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CURSO TEORÍA CUÁNTICA DE CAMPOS

J. GARCÍA - EJERCICIOS CAP. 5

HOJA Nº 1

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EJERCICIO 5.4. Comprobar que dada la acción $S_{(\phi)} = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$ la ecuación de Schwinger - Dyson es: $m^2 Z'_{[J]} + \frac{\lambda}{6} Z'''_{[J]} = J Z[J]$

$$Z_{[J]} = \int_{-\infty}^{\infty} e^{-S+J\phi}\, d\phi$$

$$Z'_{[J]} = \int_{-\infty}^{\infty} e^{-S+J\phi}\, \phi\, d\phi$$

$$Z''_{[J]} = \int_{-\infty}^{\infty} e^{-S+J\phi}\, \phi^2\, d\phi$$

$$Z'''_{[J]} = \int_{-\infty}^{\infty} e^{-S+J\phi}\, \phi^3\, d\phi$$

Si $S(\phi) = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$ entonces

$$S'(\phi) = m^2\phi + \frac{\lambda}{6}\phi^3$$

$$\int_{-\infty}^{\infty} e^{-S+J\phi} \cdot S'_{(\phi)}\, d\phi = \int e^{-S+J\phi}\, m^2\phi\, d\phi + \int e^{-S+J\phi}\, \frac{\lambda}{6}\phi^3\, d\phi$$

$$= m^2 \underbrace{\int_{-\infty}^{\infty} e^{-S+J\phi}\, \phi\, d\phi}_{Z'_{[J]}} + \frac{\lambda}{6} \underbrace{\int_{-\infty}^{\infty} e^{-S+J\phi}\, \phi^3\, d\phi}_{Z'''_{[J]}}$$

① $$\int_{-\infty}^{\infty} e^{-S+J\phi} \cdot S'_{(\phi)} \cdot d\phi = m^2 Z'_{[J]} + \lambda/6 \cdot Z'''[J]$$

Dado $e^{-S+J\phi}$ entonces

② $$\frac{\partial}{\partial\phi}\left(e^{-S+J\phi}\right) = e^{-S+J\phi}\left(-S'_{(\phi)} + J\right)$$

Volviendo a

$$\int_{-\infty}^{\infty} e^{-S+J\phi}\, S'_{(\phi)}\, d\phi = -\int e^{-S+J\phi}(-1)\, S'_{(\phi)}\,(+J-J)\, d\phi$$

$$= -\int e^{-S+J\phi}\left(-S'_{(\phi)} + J\right) d\phi + \int e^{-S+J\phi}\, J\, d\phi$$

NOTA

$$Z_{[J]} = \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi}\, d\phi$$

$$= \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi} \cdot e^{-\frac{\lambda}{24}\phi^4}\, d\phi$$

Siendo $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$

$$e^{-\frac{\lambda}{24}\phi^4} \approx 1 + \left(-\frac{\lambda}{24}\phi^4\right) + \frac{1}{2}\left(-\frac{\lambda}{24}\phi^4\right)^2 + \frac{1}{6}\left(-\frac{\lambda}{24}\phi^4\right)^3 + \frac{1}{24}\left(-\frac{\lambda}{24}\phi^4\right)^4$$

$$\approx 1 - \frac{\lambda}{24}\phi^4 + \frac{\lambda^2}{2.24^2}\phi^8 - \frac{\lambda^3}{6.24^3}\phi^{12} + \frac{\lambda^4}{24.24^4}\phi^{16}$$

$$Z_{[J]} \approx \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\left(1 - \frac{\lambda}{24}\phi^4 + \frac{\lambda^2}{2.24^2}\phi^8 - \frac{\lambda^3}{6.24^3}\phi^{12} + \frac{\lambda^4}{24.24^4}\phi^{16}\right) d\phi$$

abreviar - $A \equiv -\frac{m^2}{2}\phi^2 + J\phi$ entonces

$$Z_{[J]} \approx \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi - \frac{\lambda}{24}\int_{-\infty}^{\infty} e^{A}\phi^4\, d\phi + \frac{\lambda^2}{2.24^2}\int_{-\infty}^{\infty} e^{A}\phi^8\, d\phi -$$

