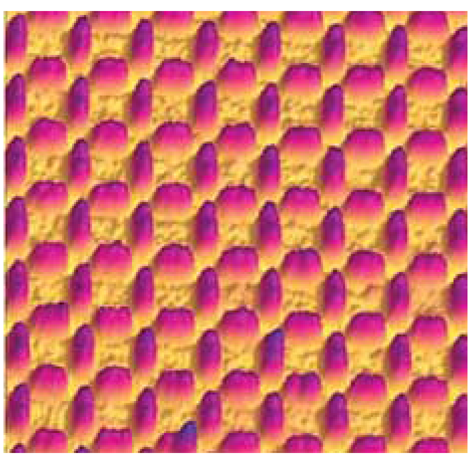
Unfortunately it is extremely difficult to obtain a real space image of the spin configurations of these 3-D magnetic pyrochlore oxide structures without altering the state the system is in. However there have been numerous experiments recently concentrating on 'artificial' 2-D spin ice and also simulations. These materials are artificially produced and lithographically fabricated isolated single-domain nano-magnets arranged in regular lattice such as the square lattice and the kagome lattice which I will be concentrating on in this report.[13]

3.1 Square Lattice

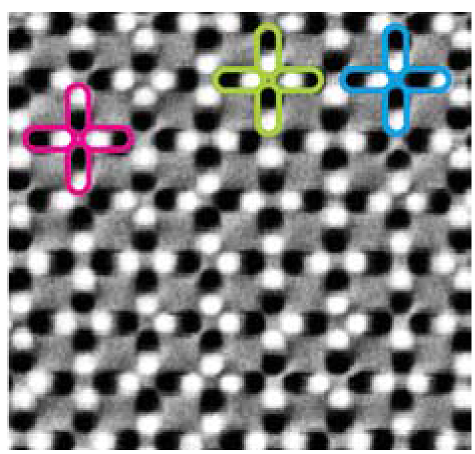
In the case of the square lattice, the islands have intrinsic magnetic moments and the shape anisotropy of the islands effectively force these moments to point along the long axis of the islands as opposed to 3-D spin ice where it is the magneto crystalline anisotropy which determines the direction of the moments.

We can now consider the moments to be Ising like as they only have two states available to them, parallel or anti-parallel to their long axis. This lattice construction enables the study of dipole interactions between islands creating a simpler 2-D analogue to the more complicated and difficult to study 3-D pyrochlore oxides.

Important notes to make here are that the islands are isolated in that they are not physically connected to one another and considering a single vertex of the lattice it obeys the ice rules of 2 in 2 out. The advantage of working with artificial spin ice is that it's possible to experimentally image the lattices using imaging techniques such as Atomic force microscopy, Magnetic Force Microscopy or X-ray Magnetic Circular Dichroism and that in the majority of cases these imaging processes can be performed at room temperature.



(a) Atomic force microscopy



(b) Magnetic force microscopy

Figure 6: Square spin ice lattice images

Some such images from the experiments carried out by R.F. Wang et al. on a square lattice are shown in Figure 6. The lattice spacing in both images is 400nm and the islands are 220nm $\times$ 80nm in size.[13]

3.2 Frustration on the Kagome Lattice

Unfortunately the origin of frustration in the square lattice is not geometric like it is in the 3-D pyrochlore lattice. There is, however, a structure in which the frustration is an inherent property due to the geometry and therefore creates a more accurate 2-D analogue to the 3-D pyrochlore lattice and this structure is the 2-D kagome lattice.

