RODOLFO
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EJ. 1 Probar que $[a_p^+ a_n, a_n^+ a_p] = a_p^+ a_p - a_n^+ a_n$

Fermiones $\begin{cases} a_i a_j^+ + a_j^+ a_i = \delta_{ij} \\ a_i a_j + a_j a_i = 0 \end{cases}$

Corchetes de Poisson (cap. 20, curso mecánica teórica de Javier)

$$[AB, C] = A[B,C] + [A,C]B$$

$$[AB] = -[BA]$$

$$[\alpha A + \beta B, C] = \alpha [A,C] + \beta [B,C]$$

$$[a_p^+ a_n, a_n^+ a_p] = a_p^+ [a_n, a_n^+ a_p] + [a_p^+, a_n^+ a_p] a_n$$

$$= -a_p^+ [a_n^+ a_p, a_n] - [a_n^+ a_p, a_p^+] a_n$$

$$= -a_p^+ \left(a_n^+ [a_p, a_n] + [a_n^+, a_n] a_p \right)$$

$$- \left(a_n^+ [a_p, a_p^+] + [a_n^+, a_p^+] a_p \right) a_n$$

$$= -a_p^+ a_n^+ (a_p a_n - a_n a_p) - a_p^+ (a_n^+ a_n - a_n a_n^+) a_p$$

$$- a_n^+ (a_p a_p^+ - a_p^+ a_p) a_n - (a_n^+ a_p^+ - a_p^+ a_n^+) a_p a_n$$

$$= \cancel{- a_p^+ a_n^+ (2 a_p a_n)} - a_p^+ (a_n^+ a_n - a_n a_n^+) a_p$$

$$- a_n^+ (a_p a_p^+ - a_p^+ a_p) a_n - \cancel{(\underbrace{2 a_n^+ a_p^+}_{= (-2 a_p^+ a_n^+)}) a_p a_n}$$

$$= - a_p^+ (\underbrace{a_n^+ a_n - a_n a_n^+}_{= (-1 + 2 a_n^+ a_n)}) a_p - a_n^+ (\underbrace{a_p a_p^+ - a_p^+ a_p}_{(1 - 2 a_p^+ a_p)}) a_n$$

$$= a_p^+ a_p - \cancel{2 a_p^+ a_n^+ a_n a_p} - a_n^+ a_n + 2 \underbrace{a_n^+ a_p^+}_{= (-a_p^+ a_n^+)} \underbrace{a_p a_n}_{= (-a_n a_p)}$$

$$= a_p^+ a_p - a_n^+ a_n$$

$$\boxed{[a_p^+ a_n, a_n^+ a_p] = a_p^+ a_p - a_n^+ a_n}$$

Ej. 2 Dados $I_1 \equiv 1/2\,(a_p^+ a_n + a_n^+ a_p)$ probar que

$I_2 \equiv 1/2i\,(a_p^+ a_n - a_n^+ a_p)$ $\qquad [I_i, I_j] = i\,\varepsilon_{ijk}\, I_k$

$I_3 \equiv 1/2\,(a_p^+ a_p - a_n^+ a_n)$

$$[I_1, I_1] = \left[\frac{1}{2}(a_p^+ a_n + a_n^+ a_p), \frac{1}{2}(a_p^+ a_n + a_n^+ a_p)\right]$$

$$= \frac{1}{4}\left[a_p^+ a_n, (a_p^+ a_n + a_n^+ a_p)\right] + \frac{1}{4}\left[a_n^+ a_p, (a_p^+ a_n + a_n^+ a_p)\right]$$

$$= \underbrace{\frac{1}{4}\left[a_p^+ a_n, a_p^+ a_n\right]}_{=0} + \frac{1}{4}\left[a_p^+ a_n, a_n^+ a_p\right] + \frac{1}{4}\left[a_n^+ a_p, a_p^+ a_n\right] + \underbrace{\frac{1}{4}\left[a_n^+ a_p, a_n^+ a_p\right]}_{=0}$$

$$= \frac{1}{4}\left[a_p^+ a_n, a_n^+ a_p\right] + \frac{1}{4}\left[a_n^+ a_p, a_p^+ a_n\right]$$

$$= \frac{1}{4}\left[a_p^+ a_n, a_n^+ a_p\right] - \frac{1}{4}\left[a_p^+ a_n, a_n^+ a_p\right]$$

