RODOLFO GUIDOBONO | CURSO TEORÍA CUÁNTICA DE CAMPOS J. GARCÍA - EJERCICIO CAP. 8 | HOJA N° 1 | FECHA 8/3/19

Calcular el valor esperado de $\phi_a \phi_b \phi_c \phi_d$

$$\langle \phi_a \phi_b \phi_c \phi_d \rangle = \frac{1}{Z[0]} \left. \frac{\partial}{\partial J_a} \frac{\partial}{\partial J_b} \frac{\partial}{\partial J_c} \frac{\partial}{\partial J_d} Z[J] \right|_{J=0}$$

$$Z[J] = Z[0] \exp\left(\frac{1}{2m^2} J^T A^{-1} J \right)$$

$$f(J) = \frac{1}{2m^2} A^{-1}_{ij} J_i J_j \longrightarrow Z[J] = Z[0]\, e^{f(J)} \quad (1)$$

$$\frac{\partial f(J)}{\partial J_k} = \frac{1}{2m^2} \left(2 A^{-1}_{kk} J_k + A^{-1}_{ij} \frac{\partial J_i}{\partial J_k} J_j + A^{-1}_{ij} J_i \frac{\partial J_j}{\partial J_k} \right)$$

$$= \frac{1}{2m^2} \left(2 A^{-1}_{kk} J_k + A^{-1}_{ij} \delta^i_k J_j + A^{-1}_{ij} J_i \delta^j_k \right)$$

$$= \frac{1}{2m^2} \left(2 A^{-1}_{kk} J_k + A^{-1}_{kj} J_j + A^{-1}_{ik} J_i \right)$$

$$i = j \text{ y } A^{-1}_{kj} = A^{-1}_{jk}$$

$$= \frac{1}{2m^2} \left(2 A^{-1}_{kk} J_k + 2 A^{-1}_{kj} J_j \right) = \frac{1}{m^2} \left(A^{-1}_{kk} J_k + A^{-1}_{kj} J_j \right) \quad (2)$$

$$\frac{\partial Z(J)}{\partial J_a} = Z[0]\, e^{f(J)} \frac{\partial f(J)}{\partial J_a} = Z(0)\, e^{f(J)} \frac{1}{m^2} \left(A^{-1}_{aa} J_a + A^{-1}_{aj} J_j \right)$$

$$\frac{\partial Z(J)}{\partial J_a J_b} = Z[0] \left[e^{f(J)} \underbrace{\frac{1}{m^2} \left(A^{-1}_{bb} J_b + A^{-1}_{bi} J_i \right)}_{\text{ver } (2)} \frac{1}{m^2} \left(A^{-1}_{aa} J_a + A^{-1}_{aj} J_j \right) + \right.$$

$$\left. + e^{f(J)} \frac{1}{m^2} A^{-1}_{ab} \right]$$

$$\frac{\partial}{\partial J_b} \left(A^{-1}_{aa} J_a + A^{-1}_{aj} J_j \right) = A^{-1}_{ab}$$

$$\frac{\partial Z(J)}{\partial J_a J_b} = \frac{Z[0]}{m^2} e^{f(J)} \left[\underbrace{\frac{1}{m^2} \left(A^{-1}_{bb} J_b + A^{-1}_{bi} J_i \right) \left(A^{-1}_{aa} J_a + A^{-1}_{aj} J_j \right) + A^{-1}_{ab}}_{\text{llamemos a esto } P_{n1}(J)} \right]$$

NOTA

(3) $P_{n1}(J) = A^{-1}_{bb} J_b A^{-1}_{ac} J_a + A^{-1}_{bb} J_b A^{-1}_{aj} J_j + A^{-1}_{bi} J_i A^{-1}_{aa} J_a + A^{-1}_{bi} J_i \times A^{-1}_{aj} J_j$

$$\frac{\partial Z(J)}{\partial J_a J_b} = \frac{Z(0)}{m^2} e^{f(J)} \left[\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab} \right]$$

$$\frac{\partial Z(J)}{\partial J_a J_b J_c} = \frac{Z(0)}{m^2} \left[e^{f(J)} \frac{1}{m^2} \left(A^{-1}_{cc} J_c + A^{-1}_{ci} J_i \right) \left(\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab} \right) + \right.$$
$$\left. + e^{f(J)} \frac{1}{m^2} \frac{\partial P_{n1}(J)}{\partial J_c} \right]$$

$$\frac{\partial Z(J)}{\partial J_a J_b J_c} = \frac{Z(0)}{m^4} e^{f(J)} \left[\underbrace{\left(A^{-1}_{cc} J_c + A^{-1}_{ci} J_i \right) \left(\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab} \right) + \frac{\partial P_{n1}(J)}{\partial J_c}}_{\text{llamemos a esto } P_{n2}(J)} \right]$$

(4) $$P_{n2}(J) = \left(A^{-1}_{cc} J_c + A^{-1}_{ci} J_i \right) \left(\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab} \right) + \frac{\partial P_{n1}(J)}{\partial J_c}$$

$$\frac{\partial Z(J)}{\partial J_a J_b J_c} = \frac{Z(0)}{m^4} e^{f(J)} P_{n2}(J)$$

