RODOLFO GUIDOBONO

CURSO TEORÍA CUÁNTICA DE CAMPOS
J. GARCÍA - EJERCICIO CAP. 19

HOJA Nº 1

FECHA 13/5/19

Dada la siguiente acción:

$$S = \frac{1}{2}\int d^4x \left[\partial_\mu\phi\,\partial^\mu\phi - m^2\phi^2\right]$$

a) Demostrar que $\mathcal{L} = \frac{1}{2}\left[\partial_\mu\phi\,\partial^\mu\phi - m^2\phi^2\right]$

es invariante bajo la transformación

$$x^{0'} = \gamma x^0 - \gamma\beta x^1$$

$$x^{1'} = -\gamma\beta x^0 + \gamma x^1$$

$$x^{2'} = x^2$$

$$x^{3'} = x^3$$

$$\mathcal{L} = \frac{1}{2}\left[\partial_0\phi\,\partial^0\phi + \partial_1\phi\,\partial^1\phi + \partial_2\phi\,\partial^2\phi + \partial_3\phi\,\partial^3\phi - m^2\phi^2\right]$$

$$\partial_0 = \partial^0$$

$$\partial_x = -\partial^x$$

$$\mathcal{L} = \frac{1}{2}\left[\partial_0\phi\,\partial_0\phi - \partial_1\phi\,\partial_1\phi - \partial_2\phi\,\partial_2\phi - \partial_3\phi\,\partial_3\phi - m^2\phi^2\right]$$

$$= \frac{1}{2}\left[(\partial_0\phi)^2 - (\partial_1\phi)^2 - (\partial_2\phi)^2 - (\partial_3\phi)^2 - m^2\phi^2\right]$$

$$\partial_0\phi_{(x^{\alpha'})} = \frac{\partial\phi}{\partial x^{0'}}\underbrace{\frac{\partial x^{0'}}{\partial x^0}}_{\gamma} + \frac{\partial\phi}{\partial x^{1'}}\cdot\underbrace{\frac{\partial x^{1'}}{\partial x^0}}_{-\gamma\beta} + \frac{\partial\phi}{\partial x^{2'}}\cdot\underbrace{\frac{\partial x^{2'}}{\partial x^0}}_{=0} + \frac{\partial\phi}{\partial x^{3'}}\cdot\underbrace{\frac{\partial x^{3'}}{\partial x^0}}_{=0}$$

$$\partial_0\phi = \gamma\,\partial_{0'}\phi - \gamma\beta\,\partial_{1'}\phi$$

$$\partial_1\phi_{(x^{\alpha'})} = \frac{\partial\phi}{\partial x^{0'}}\underbrace{\frac{\partial x^{0'}}{\partial x^1}}_{-\gamma\beta} + \frac{\partial\phi}{\partial x^{1'}}\cdot\underbrace{\frac{\partial x^{1'}}{\partial x^1}}_{\gamma} + \frac{\partial\phi}{\partial x^{2'}}\cdot\underbrace{\frac{\partial x^{2'}}{\partial x^1}}_{=0} + \frac{\partial\phi}{\partial x^{3'}}\cdot\underbrace{\frac{\partial x^{3'}}{\partial x^1}}_{=0}$$

$$\partial_1\phi = -\gamma\beta\,\partial_{0'}\phi + \gamma\,\partial_{1'}\phi$$

$$\partial_2\phi_{(x^{\alpha'})} = \frac{\partial\phi}{\partial x^{0'}}\cdot\underbrace{\frac{\partial x^{0'}}{\partial x^2}}_{=0} + \frac{\partial\phi}{\partial x^{1'}}\cdot\underbrace{\frac{\partial x^{1'}}{\partial x^2}}_{=0} + \frac{\partial\phi}{\partial x^{2'}}\cdot\underbrace{\frac{\partial x^{2'}}{\partial x^2}}_{=1} + \frac{\partial\phi}{\partial x^{3'}}\cdot\underbrace{\frac{\partial x^{3'}}{\partial x^2}}_{=0}$$

NOTA

HOJA N° 2

FECHA

$$\partial_2 \phi = \partial_{2'} \phi$$

$$\partial_3 \phi_{(x^{\mu'})} = \frac{\partial \phi}{\partial x^{0'}} \cdot \underbrace{\frac{\partial x^{0'}}{\partial x^3}}_{=0} + \frac{\partial \phi}{\partial x^{1'}} \cdot \underbrace{\frac{\partial x^{1'}}{\partial x^3}}_{=0} + \frac{\partial \phi}{\partial x^{2'}} \cdot \underbrace{\frac{\partial x^{2'}}{\partial x^3}}_{=0} + \frac{\partial \phi}{\partial x^{3'}} \cdot \underbrace{\frac{\partial x^{3'}}{\partial x^3}}_{=1}$$