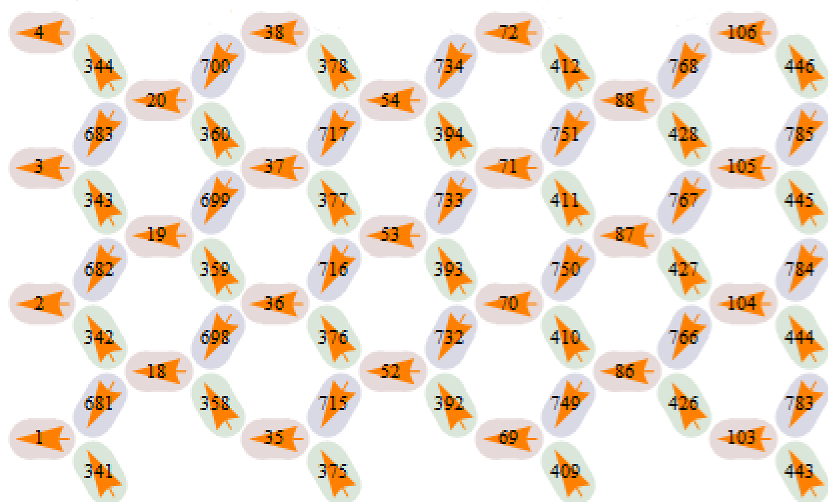


Figure 7: Kagome lattice sample

The Monte Carlo simulations in this experiment were performed on a 2-D Kagome lattice such as that shown in Figure 6 above. It is important to note that, similar to the experiments on the square lattice, the islands share no connections with neighbouring islands or in other words are 'isolated' and the dipoles are limited to point in two directions parallel or anti-parallel to their long axis.

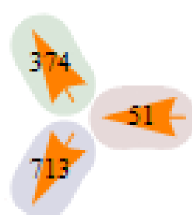


Figure 8: Basic kagome lattice unit

If we examine for a moment the basic unit of the kagome lattice, it is composed of 3 dipoles converging at a vertex as shown in Figure 8 above. Consider that each dipole's magnetic moment to be stretched into a charged dumbbell with the charges ±q residing at the opposite ends. This is known as the charge model. Then at the vertex in this basic unit the ground state can be one of two values : ±q.

These vertex have an inherently frustrated or degenerate ground state that can take one of two forms : 2 in 1 out, giving +q or 1 in 2 out, giving -q. In the case of the above figure 8 the 'charge' at the vertex would be $-$q.

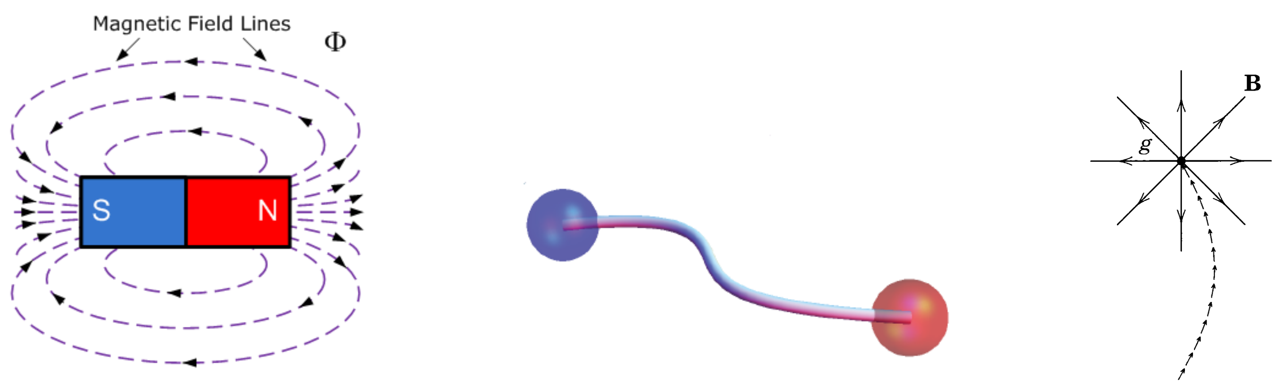
4 Dirac Strings and Monopoles

In order to progress these ideas to incorporate the overall picture of entire lattice it is necessary to motivate the language from the concepts of Dirac strings and monopoles. In his 1931 Paper entitled 'Quantised Singularities in the Electromagnetic Field', Dirac postulated that the existence of magnetic monopoles would automatically imply that electric charge must be quantised, specifically that :

$$e = \frac{\hbar c}{2g} n \ , \tag{1}$$

where $n = 0, \pm 1, \ \dots$ e is the fundamental unit of electric charge and g is the magnetic charge of a monopole. [1]

Dirac started with a pair of magnetic poles connected by an ideal string. He then took one of the poles and sent it off to infinity. What he was left with was a magnetic monopole who's magnetic flux was solely due to the string. This string could be shrunk to a line, thus becoming a remnant of one end of Coulomb's bar magnet. Dirac thus had produced an object that created the magnetic field of a single magnetic pole everywhere in space except on the string. This is known as a Dirac string.



