Ejercicios Teoría Cuántica de Campos. Capítulo 29

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1. Calcular $\sigma_{\hat{p}}$

Consideremos el estado coherente $|\alpha\rangle$, queremos calcular

$$\sigma_{\hat{p}} = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2} \tag{1}$$

Empecemos por $\langle \hat{p} \rangle$. Usando las siguientes ecuaciones del formulario de Crul:

$$\hat{p} = -i\sqrt{\frac{\hbar m \omega}{2}} \left(\hat{a} - \hat{a}^\dagger \right) \tag{28.2.4}$$

$$N = \hat{a}^\dagger \hat{a} \tag{28.3.5}$$

$$\left[\hat{a}, \hat{a}^\dagger \right] = 1 \implies \hat{a}\hat{a}^\dagger = 1 + \hat{a}^\dagger \hat{a} = 1 + N \tag{28.3.2}$$

$$\hat{a} |\alpha\rangle = \alpha |\alpha\rangle \implies \langle \alpha | \hat{a}^\dagger = \alpha^* \langle \alpha | \tag{29.1.1}$$

$$\langle \alpha | \alpha \rangle = 1 \tag{29.1.4}$$

$$\langle \alpha | N |\alpha\rangle = |\alpha|^2 \tag{29.3.1}$$

Entonces tenemos que,

$$\langle \hat{p} \rangle = \langle \alpha | \hat{p} |\alpha\rangle = -i\sqrt{\frac{\hbar m \omega}{2}} \left(\langle \alpha | \hat{a} |\alpha\rangle - \langle \alpha | \hat{a}^\dagger |\alpha\rangle \right) = -i\sqrt{\frac{\hbar m \omega}{2}} \left(\alpha - \alpha^* \right) \tag{2}$$

Para calcular $\left\langle \hat{p}^2 \right\rangle$ procedemos de la misma forma;

$$\left\langle \hat{p}^2 \right\rangle = \langle \alpha | \, \hat{p}^2 \, | \alpha \rangle = -\frac{\hbar m \omega}{2} \langle \alpha | \left(\hat{a}^2 + \left(\hat{a}^\dagger \right)^2 - \hat{a}\hat{a}^\dagger - \hat{a}^\dagger \hat{a} \right) | \alpha \rangle \tag{3}$$

$$= -\frac{\hbar m \omega}{2} \langle \alpha | \left(\hat{a}^2 + \left(\hat{a}^\dagger \right)^2 - 1 - 2N \right) | \alpha \rangle \tag{4}$$

$$= -\frac{\hbar m \omega}{2} \left(\langle \alpha | \, \hat{a}^2 \, | \alpha \rangle + \langle \alpha | \left(\hat{a}^\dagger \right)^2 | \alpha \rangle - \langle \alpha | \alpha \rangle - 2 \langle \alpha | \, N \, | \alpha \rangle \right) \tag{5}$$

$$= -\frac{\hbar m \omega}{2} \left(\alpha^2 + (\alpha^*)^2 - 1 - 2|\alpha|^2 \right) \tag{6}$$

Finalmente

$$\left\langle \hat{p}^2 \right\rangle - \langle \hat{p} \rangle^2 = -\frac{\hbar m \omega}{2} \left(\alpha^2 + (\alpha^*)^2 - 1 - 2|\alpha|^2 \right) + \frac{\hbar m \omega}{2} \left(\alpha^2 + (\alpha^*)^2 - 2|\alpha|^2 \right) \tag{7}$$

$$= \frac{\hbar m \omega}{2} \implies \sigma_{\hat{p}} = \sqrt{\frac{m \hbar \omega}{2}} \tag{8}$$