RODOLFO GUIDOBONO	CURSO TEORÍA CUÁNTICA DE CAMPOS J. GARCÍA - EJERCICIO CAP. 17	HOJA Nº 1 FECHA 22/4/19

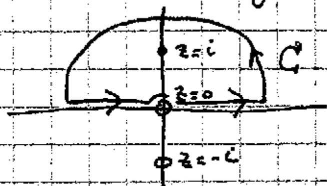
Calcular la parte principal de la integral

$$\int_{-\infty}^{\infty} \frac{\operatorname{sen} x}{x(x^2+1)}\, dx$$

Definimos una función $g(z) = \dfrac{e^{iz}}{z(z^2+1)} = \dfrac{e^{iz}}{z(z-i)(z+i)}$

con singularidades en $z=0$; $z=i$; $z=-i$

Se calcula la integral de $g(z)$ a lo largo del siguiente contorno cerrado.



$g(z)$ puede expresarse como $g(z) = \dfrac{f(z)}{z-i}$

donde $f(z) = \dfrac{e^{iz}}{z(z+i)}$, la cual es holomorfa dentro del contorno C

pudiendo aplicarse la fórmula integral de Cauchy.

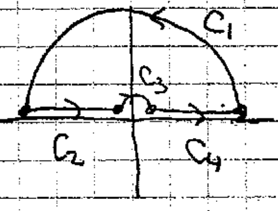
$$\oint_C \frac{f(z)}{(z-z_0)^{n+1}}\, dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

$$\oint_C \frac{f(z)}{(z-i)^{0+1}}\, dz = \frac{2\pi i}{0!} f^{(0)}(i) = 2\pi i \frac{e^{i \cdot i}}{i(i+1)} = \frac{2\pi i\, e^{-1}}{i\, 2i}$$

$$= \frac{\pi e^{-1}}{i} = -\pi e^{-1} i$$

A su vez

$$\oint_C g(z)\, dz = \int_{C_1} g(z)\, dz + \int_{C_2} g(z)\, dz + \int_{C_3} g(z)\, dz + \int_{C_4} g(z)\, dz$$



NOTA

HOJA Nº 2

FECHA

$$\int_{C_1} \frac{e^{iz}}{z(z^2+1)}\,dz \qquad C_1 \rightarrow z = Re^{i\theta}$$

$$dz = Re^{i\theta}\, i\, d\theta$$

$$\int_{C_1} \frac{e^{iz}}{z(z^2+1)}\,dz = \int_0^{\pi} \frac{e^{i(R\cos\theta + iR\,\mathrm{sen}\,\theta)}}{Re^{i\theta}\left(R^2 e^{2i\theta}+1\right)} \cdot R \cdot e^{i\theta} \cdot i\, d\theta$$

$$= \int_0^{\pi} \frac{e^{iR\cos\theta}\, e^{-R\,\mathrm{sen}\,\theta}}{R^2 e^{2i\theta}+1} \cdot i\, d\theta$$

$$\left| e^{iR\,\mathrm{sen}\,\theta} \right| = 1$$

como $\mathrm{sen}\,\theta > 0$ cuando $R \to \infty \Rightarrow e^{-R\,\mathrm{sen}\,\theta} \to 0$

y como $\dfrac{1}{aR^2+b} \to 0$ cuando $R \to \infty$

$$\lim_{R\to\infty} \int_{C_1} \frac{e^{iz}}{z(z^2+1)}\,dz = 0$$

$$\int_{C_3} \frac{e^{iz}}{z(z^2+1)}\,dz = \int_{\pi}^{0} \frac{e^{i(\varepsilon\cos\theta + \varepsilon\,\mathrm{sen}\,\theta\, i)}}{\varepsilon e^{i\theta}\left(\varepsilon^2 e^{2i\theta}+1\right)} \cdot \varepsilon \cdot e^{i\theta} \cdot i \cdot d\theta$$

$$C_3 \rightarrow z = \varepsilon e^{i\theta}$$

$$= \int_{\pi}^{0} \frac{e^{i\varepsilon(\cos\theta + i\,\mathrm{sen}\,\theta)}}{\varepsilon^2 e^{2i\theta}+1}\, i\, d\theta$$

$$= i\int_{\pi}^{0} \frac{e^{i\varepsilon(\cos\theta + i\,\mathrm{sen}\,\theta)}}{\varepsilon^2 e^{2i\theta}+1}\, d\theta$$

$$\lim_{\varepsilon\to 0} \int_{C_3} \frac{e^{iz}}{z(z^2+1)}\,dz = i \lim_{\varepsilon\to 0} \int_{\pi}^{0} \frac{e^{\varepsilon[i(\cos\theta + i\,\mathrm{sen}\,\theta)]}}{\varepsilon^2 e^{2i\theta}+1}\, d\theta$$

$$= i\int_{\pi}^{0} \frac{1}{0+1}\, d\theta = i\left[0-\pi\right] = -i\pi$$

NOTA

HOJA Nº 3

FECHA

$$\oint_C g(z)\,dz = \int_{C_1} g(z)\,dz + \int_{C_2} g(z)\,dz + \int_{C_3} g(z)\,dz + \int_{C_4} g(z)\,dz$$

$$-\pi e^{-1} i = \underbrace{0}_{R\to\infty} + \underbrace{\int_{C_2} g(z)\,dz}_{\int_{-\infty}^{-\varepsilon} g(z)\,dz \;=\; \int_{-\infty}^{-\varepsilon} \frac{e^{ix}}{x(x^2+1)}dx} + \underbrace{(-i\pi)}_{\varepsilon\to 0} + \underbrace{\int_{C_4} g(z)\,dz}_{\int_{\varepsilon}^{\infty} g(z)\,dz \;=\; \int_{\varepsilon}^{\infty} \frac{e^{ix}}{x(x^2+1)}dx}$$

$$\lim_{\varepsilon\to 0} \int_{-\infty}^{-\varepsilon} \frac{e^{ix}}{x(x^2+1)}dx + \int_{\varepsilon}^{\infty} \frac{e^{ix}}{x(x^2+1)}dx = \int_{-\infty}^{\infty} \frac{e^{ix}}{x(x^2+1)}dx$$

$$-\pi e^{-1} i = -i\pi + \int_{-\infty}^{\infty} \frac{e^{ix}}{x(x^2+1)}dx$$

$$\pi(1-e^{-1})\cdot i = \int_{-\infty}^{\infty} \frac{\cos x}{x(x^2+1)} + i\,\frac{\operatorname{sen} x}{x(x^2+1)}\,dx$$

por ser $\frac{\cos x}{x^2+1}$ función PAR y $\frac{1}{x}$ función IMPAR $\int_{-\infty}^{\infty} \frac{\cos x}{x(x^2+1)}dx = 0$

por ser sen x y x funciones IMPARES, $\frac{\operatorname{sen} x}{x}$ es par entonces

$$\int_{-\infty}^{\infty} \frac{\operatorname{sen} x}{x(x^2+1)}dx \neq 0$$

$$i\left[\pi(1-e^{-1})\right] = i\int_{-\infty}^{\infty} \frac{\operatorname{sen} x}{x(x^2+1)}dx$$

$$\boxed{\int_{-\infty}^{\infty} \frac{\operatorname{sen} x}{x(x^2+1)}dx = \pi\left(1 - {}^{1}/_{e}\right)}$$

NOTA