(a) Classical bar magnet (b) Monopoles connected by a string (c) Dirac monopole

Figure 9: Examples of types of magnets

The magnetic flux due to the monopole is exactly compensated by the flux fed into the monopole via the string, so the string is an inseparable part of Dirac's monopole. This means that the net flux through any closed surface is still zero, satisfying Maxwell's Equation $\Delta \cdot B = 0$.

4.1 The Charge Model

The islands interact via long-range pair wise dipolar interactions. There are two different, but equivalent, ways in which one can model the interactions between the islands, the pure dipole model and the charge model. The emergence of the quasi-particles is more easily seen in the charge formalization.

In the charge model the magnetic dipoles are imagined to be stretched into charged dumbbells which carry opposite charges, ±q, at their ends. The magnitude of the charges are q = $\frac{\mu}{l}$, l is the length of the island. We then take the limit $l \to a$, where a is the lattice spacing. For the interaction between two vertices, given by the magnetic Coulomb law:

$$V(r_{\alpha\beta}) = \frac{\mu_0}{4\pi}\frac{Q_\alpha Q_\beta}{r_{\alpha\beta}} \quad ; \alpha \neq \beta \ , \tag{2}$$

$$V(r_{\alpha\beta}) = \frac{1}{2}v_0 Q_\alpha^2 \quad ; \alpha = \beta \ , \tag{3}$$

where Q_α is the total magnetic charge at vertex α, $r_{\alpha\beta}$ is the distance between two vertices α and β and v_0 is a 'self energy' resulting from the interaction of charges at the same vertex.

4.2 Emergence of Fractional Quasi-particles

When several Kagome base units are constructed into a lattice the magnetic monopoles may be realised as emergent fractional quasi-particles in the structure. An important note to make here is that these fractional quasi-particles cannot be constructed out of combinations of the elementary particles, as opposed to quasi-particles like Cooper pairs which are a combination of two electrons.

We get emergent 'monopoles' due to the cooperation of base units which are arranged differently. The emergence is unlike its components insofar as they are incommensurable but is instead a result of combinations of fractional parts of these base units. This meaning is easily seen when you look at an adjacent series of overturned dipoles. The flipping of one dipole will change the two vertices it is contained by.

The ground state of the Kagome lattice in which all dipoles point in one global direction, consists of vertices with ±q net charge. If we now consider an excited state in which a continuous chain of dipoles(or charged dumb bell) has been flipped there will now at either end be a vertex of charge -3q marked in red and +3q for the vertex marked in blue.

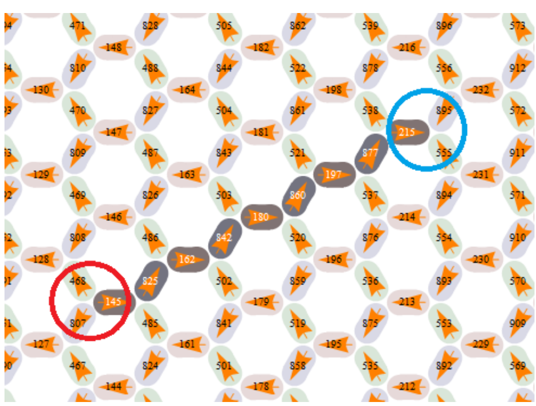


Figure 10: Two Monopoles connected by a string of overturned dipoles

In the above Figure 10. the string of overturned dipoles is marked darker than it's surrounding dipoles. The 'charges' at the marked vertices can be viewed as a 'monopole-antimonopole' pair interacting via the magnetic Coulomb interaction given by the equations 2 & 3.

