MECHANICS

GRAVITATION

1. MOTION OF PLANETS

Our solar system consists of a Sun which is stationary at the centre of the universe and nine planets which revolve around the sun in separate orbits. The names of these planets are : Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto. The planet Mercury is closest to the Sun and Pluto is farthest.

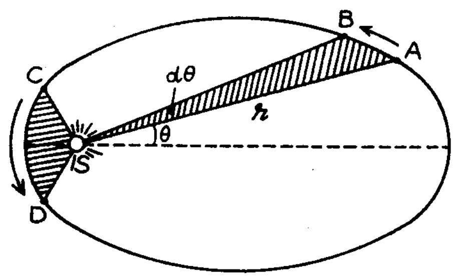
There are certain celestial bodies which revolve around the planets. These are called 'satellites'. For example, moon revolves around the earth, hence moon is a satellite of the earth. Similarly, Mars has two satellites Jupiter has twelve satellites, Saturn has ten satellites, and so on.

1.1 Kepler's Laws : Kepler found important regularities in the motion of the planets. These regularities are known as 'Kepler's three laws of planetary motion'.

(a) First Law (Law of Orbits) : All planets move around the sun in elliptical orbits, having the sun at one focus of the orbit.

(b) Second Law (Law of Areas) : A line joining any planet to the sun sweeps out equal areas in equal times, that is, the areal speed of the planet remains constant.

According to the second law, when the planet is nearest the sun, then its speed is maximum and when it is farthest from the sun, then its speed is minimum. In Fig. if a planet moves from A to B in a given time-interval, and from C to D in the same time-interval, then the areas ASB and CSD will be equal.



dA = area of the curved triangle SAB

$$= \frac{1}{2}(AB \times SA) = \frac{1}{2}(r\, d\theta \times r) = \frac{1}{2} r^2\, d\theta$$

Thus, the instantaneous areal speed of the planet is

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} r^2 \omega, \qquad \text{...(i)}$$

where ω is the angular speed of the planet. Let I be the angular momentum of the planet about the sun S and m the mass of the planet. Then

$$J = I\omega = mr^2\, \omega, \qquad \text{...(ii)}$$

where $I\ (= mr^2)$ is the instantaneous moment of inertia of the planet about the sun S.
From eq. (i) and (ii), we have

$$\frac{dA}{dt} = \frac{J}{2m} \qquad \text{...(iii)}$$

Now, the areal speed dA/dt of the planet is constant, according to Kepler's second law. Therefore, according to eq. (iii), the angular momentum J of the planet is also constant, that is, the angular momentum of the planet is conserved. Thus, Kepler's second law is equivalent to conservation of angular momentum.

(c) Third Law : (Law of Periods) : The square of the period of revolution (time of one complete revolution) of any planet around the sun is directly proportional to the cube of the semi-major axis of us elliptical orbit.

Proof : If a and b are the semimajor and the semi-minor axes of the ellipse, then the area of the ellipse will be p ab. Hence if T be the period of revolution of the planet, then

$$T = \frac{\text{area of the ellipse}}{\text{areal speed}} = \frac{\pi ab}{J/2m} \qquad \text{or} \qquad T^2 = \frac{4\pi^2 m^2 a^2 b^2}{J^2}$$

Let I be the semi-latus rectum of the elliptical orbit. Then $l = \dfrac{b^2}{a}$

$$\therefore \qquad T^2 = \frac{4\pi^2 m^2 a^3 l}{J^2} \qquad \text{or} \qquad T^2 \propto a^3$$

The period of revolution of the closest planet Mercury is 88 days, while that of the farthest planet pluto is 248 years.

1.2 Newton's Conclusions from Kepler's Laws : Newton found that the orbits of most of the planets (except Mercury and Pluto) are nearly circular. According to Kepler's second law, the areal speed of a planet remains constant. This means that in a circular orbit the linear speed of the planet will be constant. Since: the planet is moving on a circular path; it is being acted upon by a centripetal force directed towards, the centre (sun). This force is given by

$$F = mv^2/r$$

where m is the mass of the planet, v is its linear speed and r is the radius of its circular orbit. If T is the period of revolution of the planet, then

$$v = \frac{\text{linear distance travelled in one revolution}}{\text{period of revolution}} = \frac{2\pi r}{T}$$

$$\therefore \quad F = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 m r}{T^2}$$