$[I_1, I_1] = 0$ del mismo modo $[I_2, I_2] = 0$ $\quad [I_3, I_3] = 0$

$$[I_1, I_2] = \left[\frac{1}{2}(a_p^+ a_n + a_n^+ a_p), \frac{1}{2i}(a_p^+ a_n - a_n^+ a_p)\right]$$

$$= \frac{1}{4i}\left\{\left[a_p^+ a_n, (a_p^+ a_n - a_n^+ a_p)\right] + \left[a_n^+ a_p, (a_p^+ a_n - a_n^+ a_p)\right]\right\}$$

$$= \frac{1}{4i}\left\{\underbrace{\left[a_p^+ a_n, a_p^+ a_n\right]}_{=0} - \left[a_p^+ a_n, a_n^+ a_p\right] + \left[a_n^+ a_p, a_p^+ a_n\right] - \underbrace{\left[a_n^+ a_p, a_n^+ a_p\right]}_{=0}\right\}$$

$$= \frac{1}{4i}\left\{-\left[a_p^+ a_n, a_n^+ a_p\right] + \underbrace{\left[a_n^+ a_p, a_p^+ a_n\right]}_{-[a_p^+ a_n,\, a_n^+ a_p]}\right\} = -\frac{2}{4i}\underbrace{\left[a_p^+ a_n, a_n^+ a_p\right]}_{\overset{(ej.1)}{=}\, a_p^+ a_p - a_n^+ a_n}$$

$$= -\frac{1}{2i}(a_p^+ a_p - a_n^+ a_n) = \frac{1}{2} i\,(a_p^+ a_p - a_n^+ a_n)$$

$$\boxed{[I_1, I_2] = i\, I_3}$$

$$[I_1, I_3] = \left[\tfrac{1}{2}(a_p^+ a_n + a_n^+ a_p), \tfrac{1}{2}(a_p^+ a_p - a_n^+ a_n)\right]$$

$$= \frac{1}{4}\left\{ [a_p^+ a_n, (a_p^+ a_p - a_n^+ a_n)] + [a_n^+ a_p, (a_p^+ a_p - a_n^+ a_n)] \right\}$$

$$= \frac{1}{4}\left\{ [a_p^+ a_n, a_p^+ a_p] - [a_p^+ a_n, a_n^+ a_n] + [a_n^+ a_p, a_p^+ a_p] - [a_n^+ a_p, a_n^+ a_n] \right\}$$

$\rightarrow$ $$[a_p^+ a_n, a_p^+ a_p] = a_p^+ [a_n, a_p^+ a_p] + [a_p^+, a_p^+ a_p] a_n$$

$$= -a_p^+ [a_p^+ a_p, a_n] - [a_p^+ a_p, a_p^+] a_n$$

$$= -a_p^+ \left(a_p^+ [a_p, a_n] + [a_p^+, a_n] a_p \right) - \left(a_p^+ [a_p, a_p^+] + \underbrace{[a_p^+, a_p^+]}_{=0} a_p \right) a_n$$

$$= -\underbrace{a_p^+ a_p^+}_{=0} [a_p, a_n] - a_p^+ [a_p^+, a_n] a_p - a_p^+ [a_p, a_p^+] a_n$$

$$= -a_p^+ [a_p^+, a_n] a_p - a_p^+ [a_p, a_p^+] a_n$$

$\rightarrow$ $$[a_p^+ a_n, a_n^+ a_n] = a_p^+ [a_n, a_n^+ a_n] + [a_p^+, a_n^+ a_n] a_n$$

$$= -a_p^+ [a_n^+ a_n, a_n] - [a_n^+ a_n, a_p^+] a_n$$

$$= -a_p^+ \left(a_n^+ \underbrace{[a_n, a_n]}_{=0} + [a_n^+, a_n] a_n \right) - \left(a_n^+ [a_n, a_p^+] + [a_n, a_p^+] a_n \right)$$

$$= -a_p^+ [a_n^+, a_n] a_n - a_n^+ [a_n, a_p^+] a_n + [a_n, a_p^+] \underbrace{a_n a_n}_{=0} \quad \times a_n$$

$$= -a_p^+ [a_n^+, a_n] a_n - a_n^+ [a_n, a_p^+] a_n$$

$\rightarrow$ $$[a_n^+ a_p, a_p^+ a_p] = a_n^+ [a_p, a_p^+ a_p] + [a_n^+, a_p^+ a_p] a_p$$

$$= -a_n^+ [a_p^+ a_p, a_p] - [a_p^+ a_p, a_n^+] a_p$$

$$= -a_n^+ \left(a_p^+ \underbrace{[a_p, a_p]}_{=0} + [a_p^+, a_p] a_p \right) - \left(a_p^+ [a_p, a_n^+] + [a_p, a_n^+] a_p \right) a_p$$