5 Computational Methods

5.1 Monte-Carlo Procedure

The Monte Carlo simulations were carried out in Mathematica using code provided by the UCD Theoretical Condensed Matter group. The majority of these simulations were carried out on a 16 * 19 lattice giving 1024 islands total. The Monte Carlo procedure is as follows:

1. Initialisation step where every island is assigned a switching value before the Monte Carlo procedure, given by $H_S = H_0\epsilon$, where ϵ is a dimensionless Gaussian random variable with mean 1 and variance σ. The variance, external field magnitude H_{Ext} and external field angle are input parameters.

2. The dipolar field H_{dip} is calculated before running the Monte Carlo selection using the Ewald summation technique.

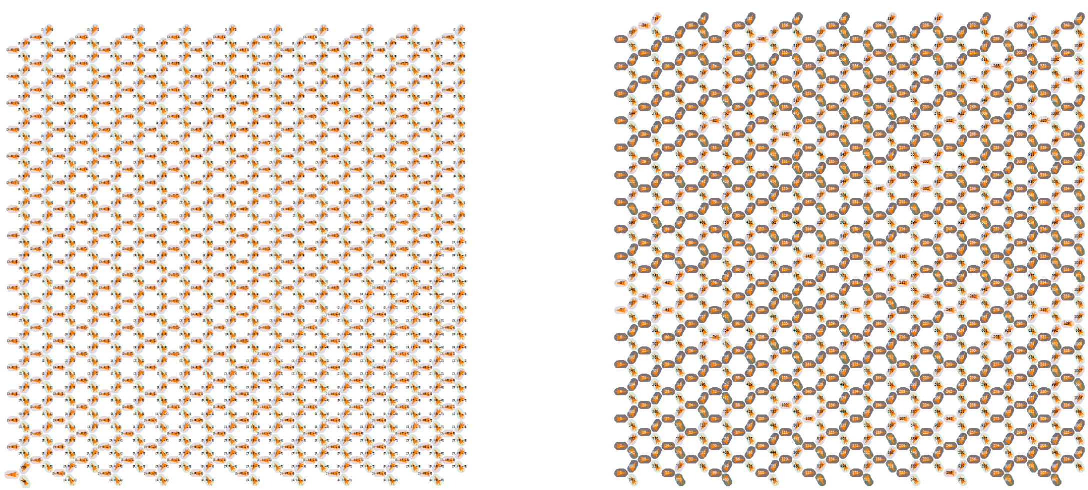
3. An island in the lattice is randomly selected using a pseudo-random number generator.

4. The local field at the island is given by:

$$H_L = \hat{e}_i(H_{dip} + H_{ext}) \ , \tag{4}$$

   where $\hat{e}_i$ is the unit vector pointing along the anisotropy direction of the island.

5. If the local field at the island exceeds the switching value the island's magnetic moment is flipped. Otherwise, the moment remains unchanged and the process is repeated from step 2.



(a) Initial test run state (b) Final test run state

Figure 11: Initial and final test run states

Steps 2 - 5 are repeated 15n times, where n is the number of islands in the lattice. The above figure 11(a) shows a typical initial lattice saturated with an external field in the $\theta = 0$ direction. Figure 11(b) is the result of running the Monte Carlo procedure with an external field of 2.7 in the $\theta = 0$ direction and a value for σ of 0.13. The dark-coloured islands are ones which have had their magnetic moment flipped.

The parameter ϵ gives a measure of the strength of each island's anisotropy. Islands with a large value of ϵ will require a higher field to flip, corresponding physically to islands with a strong anisotropy. Similarly islands with a low ϵ value correspond physically to islands with a low anisotropy. Thus the variance of ϵ is a measure of the disorder in the lattice. If the anisotropy of each island was the same then the lattice would be isotropic and all dipoles would flip at the same value for the magnetic field.

Before each simulation a number of input parameters need to be initialised, such as the magnitude and angle of the external field, the variance of the anisotropies, the initial lattice configuration, etc. Furthermore it was possible to run several simulations consecutively, with a specific parameter varying between a given initial and final value in a chosen step size.