$$- \frac{\lambda^3}{6.24^3}\int_{-\infty}^{\infty} e^{A}\phi^{12}\, d\phi + \frac{\lambda^4}{24.24^4}\int_{-\infty}^{\infty} e^{A}\, d\phi \,.\, \phi^{16}$$

con $Z^{(i)}_{0[J]} = \int_{-\infty}^{\infty} e^{-m^2/2\,\phi^2 + J\phi}\,\phi^i\, d\phi \equiv \int_{-\infty}^{\infty} e^{A}\phi^i\, d\phi$

$$Z_{[J]} = Z_{0[J]} - \frac{\lambda}{24} Z^{(4)}_{0[J]} + \frac{\lambda^2}{2.24^2} Z^{(8)}_{0[J]} - \frac{\lambda^3}{6.24^3} Z^{(12)}_{0[J]} + \frac{\lambda^4}{24.24^4} Z^{(16)}_{0[J]}$$

$$Z_{[J]} = Z_{0[J]}\left(1 - \frac{\lambda}{24}\underbrace{\frac{Z^{(4)}_{0[J]}}{Z_{0[J]}}}_{\langle\phi^4\rangle_0} + \frac{\lambda^2}{2.24^2}\underbrace{\frac{Z^{(8)}_{0[J]}}{Z_{0[J]}}}_{\langle\phi^8\rangle_0} - \frac{\lambda^3}{6.24^3}\underbrace{\frac{Z^{(12)}_{0[J]}}{Z_{0[J]}}}_{\langle\phi^{12}\rangle_0} + \frac{\lambda^4}{24.24^4}\underbrace{\frac{Z^{(16)}_{0[J]}}{Z_{0[J]}}}_{\langle\phi^{16}\rangle_0}\right)$$

$J = 0 \Rightarrow$ fórmula 4.3 Cruel

$$Z_{[0]} = Z_{0[0]}\left(1 - \frac{\lambda}{24}\langle\phi^4\rangle_0 + \frac{\lambda^2}{2.24^2}\langle\phi^8\rangle_0 - \frac{\lambda^3}{6.24^3}\langle\phi^{12}\rangle_0 + \frac{\lambda^4}{24.24^4}\langle\phi^{16}\rangle_0\right)$$

$$Z''_{[J]} = Z''_0[J] - \frac{\lambda}{24} Z^{(6)}_{0[J]} + \frac{\lambda^2}{2.24^2} Z^{(10)}_{0[J]} - \frac{\lambda^3}{6.24^3} Z^{(14)}_{0[J]} + \frac{\lambda^4}{24.24^4} Z^{(18)}_{0[J]}$$

NOTA

Pn ② $\int_{-\infty}^{\infty} e^{-S+J\phi} \left(-S'_{(\phi)} + J\right) d\phi = e^{-S+J\phi}\Big|_{-\infty}^{\infty} =$

$= e^{-(\infty^2)+\infty} - e^{-(-\infty)^2-\infty} \longrightarrow 0$

tiende a cero por haber siempre un $e^{-\infty^2}$

$$\int_{-\infty}^{\infty} e^{-S+J\phi}\, S'_{(\phi)}\, d\phi = 0 + J\int_{-\infty}^{\infty} e^{-S+J\phi}\, d\phi$$

$$= J\; Z[J]$$

De ①

$$\boxed{m^2 Z'[J] + \frac{\lambda}{6} Z'''[J] = J\, Z[J]} \qquad \text{Q.E.D.}$$

EJERCICIO 5.7 Calcular $\langle\phi^2\rangle$ a segundo orden

c) Con los diagramas de Feynman

b) Cálculo directo

CÁLCULO DIRECTO

$$\langle\phi^2\rangle = \frac{Z''(0)}{Z(0)}$$

Vamos a necesitar $\langle\phi^2\rangle_0 = 1/m^2$ $\qquad Z_0[J] = \int_{-\infty}^{\infty} e^{-\frac{m^2}{2}\phi^2 + J\phi}\, d\phi$

