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CALCULAR σ_p en el SQUEZEED VACUUM

$$\sigma_{p_{SV}} = \sqrt{\langle b|\hat{p}^2|b\rangle - \langle b|p|b\rangle^2} \qquad b = r e^{i\theta}$$

$$|b\rangle = S_{(b)}|0\rangle \qquad \langle b| = \langle 0|S^{+}_{(b)}$$

$$\langle b|p|b\rangle = \langle 0|S^{+}_{(b)}\, p\, S_{(b)}|0\rangle \qquad S^{+}_{(b)} = S(-b)$$

$$= \langle 0|S_{(-b)}\, p\, S_{(b)}|0\rangle$$

$$S_{(b)}\hat{p}\,S^{+}_{(b)} = -\operatorname{sen}\theta \operatorname{senh} r\, m\omega\,\hat{x} + (\cosh r - \cos\theta \operatorname{senh} r)\,\hat{p}$$

$$= S_{(b)}\,\hat{p}\,S_{(-b)}$$

$$S_{(-b)}\hat{p}\,S_{(b)} = -\operatorname{sen}\theta \operatorname{senh}(-r)\, m\omega\,\hat{x} + (\cosh(-r) - \cos\theta \operatorname{senh}(-r))\,\hat{p}$$

$$= \operatorname{sen}\theta \operatorname{senh} r\, m\omega\,\hat{x} + (\cosh r + \cos\theta \operatorname{senh} r)\,\hat{p}$$

$$\langle b|p|b\rangle = \langle 0|\operatorname{sen}\theta \operatorname{senh} r\, m\omega\,\hat{x} + (\cosh r + \cos\theta \operatorname{senh} r)\,\hat{p}|0\rangle$$

pero $\langle 0|\hat{x}|0\rangle = 0$ y $\langle 0|\hat{p}|0\rangle = 0$ entonces

$$\langle b|p|b\rangle = 0$$

$$\langle b|\hat{p}^2|b\rangle = \langle b|\hat{p}\cdot\hat{p}|b\rangle = \langle 0|S^{+}_{(b)}\,\hat{p}\,\mathbb{I}\,\hat{p}\,S_{(b)}|0\rangle$$

$$= \langle 0|\underbrace{S^{+}_{(b)}\hat{p}\,S_{(b)}}_{= S_{(-b)}\hat{p}\,S_{(b)}}\, S^{+}_{(b)}\,\hat{p}\,S_{(b)}|0\rangle$$

$$S^{+}_{(b)}\hat{p}\,S_{(b)} = \operatorname{sen}\theta \operatorname{senh} r\, m\omega\,\hat{x} + (\cosh r + \cos\theta \operatorname{senh} r)\,\hat{p}$$

$$= A_x\,\hat{x} + A_p\,\hat{p}$$

$$S^{+}_{b}\,\hat{p}\,S_{(b)}\,S^{+}_{(b)}\,\hat{p}\,S_{(b)} = (A_x\hat{x} + A_p\hat{p})(A_x\hat{x} + A_p\hat{p})$$

$$= A_x^2\hat{x}^2 + A_xA_p\,\hat{x}\hat{p} + A_pA_x\,\hat{p}\hat{x} + A_p^2\hat{p}^2$$

$$= A_x^2\hat{x}^2 + A_xA_p(\hat{x}\hat{p} + \hat{p}\hat{x}) + A_p^2\hat{p}^2$$

$$\langle b|p^2|b\rangle = \langle 0|A_x^2\hat{x}^2 + A_xA_p(\hat{x}\hat{p} + \hat{p}\hat{x}) + A_p^2\hat{p}^2|0\rangle$$

$$= A_x^2\langle 0|\hat{x}^2|0\rangle + A_xA_p\langle 0|\hat{x}\hat{p} + \hat{p}\hat{x}|0\rangle + A_p^2\langle 0|\hat{p}^2|0\rangle$$

NOTA

$$\langle 0|\hat{x}^2|0\rangle = \frac{\hbar}{2m\omega} \qquad \langle 0|\hat{p}^2|0\rangle = \frac{m\hbar\omega}{2} \qquad (29.4)$$