5.2 Ewald Summation

The Ewald summation technique was used to calculate the dipolar fields. It's a standard technique used in the calculation of long range forces on a periodic lattice and is a special case of the Poisson summation formula:

$$\sum_{n \epsilon \mathbb{R}} f(n) = \sum_{k \epsilon \mathbb{R}} \hat{f}(k) \ , \tag{5}$$

where $\hat{f}$ is the Fourier transform of the function f. The sums over $\mathbb{R}$ can be replaced by discrete sums over $\mathbb{Z}$ and the formula can be generalised to higher dimensions. For example the sums could be taken over a discrete 2-D periodic lattice, as was the case for the simulations performed here. However, it is then necessary to include a geometric factor γ on the right hand side of equation 5.

This depends on the structure of the unit cell used. We shall denote the dipole field at an island i due to a dipole j as H_{ij} . The patch used in the simulation (a 17×20 unit kagome lattice with each unit having 3 dipoles amounting to 1020 islands) is replicated periodically, extending the lattice to an infinite size. Thus the dipole field at site i due to site j(m,n), where (m,n) is the location of the island in the infinite lattice, is now denoted $H_{ij}(m,n)$. The total field at i due to dipole j is then given by:

$$H_{ij} = \sum_{m,n \, \epsilon \, \mathbb{Z}^2} H_{ij}(m,n) \ , \tag{6}$$

where the sum is over all periodic patches in the infinite lattice. The total dipolar field at i is thus simply:

$$H_i = \sum_{j=1}^{1020} \left(\sum_{m,n \, \epsilon \, \mathbb{Z}^2} H_{ij}(m,n) \right) \ . \tag{7}$$

This method is slowly convergent in real space and the need to take a sufficiently large number of terms in order to obtain an accurate result is computationally inefficient. There is another problem with this as $H_{i,j}$ is dependent on 1/r the convergence is conditional insofar as the order in which the sum is carried out. We can overcome this problem by using the Poisson Summation to replace equation 6 with:

$$H_{ij} = \sum_{k,q \, \epsilon \, \mathbb{Z}^2} \hat{H}_{ij}(k,q) \ , \tag{8}$$

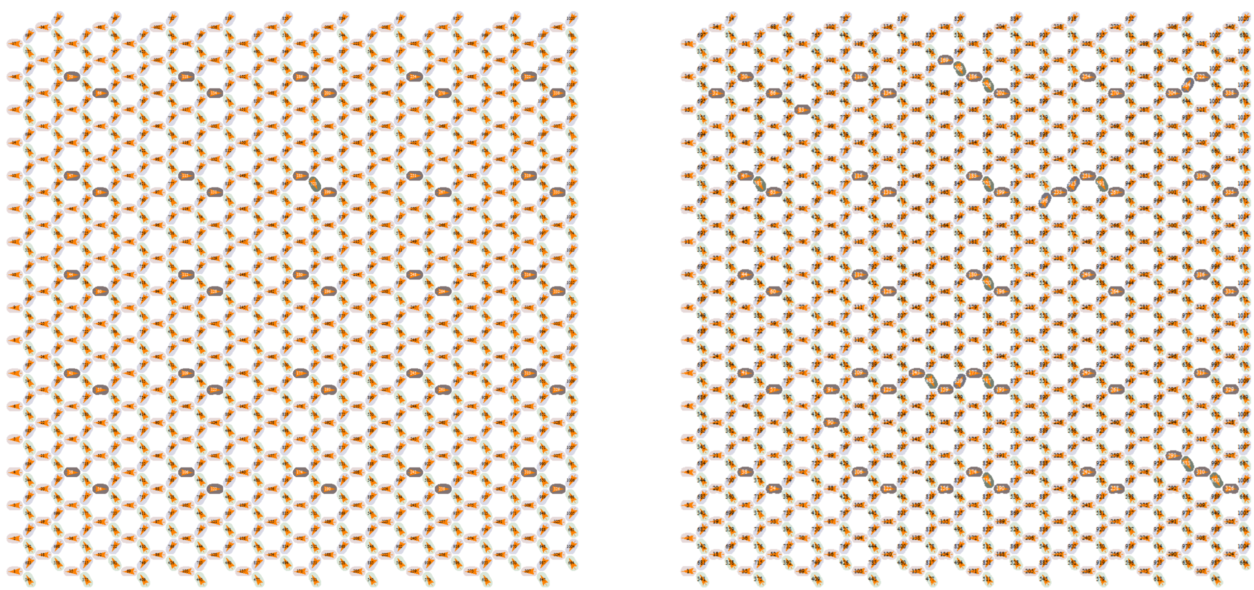
where $\hat{H}_{ij}$ is the Fourier transform of $H_{ij}(m,n)$. It is also worth mentioning that we can absorb the geometric factor into $\hat{H}_{ij}$. The advantage of summing over $\hat{H}_{ij}$ is that the sum is rapidly convergent so an accuracy obtained with previously using equation 7 is obtained with fewer terms to be computed using equation 8. This is therefore a reduction in computation time to compute accurate fields.