But, for circular orbit, according to Kepler's third law, $T^2 = Kr^3$, where K is some constant.

$$\therefore \quad F = \frac{4\pi^2 m r}{K r^3} = \frac{4\pi^2}{K}\left(\frac{m}{r^2}\right) \qquad .....(i)$$

or $\quad F \propto m/r^2$

1.3 Thus, on the basis of Kepler's laws, Newton drew the following conclusions:

- A planet is acted upon by a centripetal force which is directed towards the sun.
- This force is inversely proportional to the square of the distance between the planet and the sun ($F \propto 1/r^2$).
- This force is directly proportional to the mass of the planet ($F \propto m$). Since the force between the planet and the sun is mutual, the force F is also proportional to the mass 'M of the sun ($F \propto M$). Now, we can replace the constant $4\pi^2/K$ in eq. (i) by GM, where G is another constant. Then, we have

$$F = G\frac{Mm}{r^2}$$

Newton stated that the above formula is not only applied between sun and planets, but also between any two bodies (or particles) of the universe. If m_1 and m_2 be the masses of two panicles.

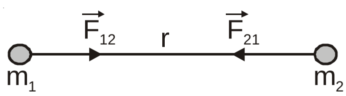
2. NEWTON'S LAW OF GRAVITATION

Every two objects in the universe attract each other. This force of attraction is called 'Gravitational force'.

The force of attraction between ay two material particles is directly proportional to the product of the mass of the particles and inversely proportional to the square of the distance between them. It acts along the line joining the two particles.

$$\therefore \quad F \propto m_1 m_2$$

and $F \propto \frac{1}{r^2}$



or $F = \frac{Gm_1 m_2}{r^2}$

G is the constant of proportionality which is called 'Newton's gravitation constant.

$G = 6.67 \times 10^{-11}$ Nm^2 /kg^2

$G = 6.67 \times 10^{-8}$ dyne cm^2/gm^2

Dimensional formula of G is $[M^{-1}L^3T^{-2}]$

In vector of from -

The force exerted by point mass (2) on points mass (1) will be

$$\vec{F}_{12} = \frac{Gm_1m_2}{r_{12}^2}\hat{r}_{12}$$

direction of $\hat{r}_{12}$ is from 1 to 2.

$$\vec{F}_{21} = \frac{Gm_1m_2}{r_{12}^3}\hat{r}_{21}, \qquad \text{similarly} \qquad \vec{F}_{12} = \frac{Gm_1m_2}{r_{21}}\hat{r}_{21}$$

$$\vec{F}_{21} = \vec{F}_{12} \qquad \text{but,} \qquad |\vec{F}_{21}| = |\vec{F}_{12}|$$

From above two expression we can conclude the force exerted between two particles is equal in magnitude but opposite in direction.

Gravitational force is the weakest force in nature.

Note :

The ratio of gravitational to electrostatic force between two electrons is of the other.

$$\frac{F_g}{F_e} = 10^{-43}$$

The range of this force is maximum upto infinity.

It is due to very small value of G that we do not experience the gravitational force in out daily life. But masses of celestial bodies are so large that the magnitude of the force of attraction between them is appreciable. In the motion of planets and satellites, this force supplies the necessary centripetal force due to which earth revolves around the sun and moon around the earth.

If the density of the earth is assumed to be uniform and a particle moves inside the earth then the gravitational force decreases because the shell of the material lying outside the particle's radial position would not exert any force on the particle.

Example based on Newton's law of gravitation

Ex. A mass m splits into two parts m and (M – m), which are then separated by a certain distance. What ratio (m / M) maximizes the gravitational force between the parts?

Sol. If r the distance between m and (M – m), the gravitational force will be

$$F = G\,\frac{m(M-m)}{r^2} = \frac{G}{r^2}(mM - m^2)$$

The force will be maximum if, $\frac{dF}{dm} = 0$

i.e. $\frac{d}{dm}\left[\frac{G}{r^2}(mM - m^2)\right] = 0$ or $\frac{m}{M} = \frac{1}{2}$ (as M and r are constants)

3. GRAVITATIONAL AND INERTIAL MASS

(a) **Inertial mass :** When mass is defined on the property of inertia, it is termed as inertial mass

(b) **Gravitational mass :** When mass is defined on the property of gravity, it is called gravitational mass.