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^{+}) \qquad \hat{p} = -i\sqrt{\frac{\hbar m\omega}{2}}\,(a - a^{+})$$

$$\left.\begin{aligned} \hat{x}\hat{p} &= \frac{\hbar}{2}(-i)\,(a^2 - aa^{+} + a^{+}a - a^{+2}) \\ \hat{p}\hat{x} &= \frac{\hbar}{2}(-i)\,(a^2 + aa^{+} - a^{+}a - a^{+2}) \end{aligned}\right\} \hat{x}\hat{p} + \hat{p}\hat{x} = -i\hbar\,(a^2 - a^{+2})$$

$$\langle 0|\hat{x}\hat{p} + \hat{p}\hat{x}|0\rangle = -i\hbar\,\langle 0|a^2|0\rangle + i\hbar\,\langle 0|a^{+2}|0\rangle$$

$$(28.5) \quad a|n\rangle = \sqrt{n}\,|n-1\rangle \qquad a|0\rangle = 0 \qquad a^2|0\rangle = 0$$

$$a^{+}|n\rangle = \sqrt{n+1}\,|n+1\rangle \qquad a^{+}|0\rangle = |1\rangle \qquad a^{+}|1\rangle = \sqrt{2}\,|2\rangle$$

$$\langle 0|\hat{x}\hat{p} + \hat{p}\hat{x}|0\rangle = 0 + i\hbar\,\langle 0|a^{+2}|0\rangle = i\hbar\sqrt{2}\,\langle 0|2\rangle$$

$$(28.4) \quad \langle n_1|n_2\rangle = \delta_{n_1,n_2} \Rightarrow \langle 0|2\rangle = 0$$

$$\langle b|\hat{p}^2|b\rangle = A_x^2 \cdot \frac{\hbar}{2m\omega} + A_p^2\,\frac{m\hbar\omega}{2}$$

$$= \operatorname{sen}^2\theta\,\operatorname{senh}^2 r\, m^2\omega^2 \frac{\hbar}{2m\omega} + (\cosh^2 r + \cos^2\theta\,\operatorname{senh}^2 r + 2\cosh r\,\operatorname{senh} r\,\cos\theta)\frac{m\hbar\omega}{2}$$

$$= \frac{\hbar}{2}\big(\underbrace{\operatorname{sen}^2\theta\,\operatorname{senh}^2 r\, m\omega + m\omega\cos^2\theta\,\operatorname{senh}^2 r}_{m\omega\,\operatorname{senh}^2 r} + m\omega\cosh^2 r + m\omega\, 2\,\mathrm{ch}\, r\,\mathrm{sh}\, r\cos\theta\big)$$

$$= \frac{\hbar m\omega}{2}\big(\underbrace{\operatorname{senh}^2 r + \cosh^2 r}_{\cosh 2r} + \underbrace{2\cosh r\,\operatorname{senh} r}_{\operatorname{senh} 2r}\cos\theta\big)$$

$$= \frac{\hbar m\omega}{2}\,(\cosh 2r + \operatorname{senh} 2r\,\cos\theta)$$

$$\sigma_{p_{sv}} = \sqrt{\frac{\hbar m\omega}{2}}\,\sqrt{\cosh 2r + \operatorname{senh} 2r\,\cos\theta} \qquad \text{si } \sigma_{p_{(0)}} = \sqrt{\frac{\hbar m\omega}{2}}$$

$$\boxed{\sigma_{p_{sv}} = \sigma_{p_{(0)}}\sqrt{\cosh 2r + \operatorname{senh} 2r \cdot \cos\theta}}$$

para $\theta = 0$ $\quad \cosh 2r + \operatorname{senh} 2r = \frac{e^{2r} + e^{-2r}}{2} + \frac{e^{2r} - e^{-2r}}{2} = e^{2r}$

$\sigma_{p_{sv}} = \sigma_{p_{(0)}}\sqrt{e^{2r}} = e^{r}\,\sigma_{p_{(0)}}$ $\quad$ como $r > 0$ $\;e^{r} > 1 \Rightarrow \boxed{\sigma_{p_{sv}} > \sigma_{p_{(0)}}}$