Ejercicio:

Demostrar que la siguiente "ley" no puede ser correcta $\Leftrightarrow$ No es invariante Lorentz

$$(\partial_0 - \partial_1^2)\phi = 0$$

Tiene que ser invariante bajo la siguiente transformación (Lorentz).

$$x^{0'} = \gamma x^0 - \gamma\beta x^1$$
$$x^{1'} = -\gamma\beta x^0 + \gamma x^1$$

Si fuera invariante, deberíamos obtener

$$(\partial_0 - \partial_1^2)\phi = (\partial_{0'} - \partial_{1'}^2)\phi$$

$$\boxed{\partial_0 \phi(x^{0'}, x^{1'})} = \frac{\partial \phi}{\partial x^{0'}} \cdot \frac{\partial x^{0'}}{\partial x^0} + \frac{\partial \phi}{\partial x^{1'}} \frac{\partial x^{1'}}{\partial x^0}$$

$\partial_0 \phi$

Regla de la cadena

$\partial_{0'}\phi$

$$= \boxed{\partial_{0'}\phi \cdot (\gamma) + \partial_{1'}\phi\,(-\gamma\beta)}$$

$$\partial_1 \phi(x^{0'}, x^{1'}) = \frac{\partial \phi}{\partial x^{0'}} \frac{\partial x^{0'}}{\partial x^1} + \frac{\partial \phi}{\partial x^{1'}} \frac{\partial x^{1'}}{\partial x^1}$$

$$= \partial_{0'}\phi \cdot (-\gamma\beta) + \partial_{1'}\phi\,(\gamma)$$

$$= (-\gamma\beta\,\partial_{0'} + \gamma\,\partial_{1'})\phi = \partial_1 \phi$$

Son la misma operación

$$\boxed{\partial_1^2 \phi} = \partial_1 (\partial_1 \phi) = (-\gamma\beta\,\partial_{0'} + \gamma\,\partial_{1'}) \cdot (-\gamma\beta\,\partial_{0'} + \gamma\,\partial_{1'})\,\phi$$

$$= (\gamma^2\beta^2\,\partial_{0'}^2 + \gamma^2\,\partial_{1'}^2 - 2\gamma^2\beta\,\partial_{0'}\partial_{1'})\,\phi$$

$$= \boxed{\gamma^2 (\beta^2\,\partial_{0'}^2 + \partial_{1'}^2 - 2\beta\,\partial_{0'}\partial_{1'})\,\phi}$$

$$\boxed{(\partial_0 - \partial_1^2)\,\phi} = \gamma\,(\partial_{0'} - \beta\,\partial_{1'})\,\phi - \gamma^2(\beta^2\,\partial_{0'}^2 + \partial_{1'}^2 - 2\beta\,\partial_{0'}\partial_{1'})\,\phi$$

$$\boxed{\neq (\partial_{0'} - \partial_{1'}^2)\,\phi}$$

$-2\beta\,\partial_{0'}\partial_{1'}$ no se anula con nada.

γ y γ^2 no se anularán.

β y β^2 tampoco...

Hay un $\partial_{0'}^2$ que tampoco se anula...

Nada, que no hay manera. NO es invariante Lorentz.