# Bayesian multimodeling: multitask learning

MIPT

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#### Multitask learning

#### Wiki

Multitask Learning is an approach to inductive transfer that improves generalization by using the domain information contained in the training signals of related tasks as an inductive bias. It does this by learning tasks in parallel while using a shared representation; what is learned for each task can help other tasks be learned better.

## Task Clustering and Gating for Bayesian Multitask Learning

Consider a 1-layer neural network:

$$y^i = W^i_{\mathsf{task}} \sigma(W_{\mathsf{shared}} x)$$

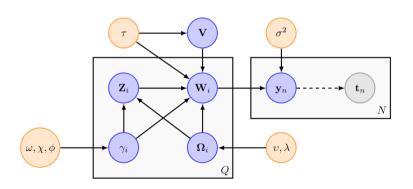
Can we set an interconnection of tasks?

## Task Clustering and Gating for Bayesian Multitask Learning

#### Interconnection of tasks:

- No:  $W_{task}^i \sim \mathcal{N}(\boldsymbol{\mu}_i, \boldsymbol{\Sigma});$
- Gaussian mixture:  $W_{task}^i \sim \sum_j \alpha_j \mathcal{N}(\boldsymbol{\mu}_j, \boldsymbol{\Sigma});$
- Using gating function:  $W_{\mathsf{task}}^i \sim \sum_j \alpha_j \mathcal{N}(\boldsymbol{\mu}_j, \boldsymbol{\Sigma});$

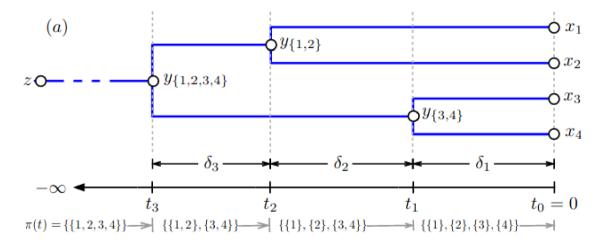
$$\mathsf{y}_n = \mathsf{Wx}_n + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_n;$$
  $\boldsymbol{\varepsilon}_n \sim \mathcal{N}(0, \boldsymbol{\Sigma}).$ 



### Bayesian Multitask Learning with Latent Hierarchies

#### Daume III, 2009

We exploit the intuition that for domain adaptation, we wish to share classifier structure, but for multitask learning, we wish to share covariance structure.



- 1. Choose a global mean and covariance  $(\boldsymbol{\mu}^{(0)}, \boldsymbol{\Lambda}) \sim \mathcal{N}or\mathcal{I}\mathcal{W}(0, \sigma^2 \mathbf{I}, D+1)$ .
- 2. Choose a tree structure  $(\pi, \delta) \sim Coalescent$  over K leaves.
- 3. For each non-root node i in  $\pi$  (top-down):
  - (a) Choose  $\boldsymbol{\mu}^{(i)} \sim \mathcal{N}or(\boldsymbol{\mu}^{(p_{\pi}(i))}, \delta_{i}\boldsymbol{\Lambda})$ , where  $p_{\pi}(i)$  is the parent of i in  $\pi$ .
- 4. For each domain  $k \in [K]$ :
  - (a) Denote by  $\boldsymbol{w}^{(k)} = \boldsymbol{\mu}^{(i)}$  where i is the leaf in  $\pi$  corresponding to domain k.
  - (b) For each example  $n \in [N_k]$ :
    - i. Choose input  $x_n^{(k)} \sim \mathcal{D}^{(k)}$ .
    - ii. Choose output  $y_n^{(k)}$  by:

Regression: 
$$Nor(\boldsymbol{w}^{(k)\top}\boldsymbol{x}_n^{(k)}, \rho^2)$$
  
Classification:  $Bin(1/(1 + e^{-\boldsymbol{w}^{(k)\top}\boldsymbol{x}_n^{(k)}}))$ 