$\langle\phi^4\rangle_0 = 3/m^4$

$\langle\phi^6\rangle_0 = \frac{1}{m^6}\, 5.3$

$\langle\phi^8\rangle_0 = 1/m^8\; 7.5.3$

$\langle\phi^{10}\rangle_0 = 1/m^{10}\; 9.7.5.3$

$\langle\phi^{12}\rangle_0 = 1/m^{12}\; 11.9.7.5.3$

$\langle\phi^{14}\rangle_0 = 1/m^{14}\; 13.11.9.7.5.3$

$\langle\phi^{16}\rangle_0 = 1/m^{16}\; 15.13.11.9.7.5.3$

$\langle\phi^{18}\rangle_0 = 1/m^{18}\; 17.15.13.11.9.7.5.3$

NOTA

$$Z''_{[J]} = Z_0[J]\left(\underbrace{\frac{Z_0''[J]}{Z_0[J]}} - \frac{\lambda}{24}\underbrace{\frac{Z_0^{(6)}[J]}{Z_0[J]}} + \frac{\lambda^2}{2.24^2}\underbrace{\frac{Z_0^{(10)}[J]}{Z_0[J]}} - \frac{\lambda^3}{6.24^3}\underbrace{\frac{Z_0^{(14)}[J]}{Z_0[J]}} + \frac{\lambda^4}{24.24^4}\underbrace{\frac{Z_0^{(18)}[J]}{Z_0[J]}}\right)$$

$J=0 \Rightarrow$ f. 43 Cruz $\langle\phi^2\rangle_0$ $\quad\langle\phi^6\rangle_0\quad$ $\langle\phi^{10}\rangle_0\quad$ $\langle\phi^{14}\rangle_0\quad$ $\langle\phi^{18}\rangle_0$

$$Z''_{(0)} = Z_0(0)\left(\langle\phi^2\rangle_0 - \frac{\lambda}{24}\langle\phi^6\rangle_0 + \frac{\lambda^2}{2.24^2}\langle\phi^{10}\rangle_0 - \frac{\lambda^3}{6.24^3}\langle\phi^{14}\rangle_0 + \frac{\lambda}{24.24^4}\langle\phi^{18}\rangle_0\right)$$

$$\langle\phi^2\rangle = \frac{Z''[0]}{Z[0]}$$

$$\langle\phi^2\rangle = \frac{\langle\phi^2\rangle_0 - \frac{\lambda}{24}\langle\phi^6\rangle_0 + \frac{\lambda^2}{2.24^2}\langle\phi^{10}\rangle_0 - \frac{\lambda^3}{6.24^3}\langle\phi^{14}\rangle_0 + \frac{\lambda^4}{24.24^4}\langle\phi^{18}\rangle_0}{1 - \frac{\lambda}{24}\langle\phi^4\rangle_0 + \frac{\lambda^2}{2.24^2}\langle\phi^8\rangle_0 - \frac{\lambda^3}{6.24^3}\langle\phi^{12}\rangle_0 + \frac{\lambda^4}{24.24^4}\langle\phi^{16}\rangle_0}$$

(3) $$\langle\phi^2\rangle = \frac{\frac{1}{m^2} - \frac{\lambda}{24}\frac{5.3}{m^6} + \frac{\lambda^2}{2.24^2}\frac{9.7.5.3}{m^{10}} - \frac{\lambda^3}{6.24^3}\frac{13.11.9.7.5.3}{m^{14}} + \frac{\lambda^4}{24.24^4}\frac{17.15.13.11.9.7.5.3}{m^{18}}}{1 - \frac{\lambda}{24}\frac{3}{m^4} + \frac{\lambda^2}{2.24^2}\frac{7.5.3}{m^8} - \frac{\lambda^3}{6.24^3}\frac{11.9.7.5.3}{m^{12}} + \frac{\lambda^4}{24.24^4}\frac{15.13.11.9.7.5.3}{m^{16}}}$$