(c) **Properties of inertial mass :** It is equal to the ratio of magnitude of external force applied on the body of the acceleration produced in it by that force.

$m = F/a$

- It is proportional to the quantity of matter present in the body.
- It is independent of shape, size and state of the body.
- It is not affected by the presence of other bodies near it.
- When various masses are put together, the inertial masses add according to the scalar laws irrespective of the material of the bodies involved.

- It increases with increase in velocity according to the relation m = $\frac{m_0}{\sqrt{1-(v^2/v^2)}}$, where m_0 = rest mass of the body. C = velocity of light and v is the velocity of particle.

Point of remember :

- It is found that the ratio of two gravitational masses is same as the radio of their inertial mass.
- If a ball is dropped from the hole passing though the two poles and the centre of earth then it will do S.H.M. in the tunnel.

Example based on Gravitational inertial mass

Ex. If an object having mass $\sqrt{3}$ kg in moving with a velocity that is 1/2 the velocity of light then inertial mass will be

Sol. $m_g = \sqrt{2}$ kg, $v = \frac{c}{2}$

$$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}} = \frac{\sqrt{m_3}}{\sqrt{1-\frac{c^2}{4xc^2}}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}\text{kg}$$

4. ACCELERATION DUE TO GRAVITY

- Acceleration produced in a body due to the force of gravity is termed as acceleration due to gravity.
- The acceleration due to gravity is the rate of increase of velocity of a body falling towards the earth.
- The acceleration due to gravity is equal to the force by which earth attracts a body of unit mass towards its centre.
- Let 'm' be the mass of body and F' be the force of attraction at a distance 'r' from the centre of earth then acceleration due to gravity (g) at that place will be

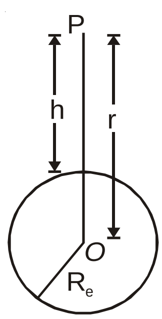
$$g = \frac{F}{m} = \frac{GM_e}{r^2}$$, where M_e = mass of earth

- The expression $g = \frac{GM_e}{r^2}$ is free from 'm' (mass of body). This means that the value of 'g' does not depend upon the shape, size and mass of the body. Hence if two bodies of different masses, shapes and sizes are allowed to fall freely, they will have the same acceleration. If they are allowed to fall form the same height, they will reach the earth simultaneously.
- The acceleration of a body on the surface of the earth is g = 9.80 m/s^2 or 981 cm/s^2.
- Dimensional formula of g is $[M^oL^1T^{-2}]$
- The value of acceleration due to gravity depends on the following factors.

(a) Height above the earth surface (b) Depth below the earth surface

(c) Shape of the earth (d) Axial rotation of the earth

(a) Height above the surface of earth :



- As we go above the surface of the earth, the value of 'g' decreases.

Consider a point P at a distance r form the centre of earth.

The acceleration due to gravity at point P is

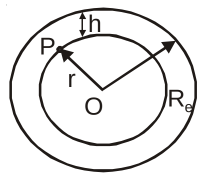
$$g = \frac{g}{\left(1+\frac{h}{R_e}\right)^2} \Rightarrow g = g\left(\frac{R_e}{R_e+h}\right) = \frac{gR_e^2}{r^2}$$ $\therefore$ g' < g

- As we go above the surface of the earth, the value of 'g' decreases. $g \propto \frac{1}{r^2}$ for $r > R_e$
- If $h << R_e$, (according to binomial expansion) $g = g\left(1 - \frac{2h}{R_e}\right)$
- If $r = \infty$, $g = 0$. At infinite distance form the earth, the of 'g' becomes zero
- Value of g at the surface of earth (h – 0) $\Rightarrow$ $g = \frac{GM_e}{R_e^2}$

(b) Below the surface of earth :

