EJERCICIO 1 (20:08)

1.a) Calcular $(|\vec{p_c}|^*)^2$

$$f_{(|\vec{p_c}|)} = E_a + E_b - \sqrt{(|\vec{p_c}|)^2 + {m_c}^2} - \sqrt{(|\vec{p_c}|)^2 + {m_d}^2} = 0$$

$$(E_a + E_b)^2 = \left(\sqrt{(|\vec{p_c}|)^2 + {m_c}^2} + \sqrt{(|\vec{p_c}|)^2 + {m_d}^2}\right)^2$$

$$(E_a + E_b)^2 = (|\vec{p_c}|)^2 + {m_c}^2 + (|\vec{p_c}|)^2 + {m_d}^2 + 2\sqrt{(|\vec{p_c}|)^2 + {m_c}^2}\sqrt{(|\vec{p_c}|)^2 + {m_d}^2}$$

$$\frac{(E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)}{2} = (|\vec{p_c}|)^2 + \sqrt{(|\vec{p_c}|)^2 + {m_c}^2}\sqrt{(|\vec{p_c}|)^2 + {m_d}^2}$$

$$\left(\frac{(E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)}{2} - (|\vec{p_c}|)^2\right)^2 = ((|\vec{p_c}|)^2 + {m_c}^2)((|\vec{p_c}|)^2 + {m_d}^2)$$

$$(|\vec{p_c}|)^2 = X$$

$$\frac{(E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)}{2} = A$$

$$(A - X)^2 = (X + {m_c}^2)(X + {m_d}^2)$$

$$A^2 + X^2 - 2AX = X^2 + ({m_c}^2 + {m_d}^2)X + {m_c}^2{m_d}^2$$

$$A^2 - 2AX = ({m_c}^2 + {m_d}^2)X + {m_c}^2{m_d}^2$$

$$A^2 - {m_c}^2{m_d}^2 = ({m_c}^2 + {m_d}^2 + 2A)X$$

$$X = \frac{A^2 - {m_c}^2{m_d}^2}{{m_c}^2 + {m_d}^2 + 2A} = \frac{\left(\frac{(E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)}{2}\right)^2 - {m_c}^2{m_d}^2}{{m_c}^2 + {m_d}^2 + 2\left(\frac{(E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)}{2}\right)}$$

$$X = \frac{1}{4}\ \frac{\left((E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)\right)^2 - 4{m_c}^2{m_d}^2}{({m_c}^2 + {m_d}^2) + (E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)} = \frac{1}{4}\ \frac{\left((E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)\right)^2 - 4{m_c}^2{m_d}^2}{(E_a + E_b)^2}$$

$$X = \frac{1}{4}\left((E_a + E_b) - \frac{({m_c}^2 + {m_d}^2)}{(E_a + E_b)}\right)^2 - \left(\frac{m_c m_d}{(E_a + E_b)}\right)^2$$

$$X = \frac{(E_a + E_b)^2}{4}\left(1 - \frac{({m_c}^2 + {m_d}^2)}{(E_a + E_b)^2}\right)^2 - \left(\frac{m_c m_d}{(E_a + E_b)}\right)^2$$

$$\boxed{(|\vec{p_c}|)^2 = \frac{(E_a + E_b)^2}{4}\left\{\left(1 - \frac{({m_c}^2 + {m_d}^2)}{(E_a + E_b)^2}\right)^2 - \left(\frac{2m_c m_d}{(E_a + E_b)^2}\right)^2\right\}}$$

Verificación

$$(|\vec{p_c}|)^2 + {m_c}^2 = \frac{(E_a + E_b)^2}{4}\left\{\left(1 - \frac{({m_c}^2 + {m_d}^2)}{(E_a + E_b)^2}\right)^2 - \left(\frac{2m_c m_d}{(E_a + E_b)^2}\right)^2\right\} + {m_c}^2$$

$$(|\vec{p_c}|)^2 + {m_c}^2 = \frac{1}{4(E_a + E_b)^2}\left\{\left((E_a + E_b)^2 - ({m_c}^2 + {m_d}^2)\right)^2 - 4{m_c}^2{m_d}^2 + 4{m_c}^2(E_a + E_b)^2\right\}$$

