Adaptive Mesh Refinment for Smooth Boundary

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1 Finite Difference

The Diffusion equation:

$$\frac{\mathrm{d}C}{\mathrm{d}t} = D\nabla^2 \tag{1}$$

Central Discretization (1D):

$$\frac{C_i^{n+1} - C_i^n}{\Delta t} = D\frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{\Delta x^2} \tag{2}$$

Solving for C_i^{n+1}:

$$C_i^{n+1} = D\Delta t\frac{C_{i+1}^n - 2C_i^n + C_{i-1}^n}{\Delta x^2} + C_i^n \tag{3}$$

Von Neumann Stability Condition:

$$\Delta t \leq \frac{\Delta x^2}{D} \tag{4}$$

Central Discretization (2D):

$$\frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t} = D(\frac{C_{i,j+1}^n - 2C_{i,j}^n + C_{i,j-1}^n}{\Delta x^2} + \frac{C_{i+1,j}^n - 2C_{i,j}^n + C_{i-1,j}^n}{\Delta y^2}) \tag{5}$$

Solving for $C_{i,j}^{n+1}$:

$$C_{i,j}^{n+1} = D\Delta t(\frac{C_{i,j+1}^n - 2C_{i,j}^n + C_{i,j-1}^n}{\Delta x^2} + \frac{C_{i+1,j}^n - 2C_{i,j}^n + C_{i-1,j}^n}{\Delta y^2}) + C_{i,j}^n \tag{6}$$