- The value of 'g' decreases on going below the surface of the earth.
- The value of 'g' at a distance h below this earth's surface be g_h and 'g' at the earth's surface then

$$g_h = g\left(1 - \frac{h}{R_e}\right) = g(R_e - h)/R_e = \frac{gr}{R_e}$$



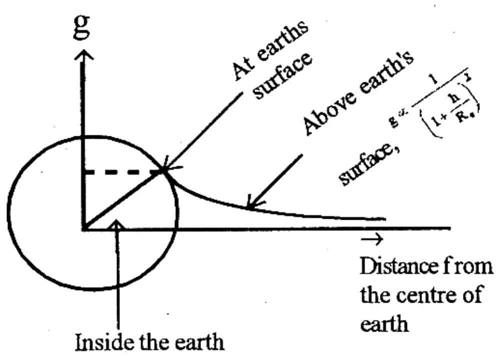
i.e. $g_h < g$

r is the distance form the centre of the earth ($r < R_e$), $r \approx R_e - h$

- If d is the density of the earth then the force on pt. P is

$$g_h = \frac{M_r G}{r^2} \text{ where } M_r = \frac{4}{3}\pi(R_e - h)^3 d;\quad r^2 = (R_e - h)^2 \quad \Rightarrow \quad g_h = \frac{4}{3}\pi G(R_e - h)d$$

- At the centre of the earth, $h = R_e$ (i.e. $r = 0$) so $g = 0$
- Value of 'g' is maximum at the surface of earth
- Graphical representation of variation in the value of g



(c) Variation in value of 'g' on the surface of earth :

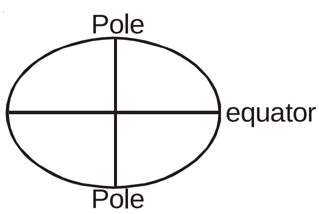
It is due to two reasons :

(1) Due to shape of earth :

(i) The earth is elliptical in shape, it is flattened at the poles and bulged out at the equator. Now, we know that $g \propto 1/R_e^2$, therefore the value of g at the equator is minimum and the value of g at the poles is maximum ($\because$ Radius at poles is < Radius at equator)

$$g_e = \frac{GM}{R_e^2} \text{ (At equator)}$$

$$g_p = \frac{GM}{R_p^2} \text{ (At pole)}$$



$\because$ $R_e > R_p$

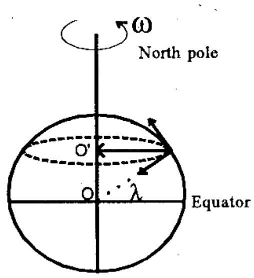
Hence, $g_e < g_p$

(ii) $\frac{g_p}{g_e} = \frac{R_e^2}{R_p^2}$

(iii) Numerical value of R_p is twenty one kilometer less than R_e. Therefore $g_p - g_e = 0.02 m/s^2$.

(2) Due to rotation of earth :

- Earth is rotating about its own axis form west to earth with an angular velocity ω.
- On a latitude λ, point P is moving in circle with radius 'r'. If we keep a body then some part of its gravitation force will be used up for providing centripetal force, therefore there is reduction in total gravitational force. As a result of this, value of 'g' decreases.
- If ω is the angular velocity of rotation of the earth, R_e is radius of the earth, then the observed value of g at the latitude λ is represented by g' then.



$g' = g_0 - w^2 R_e \cos^2 \lambda$

or $g' = g_0 - 0.0337 \cos^2 \lambda$

where g_0 is value of 'g' at the poles.

- At equator, $\lambda = o$

  $g' = g_0 - w^2 R_e$ (Minimum value)

  $= g - 0.0337$

- At poles, $\lambda = 90^o$

  $\therefore \cos \lambda = 0$

  $g' = g$ (Maximum value)

- From the above expressions we can conclude that the value of 'g' at the surface of earth is maximum at poles and minimum at the equator. Therefore weight of bodies is maximum at the poles and will go on decreasing towards the equator. (it is minimum at the equator).
- If earth stops rotating about its axis ($\omega = 0$), the value of g will increases everywhere, except at the poles. On the contrary, if there is increase in the angular velocity of earth, then except at the poles the value of 'g' will decrease at all places.
- Maximum effect of rotation takes place at the equator while at poles, there is no effect.
- If $\omega = \sqrt{\frac{g}{R_e}}$ then, at equator weight of body will become zero but its mass remains unaltered.

  (a) That means if the earth starts rotating with an angular speed 17 times the present,

  (b) If $g_{equator} = 0$, in this condition, times period of earth's rotation will become 1.41 hours instead of 24 hours.

Example based on Acceleration due to gravity

Ex. What would be the angular speed of earth, so that bodies lying on equator may appear weightless ?

