$$\langle \phi^2 \rangle = \frac{Z^{(2)}[0]}{Z[0]} \qquad S[\phi] = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

$$Z[J] = \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi}\, d\phi = \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\, e^{-\frac{\lambda}{24}\phi^4}\, d\phi$$

Como $e^x \cong 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ Reemplazando x por $-\frac{\lambda}{24}\phi^4$

$$Z[J] \simeq \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\left(1 - \frac{\lambda}{24}\phi^4 + \frac{1}{2}\frac{\lambda^2}{24^2}\phi^8 - \frac{1}{6}\frac{\lambda^3}{24^3}\phi^{12}\right) d\phi$$

Y como λ es constante se lo puede sacar de las Integrales

$$Z[J] \simeq \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi - \frac{\lambda}{24}\int_{-\infty}^{\infty} \phi^4 e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi + \frac{1}{2}\frac{\lambda^2}{24^2}\int_{-\infty}^{\infty} \phi^8 e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi$$
$$- \frac{1}{6}\frac{\lambda^3}{24^3}\int_{-\infty}^{\infty} \phi^{12} e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi$$

$$Z[J] \simeq Z_0[J] - \frac{\lambda}{24} Z_0^{(4)}[J] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(8)}[J] - \frac{1}{6}\frac{\lambda^3}{24^3} \underline{Z_0^{(12)}[J]} \quad (1)$$

Sacando factor común $Z_0[J]$

$$Z[J] = Z_0[J]\left[1 - \frac{\lambda}{24}\frac{Z_0^{(4)}[J]}{Z_0[J]} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{Z_0^{(8)}[J]}{Z_0[J]} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{Z_0^{(12)}[J]}{Z_0[J]}\right]$$

Poniendo $J = 0$

$$Z[0] = Z_0[0]\left[1 - \frac{\lambda}{24}\frac{Z_0^{(4)}[0]}{Z_0[0]} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{Z_0^{(8)}[0]}{Z_0[0]} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{Z_0^{(12)}[0]}{Z_0[0]}\right] \quad \text{o sea}$$

$$Z[0] = Z_0[0]\left[1 - \frac{\lambda}{24}\langle\phi^4\rangle_0 + \frac{1}{2}\frac{\lambda^2}{24^2}\langle\phi^8\rangle_0 - \frac{1}{6}\frac{\lambda^3}{24^3}\langle\phi^{12}\rangle\right]$$

$$Z[0] = Z_0[0]\left[1 - \frac{\lambda}{24}\frac{1}{m^4}\,3.5 + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^8}\,7.5.3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{11.9.7.5.3}{m^{12}}\right]$$

Derivando 2 veces la ecuación (1)

$$Z^2[J] = Z_0^{(2)}[J] - \frac{\lambda}{24} Z_0^{(6)}[J] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(10)} - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(14)}[J]$$

Entonces

$$Z^{(2)}[J] = Z_0^{(2)}[J] - \frac{\lambda}{24} Z_0^{(6)}[J] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(10)}[J] - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(14)}[J]$$

Multiplicando y dividiendo el segundo miembro por $Z_0[J]$

$$Z^{(2)}[J] = Z_0[J]\left[\frac{Z_0^{(2)}[J]}{Z_0[J]} - \frac{\lambda}{24}\frac{Z_0^{(6)}[J]}{Z_0[J]} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{Z_0^{(10)}[J]}{Z_0[J]} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{Z_0^{(14)}[J]}{Z_0[J]}\right]$$

$$Z^{(2)}[0] = Z_0[0]\left[\langle\phi^2\rangle_0 - \frac{\lambda}{24}\langle\phi^6\rangle_0 + \frac{1}{2}\frac{\lambda^2}{24^2}\langle\phi^{10}\rangle_0 - \frac{1}{6}\frac{\lambda^3}{24^3}\langle\phi^{14}\rangle_0\right]$$

$$Z^{(2)}[0] = Z_0[0]\left[\frac{1}{m^2} - \frac{\lambda}{24}\frac{1}{m^6}5\cdot3 + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^{10}}9\cdot7\cdot5\cdot3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{1}{m^{14}}13\cdot11\cdot9\cdot7\cdot5\cdot3\right]$$

Y como $\langle\phi^2\rangle = \dfrac{Z^{(2)}[0]}{Z[0]}$

$$\langle\phi^2\rangle = \frac{\left[\frac{1}{m^2} - \frac{\lambda}{24}\frac{1}{m^6}5\cdot3 + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^{10}}9\cdot7\cdot5\cdot3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{1}{m^{14}}13\cdot11\cdot9\cdot7\cdot5\cdot3\right]}{\left[1 - \frac{\lambda}{24}\frac{1}{m^4}3 + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^8}7\cdot5\cdot3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{1}{m^{12}}11\cdot9\cdot7\cdot5\cdot3\right]}$$

