Simple test examples

Simon Mathis, Nuriya Nurgalieva, Lídia del Rio, and Renato Renner

Institute for Theoretical Physics, ETH Zürich, 8049 Zürich, Switzerland September 27, 2019

Here we describe few simple protocols which can be used to test the rules of consistency and the theory employed for an ability to yield simplest predictions.

Example 1. Suppose that Alice and Bob share a Bell pair $(|00\rangle_{SR} + |11\rangle_{SR})/\sqrt{2}$, where Alice has access to the system S, and Bob has access to the system R. The experiment proceeds as follows:

- t=1. Alice measures her qubit S in basis $\{|0\rangle_S, |1\rangle_S\}$, and records the result in her memory A.
- t=2. Alice makes a prediction about Bob's outcome.
- t=3. Bob measures his qubit R in basis $\{|0\rangle_R, |1\rangle_R\}$, and records the result in his memory B.

Using her knowledge of quantum theory, Alice can run a simulation of the whole experiment (before t_1), and update her instruction registers to reflect the following: "if my measurement outcome is 0, I should predict that Bob will obtain 0; if my measurement outcome is 1, I should predict that Bob will obtain 1." This can be economically encoded by initializing her prediction qubit to $|0\rangle$ (the default prediction), setting her first instruction qubit to $|0\rangle$ ("if I see 0, I should not change my prediction") and her second instruction qubit to $|1\rangle$ ("if I see 1, transform the prediction"). When the experiment actually runs, one can simulate Alice's reasoning by running her circuit between times t_1 and t_2 , obtain her prediction, and correlate it with Bob's outcome at time t_3 .

The way this example is implemented, is explained at length in the corresponding Jupyter Notebook in the folder *simpleExamples*.

- **Example II.** The setting consists of two agents: Alice and Bob. Each experimenter is equipped with a memory qubit. Additionally, there is another system qubit S. The initial state of the S is $\frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S)$. The initial state of the relevant subsystems of the agents' memories is $|0\rangle_A$ and $|0\rangle_B$. The experiment proceeds as follows:
- t=1. Alice measures system S in basis $\{|0\rangle_S, |1\rangle_S\}$, and records the result in her memory A.
- t=2. Bob measures system S in basis $\{|0\rangle_S, |1\rangle_S\}$, and records the result in his memory B.
- t=3. Alice and Bob reason about each other's outcomes.

According to the laws of quantum theory, their results should be the same, and no contradiction should arise. This is implemented in $simple\ example\ II$.

References