RODOLFO GUIDOBONO	CURSO TEORÍA CUÁNTICA DE CAMPOS - EJERCICIO	HOJA Nº 1 FECHA 8/2/19

Definida la "COSA" $= -6\phi_1^2 - 6\phi_2^2 - 6\phi_3^2 - \sqrt{2}\,\phi_1\phi_2 - \sqrt{2}\,\phi_2\phi_3$

a) Encontrar la matriz A tal que:

$$(\phi_1\ \phi_2\ \phi_3)\,(A)\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix} = \text{"COSA"}$$

$$\text{COSA} = (\phi_1\ \phi_2\ \phi_3)\begin{pmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{pmatrix}\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix} = \phi_i A_{ij}\phi_j \quad (1)$$

$$\text{COSA} = \phi_1(-6)\phi_1 + \phi_2(-6)\phi_2 + \phi_3(-6)\phi_3 +$$
$$+ \phi_1(-\sqrt{2})\phi_2 + \phi_2(-\sqrt{2})\phi_3 \quad (2)$$

Comparando (1) y (2)

$-6 = A_{11}$

$-6 = A_{22}$

$-6 = A_{33}$

$$\left.\begin{array}{l} -\sqrt{2} = A_{12} + A_{21}\\ -\sqrt{2} = A_{23} + A_{32}\end{array}\right\} \xrightarrow{\text{asumiendo simetría}} \begin{array}{l} A_{12} = A_{21}\\ A_{23} = A_{32}\end{array}$$

entonces $A_{12} = A_{21} = -\sqrt{2}/2$

$A_{23} = A_{32} = -\sqrt{2}/2$

$$A = \begin{pmatrix} -6 & -\sqrt{2}/2 & 0\\ -\sqrt{2}/2 & -6 & -\sqrt{2}/2\\ 0 & -\sqrt{2}/2 & -6\end{pmatrix}$$

NOTA

HOJA Nº 2

FECHA

b) Diagonalizar A

1. Valores propios $\Rightarrow \det(A-\lambda I)=0$

$$\det\begin{pmatrix} (-6-\lambda) & -\sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & (-6-\lambda) & -\sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & (-6-\lambda) \end{pmatrix}=0$$

$$(-6-\lambda)\left[(-6-\lambda)^2-1/2\right]-\left(-\sqrt{2}/2\right)\left[-\sqrt{2}/2\,(-6-\lambda)-0\right]=0$$

$$(-6-\lambda)^3-\frac{1}{2}(-6-\lambda)-\frac{1}{2}(-6-\lambda)=0$$

$$(-6-\lambda)^3-(-6-\lambda)=0$$

$$(-6-\lambda)\left((-6-\lambda)^2-1\right)=0$$

$\hookrightarrow -6-\lambda=0 \rightarrow \boxed{\lambda=6}$

$\hookrightarrow (-6-\lambda)^2-1=0$

$-6-\lambda=\pm 1 \quad \boxed{\lambda=5} \quad \boxed{\lambda=7}$

2. Vectores propios $\Rightarrow (A-\lambda I)(v)=0$

$$\begin{pmatrix} -6-\lambda & -\sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & -6-\lambda & -\sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & -6-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

2.1 $\underline{\lambda_1=5}$

$$\begin{pmatrix} -1 & -\sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & -1 & -\sqrt{2}/2 \\ 0 & -\sqrt{2}/2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -x_1-\sqrt{2}/2\,x_2=0 \\ -\sqrt{2}/2\,x_1-x_2-\sqrt{2}/2\,x_3=0 \\ -\sqrt{2}/2\,x_2-x_3=0 \end{cases}$$

$\longrightarrow -x_1+x_3=0 \Rightarrow x_1=x_3$

$\hookrightarrow -\sqrt{2}\left(x_1/2+x_3/2\right)-x_2=0 \longrightarrow -\sqrt{2}\,x_1-x_2=0$

$$v_1=\begin{pmatrix} x_1 \\ -\sqrt{2}\,x_1 \\ x_1 \end{pmatrix}=x_1\begin{pmatrix} 1 \\ -\sqrt{2} \\ 1 \end{pmatrix}$$ para que $\|v_1\|=1$

