Chapter 2: Polynomial Interpolation

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A.

Answer: Construct function to calculate divided differences

```
std::vector<double> dividedDifferences(const std::vector<double>& x, const std::vector<double>& y) {
    int n = x.size();
    std::vector<double> coeffs = y; // Start with y values
    for (int j = 1; j < n; ++j) {
        for (int i = n - 1; i >= j; --i) {
            coeffs[i] = (coeffs[i] - coeffs[i - 1]) / (x[i] - x[i - j]);
        }
    }
    return coeffs;
}
```

In this function, the size of vector x need to be same as that of vector y.

Then, we construct function to calculate the Newton interpolation by accumulating.

```
    double newtonInterpolation(double x, const std::vector<double>& x_points, const std::vector<doubl
    double result = coeffs[0];
    double product = 1.0;
    for (int i = 1; i < coeffs.size(); ++i) {
        product *= (x - x_points[i - 1]);
        result += coeffs[i] * product;
    }
    return result;
}
```

B.

Answer: Construct the function we need:

```
    double f(double x) {
    return 1.0 / (1 + x * x);
}
```

The different divisions can be realized by the function below:

```
 for (int n : n_values) {
        // Generate x_i points
        std::vector<double> x_i, y_i;
        for (int i = 0; i <= n; ++i) {
            double x_val = x_min + i * (x_max - x_min) / n;
            x_i.push_back(x_val);
            y_i.push_back(f(x_val));
        }
```

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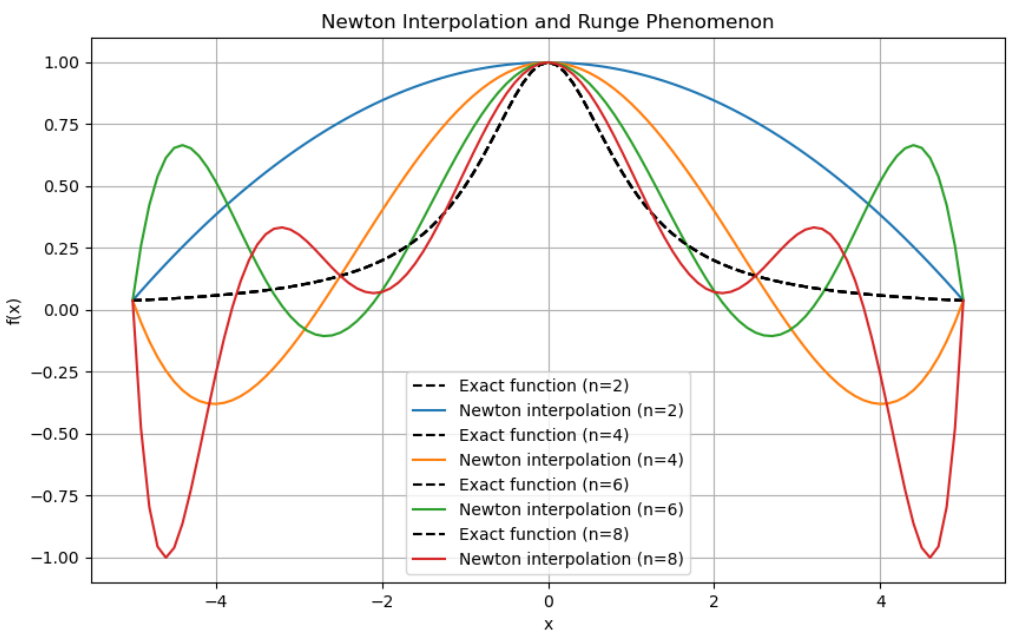
Using the functions constructed above, we can implement Newton interpolate.

```
    std::cout << "n = " << n << ":\n";
        for (int i = 0; i <= num_points; ++i) {
            double x = x_min + i * (x_max - x_min) / num_points;
            double y_exact = f(x);
            double y_interp = newtonInterpolation(x, x_i, coeffs);
            std::cout << std::fixed << std::setprecision(5)
                      << "x: " << x << " Exact: " << y_exact << " Interpolated: " << y_interp << "\n"
        }
        std::cout << "\n";
```

To visualize the result, in the real code we change the output into a *txt* file. The figure output lies below,

