The natural logarithm function

Tobias Rasmussen

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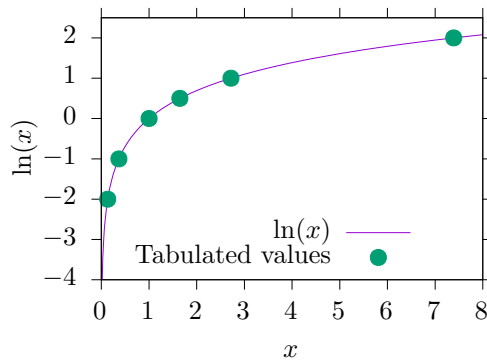

Figure 1: Graph of the natural logarithm function along with some tabulated values.

1 Introduction

The natural logarithm of a number is its logarithm to the base of Euler's number $e \approx 2.718281828459$. The natural logarithm is usually referred to as either $\ln(x)$ or $\log_e(x)$.

The natural logarithm maps multiplication into addition as all other logarithms do:

$$\ln xy = \ln x + \ln y \tag{1}$$

2 Definition

The natural logarithm function can be defined for some $x \in \mathbb{R}_{>0}$ by the integral

$$\ln(x) = \int_1^x \frac{1}{t} dt \tag{2}$$

If $x < 1$, this integral is negative.

3 Properties

The natural logarithm satisfies the following properties, among others:

$$\ln 1 = 0 \tag{3}$$

$$\ln e = 1 \tag{4}$$

$$\ln(xy) = \ln x + \ln y, \quad x, y > 0 \tag{5}$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1 \tag{6}$$

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