Teoria Cuantica de Campos.

Ejercicio:

Definición: $$\langle \blacksquare \rangle = \frac{\int_{-\infty}^{+\infty} dx \, \blacksquare \, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x^2}}$$

a) $\langle x \rangle$

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} dx \, x e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x^2}} = \frac{-\frac{1}{a}\int_{-\infty}^{+\infty} dx \, x(-a) \, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x^2}}$$

$$\int_{-\infty}^{+\infty} dx \, \blacksquare' e^{\blacksquare} = e^{\blacksquare} \quad ; \quad \int_{-\infty}^{+\infty} dx \, e^{-\frac{ax^2}{2}} = \sqrt{\frac{\pi}{a/2}} = \sqrt{2\pi} \, . \, a^{-1/2}$$

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} dx \, (-ax) \, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x}} = \frac{\left(e^{-\frac{a}{2}x^2}\right|_{-\infty}^{+\infty}}{\sqrt{2\pi} \, . \, a^{-1/2}} = \frac{0}{\sqrt{2\pi} \, . \, a^{-1/2}} = 0$$

b) $\langle x^2 \rangle$

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{+\infty} dx \, x^2 e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x}}$$

$$\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x^2} = \sqrt{2\pi} \, . \, a^{-1/2}$$

derivamos con respecto a "a"

$$\int_{-\infty}^{+\infty} dx \, e^{-\frac{a}{2}x^2} \cdot \left(-\frac{x^2}{2}\right) = -\frac{1}{2} . \sqrt{2\pi} \, . \, a^{-3/2}$$

$$\int_{-\infty}^{+\infty} dx \, x^2 e^{-\frac{a}{2}x^2} = 1 . \sqrt{2\pi} \, . \, a^{-3/2}$$

$$\langle x^2 \rangle = \frac{\sqrt{2\pi} \, a^{-3/2}}{\sqrt{2\pi} \, a^{-1/2}}$$

$$\langle x^2 \rangle = a^{-1}$$

$$\langle x^2 \rangle = \frac{1}{a}$$

c)

$$\langle x^{2n}\rangle = \frac{\int_{-\infty}^{+\infty} dx\, x^{2n} e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2}}$$

Derivamos con respecto a "a"

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} = \sqrt{2\pi}\, a^{-1/2}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2}\left(-\frac{1}{2}x^2\right) = \sqrt{2\pi}\left(\frac{-1}{2}\right). a^{-3/2}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} x^2 = \sqrt{2\pi}. (1). a^{-3/2}$$

Derivamos otra vez.

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} x^2\left(\frac{-1}{2}x^2\right) = \sqrt{2\pi}. 1. \left(-\frac{3}{2}\right) a^{-5/2}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} x^4 = \sqrt{2\pi}. 1. 3. a^{-5/2}$$

Derivamos otra vez.

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} x^4\left(\frac{-1}{2}x^2\right) = \sqrt{2\pi}. 1. 3. \left(\frac{-5}{2}\right). a^{-7/2}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{ax^2}{2}} x^6 = \sqrt{2\pi}. 1. 3. 5. a^{-7/2}$$

Derivamos otra vez.

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{ax^2}{2}} x^6\left(-\frac{1}{2}x^2\right) = \sqrt{2\pi}. 1. 3. 5. \left(-\frac{7}{2}\right). a^{-9/2}$$

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{ax^2}{2}} x^8 = \sqrt{2\pi}. 1. 3. 5. 7. a^{-9/2}$$

Entonces

$$\int_{-\infty}^{+\infty} dx\, e^{-\frac{a}{2}x^2} x^{2n} = \sqrt{2\pi}\,(2n-1)(2n-3)(2n-5)\ldots 5.3.1.\, a^{-(2n+1)/2}$$

$$\langle x^{2n} \rangle = \frac{\int_{-\infty}^{+\infty} dx \; x^{2n} \, e^{-\frac{a}{2}x^2}}{\int_{-\infty}^{+\infty} dx \; e^{-\frac{a}{2}x^2}}$$

$$= \frac{\sqrt{2\pi}\,(2n-1)(2n-3)(2n-5)\cdots 5.3.1.a^{-(2n+1)/2}}{\sqrt{2n}\,.\,a^{-1/2}}$$

$$= a^{-\frac{2n}{2}-\frac{1}{2}+\frac{1}{2}} \cdot (2n+1)(2n-3)(2n-5)\cdots 5.3.1$$

$$= a^{-n}\,(2n+1)(2n-3)(2n-5)\cdots 5.3.1$$

$$\langle x^{2n} \rangle = \frac{1}{a^n}\,(2n+1)(2n-3)(2n-5)\cdots 5.3.1$$