Description of Frauchiger-Renner example

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September 16, 2019

The aim of this document is to give the reader the idea and intuition behind the Frauchiger-Renner thought experiment [1], which can be used as an example of a possible protocol for software. The step-by-step explanation of the implementation of the protocol can be found in the Jupyter notebooks in this folder: one with the agents using what we call Copenhagen interpretation, and another in the view of the collapse theories. Here we describe the setting in terms of Copenhagen interpretation to pinpoint the arising paradox.

The setting of the experiment consists of four agents: Alice, Bob, Ursula and Wigner. Each experimenter is equipped with a memory qubit (A, B, U and W, respectively). Additionally, there are two other systems - qubits R and S.

The initial state of the R is $\sqrt{\frac{1}{3}}|0\rangle_R + \sqrt{\frac{2}{3}}|1\rangle_R$. The initial state of S is $|0\rangle_S$. The initial state of the relevant subsystems of the agents' memories is $|0\rangle_A, |0\rangle_B, |0\rangle_U$ and $|0\rangle_W$. The experiment proceeds as follows:

$t = 1$. Alice measures system R in basis $\{|0\rangle_R, |1\rangle_R\}$. She records the result in her memory A, and prepares S accordingly: if she obtains outcome $a = 0$ she keeps her memory in state $|0\rangle_A$ and S in state $|0\rangle_S$; if she obtains outcome $a = 1$ she changes her memory to $|1\rangle_A$ and S to $\frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S)$. Thus, first the joint state of R and her memory A becomes entangled,

$$\left(\sqrt{\frac{1}{3}}|0\rangle_R + \sqrt{\frac{2}{3}}|1\rangle_R\right)|0\rangle_A \quad \longrightarrow \quad \sqrt{\frac{1}{3}}|0\rangle_R|0\rangle_A + \sqrt{\frac{2}{3}}|1\rangle_R|1\rangle_A.$$

When Alice prepares S, it too becomes entangled with R and A,

$$\sqrt{\frac{1}{3}}|0\rangle_R|0\rangle_A|0\rangle_S + \sqrt{\frac{2}{3}}|1\rangle_R|1\rangle_A\ \frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S).$$

Alice sends S to Bob.

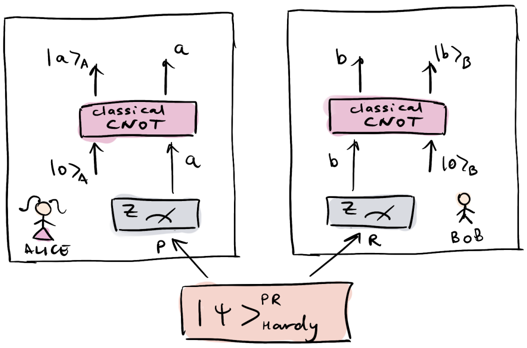
$t = 2$. Bob measures system S in basis $\{|0\rangle_S, |1\rangle_S\}$ and records the outcome b in his memory B, similarly to Alice. The global state of R, A, S and B becomes

$$|\psi\rangle_{RASB} = \frac{1}{\sqrt{3}}\left(|0\rangle_R|0\rangle_A|0\rangle_S|0\rangle_B + |1\rangle_R|1\rangle_A|0\rangle_S|0\rangle_B + |1\rangle_R|1\rangle_A|1\rangle_S|1\rangle_B\right).$$

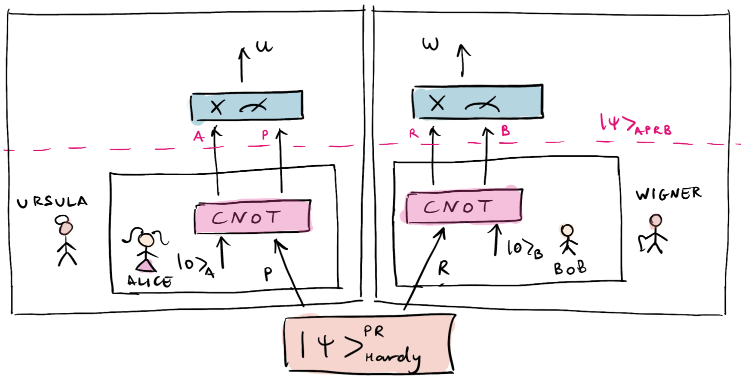
$t = 3$. Ursula measures Alice's lab (consisting of R and the memory A) in basis $\{|ok\rangle_{RA}, |fail\rangle_{RA}\}$, where

$$|ok\rangle_{RA} = \sqrt{\frac{1}{2}}(|0\rangle_R|0\rangle_A - |1\rangle_R|1\rangle_A)$$

$$|fail\rangle_{RA} = \sqrt{\frac{1}{2}}(|0\rangle_R|0\rangle_A + |1\rangle_R|1\rangle_A).$$



