- Función promedio $\equiv \langle \square \rangle = \dfrac{\int_{-\infty}^{+\infty} dx\, \square\, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx\, e^{-a/2\, x^2}}$

a) $\langle x \rangle$

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} dx\, x\, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx\, e^{-a/2\, x^2}}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} = \sqrt{\frac{2\pi}{a}} \quad ; \quad \int_{-\infty}^{+\infty} dx\, x\, e^{-\frac{a}{2}x^2} = \quad ; \quad u = \frac{-ax^2}{2} \; ; \; x\,dx = -\frac{1}{a}du$$

$$= \frac{-1}{a}\int_{-\infty}^{+\infty} e^u\, du = -\frac{1}{a}e^u = \frac{-1}{a}e^{-\frac{a}{2}x^2}\Big]_{-\infty}^{+\infty} = -\frac{1}{a}(0-0)$$

$$\boxed{\langle x \rangle = 0}$$

b) $$\int_{-\infty}^{+\infty} dx\, x^2 e^{-\frac{a}{2}x^2} = \frac{\sqrt{\pi}}{2\, a^{3/2}}\, 2^{3/2} = \frac{\sqrt{2\pi}}{a^{3/2}}$$

$$\boxed{\langle x^2 \rangle = \frac{\sqrt{2\pi}}{a^{3/2}}\sqrt{\frac{a}{2\pi}} = \frac{1}{a}}$$

c) $\langle x^{2n} \rangle$

$$\int_{-\infty}^{+\infty} dx\, x^4 e^{-\frac{ax^2}{2}} \quad \text{por parts} \quad \left| \begin{array}{l} u = x^3 \Rightarrow u' = 3x^2 \\ dv = x\, e^{-\frac{ax^2}{2}} \Rightarrow v = -\frac{1}{a}e^{-\frac{ax^2}{2}} \end{array} \right.$$

$$= -\left[\underbrace{\frac{1}{a}x^3 e^{-\frac{ax^2}{2}}}_{0}\right]_{-\infty}^{+\infty} - \underbrace{\int_{-\infty}^{+\infty}\left(-\frac{1}{a}\right)3x^2 e^{-\frac{ax^2}{2}}dx}_{\frac{3}{a}\frac{\sqrt{2\pi}}{a^{3/2}}}$$

$$\langle x^4 \rangle = \frac{\cancel{\sqrt{2\pi}}}{a^{3/2}}\,\frac{3}{a}\cdot\sqrt{\frac{a}{\cancel{2\pi}}} = \frac{3}{a^2}$$

$$\int_{-\infty}^{+\infty} dx\, x^6\, e^{-\frac{a}{2}x^2} \qquad \text{por parts} \begin{cases} u = x^5 & \Longrightarrow\ u' = 5x^4\,dx \\ v' = x\, e^{-\frac{a}{2}x^2} & \Longrightarrow\ v = -\frac{1}{a}\, e^{-\frac{a}{2}x^2} \end{cases}$$

$$= \cancel{\left[\left(-\frac{1}{a}\right) x^5 e^{-\frac{a}{2}x^2}\right]_{-\infty}^{+\infty}} - 5\left(-\frac{1}{a}\right)\int_{-\infty}^{+\infty} x^4 e^{-\frac{a}{2}x^2}\,dx = 5\left(\frac{1}{a}\right)\cdot 3\left(\frac{1}{a}\right)\frac{\sqrt{2\pi}}{a^{3/2}}$$

$$\langle x^6\rangle = \frac{1}{a^2}\, 5.3\, \frac{\sqrt{2\pi}}{a^{3/2}}\cdot\frac{\sqrt{a}}{\cancel{\sqrt{2\pi}}} = 5.3\,\frac{1}{a^3}$$

$$\int_{-\infty}^{+\infty} dx\, x^8\, e^{-\frac{a}{2}x^2} \qquad \text{por parts} \begin{cases} u = x^7 \\ v = x\, e^{-\frac{a}{2}x^2} \end{cases} \longrightarrow$$

$$= \cancel{} + 7\left(\frac{1}{a}\right)\int_{-\infty}^{+\infty} x^6 e^{-\frac{a}{2}x^2}\,dx = 7\left(\frac{1}{a}\right)\cdot 5\left(\frac{1}{a}\right)\cdot 3\left(\frac{1}{a}\right)\frac{\sqrt{2\pi}}{a^{3/2}}$$

En general $\displaystyle\int_{-\infty}^{+\infty} dx\, x^{2n-2}\, e^{-\frac{a}{2}x^2}$ por parts $\left|\begin{array}{l} u = x^{2n-1} \\ v' = x\, e^{-\frac{a}{2}x^2} \end{array}\right.$

$$= \cancel{\left(-\frac{1}{a}\right) x^{2n-1} e^{-\frac{a}{2}x^2}}\Big|_{-\infty}^{+\infty} + (2n-1)\frac{1}{a}\,(2n-3)\frac{1}{a}\,(2n-5)\frac{1}{a}\cdots 5\frac{1}{a}\,3\frac{1}{a}\,1\,\frac{\sqrt{2\pi}}{a^{3/2}}$$

$$\langle x^{2n}\rangle = (2n-1)\frac{1}{a}\cdot(2n-3)\frac{1}{a}\cdot(2n-5)\frac{1}{a}\cdots 5\frac{1}{a}\cdot 3\frac{1}{a}\,\frac{\cancel{\sqrt{2\pi}}}{a^{3/2}}\,\frac{\sqrt{a}}{\cancel{\sqrt{2\pi}}} =$$

$$\boxed{\langle x^{2n}\rangle = \frac{1}{a^n}(2n-1)(2n-3)(2n-5)\cdots 5\cdot 3\cdot 1}$$

Y todos los impares el promedio es 0.

$$I = \int_{-\infty}^{+\infty} dx\, x^3\, e^{-\frac{a}{2}x^2} \qquad \text{por parts} \left|\begin{array}{l} u = x^2 \\ v' = x\, e^{-\frac{a}{2}x^2} \end{array}\right. \Longrightarrow \begin{array}{l} u' = 2x\,dx \\ v = \int dx\, x\, e^{-\frac{a}{2}x^2} = -\frac{1}{a}\, e^{-\frac{a}{2}x^2} \end{array}$$

$$I = \cancel{\left[x^2\left(-\frac{1}{a}\right)e^{-\frac{a}{2}x^2}\right]_{-\infty}^{+\infty}} - \left(-\frac{1}{a}\right)\cdot 2\underbrace{\int_{-\infty}^{+\infty} dx\, x\, e^{-a/2\, x^2}}_{=0 \text{ apartado a)}}$$