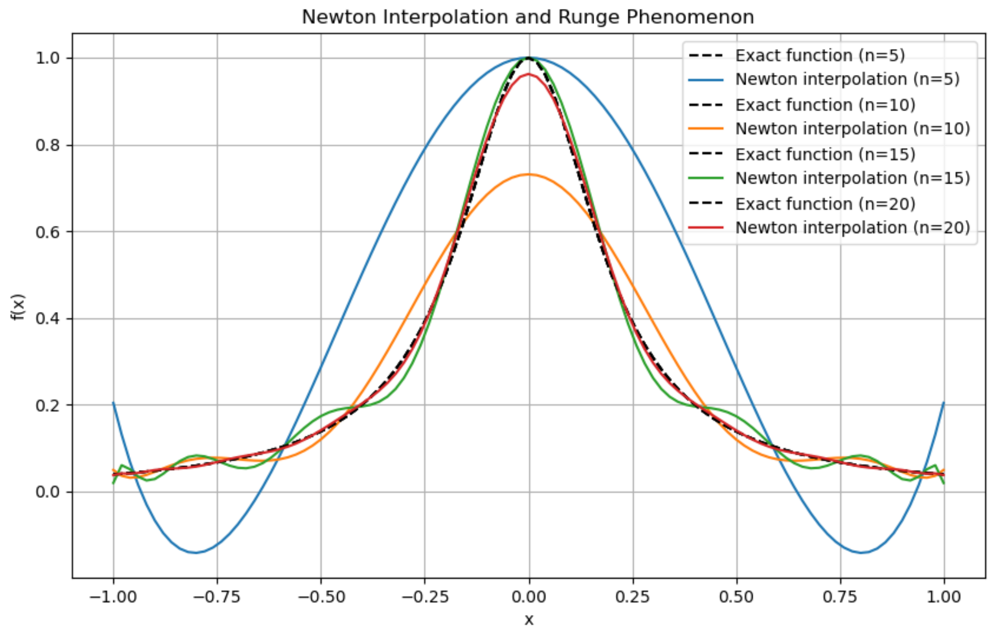


C.

Answer:
Construct function to calculate Chebyshev nodes for interpolation

```
std::vector<double> chebyshevNodes(int n) {
    std::vector<double> nodes;
    for (int k = 0; k < n; ++k) {
        double node = cos(M_PI * (2 * k + 1) / (2 * n));
        nodes.push_back(node);
    }
    return nodes;
}
```

Similar to the problem B, the output can be showed as below: We can easily see that the accuracy is significantly improved when n increasing.



D.

Answer: The divided difference can be calculated by

$$dividedDifferences[i] = \frac{dividedDifferences[i] - dividedDifferences[i-1]}{timePoints[i] - timePoints[i-j]}$$

If the division is 0, the divided difference can be calculated by

$$dividedDifferences[i] = velocities[i/2]$$

The interpolation can be solved through using the Newton accumulation method.
While the result turns out wrong as below,

```
Predicted position at t = 10: -977.897 feet
Predicted speed at t = 10: 6.75285e+08 feet/sec
Did the car exceed 81 feet/sec? Yes
```

E.

Answer: Using the interpolation code mentioned at problem A, we can implement Newton interpolation.

```
Predicted weight for Sp1 at day 43: 14640.3 grams
Predicted weight for Sp2 at day 43: 2981.48 grams
Sp1 larvae might survive after another 15 days.
Sp2 larvae might survive after another 15 days.
```

But the prediction result deviate greatly from the original numbers.

F.

The result can be showed as below

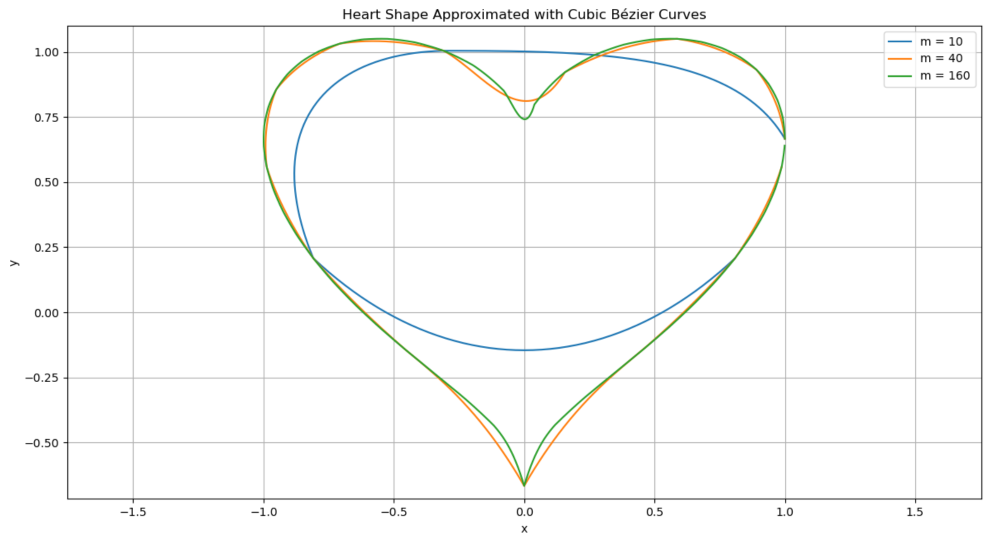


Figure 1: Problem F

Acknowledgments

During the preparation of this work the author used ChatGPT to solve the questions and polish the language. In these work, GPT helped me a lot in transferring algorithms to codes and visualization.