(a) Inside perspective (Alice and Bob)



(b) Outside perspective (Ursula and Wigner)

Figure 1: **An entanglement-based version of the Frauchiger-Renner paradox [1] from different perspectives.** Alice and Bob (inside agents) share a Hardy state $|\Psi\rangle_{RS} = (|00\rangle + |10\rangle + |11\rangle)/\sqrt{3}$, measure each their qubit (R and S respectively) and update their memories A and B accordingly. Their labs are contained inside the labs of the outside observers Ursula and Wigner, who can measure the systems RA and SB respectively. The paradox arises when one tries to combine the inside and outside perspectives of quantum measurements on an entangled system into a single perspective. **a)** Alice and Bob measure their halves of $|\Psi\rangle_{RS}$ in the $\{|0\rangle, |1\rangle\}$ (denoted Z) basis to obtain the classical outcomes a and b. They then perform a classical CNOT (i.e., classical copy) to copy their classical outcome into their memories A and B both initialised to $|0\rangle$. **b)** Ursula and Wigner perceive the memory updates as implementing quantum CNOTs on A controlled by R and B controlled by S respectively. The resultant joint state after the memory updates in this case would be $|\Psi\rangle_{ARSB} = 1/\sqrt{3}(|0000\rangle + |1100\rangle + |1111\rangle)$. Hence, they see quantum correlations between the systems and memories of the inside agents. They then measure the joint systems AR and SB in the basis $\{|ok\rangle_{AR} = 1/\sqrt{2}(|00\rangle_{AR} - |11\rangle_{AR}), |fail\rangle_{AR} = 1/\sqrt{2}(|00\rangle_{AR} + |11\rangle_{AR})\}$ (denoted X) to obtain the outcomes u and w. If they obtain $u = w = ok$, the agents can reason about each others' knowledge to arrive at the paradoxical chain of statements $u = w = ok \Rightarrow b = 1 \Rightarrow a = 1 \Rightarrow w = fail$.

$t = 4$. Wigner measures Bob's lab (consisting of S and the memory B) in basis $\{|ok\rangle_{SB}, |fail\rangle_{SB}\}$, where

$$|ok\rangle_{SB} = \sqrt{\frac{1}{2}}(|0\rangle_S|0\rangle_B - |1\rangle_S|1\rangle_B)$$

$$|fail\rangle_{SB} = \sqrt{\frac{1}{2}}(|0\rangle_S|0\rangle_B + |1\rangle_S|1\rangle_B).$$

$t = 5$. Ursula and Wigner compare the outcomes of their measurements. If they were both "ok", they halt the experiment. Otherwise, they reset the timer and all systems to the initial conditions, and repeat the experiment. If we follow the Born rule, this experiment will at some point halt, as the overlap of the global state at time $t = 2.5$ with $|ok\rangle_{RA}|ok\rangle_{SB}$ gives us a probability of 1/12.

It can be shown that since the global state $|\psi\rangle_{RASB}$ can be rewritten as

$$\frac{1}{\sqrt{3}}|0\rangle_R|0\rangle_A|0\rangle_S|0\rangle_B + \sqrt{\frac{2}{3}}|1\rangle_R|1\rangle_A|fail\rangle_{SB},$$

Alice can reason that whenever she finds R in state $|1\rangle_R$, Wigner will obtain outcome "fail" when he measures Bob's lab. In other words, the inference "$a = 1 \implies w = fail$" can be made. Analogously, one can derive inferences "$b = 1 \implies a = 1$" and "$u = ok \implies b = 1$" made by Bob and Ursula respectively. Together with both Ursula and Wigner eventually obtaining "ok", chaining these statements leads to a contradiction: given that Wigner got an outcome "ok", he with certainty predicts that he got outcome "fail". The description of the thought experiment in terms of agents having different perspectives is shown on Figure 1.

The exact assumptions under which this result is derived, can be found in the description file of the folder *Interpretations*.

References

[1] Frauchiger, D. & Renner, R. Quantum theory cannot consistently describe the use of itself. *Nature Communications* **9**, 3711 (2018). URL https://doi.org/10.1038/s41467-018-05739-8.