- $\rightarrow$  1. Choose **R** by Eq (2) and deviation covariance  $\Lambda \sim \mathcal{IW}(\sigma^2 \mathbf{I}, D+1)$ .
  - 2. Choose a tree structure  $(\pi, \delta) \sim Coalescent$  over K leaves.
  - 3. For each non-root node i in  $\pi$  (top-down):
  - $\rightarrow$  (a) Choose  $\mathbf{S}^{(i)} \sim \mathcal{N}or(\mathbf{S}^{(p_{\pi}(i))}, \delta_{i}\mathbf{\Lambda})$ , where  $p_{\pi}(i)$  is the parent of i in  $\pi$ .
  - 4. For each task  $k \in [K]$ :
  - $\rightarrow$  (a) Choose  $w^{(k)}$  by (*i* is the leaf associated with task *k*):  $\mathcal{N}or(0, (\exp \mathbf{S}^{(i)})\mathbf{R}(\exp \mathbf{S}^{(i)}))$ 
    - (b) For each example  $n \in [N_k]$ :
      - $\rightarrow$  i. Choose input  $\boldsymbol{x}_n^{(k)} \sim \mathcal{D}$ .
        - ii. Choose output  $y_n^{(k)}$  by:

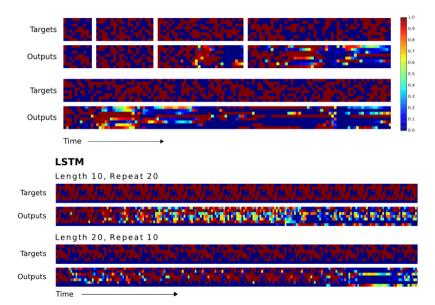
Regression:  $Nor(\boldsymbol{w}^{(k)\top}\boldsymbol{x}_n^{(k)}, \rho^2)$ Classification:  $Bin(1/(1 + e^{-\boldsymbol{w}^{(k)\top}\boldsymbol{x}_n^{(k)}}))$ 

#### Automated Curriculum Learning, 2017

#### Algorithm 1 Intrinsically Motivated Curriculum Learning

$$\begin{aligned} & \textbf{Initially:} \ \ w_i = 0 \ \text{for} \ i \in [N] \\ & \textbf{for} \ t = 1 \dots T \ \textbf{do} \\ & \pi(k) := (1 - \epsilon) \frac{e^{w_k}}{\sum_i e^{w_i}} + \frac{\epsilon}{N} \\ & \text{Draw task index} \ k \ \text{from} \ \pi \\ & \text{Draw training sample} \ \mathbf{x} \ \text{from} \ D_k \\ & \text{Train network} \ p_\theta \ \text{on} \ \mathbf{x} \\ & \text{Compute learning progress} \ \nu \ (\text{Sections 3.1 \& 3.2}) \\ & \text{Map} \ \hat{r} = \nu/\tau(\mathbf{x}) \ \text{to} \ r \in [-1,1] \ (\text{Section 2.3}) \\ & \text{Update} \ w_i \ \text{with reward} \ r \ \text{using Exp3.S} \ (1) \\ & \textbf{end for} \end{aligned}$$

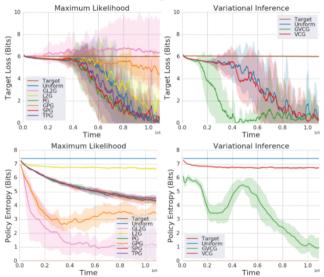
#### Repeat-copy task, 2014



#### Automated Curriculum Learning, 2017

- Loss-driven Progress
  - ▶ Prediction Gain: L(w',x) L(w,x)
  - ► Gradient prediction gain
  - ► Self prediction gain: sampling x
  - ► Mean prediction gain: averagin across the tasks
- Complexity-driven Progress
  - ▶ Variational complexity gain: KL(q|p) KL(q|p)
  - ► Gradient Variational complexity gain
  - ► L2G: difference in *l*<sub>2</sub> regularization
  - ► GL2G: L2G gradient

#### Automated Curriculum Learning, 2017



#### Continual learning

#### **Continual learning**

Continual Learning is a concept to learn a model for a large number of tasks sequentially without forgetting knowledge obtained from the preceding tasks, where the data in the old tasks are not available any more during training new ones.

