Solucion al ejercicio propuesto en el Cap 8 Sebastián C. Higuera Mesa.

Sabemos que:

$$\partial_a\partial_b \operatorname{Exp}(a_{ij}x^ix^j) = \operatorname{Exp}(a_{ij}x^ix^j)\overbrace{\partial_a(a_{ij}x^ix^j)}^{(a_{aj}x^j+a_{ia}x^i)}(a_{bj}x^j+a_{ib}x^i)$$
$$+\operatorname{Exp}(a_{ij}x^ix^j)(2a_{ab})$$

derivando con respecto a c tenemos:

$$\partial_a\partial_b\partial_c \operatorname{Exp}(a_{ij}x^ix^j) = \left[\partial_c \operatorname{Exp}(a_{ij}x^ix^j)\right](a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)$$
$$+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_c\left[(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)\right]$$
$$+2a_{ab}\operatorname{Exp}(a_{ij}x^ix^j)\underbrace{\partial_c(a_{ij}x^ix^j)}_{(a_{cj}x^j+a_{ic}x^i)}$$

$$= \operatorname{Exp}(a_{ij}x^ix^j)(a_{cj}x^j+a_{ic}x^i)(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)$$
$$+\operatorname{Exp}(a_{ij}x^ix^j)\left[(a_{ac}+a_{ca})(a_{bj}x^j+a_{ib}x^i)+(a_{aj}x^j+a_{ia}x^i)(a_{bc}+a_{cb})\right]$$
$$+\operatorname{Exp}(a_{ij}x^ix^j)\left[2a_{ab}(a_{cj}x^j+a_{ic}x^i)\right]$$

Como puede observarse, al evaluar ésto en $x=0$ todos los terminos se hacen cero.

Al derivar con respecto a d tenemos $\partial_a\partial_b\partial_c\partial_d \operatorname{Exp}(a_{ij}x^ix^j) =$

$$= \operatorname{Exp}(a_{ij}x^ix^j)(a_{dj}x^j+a_{id}x^i)(a_{cj}x^j+a_{ic}x^i)(a_{bj}x^j+a_{ib}x^i)(a_{aj}x^j+a_{ia}x^i)$$

① $+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_d\left[(a_{cj}x^j+a_{ic}x^i)(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)\right]$

② $+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_d(a_{ij}x^ix^j)\left[(a_{ac}+a_{ca})(a_{bj}x^j+a_{ib}x^i)+(a_{aj}x^j+a_{ia}x^i)(a_{bc}+a_{c}\right.$

③ $+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_d\left[\overbrace{(a_{ac}+a_{ca})}^{2a_{ac}}(a_{bj}x^j+a_{ib}x^i)+(a_{aj}x^j+a_{ia}x^i)\underbrace{(a_{bc}+a_{cb})}_{2a_{bc}}\right.$

④ $+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_d(a_{ij}x^ix^j)\left(2a_{ab}(a_{cj}x^j+a_{ic}x^i)\right)$

⑤ $+\operatorname{Exp}(a_{ij}x^ix^j)\,\partial_d\left[2a_{ab}(a_{cj}x^j+a_{ic}x^i)\right]$

debemos resolver los términos ①, ②, ③, ④ y ⑤

$$① = \mathrm{Exp}(a_{ij}x^ix^j)\Big[(a_{cd}+a_{dc})(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)$$

$$+(a_{cj}x^j+a_{ic}x^i)\,\underbrace{\partial_d\big[(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)\big]}\Big]$$

$$\Downarrow$$

$$(a_{ad}+a_{da})(a_{bj}x^j+a_{ib}x^i)+(a_{aj}x^j+a_{ia}x^i)(a_{bd}+a_{d}$$

$$① = \mathrm{Exp}(a_{ij}x^ix^j)\Big[2a_{cd}(a_{aj}x^j+a_{ia}x^i)(a_{bj}x^j+a_{ib}x^i)$$

$$+2a_{ad}(a_{bj}x^j+a_{ib}x^i)(a_{cj}x^j+a_{ic}x^i)$$

$$+2a_{bd}(a_{cj}x^j+a_{ic}x^i)(a_{aj}x^j+a_{ia}x^i)\Big]$$

$$② = \mathrm{Exp}(a_{ij}x^ix^j)(a_{dj}x^j+a_{id}x^i)\big[2a_{ac}(a_{bj}x^j+a_{ib}x^i)+2a_{bc}(a_{aj}x^j+a_{ia}x^i)$$

$$③ = \mathrm{Exp}(a_{ij}x^ix^j)\big[2a_{ac}\underbrace{(a_{bd}+a_{db})}_{2a_{bd}}+2a_{bc}\underbrace{(a_{ad}+a_{da})}_{2a_{ad}}\big]$$

$$= \mathrm{Exp}(a_{ij}x^ix^j)\big[4a_{ac}a_{bd}+4a_{bc}a_{ad}\big]$$

$$④ = \mathrm{Exp}(a_{ij}x^ix^j)(a_{dj}x^j+a_{id}x^i)\big(2a_{ab}(a_{cj}x^i+a_{ic}x^i)\big)$$

$$⑤ = \mathrm{Exp}(a_{ij}x^ix^j)\big(\underbrace{2a_{ab}a_{cd}+2a_{ab}a_{dc}}_{4a_{ab}a_{cd}}\big)$$

Al evaluar estos resultados en $J=0$ es decir en $x=0$ obtenemos

$$\partial_a\partial_b\partial_c\partial_d\,\mathrm{Exp}(a_{ij}x^ix^j)\Big|_{x=0} = 4a_{ac}a_{bd}+4a_{bc}a_{ad}+4a_{ab}a_{cd}$$

es decir que

$$\langle\phi_1\phi_2\phi_3\phi_4\rangle = \cancel{4}\,\frac{A^{-1}_{ac}}{\cancel{2}m^2}\frac{A^{-1}_{bd}}{\cancel{2}m^2}+\cancel{4}\,\frac{A^{-1}_{bc}}{\cancel{2}m^2}\frac{A^{-1}_{ad}}{\cancel{2}m^2}+\cancel{4}\,\frac{A^{-1}_{ab}}{\cancel{2}m^2}\frac{A^{-1}_{cd}}{\cancel{2}m^2}$$

$$= \frac{1}{m^4}\left[A^{-1}_{ac}A^{-1}_{bd}+A^{-1}_{bc}A^{-1}_{ad}+A^{-1}_{ab}A^{-1}_{cd}\right]$$