Teoria Cuantica de Campos.

) Calculo directo.

$$Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 - \frac{\lambda}{24}\phi^4 + J\phi} \Big/ \quad S[\phi] = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$$

$$Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 + J\phi} \cdot e^{-\frac{\lambda}{24}\phi^4}$$

TRUCO

$f(x) = e^x \to f(0) = 1$

$f'(x) = e^x \to f'(0) = 1$

$f''(x) = e^x \to f''(0) = 1$

$$e^{-\frac{\lambda}{24}\phi^4} = 1 + \left(-\frac{\lambda}{24}\phi^4\right) + \frac{1}{2}\left(-\frac{\lambda}{24}\phi^4\right)^2 + \frac{1}{6}\left(-\frac{\lambda}{24}\phi^4\right)^3$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$Z[J] = \int_{-\infty}^{\infty} d\phi\, e^{-\frac{m^2}{2}\phi^2 + J\phi} \cdot \left(1 - \frac{\lambda}{24}\phi^4 + \frac{1}{2}\frac{\lambda^2}{24^2}\phi^8 - \frac{1}{6}\cdot\frac{\lambda^3}{24^3}\phi^{12}\right.$$

$$Z[J] = Z_0[J] - \frac{\lambda}{24} Z_0^{(4)}[J] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(8)}[J] - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(12)}[J] \quad \dots (1)$$

$$Z[0] = Z_0[0] - \frac{\lambda}{24} Z_0^{(4)}[0] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(8)}[0] - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(12)}[0]$$

$$Z[0] = Z_0[0]\left(1 - \frac{\lambda}{24}\langle\phi^4\rangle_0 + \frac{1}{2}\frac{\lambda^2}{24^2}\langle\phi^8\rangle_0 - \frac{1}{6}\frac{\lambda^3}{24^3}\langle\phi^{12}\rangle_0\right)$$

$$Z[0] = Z_0[0]\left(1 - \frac{\lambda}{24}\frac{3}{m^4} + \frac{1}{2}\frac{\lambda^2}{24}\cdot\frac{7.5.3}{m^8} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{11.9.7.5.3}{m^{12}}\right) \quad \dots (2)$$

Derivamos dos veces la ecuación (1)

$$Z^{(2)}[J] = Z_0^{(2)}[J] - \frac{\lambda}{24} Z_0^{(6)}[J] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(10)}[J] - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(14)}[J]$$

$$Z^{(2)}[0] = Z^{(2)}[0] - \frac{\lambda}{24} Z_0^{(6)}[0] + \frac{1}{2}\frac{\lambda^2}{24^2} Z_0^{(10)}[0] - \frac{1}{6}\frac{\lambda^3}{24^3} Z_0^{(14)}[0]$$

$$Z^{(2)}[0] = Z_0[0]\left(\frac{Z^{(2)}[0] - \frac{\lambda}{24}Z_0^{(6)}[0] + \frac{1}{2}\frac{\lambda^2}{24^2}Z_0^{(10)}[0] - \frac{1}{6}\frac{\lambda^3}{24^3}Z_0^{(14)}[0]}{Z_0[0]}\right)$$

$$Z^{(2)}[0] = Z_0[0]\left(\langle\phi^2\rangle_0 - \frac{\lambda}{24}\langle\phi^6\rangle_0 + \frac{1}{2}\frac{\lambda^2}{24^2}\langle\phi^{10}\rangle_0 - \frac{1}{6}\frac{\lambda^3}{24^3}\langle\phi^{14}\rangle_0\right)$$

$$Z^{(2)}[0] = Z_0[0]\left(\frac{1}{m^2} - \frac{\lambda}{24}\frac{5.3}{m^6} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{9.7.5.3}{m^{10}} - \frac{1}{3!}\frac{\lambda^3}{24^3}\frac{13.11.9.7.5.3}{m^{14}}\right)$$

$\dots (3)$

Entonces lo que queremos.

$$\langle \phi^2 \rangle = \frac{Z^{(2)}[0]}{Z[0]} \quad \ldots (4)$$

Reemplazando (2) y (3) en (4) se tiene.

$$\langle \phi^2 \rangle = \underbrace{\frac{\frac{1}{m^2} - \frac{\lambda}{24}\frac{5.3}{m^6} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{9.7.5.3}{m^{10}} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{13.11.9.7.5.3}{m^{14}}}{1 - \frac{\lambda}{24}\frac{3}{m^4} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{7.5.3}{m^8} - \frac{1}{3!}\frac{\lambda^3}{24^3}\frac{11.9.7.5.3}{m^{12}}}}_{f(\lambda)}$$

Donde.

$$f(0) = \frac{1}{m^2}$$

$$f'(0) = -\frac{1}{2m^6}$$

$$f''(0) = \frac{4}{3m^{10}}$$

Por lo tanto:

$$\langle \phi^2 \rangle = \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{4}{3m^{10}}\lambda^2 . \frac{1}{2!}$$

$$\langle \phi^2 \rangle = \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{4}{6m^{10}}\lambda^2$$

$$\langle \phi^2 \rangle = \frac{1}{m^2} - \frac{1}{2m^6}\lambda + \frac{2}{3m^{10}}\lambda^2$$

$$f(\lambda) = \frac{\frac{1}{m^2} - \frac{\lambda}{24}\frac{5*3}{m^6} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{9*7*5*3}{m^{10}} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{13*11*9*7*5*3}{m^{14}}}{1 - \frac{\lambda}{24}\frac{3}{m^4} + \frac{1}{2}\frac{\lambda^2}{24^2}\frac{7*5*3}{m^8} - \frac{1}{6}\frac{\lambda^3}{24^3}\frac{11*9*7*5*3}{m^{12}}}$$

$$f'(0) = -\frac{1}{2m^6}$$

$$f(0) = \frac{1}{m^2}$$

$$f''(0) = \frac{4}{3m^{10}}$$