Diagramas de Feinman — Teoría Cuántica de Campos — Ejercicio capítulo 5

$$S[\phi] = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

$$Z[J] = \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi}\, d\phi$$

$$Z[0] \cong Z_0[0]\left(1 - \frac{\lambda}{24}\frac{3}{m^4} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{7\cdot5\cdot3}{m^8}\right) = Z_0[0]\left(1 - \frac{\lambda}{8}\frac{1}{m^4} + \frac{\lambda^2}{8^2}\frac{35}{6}\frac{1}{m^8}\right)$$

$$Z^{(2)}[0] \cong Z_0[0]\left(\frac{1}{m^2} - \frac{\lambda}{24}\frac{5\cdot3}{m^6} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{9\cdot7\cdot5\cdot3}{m^{10}}\right) = Z_0[0]\left(\frac{1}{m^2} - \frac{\lambda}{8}\frac{5}{m^6} + \frac{\lambda^2}{8^2}\frac{105}{2}\frac{1}{m^{10}}\right)$$

Así que $\langle\phi^2\rangle = \dfrac{Z^{(2)}[0]}{Z[0]} \cong \dfrac{p_2(\lambda)}{p_1(\lambda)} = p_2(\lambda)\, p_1^{-1}(\lambda) \cong \langle\phi^2\rangle(\lambda)$

B) Hagámoslo por cálculo directo de principio

$$p_1(\lambda) = a_1\lambda^2 + b_1\lambda + c_1 \qquad \left(a_1 = \frac{35}{384 m^8}\ ;\ b_1 = -\frac{1}{8m^4}\ ;\ c_1 = 1\right)$$

$$p_2(\lambda) = a_2\lambda^2 + b_2\lambda + c_2 \qquad \left(a_2 = \frac{105}{128 m^{10}}\ ;\ b_2 = -\frac{5}{8m^6}\ ;\ c_2 = \frac{1}{m^2}\right)$$

$$p_1'(\lambda) = 2a_1\lambda + b_1 \quad ; \quad p_1''(\lambda) = 2a_1$$

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$$\langle\phi^2\rangle(\lambda)\Big|_{\lambda=0} = \frac{p_2(0)}{p_1(0)} = \frac{c_2}{c_1} = \boxed{\frac{1}{m^2} = \langle\phi^2\rangle(\lambda=0)}$$

$$\langle\phi^2\rangle'(\lambda)\Big|_{\lambda=0} \cong \Big[p_2'(\lambda)\, p_1^{-1}(\lambda) - p_2(\lambda)\, p_1^{-2}(\lambda)\, p_1'(\lambda)\Big]_{\lambda=0} = \frac{b_2}{c_1} - \frac{c_2}{c_1^2} b_1 =$$

$$= \frac{-5}{8m^6}\cdot 1 - \frac{1}{m^2}\left(-\frac{1}{8m^4}\right) = -\frac{1}{8m^6}(5-1) = \boxed{-\frac{1}{2m^6} = \langle\phi^2\rangle'(\lambda=0)}$$

$$\langle\phi^2\rangle''(\lambda)\Big|_{\lambda=0} \cong \Big[p_2'' p_1^{-1} - p_2' p_1^{-2} p_1' - p_2' p_1^{-2} p_1' + 2 p_2 p_1^{-3} p_1' p_1' - p_2 p_1^{-2} p_1''\Big]_{\lambda=0} =$$

$$= \Big[p_2'' p_1^{-1} - 2 p_2' p_1' p_1^{-2} + 2 p_2 (p_1')^2 p_1^{-3} - p_2 p_1'' p_1^{-2}\Big]_{\lambda=0} =$$

$$= \frac{2a_2}{c_1} - \frac{2b_2 b_1}{c_1^2} + 2\frac{c_2 b_1^2}{c_1^3} - \frac{2c_2 a_1}{c_1^2} \underset{(c_1=1)}{=} 2a_2 - 2b_2b_1 + 2c_2b_1^2 - 2c_2a_1 =$$

$$= \frac{2\cdot105}{128\, m^{10}} - 2\left(\frac{-5}{8m^6}\right)\left(\frac{-1}{8m^4}\right) + 2\left(\frac{1}{m^2}\right)\left(-\frac{1}{8m^4}\right)^2 - 2\left(\frac{1}{m^2}\right)\left(\frac{35}{384}\frac{1}{m^8}\right)$$

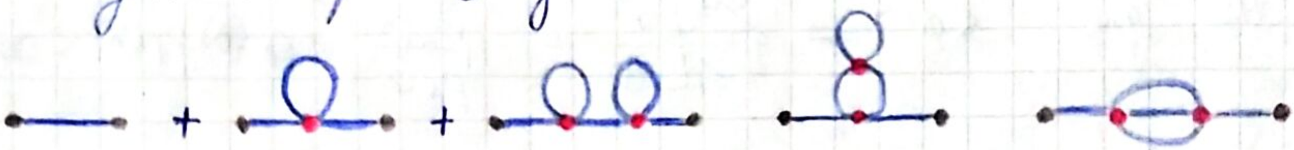
$$\boxed{\langle \phi^2 \rangle(\lambda=0) = \frac{1}{m^{10}}\frac{1}{8^2}\left(105 - 10 + 2 - \frac{35}{3}\right) = \frac{4}{3}\cdot\frac{1}{m^{10}}}$$

Entonces dado que por expansión de Taylor en $\lambda = 0$

$$\langle \phi^2 \rangle(\lambda) \cong \langle \phi^2 \rangle(\lambda=0) + \langle \phi^2 \rangle'(\lambda=0)\cdot\lambda + \frac{1}{2}\langle \phi^2 \rangle''(\lambda=0)\cdot\lambda^2$$

$$\boxed{\langle \phi^2 \rangle(\lambda) \cong \frac{1}{m^2} - \frac{1}{2}\frac{1}{m^6}\lambda + \frac{2}{3}\frac{1}{m^{10}}\lambda^2}$$

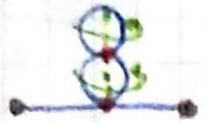
B) Ahora hagámoslo por diagramas de Feinman

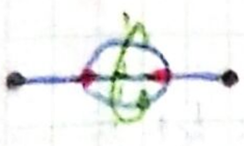
$\langle \phi^2 \rangle \cong$ 

 1 solo línea libre $\Rightarrow \frac{1}{m^2}$

 3 líneas, 1 interacción, 1 eje de simetría $\Rightarrow \left(\frac{1}{m^2}\right)^3\cdot(-\lambda)\left(\frac{1}{2}\right)$

 5 líneas 2 interacciones, 2 ejes de simetría $\Rightarrow \left(\frac{1}{m^2}\right)^5(-\lambda)^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

 5 líneas, 2 interacciones, 2 ejes de simetría $\Rightarrow \left(\frac{1}{m^2}\right)^5(-\lambda)^2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

 5 líneas, 2 interacciones, 3·2 intercambio de líneas $\Rightarrow \left(\frac{1}{m^2}\right)^5(-\lambda)^2\left(\frac{1}{3\cdot 2}\right)$

Así que $\langle \phi^2 \rangle \cong \frac{1}{m^2} - \frac{1}{2}\lambda\left(\frac{1}{m^2}\right)^3 + \lambda^2\left(\frac{1}{m^2}\right)^5\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{6}\right)$

$$\boxed{\langle \phi^2 \rangle \cong \frac{1}{m^2} - \frac{1}{2}\frac{1}{m^6}\lambda + \frac{2}{3}\frac{1}{m^{10}}\lambda^2}$$