

# Simple test examples

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Here we describe few simple protocols which can be used to test the rules of consistency and the theory employed for an ability to yield simplest predictions.

**Example 1.** The setting consists of two agents: Alice and Bob. Each experimenter is equipped with a memory qubit. Additionally, there is another system - qubit  $S$ . The initial state of the  $S$  is  $\frac{1}{\sqrt{2}}(|0\rangle_S + |1\rangle_S)$ . The initial state of the relevant subsystems of the agents' memories is  $|0\rangle_A$  and  $|0\rangle_B$ . The experiment proceeds as follows:

- $t = 1$ . Alice measures system  $S$  in basis  $\{|0\rangle_S, |1\rangle_S\}$ , and records the result in her memory  $A$ .
- $t = 2$ . Bob measures system  $S$  in basis  $\{|0\rangle_S, |1\rangle_S\}$ , and records the result in his memory  $B$ .
- $t = 3$ . Alice and Bob reason about each other's outcomes.

According to the laws of quantum theory, their results should be the same, and no contradiction should arise.

**Example 2.** Suppose that Alice and Bob share a Bell pair  $(|00\rangle_{SR} + |11\rangle_{SR})/\sqrt{2}$ , where Alice has access to the system  $S$ , and Bob has access to the system  $R$ . The experiment proceeds as follows:

- $t = 1$ . Alice measures her qubit  $S$  in basis  $\{|0\rangle_S, |1\rangle_S\}$ , and records the result in her memory  $A$ .
- $t = 2$ . Alice makes a prediction about Bob's outcome.
- $t = 3$ . Bob measures his qubit  $R$  in basis  $\{|0\rangle_R, |1\rangle_R\}$ , and records the result in his memory  $B$ .

Using her knowledge of quantum theory, Alice can run a simulation of the whole experiment (before  $t_1$ ), and update her instruction registers to reflect the following: "if my measurement outcome is 0, I should predict that Bob will obtain 0; if my measurement outcome is 1, I should predict that Bob will obtain 1." This can be economically encoded by initializing her prediction qubit to  $|0\rangle$  (the default prediction), setting her first instruction qubit to  $|0\rangle$  ("if I see 0, I should not change my prediction") and her second instruction qubit to  $|1\rangle$  ("if I see 1, transform the prediction"). When the experiment actually runs, one can simulate Alice's reasoning by running her circuit between times  $t_1$  and  $t_2$ , obtain her prediction, and correlate it with Bob's outcome at time  $t_3$ .

## References