$$\partial_3 \phi = \partial_{3'} \phi$$

$$\mathcal{L} = \frac{1}{2}\Big[(\gamma \partial_{0'} \phi - \gamma\beta \partial_{1'} \phi)^2 - (-\gamma\beta \partial_{0'} \phi + \gamma \partial_{1'} \phi)^2 - (\partial_{2'} \phi)^2 - (\partial_{3'} \phi)^2 -$$
$$- m^2 \phi^2 \Big]$$

$$\mathcal{L} = \frac{1}{2}\Big[\left(\gamma^2 (\partial_{0'} \phi)^2 - 2\gamma^2 \beta \partial_{0'} \phi \, \partial_{1'} \phi + \gamma^2 \beta^2 (\partial_{1'} \phi)^2\right) -$$
$$- \left(\gamma^2 \beta^2 (\partial_{0'} \phi)^2 - 2\gamma^2 \beta \partial_{0'} \phi \, \partial_{1'} \phi + \gamma^2 (\partial_{1'} \phi)^2\right) -$$
$$- (\partial_{2'} \phi)^2 - (\partial_{3'} \phi)^2 - m^2 \phi^2 \Big]$$

$$\mathcal{L} = \frac{1}{2}\Big[(\gamma^2 - \gamma^2 \beta^2)(\partial_{0'} \phi)^2 + (\gamma^2 \beta^2 - \gamma^2)(\partial_{1'} \phi)^2 - (\partial_{2'} \phi)^2 -$$
$$- (\partial_{3'} \phi)^2 - m^2 \phi^2 \Big]$$

$$\gamma^2 - \gamma^2 \beta^2 = \gamma^2 (1 - \beta^2) = 1$$

$$\mathcal{L} = \frac{1}{2}\Big[\partial_{0'} \phi \, \partial_{0'} \phi - \partial_{1'} \phi \, \partial_{1'} \phi - \partial_{2'} \phi \, \partial_{2'} \phi - \partial_{3'} \phi \, \partial_{3'} \phi - m^2 \phi^2 \Big]$$

$$\partial_{0'} = \partial^{0'}$$

$$\partial_{x'} = -\partial^{x'}$$

$$\mathcal{L} = \frac{1}{2}\Big[\partial_{0'} \phi \, \partial^{0'} \phi + \partial_{1'} \phi \, \partial^{1'} \phi + \partial_{2'} \phi \, \partial^{2'} \phi + \partial_{3'} \phi \, \partial^{3'} \phi - m^2 \phi^2 \Big]$$

$$\mathcal{L} = \frac{1}{2}\Big[\partial_{\mu'} \phi \, \partial^{\mu'} \phi - m^2 \phi^2 \Big] \qquad \text{Q.E.D.}$$

NOTA

b) Calcular $\dfrac{\delta S}{\delta \phi} \longrightarrow \dfrac{\delta S}{\delta \phi} = \dfrac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \left(\dfrac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right)$

$$\mathcal{L} = \frac{1}{2}\left[(\partial_0\phi)^2 - (\partial_1\phi)^2 - (\partial_2\phi)^2 - (\partial_3\phi)^2 - m^2\phi^2\right]$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -\frac{1}{2} m^2 \cdot 2\phi = -m^2\phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_0\phi)} = \frac{1}{2} 2 \cdot \partial_0\phi = \partial_0\phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_1\phi)} = -\frac{1}{2} 2\, \partial_1\phi = -\partial_1\phi$$

$$\frac{\partial \mathcal{L}}{\partial(\partial_2\phi)} = -\frac{1}{2} 2\, \partial_2\phi = -\partial_2\phi \qquad \frac{\partial \mathcal{L}}{\partial(\partial_3\phi)} = -\frac{1}{2} 2\, \partial_3\phi = -\partial_3\phi$$

$$\frac{\delta S}{\delta \phi} = \frac{\partial \mathcal{L}}{\partial \phi} - \partial_0\left(\frac{\partial \mathcal{L}}{\partial(\partial_0\phi)}\right) - \partial_1\left(\frac{\partial \mathcal{L}}{\partial(\partial_1\phi)}\right) - \partial_2\left(\frac{\partial \mathcal{L}}{\partial(\partial_2\phi)}\right) - \partial_3\left(\frac{\partial \mathcal{L}}{\partial(\partial_3\phi)}\right)$$

$$\frac{\delta S}{\delta \phi} = -m^2\phi - \partial_0(\partial_0\phi) - \partial_1(-\partial_1\phi) - \partial_2(-\partial_2\phi) - \partial_3(-\partial_3\phi)$$

$$\boxed{\frac{\delta S}{\delta \phi} = -m^2\phi - \partial_0^2\phi + \partial_1^2\phi + \partial_2^2\phi + \partial_3^2\phi}$$

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