6 Simulation Results

The main results of this project will be in the areas concerning the effect of the topology of the lattice on a single island and string collisions. Variations of these experiments were performed by modifying islands to be more likely to flip, modifying islands by manually flipping them and changing the direction and strength of the magnetic field.

6.1 Singular Island Experiments

This experiment, dubbed the sandwich test, was run without any manipulation and roughly 25 evenly spaced 'A' islands on the lattice were placed under observation. The external field was steadily increased to see what value the islands of interest flipped at (if they flipped at all) and these values were recorded. The external magnetic field applied was incident left to right.



(a) Initial modified island A sandwich run state

(b) Final modified island A sandwich run state

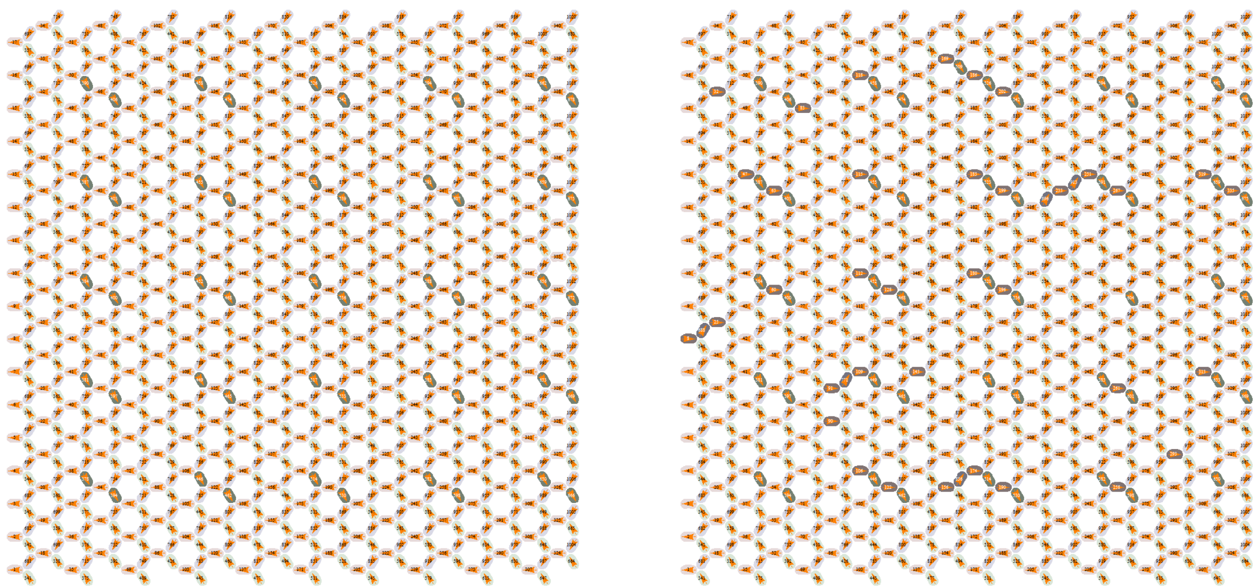
Figure 12: Modified island A sandwich run states

Then the lattice was reset and islands sandwiching the islands of interest (hereafter known as sandwich islands) had their anisotropy modified so that they were guaranteed to flip at very low field values. The experiment was then rerun to see if this had an affect on the flipping of the islands of interest. One such run can be seen above in figure 12.