$$(|\overrightarrow{p_c}|)^2+{m_c}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+({m_c}^2+{m_d}^2)^2-2(E_a+E_b)^2({m_c}^2+{m_d}^2)-4{m_c}^2{m_d}^2+4{m_c}^2(E_a+E_b)^2\}$$

$$(|\overrightarrow{p_c}|)^2+{m_c}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+{m_c}^4+2{m_c}^2{m_d}^2+{m_d}^4-2(E_a+E_b)^2{m_c}^2-2(E_a+E_b)^2{m_d}^2-4{m_c}^2{m_d}^2+4{m_c}^2(E_a+E_b)^2\}$$

$$(|\overrightarrow{p_c}|)^2+{m_c}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+{m_c}^4-2{m_c}^2{m_d}^2+{m_d}^4+2(E_a+E_b)^2{m_c}^2-2(E_a+E_b)^2{m_d}^2\}$$

$$(|\overrightarrow{p_c}|)^2+{m_c}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+({m_c}^2-{m_d}^2)^2+2(E_a+E_b)^2({m_c}^2-{m_d}^2)\}$$

$$(|\overrightarrow{p_c}|)^2+{m_c}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^2+({m_c}^2-{m_d}^2)\}^2$$

$$[1]\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}=\frac{1}{2(E_a+E_b)}\{(E_a+E_b)^2+({m_c}^2-{m_d}^2)\}$$

$$(|\overrightarrow{p_c}|)^2+{m_d}^2=\frac{(E_a+E_b)^2}{4}\left\{\left(1-\frac{({m_c}^2+{m_d}^2)}{(E_a+E_b)^2}\right)^2-\left(\frac{2m_cm_d}{(E_a+E_b)^2}\right)^2\right\}+{m_d}^2$$

$$(|\overrightarrow{p_c}|)^2+{m_d}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+({m_c}^2+{m_d}^2)^2-2(E_a+E_b)^2({m_c}^2+{m_d}^2)-4{m_c}^2{m_d}^2+4{m_d}^2(E_a+E_b)^2\}$$

$$(|\overrightarrow{p_c}|)^2+{m_d}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+{m_c}^4-2{m_c}^2{m_d}^2+{m_d}^4-2(E_a+E_b)^2{m_c}^2+2(E_a+E_b)^2{m_d}^2\}$$

$$(|\overrightarrow{p_c}|)^2+{m_d}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^4+({m_c}^2-{m_d}^2)^2-2(E_a+E_b)^2({m_c}^2-{m_d}^2)\}$$

$$(|\overrightarrow{p_c}|)^2+{m_d}^2=\frac{1}{4(E_a+E_b)^2}\{(E_a+E_b)^2-({m_c}^2-{m_d}^2)\}^2$$

$$[2]\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}=\frac{1}{2(E_a+E_b)}\{(E_a+E_b)^2-({m_c}^2-{m_d}^2)\}$$

[1] + [2]

$$\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}+\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}=\frac{1}{2(E_a+E_b)}\{(E_a+E_b)^2+({m_c}^2-{m_d}^2)\}+\frac{1}{2(E_a+E_b)}\{(E_a+E_b)^2-({m_c}^2-{m_d}^2)\}$$

$$\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}+\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}=(E_a+E_b)$$

$$f_{(|\overrightarrow{p_c}|)}=E_a+E_b-\left(\sqrt{(|\overrightarrow{p_c}|)^2+{m_c}^2}+\sqrt{(|\overrightarrow{p_c}|)^2+{m_d}^2}\right)=0$$

1.b) Calcular $f'|_{|\vec{p_c}|^*}$

$$f_{(|\vec{p_c}|)} = E_a + E_b - \sqrt{(|\vec{p_c}|)^2 + {m_c}^2} - \sqrt{(|\vec{p_c}|)^2 + {m_d}^2}$$

$$f' = -\frac{1}{2}\frac{1}{\sqrt{(|\vec{p_c}|)^2 + {m_c}^2}} 2|\vec{p_c}| - \frac{1}{2}\frac{1}{\sqrt{(|\vec{p_c}|)^2 + {m_d}^2}} 2|\vec{p_c}|$$

$$f'|_{|\vec{p_c}|^*} = -\frac{|\vec{p_c}|^*}{\sqrt{(|\vec{p_c}|^*)^2 + {m_c}^2}} - \frac{|\vec{p_c}|^*}{\sqrt{(|\vec{p_c}|^*)^2 + {m_d}^2}}$$