($g = 10$ m/s^2 and radius of earth = 6400 km)

Sol. The apparent weight of person on the equator (latitude $\lambda = 0$) is given by

$W' = W - m R_e \omega^2$,

$W' = \frac{3}{5} W = \frac{3}{5} mg$

$\frac{3}{5}$ mg = mg – mRω^2 or mRω^2 = mg – $\frac{3}{5}$ mg

$\omega = \sqrt{\frac{2}{5}\frac{g}{R}} = \sqrt{\frac{2}{5} \times \frac{98}{6400 \times 10^3}}$ rad/sec $= 7.826 \times 10^{-4}$ rad/sec

Ex. On a planet (whose size is the same as that of earth and mass 4 time to the earth) the energy neeeded to lift a 2kg mass vertically upwards through 2m distance on the planet is ($g = 10$ m/sec^2 on surface of earth)

Sol. According to question,

$g' = \frac{G \times 4M_p}{R_p^2}$ on the planet and $g = \frac{GM_e}{R_p^2}$ on the earth

$\because$ $R_p = R_e$ and $M_p = M_e$

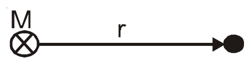
Now $\frac{g'}{g} = 4 \Rightarrow g' = 4g = 40$ m/sec^2

Energy needed to lift 2kg mass through 2m, distance = mg'h = 2 × 40 × 2 = 160 J

5. GRAVITATIONAL FIELD AND GRAVITATIONAL FIELD INTENSITY

- It is defined as the space around the attracting body, in which its attraction (gravitational) can be experienced.
- **Intensity of gravitational field gravitational field strength-** It is defined as the force experienced by unit mass place at any point in the gravitational field.
- Gravitational field is a vector quantity.
- Suppose a body of mass M is placed at a distance r, then intensity of gravitational field at point P will be

$$\vec{E} = \frac{GM_e}{r^2}(-\hat{r})$$

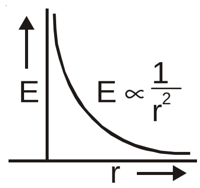


- Unit of gravitational field strength is Newton/kg or m/sec^2.

  Dimension formula is $[M^o L^1 T^{-2}]$
- As the distance (r) increases, gravitational field strength decreases.

  At r = ∞, value of intensity of gravitational field becomes zero.
- Intensity of gravitational field at a distance r form the centre of earth is

$$E = \frac{GM_e}{r^2} = g$$



Note : From this expression

$$E = \frac{GM_e}{r^2} = g$$

It is clear that the intesity of gravitational field at any place is equal to acceleration due to gravity.

- Change of intensity of gravitational field due to a point mass with respect to distance

$$E = \frac{GM_e}{r^2}$$

- Relation between gravitational field and gravitational potential.

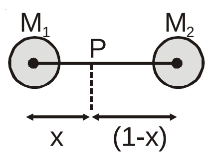
$$E = -\nabla V$$

$$E = -\frac{dV}{dr}$$

Example based on Gravitational Field and Gravitational Field Intensity

Ex. Two bodies of mass 100 kg and 10^4 kg are lying on meter apart. At what distance from 100 kg body will the intensity of gravitational field be zero

Sol. Let I_g = 0, at a distance x from 100 kg body, $\therefore \frac{GM_1}{x^2} = \frac{GM_2}{(r-x)^2}$

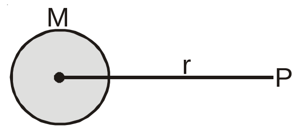


or $\frac{10^2}{x^2} = \frac{10^4}{(1-x)^2}$

or $x = \frac{1}{11}$m

6. GRAVITATIONAL POTENTIAL

- The work done in duringing a unit mass form infinity to a point in the gravitational field is called the gravitational potential at that point.
- Gravitational potential at a point P distance r form a point mass 'M' will be $V = -\frac{GM}{r}$



- Unit of Gravitational potential is Joule/kg
- Dimensional formula of Gravitational potential is $[M^0L^2T^{-2}]$
- Gravitational potential is a scalar quantity.