NOTA

		HOJA Nº 3
		FECHA

para que $\|v_1\| = 1 \Rightarrow \left(x_1^2 + (\sqrt{2}\,x_1)^2 + x_1^2\right)^{1/2} = 1 \qquad x_1 = 1/2$

$$v_1 = \frac{1}{2}\begin{pmatrix}1\\-\sqrt{2}\\1\end{pmatrix}$$

2.2. $\underline{\lambda_2 = 6}$

$$\begin{pmatrix}0 & -\sqrt{2}/2 & 0\\ -\sqrt{2}/2 & 0 & -\sqrt{2}/2\\ 0 & -\sqrt{2}/2 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left.\begin{array}{l} x_2 = 0\\ -\sqrt{2}/2\,x_1 - \sqrt{2}/2\,x_3 = 0 \qquad -x_1 = x_3\end{array}\right\} \rightarrow v_2 = \begin{pmatrix}x_1\\0\\-x_1\end{pmatrix}$$

$\|v_2\| = 1 \Rightarrow 2x_1^2 = 1 \qquad x_1 = 1/\sqrt{2}$

$$v_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}\sqrt{2}\\0\\-\sqrt{2}\end{pmatrix}$$

2.3 $\underline{\lambda_3 = 7}$

$$\begin{pmatrix}1 & -\sqrt{2}/2 & 0\\ -\sqrt{2}/2 & 1 & -\sqrt{2}/2\\ 0 & -\sqrt{2}/2 & 1\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\left.\begin{array}{l} +\text{ fila 1 y 3} \rightarrow x_1 - x_3 = 0 \qquad x_1 = x_3\\ +\text{ fila 2} \rightarrow -\sqrt{2}/2\,x_1 + x_2 + (-\sqrt{2}/2)x_3 = 0\\ \qquad x_2 - \sqrt{2}\,x_1 = 0 \longrightarrow x_2 = \sqrt{2}\,x_1\end{array}\right\} v_3 = \begin{pmatrix}x_1\\ \sqrt{2}\,x_1\\ x_1\end{pmatrix}$$

$\|v_3\| = 1 = \left(x_1^2 + 2x_1^2 + x_1^2\right)^{1/2} = 2x_1 \longrightarrow x_1 = 1/2$

$$v_3 = \frac{1}{2}\begin{pmatrix}1\\ \sqrt{2}\\ 1\end{pmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix}1 & \sqrt{2} & 1\\ -\sqrt{2} & 0 & \sqrt{2}\\ 1 & -\sqrt{2} & 1\end{pmatrix}$$

NOTA

c) Demostrar que

$$\text{"cos A"} = -5\varphi_1^2 - 6\varphi_2^2 - 7\varphi_3^2 \quad \text{donde} \quad (\phi) = (M)(\varphi) \quad (3)$$

$$\text{"cos A"} = (\phi_1 \ \phi_2 \ \phi_3)(A)\begin{pmatrix}\phi_1\\ \phi_2\\ \phi_3\end{pmatrix} = \phi^T A \phi$$

por (3) $\phi = M\varphi \Rightarrow \phi^T = \varphi^T M^T$

$$\text{"cos A"} = \phi^T A \phi = (\varphi^T M^T) A (M\varphi)$$

$$= \varphi^T \underbrace{(M^T A M)}_{= D} \varphi$$

$$\text{"cos A"} = \varphi^T D \varphi \quad \text{pero} \quad D = \begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{pmatrix}$$

$$\text{"cos A"} = (\varphi_1 \ \varphi_2 \ \varphi_3)\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{pmatrix}\begin{pmatrix}\varphi_1\\ \varphi_2\\ \varphi_3\end{pmatrix}$$

$$\text{"cos A"} = \varphi_1 \lambda_1 \varphi_1 + \varphi_2 \lambda_2 \varphi_2 + \varphi_3 \lambda_3 \varphi_3$$

$$= -5\varphi_1^2 - 6\varphi_2^2 - 7\varphi_3^2 \qquad \text{Q.E.D.}$$