Desarrollamos $\langle\phi^2\rangle$ por Taylor en torno a $\lambda=0$

$$\langle\phi^2\rangle = f_{(\lambda)} = \frac{N(\lambda)}{D(\lambda)} \longrightarrow f_{(\lambda)} = f(0) + f'_{(0)}\lambda + \frac{f''_{(0)}}{2}\lambda^2 + \dots \quad (4a)$$

(4b) $$f'_{(\lambda)} = \left[\frac{N'_{(\lambda)}}{D(\lambda)}\right] - \left[\frac{N(\lambda)}{D(\lambda)^2}D'_{(\lambda)}\right]$$

$$f''_{(\lambda)} = \left[\frac{N''_{(\lambda)}}{D(\lambda)} - \frac{N'_{(\lambda)}}{D^2_{(\lambda)}}D'_{(\lambda)}\right] - \left[\frac{N'_{(\lambda)}}{D^2_{(\lambda)}}D'_{(\lambda)} - 2\frac{N(\lambda)}{D^3_{(\lambda)}}D'_{(\lambda)}D'_{(\lambda)} + \frac{N(\lambda)}{D^2_{(\lambda)}}D''_{(\lambda)}\right]$$

(4c) $$f''_{(\lambda)} = \frac{N''_{(\lambda)}}{D(\lambda)} - \frac{N'_{(\lambda)}}{D^2_{(\lambda)}}D'_{(\lambda)} - \frac{N'_{(\lambda)}}{D^2_{(\lambda)}}D'_{(\lambda)} + 2\frac{N(\lambda)}{D^3_{(\lambda)}}D'_{(\lambda)}D'_{(\lambda)} - \frac{N(\lambda)}{D^2_{(\lambda)}}D''_{(\lambda)}$$

$N(\lambda)$ es el numerador de (3) por lo tanto

$$N'_{(\lambda)} = -\frac{5.3}{24}\frac{1}{m^6} + \frac{2.9.7.5.3}{2.24^2}\frac{\lambda}{m^{10}} - \frac{3}{6.24^3}\frac{13.11\dots3}{m^{14}}\lambda^2 + \frac{4}{24.24^4}\frac{17.15\dots3}{m^{18}}\lambda^3$$

$$N''_{(\lambda)} = \frac{2.9.7.5.3}{2.24^2}\frac{1}{m^{10}} - \frac{2.3}{6.24^3}\frac{13.11.9.7.5.3}{m^{14}}\lambda + \frac{3.4}{24.24^4}\frac{17.15.13.11.9.7.5.3}{m^{18}}\lambda^2$$

$D(\lambda)$ es el denominador de (3) por lo tanto

$$D'_{(\lambda)} = -\frac{3}{24m^4} + \frac{2.7.5.3}{2.24^2}\frac{\lambda}{m^8} - \frac{3}{6.24^3}\frac{11.9.7.5.3}{m^{12}}\lambda^2 + \frac{4.}{24.24^4}\frac{15.13\ldots 3}{m^{16}}\lambda^3$$

$$D''_{(\lambda)} = \frac{2}{2.24^2}\frac{7.5.3}{m^8} - \frac{2.3}{6.24^3}\frac{11.9.7.5.3}{m^{12}}\lambda + \frac{3.4}{24.24^4}\frac{15.13.11.9.7.5.3}{m^{16}}\lambda^2$$

Para $\lambda = 0$

$$N_{(0)} = \frac{1}{m^2} \qquad D_{(0)} = 1$$

$$N'_{(0)} = -\frac{5.3}{24}\frac{1}{m^6} \qquad D'_{(0)} = -\frac{3}{24}\frac{1}{m^4}$$

$$N''_{(0)} = \frac{2.9.7.5.3}{2.24^2}\frac{1}{m^{10}} \qquad D''_{(0)} = \frac{2.7.5.3}{2.24^2}\frac{1}{m^8}$$