(5) $$\frac{\partial Z(J)}{\partial J_a J_b J_c J_d} = \frac{Z(0)}{m^4} \left[e^{f(J)} \frac{1}{m^2} \left(A^{-1}_{dd} J_d + A^{-1}_{di} J_i \right) P_{n2}(J) + e^{f(J)} \frac{\partial P_{n2}(J)}{\partial J_d} \right]$$
$$= \frac{Z(0)}{m^4} e^{f(J)} \left[\frac{1}{m^2} \left(A^{-1}_{dd} J_d + A^{-1}_{di} J_i \right) P_{n2}(J) + \frac{\partial P_{n2}(J)}{\partial J_d} \right]$$

Derivamos (4) respecto a J_d

$$\frac{\partial P_{n2}(J)}{\partial J_d} = A^{-1}_{ci} \frac{\partial J_i}{\partial J_d} \left(\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab} \right) + \left(A^{-1}_{cc} J_c + A^{-1}_{ci} J_i \right) \frac{1}{m^2} \frac{\partial P_{n1}(J)}{\partial J_d} +$$
$$+ \frac{\partial}{\partial J_d} \left(\frac{\partial P_{n1}(J)}{\partial J_c} \right)$$

(6) $$\frac{\partial P_{n2}(J)}{\partial J_d} = A^{-1}_{cd}\left(\frac{1}{m^2} P_{n1}(J) + A^{-1}_{ab}\right) + \left(A^{-1}_{cc} J_c + A^{-1}_{ci} J_i\right)\frac{1}{m^2}\frac{\partial P_{n1}(J)}{\partial J_d} + \frac{\partial}{\partial J_d}\left(\frac{\partial P_{n1}(J)}{\partial J_c}\right)$$

Derivamos (3) respecto a J_c

$$\frac{\partial P_{n1}(J)}{\partial J_c} = A^{-1}_{bb} J_b A^{-1}_{aj}\frac{\partial J_j}{\partial J_c} + A^{-1}_{bi}\frac{\partial J_i}{\partial J_c} A^{-1}_{aa} J_a + A^{-1}_{bi}\frac{\partial J_i}{\partial J_c} A^{-1}_{aj} J_j + A^{-1}_{bi} J_i \cdot A^{-1}_{aj}\frac{\partial J_j}{\partial J_c}$$

$$\frac{\partial P_{n1}(J)}{\partial J_c} = A^{-1}_{bb} J_b A^{-1}_{ac} + A^{-1}_{bc} A^{-1}_{aa} J_a + A^{-1}_{bc} A^{-1}_{aj} J_j + A^{-1}_{bi} J_i A^{-1}_{ac}$$

(7) $$\frac{\partial}{\partial J_d}\left(\frac{\partial P_{n1}(J)}{\partial J_c}\right) = A^{-1}_{bc} A^{-1}_{aj}\frac{\partial J_j}{\partial J_d} + A^{-1}_{bi}\frac{\partial J_i}{\partial J_d} A^{-1}_{ac} = A^{-1}_{bc} A^{-1}_{ad} + A^{-1}_{bd} A^{-1}_{ac}$$

En (5) tenemos a $\dfrac{\partial Z(J)}{\partial J_a J_b J_c J_d}$.

$$\frac{\partial Z(J)}{\partial J_a \partial J_b J_c J_d} = \frac{Z(0)}{m^4} e^{f(J)}\left[\frac{1}{m^2}\left(A^{-1}_{dd} J_d + A^{-1}_{di} J_i\right) P_{n2}(J) + \frac{\partial P_{n2}(J)}{\partial J_d}\right]$$

Cuando hacemos $J=0$, el término $e^{f(J)}=1$ y el primer término del paréntesis se anula, quedando:

$$\left.\frac{\partial Z(J)}{\partial J_a J_b J_c J_d}\right|_{J=0} = \frac{Z(0)}{m^4} \cdot \left.\frac{\partial P_{n2}(J)}{\partial J_d}\right|_{J=0}$$

Si aplicamos $J=0$ a la fórmula (6) queda

$$\left.\frac{\partial P_{n2}(J)}{\partial J_d}\right|_{J=0} = A^{-1}_{cd} \cdot A^{-1}_{ab} + \underbrace{\left.\frac{\partial}{\partial J_d}\left(\frac{\partial P_{n1}(J)}{\partial J_c}\right)\right|_{J=0}}$$

$$\overset{(7)}{=} A^{-1}_{bc} A^{-1}_{ad} + A^{-1}_{bd} A^{-1}_{ac}$$

NOTA

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$$\left.\frac{\partial Z(J)}{\partial J_a J_b J_c J_d}\right|_{J=0} = \frac{Z(0)}{m^4}\left(A^{-1}_{cd} A^{-1}_{ab} + A^{-1}_{bc} A^{-1}_{ad} + A^{-1}_{bd} A^{-1}_{ac}\right)$$

$$\langle \phi_a \phi_b \phi_c \phi_d \rangle = \frac{1}{m^4}\left(A^{-1}_{cd} A^{-1}_{ab} + A^{-1}_{bc} A^{-1}_{ad} + A^{-1}_{bd} A^{-1}_{ac}\right)$$

NOTA