The results of all runs of this particular experiment are categorized as follows: Islands which previously did not flip and islands which flipped at lower values. Some notes on gathering data as the lower the variance is set the greater the number of islands that flip around the same input magnetic field value. Yet the larger the variance the more likely that islands we are not interested in will perturb results.

For these experiments the variance was kept to ±0.13 around the average. The field incidence angle was kept to 0 so A islands felt the full weight of the magnetic field to increase the possibility of recording islands flipping.

The key findings for the sandwiched A islands from this was that the roughly 30 % (with a variance of 22 %) of islands of interest flipped on each pass and all islands which were flipped in the initial scanning run were flipped at earlier field values.



(a) Initial modified island B sandwich run state

(b) Final modified island B andwich run state

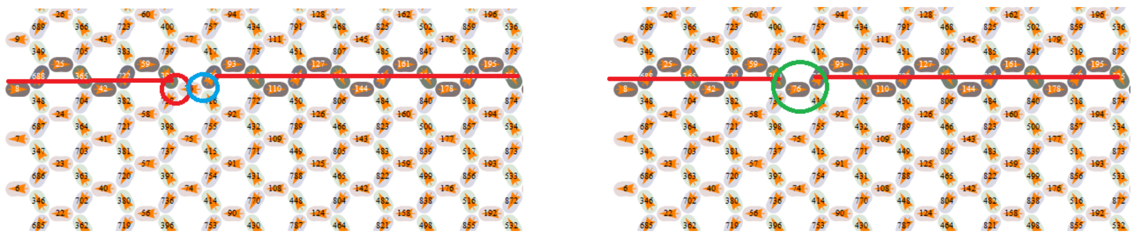
Figure 13: Modified island B sandwich run states

When the field was unbiased these numbers dropped to 16 % (with a variance of 12%) islands of interest that flipped. There is a high variance due to the size of dataset. If significant statistics were attained this value and it's variance would be more normalized. The increase in the number of flips when the anisotropy is modified is evidence that by changing the topology of the lattice, neighbouring islands can be softened or flipped easier. Similar experiments could be run on the other 'B' and 'C' islands but those results would have an equivalent A island analogue by simply changing the angle of incidence of the external magnetic field. A sample run is included above though for completeness.

6.2 String Collisions

The second part I have been able to investigate was the impact of string collisions. Several strings were embedded into the lattice in different forms to see how the strings would interact with each other.

A string was embedded aligning with the external magnetic field. The strings were generally seperated by a single unflipped island. From the data of all simulations it was found that you can increase the islands anisotropy by on average 22 % and it will still flip. This is direct evidence of the emergence of properties of vertex having an influence on islands contained by them.



(a) Initial modified A island string collision (b) Final modified sandwich island A run state

Figure 14: Modified sandwich island A run states

In figure 14 we can see two string with opposing monopoles ending beside each other red cirlce is the +q monopole and the blue is marked as the - q Monopole. The attraction between these two monopoles is such that increasing the anisotropy still results in site 76 flipping. in this particular case site 76 could have it's anisotropy increased by 17 % and the island would still flip.