  (f) At $r = \infty$, $V = 0$

Example based on Gravitational Potential

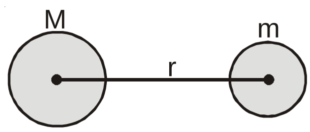
Ex. Two bodies of mass 10^2kg and 10^3 kg are lying 1m apart. The gravitational potential at the mid-point of the line joining them is

Sol. $V_g = V_{g\,1} + V_{g\,2} = -\frac{Gm_1}{r_1} - \frac{Gm_2}{r_2} = 6.67 \times 10^{-11}\left[\frac{10^2}{0.5} + \frac{10^3}{0.5}\right] = 1.47 \times 10^{-7}$ joule/kg

7. GRAVITATIONAL POTENTIAL ENERGY

- The gravitational potential energy of a body at a point is defined as the amount of work done in bringing the body from infinity to that point against the field.
- The gravitational potential energy of mass 'm' in the gravitational field of mass M at a distance r form it is

$$U = -\frac{GMm}{r}$$



where r is distance between M and m.

- At any placed in gravitational field, gravitational potential is V, then the gravitational potential energy of a mass 'm' at that place will be $U = -mV$.
- The gravitational potential energy of a particle of mass 'm' at a point distance 'r' from the centre of earth is

$$U = -\frac{GM_em}{r}, \text{ if } r > R_e$$

$$= -\frac{GM_em(3R_e^2 - r^2)}{2R_e^3}, \text{ if } r < R_e$$

- Force between two particle if their potential energy is U. is $F = -\frac{dU}{dr} = -\frac{d}{dr}\left(\frac{GMm}{r}\right) = -\frac{GMm}{r^2}$ minus sign. indicates that the force on the bodies is towards each other.

  Note : If a particle is at a height h from earth's surface and R_e be the radius of earth, $r = R_e + h$

$$U = -\frac{GM_em}{R_e + h}$$

- It is a scalar quantity and its values is always negative.
- It's unit is Joule or Erg.
- Gravitational potential energy of a mass at infinite distance from earth is zero, and at all the other points it is less than zero, i.e. it is negative.

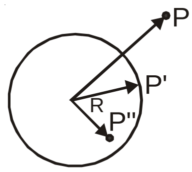
7.1 Intensity of Gravitational field and gravitational potential due to hollow sphere

(a) Hollow Shere :

(i) Let OP = r

If point P is situated outside the hollow sphere, then OP = r > R

(1) $E_{out} = -\frac{GM}{r^2}$ (2) $V_{out} = -\frac{GM}{r}$



(ii) If point P is situated on the surface of sphere, then OP' = r = R

(1) $E_{surface} = -\frac{GM}{R^2}$ (2) $V_{surface} = -\frac{GM}{R}$

(iii) If point P is inside the hollow sphere, then OP" = r < R

(1) $E_{in} = 0$ (2) $V_{in} = -\frac{GM}{R}$

Note : Gravitational field intensity inside a hollow sphere is zero but gravitation potential is constant and is equal to the potential at the surface.

(b) Solid Sphere :

Let OP = r

(i) If point P is situated outside the sphere, then OP = r > R

(1) $E_{out} = -\frac{GM}{r^2}$ (2) $V_{out} = \frac{GM}{r}$

(ii) If point P is situated on the surface of sphere, then OP = r + R

(1) $E_{surface} = -\frac{GM}{R^2}$ (2) $V_{surface} = -\frac{GM}{R}$

(iii) If point P is situated inside the sphere, then OP = r < R_e

(1) $E_{in} = -\frac{GMr}{R^3}$ (2) $V_{in} = -\frac{GM(3R^2 - r^2)}{2R^3}$

Note : $V_{centre} = 1.5\ V_{surface}$

Example based on Gravitational Potential Energy

Ex. If g is the acceleration due to gravity on the earth's surface, the gain in P.E. of an object of mass m raised from the surface of the earth to a height of the radius R of the earth is

Sol. The P.E. of the object on the surface of earth is $U_1 = -\frac{GMm}{R}$

The P.E. of object at a height R, $U_2 = -\frac{GMm}{(R+R)}$ The gain in P E is $U_2 - U_1$ $= \frac{GMm}{2R} = \frac{1}{2}mgR$

$\left[\because \quad g = \frac{GM}{R^2} \text{ on surface of earth}\right]$

8. SATELLITE

- Celestical bodies revolving round the gravitational field of the planet s is collate satellite.
- **Satellites are of two types -**

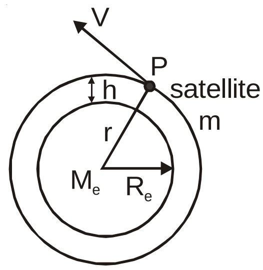
Nautral satellites - As moon is a satellite of the earth.