Recordemos:

1) $E_c = \sqrt{(|\vec{p_c}|^*)^2 + {m_c}^2}$
2) Por definición: $E_d = \sqrt{(|\vec{p_c}|^*)^2 + {m_d}^2}$

   Lo cual puede verse en el minuto 15:47 del video del capítulo, en el que Javier explicita la fórmula de σ

$$\sigma = \frac{1}{64\pi^2}\frac{1}{E_a E_b |v_a - v_b|}\int d^3p_c \frac{|\mathcal{M}|^2}{E_c E_d}\delta_{(E_a+E_b-E_c-E_d)}$$

Por lo tanto:

$$\boxed{\boldsymbol{f'|_{|\vec{p_c}|^*} = -\frac{|\vec{p_c}|^*}{E_c} - \frac{|\vec{p_c}|^*}{E_d}}}$$

EJERCICIO 2 (37:41)

Determinar las dimensiones de [σ]

$$[\boldsymbol{\sigma}] = \frac{[\boldsymbol{\lambda}]^2}{[\boldsymbol{E}]^2}$$

Partimos de $\mathcal{L}$ que es la densidad lagrangiana, igual a (38.1 del formulario de Krul – Lagrangiano de Klein Gordon)

$$\mathcal{L}(\phi,\dot{\phi},\phi') = \frac{1}{2}(\dot{\phi})^2 - \frac{1}{2}(\phi')^2 - \frac{1}{2}m^2\phi^2$$

Trabajemos con los valores de $\hbar$ y c, de modo que la densidad resulta

$$\mathcal{L}(\phi,\dot{\phi},\phi') = \frac{1}{2}(\dot{\phi})^2 - \frac{1}{2}(\phi')^2 - \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2\phi^2$$

$$[\mathcal{L}] = \frac{[E]}{[L]^3}$$

Por consistencia debe resultar

$$\frac{[E]}{[L]^3} = \left[\frac{mc}{\hbar}\right]^2 [\phi]^2$$

$$[\hbar] = [E][T]$$

$$\frac{[E]}{[L]^3} = \left[\frac{mc}{\hbar}\right]^2 [\phi]^2 = \left(\frac{[M]\,{}^{[L]}\!/_{[T]}}{[E][T]}\right)^2 [\phi]^2 = \left(\frac{[M][L]}{[E][T]^2}\right)^2 [\phi]^2$$

$$[E] = \frac{[M][L]^2}{[T]^2}$$

$$\frac{[E]}{[L]^3} = \left(\frac{{}^{[E]}\!/_{[L]}}{[E]}\right)^2 [\phi]^2$$

$$[\phi]^2 = \frac{[E]}{[L]}$$

$$\boxed{[\phi] = \sqrt{\frac{[E]}{[L]}}}$$

Volvemos a utilizar unidades naturales, c=ħ=1, para las cuales $[E] = [M]$; entonces:

$$[\phi]^2 = \frac{[M]}{[L]}$$

Para la teoría λϕ4 la densidad lagrangiana (seguimos con unidades naturales) es (capítulo 78, de este curso de Javier):

$$\mathcal{L}\left(\phi,\dot{\phi},\phi^{'}\right)=\frac{1}{2}(\dot{\phi})^{2}-\frac{1}{2}\left(\phi^{'}\right)^{2}-\frac{1}{2}m^{2}\phi^{2}-\frac{\lambda}{4!}\phi^{4}$$

Para ser dimensionalmente consistente debe cumplirse que:

$$[M]^{2}[\phi]^{2}=[\lambda][\phi]^{4}$$

$$[\lambda]=\frac{[M]^{2}}{[\phi]^{2}}=\frac{[M]^{2}}{[M]/[L]}=[M][L]$$

$$[\sigma]=\frac{[\lambda]^{2}}{[E]^{2}}=\frac{[\lambda]^{2}}{[M]^{2}}=\frac{([M][L])^{2}}{[M]^{2}}$$

$$\boxed{[\boldsymbol{\sigma}]=[\boldsymbol{L}]^{2}}$$