6.3 Unbiased Magnetic Field

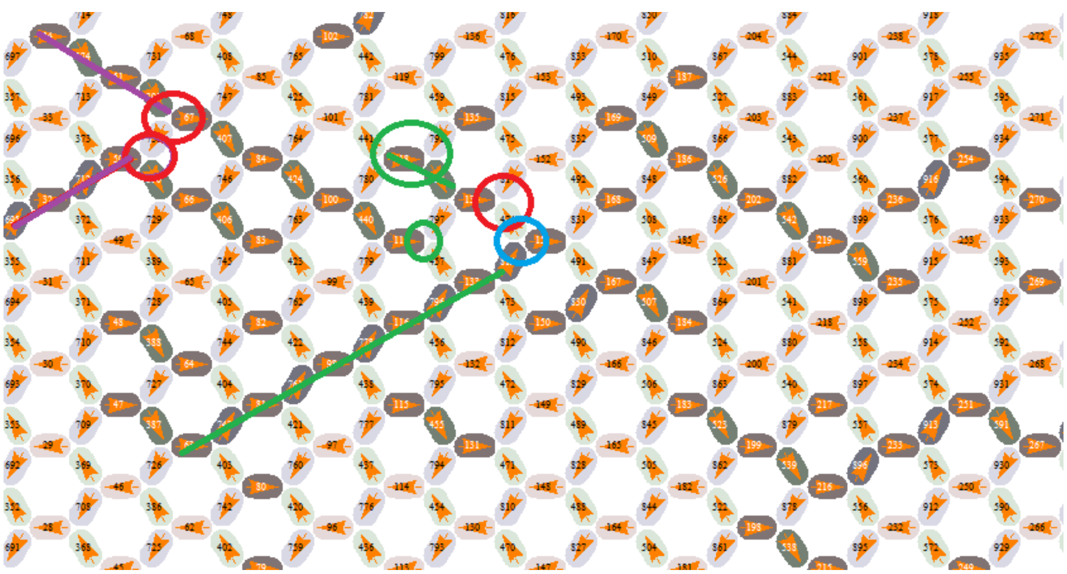


Figure 15: Unbiased magnetic field.

In addition to the two experiments outlined the last and most interesting experiment was to unbias the field and let some strings form as seen in above figure 15. The general rules obtained from the few simulations of this nature that were run was that positive monopoles preferred to travel in the same direction the field pointed and that negatively charge poles preferred to move in the opposite directions. The above figure 15 shows two independent interacting string pairs which formed in the lattice.

The green and purple markings here are the initial flipped islands and strings. If we look at the purple strings we can see at some stage in the simulation these two positively charged monopoles approached each other and repelled each other by some kind of Coulomb magnetic interaction. If we observe the green strings they have all converged onto each other. There is a possibility of the red and blue circled points to collide thus cancelling the monopoles present.

7 Conclusion

Spin ice structures have no definite application as of yet but the most likely procurement from studying these structures is that it may be possible to use them in future spintronic devices. They could potentially be used as high density data storage devices. W.R. Branford et al. recently demonstrated the ability to 'read' and 'write' patterns in a honeycomb lattice. [14]

Even given current memory standards of approximately 1.73×10^5 bits per mm^2 in domestic computers, a spin ice like the one used by W.R. Bradford et al. could potentially lead to memory devices with storage densities of order 1×10^6 bits per mm^2.

Another interesting application is in the area of logic devices. The frustration in spin ice could be harnessed to perform complex computational problems in a single calculation, by using it as a kind of neural network.

In the string collisions we see evidence of magnetic charge behaviour, with attraction between opposite charges and repulsion between same sign charges. Furthermore, the simulations show that the positive charges move in the same direction as the external magnetic field and the negative charges move in the opposite direction.

This is analogous to a semiconductor in which the positively charged holes move in the direction of an external electric field and the negatively charged electrons move opposite to the external field. Thus we can say that the charge model seems to be a valid model in the case of the square lattice. The algorithms and Monte Carlo used in this project were adapted from the kagome lattice.

Due to constraints on computational resources available the data set is limited so other interesting questions requiring a statisically significant set of data were not explored. As an extension, it would be interesting to examine the avalanche behaviour of the system could be examined in more detail, with the form of the avalanche probability P(s) (probability of an avalanche as a function of the number of spins in the avalanche) still to be investigated.

Nonetheless this project has yielded a better insight into the behaviour of the frustrated Kagome lattice with evidence for the emergence of fractional quasi-particles or 'magnetic monopoles' and Dirac strings in 2-D artificial spin ice.

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