Artifical satellites- They are launched by man such as Rohini, Aryabhatt etc.

- Let a sateelite of mass ‘m’ revolves in a circular orbit with radius ‘r’ around the earth.

  The necessary centripetal force needed for circular motion is provided by the graviational force fo the earth

  so, $\frac{GM_e m}{r^2} = \frac{mv^2}{r}$



  Where M_e = mass of earth

  v = orbitial velocity of satellite

  r = radius of satellite’s orbit = R_e + h = orbital radius

  R_e = Radius of earth

  h = The height of the satllite above the earth’s surface

  g = Acceleration due to gravity on the surface of the earth

  Orbital velocty of sateelite :

- $$v = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM_e}{R_e + h}} = \sqrt{\frac{gR_e^2}{R_e + h}}$$

- From theis expression, $\sqrt{\frac{gR_e^2}{R_e + h}}$, it is clear that orbital velocity does not depend on the mass of satellite but it depends on the height of the satellite above the eath’s surface (h). Greater the height of sateelite, smaller is the orbital velocity.

  (iii) If a sateelilte is very close to the earth surface (h << Re) then h will be negligible as compared to R than the orbitial speed of satellite is given by

  $$v = \sqrt{\frac{GM_e}{R}} = \sqrt{gR}$$

  = 7.92 km/sec. ($\cong$8km/sec)

  Period of revolution :

- The time taken by the satellite for completing one revolution of earth is called as period of revolution of satellite.

- Period or Revolution of a satellite is

  $$T = \frac{2\pi r}{v} = \frac{2\pi(R_e + h)}{v}$$

  where T is the time period of a satellite at a height ‘h’

- $$T = 2\pi\sqrt{\frac{r^3}{GM_e}} = 2\pi\sqrt{\frac{r^3}{gR_e^2}}$$

  $$= 2\pi\sqrt{\frac{(h + R_e)^3}{gR_e^2}} = 2\pi\sqrt{\frac{R_e}{g}}\left(1 + \frac{h}{R_e}\right)^{3/2}$$

- It is evident from the above expresion that T^2 r^3 i.e kepler’s III law is true for circular motion also.

- For a satellite revolving very close to the surafce of earth (h << R_e).

  $$T = 2\pi\sqrt{\frac{R_e}{g}} = 84.4\,\text{min}$$

  therefore, the minimum time period of the satellite revolving very close to the surface of earth is 84.4 min.

- From the expression $T = 2\pi\sqrt{\frac{r^3}{GM_e}}$ we can say that time period of satellite depends on its orbital radius $T^2 \propto R^3$

  As the radius increases, simultaneously time period also increase.

  Energy of satellite : When satellite is revolving in the orbit of radius 'r' then

- Potential energy of satellite : P.E = $-\frac{GM_e m}{r}$, where r = h
- Kinetic energy of satellite : K.E = $\frac{1}{2}mv^2 = \frac{GM_e m}{2r}$
- Total energy of satellite = K.E. + P.E. = $\frac{-GM_e m}{2r}$
- We can say that : Total energy of satellite = $\frac{1}{2}$ (Potential energy of satellite) = – Kinetic energy of satellite.

  Binding energy of satellite :

- Binding energy is the energy given to satellite in order that the satellite escape away form its robit.

  Binding energy = – total energy = $\frac{GM_e m}{2r}$ (i.e. equal to kineitc energy)

  If energy equals to $\frac{GM_e m}{2r}$, is provided to the satellite, it will escape away from the graviational field of the planet.

- Unless a revolving satellite gets extra energy, it would not leave its orbit. If the kinetic energy of a satellite happens to increase to two times, the satellite would escape.
- If the orbital velocity of a satellite revolving close to the earth happens to increase to $\sqrt{2}$ times, the satellite would escape, That means if orbital velocity increaeses to 41.4%, satellite would leave the orbit.
- Total energy of satellite is always negative. When the energy of the satellite is negative, it moves in either a cicular or an elliptical orbit.

  (v) Binding energy = Kinetic energy = – (Total energy) = $-\frac{(\text{Potential energy})}{2}$

Example based on Satellite

Ex. A satellite is revolving in a circular orbit at a distance of 2620 km from the surface of the earth. Calculate the orbital velocity and the period of revolution of the satellite. Radius of the earth = 6380 km, mass of the earth = 6×10^{24} kg and G = 6.67×10^{-11} Nm^{2}kg^{-2}.