## Three scenarios for continual learning

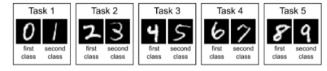
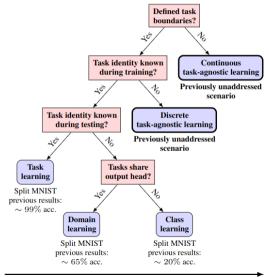


Figure 1: Schematic of split MNIST task protocol.

Table 2: Split MNIST according to each scenario.

Task-IL	With task given, is it the 1 <sup>st</sup> or 2 <sup>nd</sup> class? (e.g., 0 or 1)	
Domain-IL	With task unknown, is it a 1 <sup>st</sup> or 2 <sup>nd</sup> class? (e.g., in [0, 2, 4, 6, 8] or in [1, 3, 5, 7, 9])	
Class-IL	With task unknown, which digit is it? (i.e., choice from 0 to 9)	

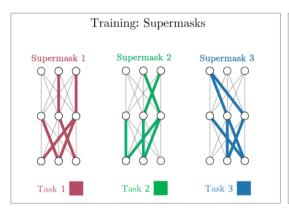
# Task Agnostic Continual Learning Using Online Variational Bayes

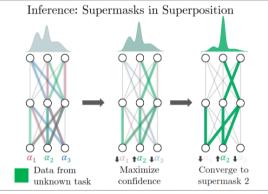


### Continual Learning: task categorization

Scenario	Description	Task space discreet or continuous?	Example methods / task names used
GG	Task Given during train and Given during inference	Either	PNN 42, BatchE 51, PSP 4, "Task learning" 55, "Task-IL" 49
GNs	Task Given during train, Not inference; shared labels	Either	EWC 23, SI 54, "Domain learning" 55, "Domain-IL" 49
GNu	Task Given during train, Not inference; unshared labels	Discrete only	"Class learning" [55], "Class-IL" [49]
NNs	Task Not given during train Nor inference; shared labels	Either	BGD, "Continuous/discrete task agnostic learning" [55]

#### Supermasks in Superposition



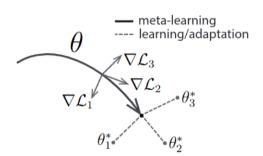


## Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks

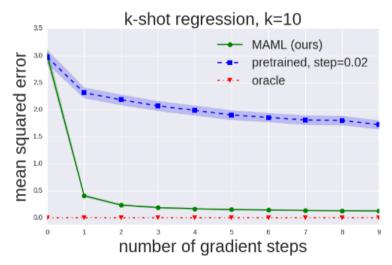
#### Algorithm 1 Model-Agnostic Meta-Learning

**Require:** p(T): distribution over tasks **Require:**  $\alpha$ ,  $\beta$ : step size hyperparameters

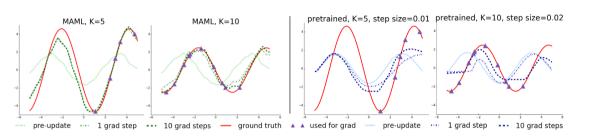
- 1: randomly initialize  $\theta$
- 2: while not done do
- 3: Sample batch of tasks  $\mathcal{T}_i \sim p(\mathcal{T})$
- 4: for all  $\mathcal{T}_i$  do
- 5: Evaluate  $\nabla_{\theta} \mathcal{L}_{\mathcal{T}_i}(f_{\theta})$  with respect to K examples
- 6: Compute adapted parameters with gradient descent:  $\theta'_i = \theta \alpha \nabla_{\theta} \mathcal{L}_{T_i}(f_{\theta})$
- 7: end for
- 8: Update  $\theta \leftarrow \theta \beta \nabla_{\theta} \sum_{\mathcal{T}_i \sim p(\mathcal{T})} \mathcal{L}_{\mathcal{T}_i}(f_{\theta_i'})$
- 9: end while



## Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks



## Model-Agnostic Meta-Learning for Fast Adaptation of Deep Networks



#### References

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