$$= -a_n^+ [a_p^+, a_p] a_p - a_p^+ [a_p, a_n^+] a_p - [a_p, a_n^+] \underbrace{a_p a_p}_{=0}$$

$$= -a_n^+ [a_p^+, a_p] a_p - a_p^+ [a_p, a_n^+] a_p$$

$\rightarrow$ $$[a_n^+ a_p, a_n^+ a_n] = a_n^+ [a_p, a_n^+ a_n] + [a_n^+, a_n^+ a_n] a_p$$

$$= -a_n^+ \left(a_n^+ [a_n, a_p] + [a_n^+, a_p] a_n \right) - \left(a_n^+ [a_n, a_n^+] + \overbrace{[a_n^+, a_n^+]}^{=0} a_n \right)$$

$$= -\underbrace{a_n^+ a_n^+}_{=0} [a_n, a_p] - a_n^+ [a_n^+, a_p] a_n - a_n^+ [a_n, a_n^+] a_p \quad \times a_p$$

$$= -a_n^+ [a_n^+, a_p] a_n - a_n^+ [a_n, a_n^+] a_p$$

$$[I_1, I_3] = \frac{1}{4}\Big\{ -a_p^+[a_p^+, a_n]a_p - a_p^+[a_p, a_p^+]a_n + a_p^+[a_n^+, a_n]a_n + a_n^+[a_n, a_p^+]a_n$$

$$- a_n^+[a_p^+, a_p]a_p - a_p^+[a_p, a_n^+]a_p + a_n^+[a_n^+, a_p]a_n + a_n^+[a_n, a_n^+]a_p \Big\}$$

$$= \tfrac{1}{4}\Big\{ -a_p^+(a_p^+ a_n - a_n a_p^+)a_p - a_p^+(a_p a_p^+ - a_p^+ a_p)a_n +$$

$$+ a_p^+(a_n^+ a_n - a_n a_n^+)a_n + a_n^+(a_n a_p^+ - a_p^+ a_n)a_n -$$

$$- a_n^+(a_p^+ a_p - a_p a_p^+)a_p - a_p^+(a_p a_n^+ - a_n^+ a_p)a_p +$$

$$+ a_n^+(a_n^+ a_p - a_p a_n^+)a_n + a_n^+(a_n a_n^+ - a_n^+ a_n)a_p \Big\}$$

$$= \tfrac{1}{4}\Big\{ a_p^+ a_n a_p^+ a_p - a_p^+ a_p a_p^+ a_n - a_p^+ a_n a_n^+ a_n + a_n^+ a_n a_p^+ a_n +$$

$$+ a_n^+ a_p a_p^+ a_p - a_p^+ a_p a_n^+ a_p - a_n^+ a_p a_n^+ a_n + a_n^+ a_n a_n^+ a_p \Big\}$$

$$= \tfrac{1}{4}\Big\{ a_p^+ a_n (a_p^+ a_p - a_n^+ a_n) - a_n^+ a_p (a_n^+ a_n - a_p^+ a_p) -$$

$$- a_p^+ a_p a_p^+ a_n + a_n^+ a_n a_p^+ a_n - a_p^+ a_p a_n^+ a_p + a_n^+ a_n a_n^+ a_p \Big\}$$

$$= \tfrac{1}{4}\Big\{ a_p^+ a_n (a_p^+ a_p - 1 + a_n a_n^+) - a_n^+ a_p (a_n^+ a_n - 1 + a_p a_p^+) - \ldots\ldots \Big\}$$

$$= \tfrac{1}{4}\Big\{ a_p^+ a_n a_p^+ a_p - a_p^+ a_n + a_p^+ \underbrace{a_n a_n}_{=0} a_n^+ - a_n^+ a_p a_n^+ a_n + a_n^+ a_p - a_n^+ \underbrace{a_p a_p}_{=0} a_p^+ \ldots \Big\}$$

$$= \tfrac{1}{4}\Big\{ -a_p^+ a_n + a_n^+ a_p \overset{T1}{+ a_p^+ a_n a_p^+ a_p} \overset{T2}{- a_n^+ a_p a_n^+ a_n} \overset{T3}{- a_p^+ a_p a_p^+ a_n} +$$

$$\overset{T4}{+ a_n^+ a_n a_p^+ a_n} \overset{T5}{- a_p^+ a_p a_n^+ a_p} \underset{T6}{+ a_n^+ a_n a_n^+ a_p} \Big\}$$

