RODOLFO GUIDOBONO - CURSO TEORÍA CUÁNTICA DE CAMPOS JAVIER GARCÍA - EJERCICIO CAP. 42

1/10/2020

Hasiendo demostrado que:

$$S[\Lambda]\cdot\begin{pmatrix}1\\0\\0\\0\end{pmatrix} = \begin{pmatrix}\mathrm{ch}\,\eta/2\\ \mathrm{sh}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}e^{-i\phi/2}\cos\theta/2\\ e^{i\phi/2}\,\mathrm{sen}\,\theta/2\end{pmatrix}$$

calcular la acción de $S(\Lambda)$ sobre los otros vectores de la base.

Siendo

$$S[\Lambda] = \underbrace{\begin{pmatrix}\mathrm{ch}\,\eta/2 & 0\\ 0 & \mathrm{sh}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}e^{-i\frac{\phi}{2}}\cos\frac{\theta}{2} & -e^{-i\frac{\phi}{2}}\,\mathrm{sen}\,\frac{\theta}{2}\\ e^{i\frac{\phi}{2}}\,\mathrm{sen}\,\frac{\theta}{2} & e^{i\frac{\phi}{2}}\cos\frac{\theta}{2}\end{pmatrix}}_{\equiv T_1} + \underbrace{\begin{pmatrix}0 & \mathrm{sh}\,\frac{\eta}{2}\\ \mathrm{sh}\,\frac{\eta}{2} & 0\end{pmatrix}\otimes\begin{pmatrix}e^{-i\frac{\phi}{2}}\cos\frac{\theta}{2} & e^{-i\frac{\phi}{2}}\,\mathrm{sen}\,\frac{\theta}{2}\\ e^{i\frac{\phi}{2}}\,\mathrm{sen}\,\frac{\theta}{2} & -e^{i\frac{\phi}{2}}\cos\frac{\theta}{2}\end{pmatrix}}_{\equiv T_2}$$

$$S[\Lambda] = T_1 + T_2$$

(I) $$S[\Lambda]\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = S[\Lambda]\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = (T_1+T_2)\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]$$

$$T_1\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = \begin{pmatrix}\mathrm{ch}\,\eta/2\\ 0\end{pmatrix}\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\phi/2}\cos\theta/2\end{pmatrix} \quad (1)$$

$$T_2\left[\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = \begin{pmatrix}0\\ \mathrm{sh}\,\eta/2\end{pmatrix}\begin{pmatrix}e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ -e^{i\phi/2}\cos\theta/2\end{pmatrix}$$

$$= \begin{pmatrix}0\\ -\mathrm{sh}\,\eta/2\end{pmatrix}\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\phi/2}\cos\theta/2\end{pmatrix} \quad (2)$$

por (1) + (2)

$$S[\Lambda]\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}\mathrm{ch}\,\eta/2\\ -\mathrm{sh}\,\eta/2\end{pmatrix}\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\theta/2}\cos\theta/2\end{pmatrix}$$

II $S[\Lambda]\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = S[\Lambda]\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right] = (T_1+T_2)\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right]$

$$T_1\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right] = \begin{pmatrix}0\\ \mathrm{ch}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}e^{-i\phi/2}\cos\theta/2\\ e^{i\phi/2}\,\mathrm{sen}\,\theta/2\end{pmatrix} \quad (3)$$

$$T_2\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right] = \begin{pmatrix}\mathrm{sh}\,\eta/2\\ 0\end{pmatrix}\otimes\begin{pmatrix}e^{-i\phi/2}\cos\theta/2\\ e^{i\phi/2}\,\mathrm{sen}\,\theta/2\end{pmatrix} \quad (4)$$

(3) + (4)

$$S[\Lambda]\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \begin{pmatrix}\mathrm{sh}\,\eta/2\\ \mathrm{ch}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}e^{-i\phi/2}\cos\theta/2\\ e^{i\phi/2}\,\mathrm{sen}\,\theta/2\end{pmatrix}$$

III $S[\Lambda]\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = S[\Lambda]\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = (T_1+T_2)\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right]$

$$T_1\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = \begin{pmatrix}0\\ \mathrm{ch}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\phi/2}\cos\theta/2\end{pmatrix} \quad (5)$$

$$T_2\left[\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right] = \begin{pmatrix}\mathrm{sh}\,\eta/2\\ 0\end{pmatrix}\otimes\begin{pmatrix}e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ -e^{i\phi/2}\cos\theta/2\end{pmatrix}$$

$$= \begin{pmatrix}-\mathrm{sh}\,\eta/2\\ 0\end{pmatrix}\otimes\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\phi/2}\cos\theta/2\end{pmatrix} \quad (6)$$

(5) + (6)

$$S[\Lambda]\begin{pmatrix}0\\0\\0\\1\end{pmatrix} = \begin{pmatrix}-\mathrm{sh}\,\eta/2\\ \mathrm{ch}\,\eta/2\end{pmatrix}\otimes\begin{pmatrix}-e^{-i\phi/2}\,\mathrm{sen}\,\theta/2\\ e^{i\phi/2}\cos\theta/2\end{pmatrix}$$