Despues de simplificar $Z_0[0]$

Ahora sea $f(\lambda) = \dfrac{P(\lambda)}{q(\lambda)}$

$P(\lambda)$ polinomio del numerador

$q(\lambda)$ polinomio del denominador

El desarrollo en serie de Taylor de $\langle\phi^2\rangle$ sería

$$\langle\phi^2\rangle \simeq f(0) + f'(0)\lambda + f''(0)\frac{\lambda^2}{2!}$$

$$P(\lambda) = \frac{1}{m^2} - \frac{\lambda}{24}\frac{1}{m^6}5.3 + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^{10}}9.7.5.3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{1}{m^{14}}13.11.9.7.5.3$$

$$P(0) = \frac{1}{m^2}$$

$$P'(\lambda) = -\frac{5.3}{24}\frac{1}{m^6} + \frac{\lambda}{24^2}\frac{1}{m^{10}}9.7.5.3 - \frac{1}{2}\frac{\lambda^2}{24^3}\frac{1}{m^{14}}13.11.9.7.5.3 \quad ; P'(0) = -\frac{5.3}{24}.\frac{1}{m^6}$$

$$P''(\lambda) = \frac{9.7.5.3}{24^2 m^{10}} - \frac{\lambda}{24^3}\frac{1}{m^{14}}13.11.9.7.5.3 \quad ; P''(0) = \frac{9.7.5.3}{24^2 m^{10}}$$

$$Q(\lambda) = 1 - \frac{3}{24}\frac{\lambda}{m^4} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{1}{m^8}7.5.3 - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{1}{m^{12}}11.9.7.5.3 \quad ; Q(0) = 1$$

$$Q'(\lambda) = -\frac{3}{24m^4} + \frac{\lambda}{24^2}\frac{1}{m^8}7.5.3 - \frac{1}{2}\frac{\lambda^2}{24^3}\frac{1}{m^{12}}11.9.7.5.3 \quad ; Q'(0) = -\frac{3}{24m^4}$$

$$Q''(\lambda) = \frac{7.5.3}{24^2 m^8} - \frac{\lambda}{24^3}\frac{1}{m^{12}}11.9.7.5.3 \quad ; Q''(0) = \frac{7.5.3}{24^2 m^8}$$

Entonces

$P(0) = \frac{1}{m^2}$	$P'(0) = -\frac{5.3}{24}\frac{1}{m^6}$	$P''(0) = \frac{9.7.5.3}{24^2 m^{10}}$
$Q(0) = 1$	$Q'(0) = -\frac{3}{24m^4}$	$Q''(0) = \frac{7.5.3}{24^2 m^8}$

Como

$$F(\lambda) = \frac{P(\lambda)}{Q(\lambda)} \Rightarrow P(\lambda) = F(\lambda)\,Q(\lambda)$$ y derivando esta igualdad

(2)

$$P'(\lambda) = F'(\lambda)Q(\lambda) + F(\lambda)Q'(\lambda)$$ Despejando $F'(\lambda)$

$$F'(\lambda) = \frac{P'(\lambda) - F(\lambda)Q'(\lambda)}{Q(\lambda)}$$

$$F(0) = \frac{P(0)}{Q(0)} = \frac{\frac{1}{m^2}}{1} = \frac{1}{m^2}$$

$$F'(0) = \frac{P'(0) - F(0)Q'(0)}{Q(0)} = \frac{-\frac{5.3}{24}\frac{1}{m^6} + \frac{1}{m^2}\frac{3}{24m^4}}{1} = \frac{-\frac{15}{24m^6} + \frac{3}{24m^6}}{1} = \frac{-12}{24m^6}$$

$$F'(0) = -\frac{1}{2m^6} \qquad = -\frac{1}{2m^6}$$

Tenemos que (De la ecuacion 2)

$$\rho'(\lambda) = f'(\lambda)\,g(\lambda) + f(\lambda)\,g'(\lambda) \qquad \text{Derivando nuevamente}$$

$$\rho''(\lambda) = f''(\lambda)\,g(\lambda) + f'(\lambda)\,g'(\lambda) + f'(\lambda)\,g'(\lambda) + f(\lambda)\,g''(\lambda)$$

$$\rho''(\lambda) = f''(\lambda)\,g(\lambda) + 2f'(\lambda)\,g'(\lambda) + f(\lambda)\,g''(\lambda) \qquad \text{Despejando } f''(\lambda)$$

$$f''(\lambda) = \frac{\rho''(\lambda) - 2f'(\lambda)\,g'(\lambda) - f(\lambda)\,g''(\lambda)}{g(\lambda)}$$

$$f''(0) = \frac{\rho''(0) - 2f'(0)\,g'(0) - f(0)\,g''(0)}{g(0)}$$

Multiplicamos y dividimos por 12 para tener 24^2 en el denominador

$$f''(0) = \frac{\frac{9.7.5.3}{24^2m^{10}} - \frac{2.1}{2m^6}\cdot\frac{3}{24m^4} - \frac{1}{m^2}\,\frac{7.5.3}{24^2m^8}}{1} = \frac{9.7.5.3}{24^2m^{10}} - \frac{24.3}{24^2m^{10}} - \frac{7.5.3}{24^2m^{10}}$$

$$= \frac{768}{24^2m^{10}}$$

$$= \frac{4}{3m^{10}}$$

o sea

$$\langle\phi^2\rangle \simeq \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{4}{3m^{10}}\cdot\frac{1}{2}\lambda^2 = \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{2}{3m^{10}}\lambda^2$$

$$\langle\phi^2\rangle = \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{2}{3m^{10}}\lambda^2$$