T2: $-a_n^+ a_p a_n^+ a_n = + a_p \underbrace{a_n^+ a_n^+}_{=0} a_n = 0$

T1: $+a_p^+ a_n a_p^+ a_p = \pm a_n \underbrace{a_p^+ a_p^+}_{=0} a_p = 0$

T5: $-a_p^+ a_p a_n^+ a_p = + a_p^+ a_p a_p a_n^+ = 0$

T4: $+a_n^+ a_n a_p^+ a_n = - a_n^+ a_n a_n a_p^+ = 0$

T3: $-a_p^+ a_p a_p^+ a_n = + a_p^+ a_p a_n a_p^+ = - a_p^+ a_n a_p a_p^+ = - a_p^+ a_n (1 - a_p^+ a_p)$

$= - a_p^+ a_n + a_p^+ a_n a_p^+ a_p = - a_p^+ a_n - a_n \underbrace{a_p^+ a_p^+}_{=0} a_p$

T6: $+a_n^+ a_n a_n^+ a_p = - a_n^+ a_n a_p a_n^+ = + a_n^+ a_p a_n a_n^+ = a_n^+ a_p (1 - a_n^+ a_n)$

$= a_n^+ a_p - a_n^+ a_p a_n^+ a_n = a_n^+ a_p + a_p \underbrace{a_n^+ a_n^+}_{=0} a_n$

$$[I_1, I_3] = \tfrac{1}{4}\Big\{ -a_p^+ a_n + a_n^+ a_p + 0 - 0 - a_p^+ a_n + 0 - 0 + a_n^+ a_p \Big\}$$

$$= \tfrac{1}{4}\Big\{ -2a_p^+ a_n + 2a_n^+ a_p \Big\} = -\tfrac{1}{2}(a_p^+ a_n - a_n^+ a_p)\cdot \tfrac{i}{i}$$

$$\boxed{[I_1, I_3] = -i\, I_2}$$

⑤

$$[I_2, I_3] = \left[\frac{1}{2i} (a_p^+ a_n - a_n^+ a_p), \frac{1}{2} (a_p^+ a_p - a_n^+ a_n) \right]$$

$$= \frac{1}{4i} \left\{ [a_p^+ a_n, a_p^+ a_p] - [a_p^+ a_n, a_n^+ a_n] - [a_n^+ a_p, a_p^+ a_p] + [a_n^+ a_p, a_n^+ a_n] \right\}$$

ver hoja ③

$$= \frac{1}{4i} \left\{ -a_p^+ [a_p^+, a_n] a_p - a_p^+ [a_p, a_p^+] a_n + a_p^+ [a_n^+, a_n] a_n + a_n^+ [a_n, a_p^+] a_n + \right.$$
$$\left. + a_n^+ [a_p^+, a_p] a_p + a_p^+ [a_p, a_n^+] a_p - a_n^+ [a_n^+, a_p] a_n - a_n^+ [a_n, a_n^+] a_p \right\}$$

$$= \frac{1}{4i} \left\{ \underline{a_p^+ a_n} a_p^+ a_p - a_p^+ a_p a_p^+ a_n - a_p^+ a_n a_n^+ a_n + a_n^+ a_n a_p^+ a_n - \right.$$
$$\left. - \underline{a_n^+ a_p} a_p^+ a_p + a_p^+ a_p a_n^+ a_p + \underline{a_n^+ a_p} a_n^+ a_n - a_n^+ a_n a_n^+ a_p \right\}$$

$$= \frac{1}{4i} \left\{ a_p^+ a_n (a_p^+ a_p - a_n^+ a_n) + a_n^+ a_p (a_n^+ a_n - a_p^+ a_p) - \right.$$
$$\left. - a_p^+ a_p a_p^+ a_n + a_n^+ a_n a_p^+ a_n + a_p^+ a_p a_n^+ a_p - a_n^+ a_n a_n^+ a_p \right\}$$

$$= \frac{1}{4i} \left\{ a_p^+ a_n (a_p^+ a_p - 1 + a_n a_n^+) + a_n^+ a_p (a_n^+ a_n - 1 + a_p a_p^+) - \dots\dots \right\}$$

$$= \frac{1}{4i} \left\{ a_p^+ a_n a_p^+ a_p - a_p^+ a_n + a_p^+ \underbrace{a_n a_n}_{=0} a_n^+ + a_n^+ a_p a_n^+ a_n - a_n^+ a_p + a_n^+ \underbrace{a_p a_p}_{=0} a_p^+ \dots \right\}$$