(4a) $\rightarrow$ $$f^*_{(0)} = \frac{N_{(0)}}{D_{(0)}} = \frac{1/m^2}{1}$$

$$\boxed{f_{(0)} = \frac{1}{m^2}}$$

(4b) $\rightarrow$ $$f'_{(0)} = \frac{-\frac{5.3}{24}\frac{1}{m^6}}{1} - \frac{1/m^2}{1}\left(-\frac{3}{24}\frac{1}{m^4}\right) = -\frac{5.3}{24}\frac{1}{m^6} + \frac{3}{24}\frac{1}{m^2 m^4}$$

$$= -\frac{(15+3)}{24}\frac{1}{m^6}$$

$$\boxed{f'_{(0)} = -\frac{1}{2}\frac{1}{m^6}}$$

(4c) $\rightarrow$ $$f''_{(0)} = \frac{\frac{2.9.7.5.3}{2.24^2}\frac{1}{m^{10}}}{1} - \frac{\left(-\frac{5.3}{24}\right)\frac{1}{m^6}}{1^2}\left(-\frac{3}{24}\frac{1}{m^4}\right) - \frac{\left(-\frac{5.3}{24}\right)\frac{1}{m^6}}{1^2}\times$$

$$\times\left(-\frac{3}{24}\frac{1}{m^4}\right) + 2.\frac{1/m^2}{1^3}\left(-\frac{3}{24}\frac{1}{m^4}\right)\left(-\frac{3}{24}\frac{1}{m^4}\right) -$$

$$-\frac{1/m^2}{1^2}\frac{2.7.5.3}{2.24^2}\frac{1}{m^8}$$

$$f''_{(0)} = \frac{9.7.5.3}{24^2}\frac{1}{m^{10}} - \frac{5.3.3}{24^2}\frac{1}{m^{10}} - \frac{5.3.3}{24^2}\frac{1}{m^{10}} + 2\frac{3.3}{24^2}\frac{1}{m^{10}} - \frac{7.5.3}{24^2}\frac{1}{m^{10}}$$

$$f''_{(0)} = \frac{1}{24^2 m^{10}}\left(9.7.5.3 - 5.3.3 - 5.3.3 + 2.3.3 - 7.5.3\right)$$

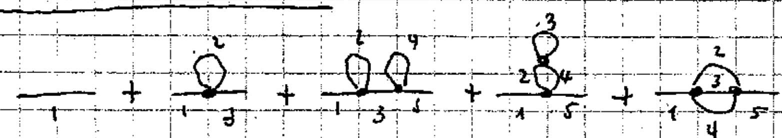
$$= \frac{768}{24^2}\frac{1}{m^{10}} = \frac{32}{24}\frac{1}{m^{10}} =$$

$$\boxed{f''_{(0)} = \frac{4}{3}\frac{1}{m^{10}}}$$

(4a) $\rightarrow \langle\phi^2\rangle \cong f_{(0)} + f'_{(0)}\lambda + f''_{(0)}\frac{\lambda^2}{2}$

$$\boxed{\langle\phi^2\rangle \approx \frac{1}{m^2} - \frac{\lambda}{2m^6} + \frac{2\lambda^2}{3m^{10}}}$$

DIAGRAMAS DE FEYNMAN

$\langle\phi^2\rangle =$ 

$\frac{1}{m^2} \rightarrow$	$\left(\frac{1}{m^2}\right)^1$	$\left(\frac{1}{m^2}\right)^3$	$\left(\frac{1}{m^2}\right)^5$	$\left(\frac{1}{m^2}\right)^5$	$\left(\frac{1}{m^2}\right)^5$
$\lambda \rightarrow$	$-$	$-\lambda$	$(-\lambda)(-\lambda)$	$(-\lambda)(\lambda)$	$(-\lambda)(-\lambda)$
simetría		$1/2$	$1/2$ $1/2$	$1/2$ $1/2$	$1/S$??

$\underbrace{\hspace{6em}}$ asumo que por haber 2 "globitos" hay 2 factores de simetría

$$\langle\phi^2\rangle = \frac{1}{m^2} - \frac{\lambda}{2}\frac{1}{m^6} + \lambda^2\frac{1}{m^{10}}\left(\frac{1}{4} + \frac{1}{4} + \frac{1}{S}\right)$$

para que ~~xxx~~ sea igual al del cálculo directo $\frac{1}{4} + \frac{1}{4} + \frac{1}{S} = \frac{2}{3}$

Se cumple para $\frac{1}{S} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$