RODOLFO GUIDOBONO | CURSO TEORÍA CUÁNTICA DE CAMPOS - J. GARCÍA - EJERCICIOS CAP. 3 | HOJA Nº 1 | FECHA 11/2/19

Dada la definición de promedio

$$\langle \square \rangle \equiv \frac{\int_{-\infty}^{\infty} dx \, \square \, e^{-a/2\, x^2}}{\int_{-\infty}^{\infty} dx \, e^{-a/2\, x^2}} \longrightarrow \text{Calcular} \quad \begin{array}{l} a) \ \langle x \rangle \\ b) \ \langle x^2 \rangle \\ c) \ \langle x^{2k} \rangle \end{array}$$

a) CÁLCULO $\langle x \rangle$

$$\int_{-\infty}^{\infty} e^{-a/2\, x^2} x\, dx = -\frac{1}{a} \int_{-\infty}^{\infty} e^{-\frac{a}{2}x^2} \underbrace{\left(-\frac{a}{2}\right) 2x\, dx}_{= -\frac{a}{2} dx^2 = d\left(-\frac{a}{2}x^2\right)}$$

$$= -\frac{1}{a} \int_{-\infty}^{\infty} e^{-\frac{a}{2}x^2} d\left(-\frac{a}{2}x^2\right) \quad \text{como} \quad \int e^y dy = e^y$$

$$= -\frac{1}{a} \, e^{-a/2\, x^2} \Big|_{-\infty}^{\infty} = -\frac{1}{a}(0-0) = 0$$

$$\boxed{\langle x \rangle = 0}$$

b) CÁLCULO $\langle x^2 \rangle$

$$N = \int_{-\infty}^{\infty} e^{-a/2\, x^2} x^2 dx \quad \text{cambio de variable} \quad y^2 = \frac{x^2}{2} \quad \sqrt{2}\, dy = dx$$

$$N = \int_{-\infty}^{\infty} e^{-ay^2} 2y^2 \sqrt{2}\, dy = 2\sqrt{2} \underbrace{\int_{-\infty}^{\infty} e^{-ay^2} y^2 dy}_{(f.3.1) = \sqrt{\pi}/2a^{3/2}}$$

$$N = 2\sqrt{2} \frac{\sqrt{\pi}}{2a^{3/2}} = \frac{\sqrt{2\pi}}{a^{3/2}}$$

$$D = \int_{-\infty}^{\infty} e^{-a/2\, x^2} dx \quad \text{cambio de variable} \quad y^2 = \frac{x^2}{2} \quad \sqrt{2}\, dy = dx$$

$$D = \int_{-\infty}^{\infty} e^{-ay^2} \sqrt{2}\, dy = \sqrt{2} \int_{-\infty}^{\infty} e^{-ay^2} dy = \sqrt{2} \cdot \sqrt{\pi/a}$$

$$D = \sqrt{\frac{2\pi}{a}}$$

NOTA

HOJA Nº 2

FECHA

$$\langle x^2\rangle = \frac{N}{D} = \frac{\frac{\sqrt{2\pi}}{a^{3/2}}}{\frac{\sqrt{2\pi}}{a^{1/2}}} = \frac{1}{a} \qquad \boxed{\langle x^2\rangle = \frac{1}{a}}$$

c) <u>CÁLCULO $\langle X^{2n}\rangle$</u>

$$\langle x^{2n}\rangle = \frac{\int_{-\infty}^{\infty} x^{2n} e^{-a/2\, x^2}\,dx}{\int_{-\infty}^{\infty} e^{-a/2\, x^2}\,dx} \quad \Big\} \longrightarrow \text{denominador } D = \sqrt{\frac{2\pi}{a}}$$

en b) calculamos $\int_{-\infty}^{\infty} e^{-a/2\, x^2}\,dx = \sqrt{\frac{2\pi}{a}} = \sqrt{2\pi}\, a^{-1/2} \equiv f(a)$

aplicando el "truco" de Javier en Cap. 3 26:29, esto es derivar $f(a)$

$$f'(a) = \int_{-\infty}^{\infty}\left(-\frac{x^2}{2}\right) e^{-\frac{a}{2}x^2}\,dx = \sqrt{2\pi}\left(-\frac{1}{2}\right) a^{-3/2}$$

$$f''(a) = \int_{-\infty}^{\infty}\left(-\frac{x^2}{2}\right)\left(-\frac{x^2}{2}\right) e^{-\frac{a}{2}x^2}\,dx = \sqrt{2\pi}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right) a^{-\frac{5}{2}}$$

$$f^{III}_{(a)} = \int_{-\infty}^{\infty}\underbrace{\left(-\frac{x^2}{2}\right)\left(-\frac{x^2}{2}\right)\left(-\frac{x^2}{2}\right)}_{3\text{ veces}} e^{-\frac{a}{2}x^2}\,dx = \sqrt{2\pi}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\overset{2\times 3-1}{5}\over 2\right) a^{-\frac{7}{2}} \; -(3+\tfrac{1}{2})$$

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$$f^{n}_{(a)} = \int_{-\infty}^{\infty}\left(-\frac{1}{2}\right)^n (x^2)^n e^{-\frac{a}{2}x^2}\,dx = \sqrt{2\pi}\left(-\frac{1}{2}\right)^n 1.3.5\ldots(2n-1)\, a^{-(n+1/2)}$$

$$\langle x^{2n}\rangle = \frac{\left(-\frac{1}{2}\right)^{-n} f^n_{(a)}}{\sqrt{2\pi}\, a^{-1/2}} = \frac{\sqrt{2\pi}\; 1.3.\ldots(2n-1)}{\sqrt{2\pi}\, a^{-1/2}\, a^{n+1/2}} \longrightarrow n+\tfrac{1}{2}-\tfrac{1}{2} = n$$

$$\boxed{\langle x^{2n}\rangle = \frac{1}{a^n}\,(2n-1)\ldots 5.3.1.}$$

NOTA