$$= \frac{1}{4i} \left\{ -a_p^+ a_n - a_n^+ a_p + \overset{T1}{a_p^+ a_n a_p^+ a_p} + \overset{T2}{a_n^+ a_p a_n^+ a_n} - \overset{T3}{a_p^+ a_p a_p^+ a_n} + \right.$$
$$\left. + \underset{T4}{a_n^+ a_n a_p^+ a_n} + \underset{T5}{a_p^+ a_p a_n^+ a_p} - \underset{T5}{a_n^+ a_n a_n^+ a_p} \right\}$$

$$= \frac{1}{4i} \left\{ -a_p^+ a_n - a_n^+ a_p + 0 + 0 - a_p^+ a_n + 0 + 0 - a_n^+ a_p \right\}$$

$$= \frac{1}{4i} \left\{ -2 a_p^+ a_n - 2 a_n^+ a_p \right\} = -\frac{1}{2i} (a_p^+ a_n + a_n^+ a_p) = i \frac{1}{2} (a_p^+ a_n + a_n^+ a_p)$$

$$\boxed{[I_2, I_3] = i I_1}$$

Ej 3 Dados $|\Delta^{++}\rangle = |\pi^+, p\rangle$

$$|\Delta^{+}\rangle = 1/\sqrt{3}\,|\pi^+, n\rangle + \sqrt{2/3}\,|\pi^0, p\rangle$$

$$|\Delta^{0}\rangle = \sqrt{2/3}\,|\pi^0, n\rangle + 1/\sqrt{3}\,|\pi^-, p\rangle$$

$$|\Delta^{-}\rangle = |\pi^-, n\rangle$$

$$|1\tfrac{1}{2}, \tfrac{1}{2}\,\tfrac{1}{2}\rangle = -\sqrt{2/3}\,|\pi^+, n\rangle + 1/\sqrt{3}\,|\pi^0, p\rangle$$

$$|1\tfrac{1}{2}, \tfrac{1}{2}\,-\tfrac{1}{2}\rangle = -1/\sqrt{3}\,|\pi^0, n\rangle + \sqrt{2/3}\,|\pi^-, p\rangle$$

encontrar $|\pi^+, p\rangle$; $|\pi^+, n\rangle$; $|\pi^0, p\rangle$; $|\pi^0, n\rangle$; $|\pi^-, p\rangle$; $|\pi^-, n\rangle$

$|\pi^+, p\rangle = |\Delta^{++}\rangle$ y $|\pi^-, n\rangle = |\Delta^-\rangle$ demostrado por Javier.

para el los otros cuatro.

$$\begin{pmatrix} |\Delta^+\rangle \\ |\Delta^0\rangle \\ |1\tfrac{1}{2}, \tfrac{1}{2}\,\tfrac{1}{2}\rangle \\ |1\tfrac{1}{2}, \tfrac{1}{2}\,-\tfrac{1}{2}\rangle \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} & \sqrt{2/3} & 0 & 0 \\ 0 & 0 & \sqrt{2/3} & 1/\sqrt{3} \\ -\sqrt{2/3} & 1/\sqrt{3} & 0 & 0 \\ 0 & 0 & -1/\sqrt{3} & \sqrt{2/3} \end{pmatrix} \begin{pmatrix} |\pi^+, n\rangle \\ |\pi^0, p\rangle \\ |\pi^0, n\rangle \\ |\pi^-, p\rangle \end{pmatrix}$$

$$\Big\downarrow \text{MATRIZ INVERSA}$$

$$\begin{pmatrix} 1/\sqrt{3} & 0 & -\sqrt{2/3} & 0 \\ \sqrt{2/3} & 0 & 1/\sqrt{3} & 0 \\ 0 & \sqrt{2/3} & 0 & -\sqrt{1/3} \\ 0 & 1/\sqrt{3} & 0 & \sqrt{2/3} \end{pmatrix}$$

$$|\pi^+, n\rangle = 1/\sqrt{3}\,|\Delta^+\rangle - \sqrt{2/3}\,|1\tfrac{1}{2}, \tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\pi^0, p\rangle = \sqrt{2/3}\,|\Delta^+\rangle + 1/\sqrt{3}\,|1\tfrac{1}{2}, \tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$$|\pi^0, n\rangle = \sqrt{2/3}\,|\Delta^0\rangle - 1/\sqrt{3}\,|1\tfrac{1}{2}, \tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$

$$|\pi^-, p\rangle = 1/\sqrt{3}\,|\Delta^0\rangle + \sqrt{2/3}\,|1\tfrac{1}{2}, \tfrac{1}{2}\,-\tfrac{1}{2}\rangle$$