Sol. The radius of the orbit of the satellite is

$(R_e + h) = 6380 + 2620 = 9000$ km $= 9 \times 10^6$ m,

mass of the earth, $M_e = 6 \times 10^{24}$ kg and G = 6.67×10^{-11} Nm^2 kg^{-2}

$\therefore$ orbital velocity of the satellite is

$$v_v = \sqrt{\frac{GM_e}{R_e + h}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})\times(6\times10^{24})}{9\times10^6}} = 6.67 \times 10^3 \text{ ms}^{-1} = 6.67 \text{ kms}^{-1}$$

Period of revolution,

$$T = \frac{2\pi(R_e + h)}{v_v} = \frac{2\times3.14\times(9.8\times10^6)}{6.4\times10^3} = 8474\,\text{s} = 2.35\,\text{hours}$$

Ex. A satellite is revolving in a circular orbit at a distance of 3400 km. Calculate the orbital velocity and the period of revolution of the satellite. Radius of the earth = 6400 km and g = 9.8 ms^{-2}.

Sol. Radius of the earth, R_e = 6400 km = 6.4×10^6 m.

Radius of the orbit of the satellite is

$(R_e + h) = 6400 + 3400 = 9800 \text{ km} = 9.8 \times 10^6$ m.

$\therefore$ orbital velocity of the satellite is

$$v_v = R_e\sqrt{\frac{g}{R_e + h}}$$

$$= 6.4 \times 10^6\sqrt{\frac{9.8}{9.8 \times 10^6}} = 6.4 \times 10^3 \text{ ms}^{-1} = 6.4 \text{ kms}^{-1}$$

Period of revolution, $T = \frac{2\pi(R_e + h)}{v_v} = \frac{2 \times 3.14 \times (9.8 \times 10^6)}{6.4 \times 10^3}$ = 9616 seconds = 2.67 hours.

Ex. (i) A satellite is revolving in an orbit close to the earth's surface. Taking the radius of the earth as 6.4×10^6 meter, find the value of the orbital speed and the period of revolution of the satellite. (g = 9.8 ms^{-2})

(ii) What is the relationship of this orbital speed to the velocity required to send a body from the earth's surface into space, never to return ?

Sol. (i) Orbital speed $v_v = \sqrt{gR_e} = \sqrt{9.8 \times (6.4 \times 10^6)}$

$= 7.92 \times 10^3 \text{ ms}^{-1} = 7.92 \text{ kms}^{-1}$

period of revolution, $T = \frac{2\pi R_e}{v_0} = \frac{2 \times 3.14 \times (6.4 \times 10^6)\text{m}}{7.92 \times 10^3 \text{ms}^{-1}} = 5075 \text{s}$ = 84.6 minutes.

(ii) $v_e = \sqrt{2}\, v_0$

Ex. An artificial satellite revolving coplanar with the equator around the earth, appears stationary to an observer on the earth. Calculate the height of the satellite above the earth. g = 9.80 ms^{-2} and $R_e = 6.37 \times 10^6$ m.

Sol. Let h be the height of satellite above the earth. The period of revolution of the satellite is

$$T = \frac{2\pi(R_e + h)^{3/2}}{R_e\sqrt{g}}$$

If the period of revolution of the satellite be equal to the axial rotation (24 hours) of the earth (and the satellite be revolving from west to east), then the satellite will appear stationary relative to the earth.

$\therefore$ T = (24 × 3600) s.

Substituting this value in eq. (i), we get

$$(24 \times 3600) \text{ s} = \frac{2\pi(R_e + h)^{3/2}}{R_e\sqrt{g}}$$

or $$(R_e + h)^3 = \frac{(24 \times 3600)^2 s^2 \times R_e^2 \times g}{4\pi^2}$$

$$= \frac{(24 \times 3600)^2 s^2 \times (6.37 \times 10^6)^2 m^2 \times 9.80 \text{m s}^{-2}}{4 \times (3.14)^2} = 75.3 \times 10^{21} m^3$$

$\therefore$ $(R_e + h) = (75.3 \times 10^{21})^{1/3} = 4.22 \times 10^7 \text{ m} = 42.2 \times 10^6$ m

$\therefore$ $h = 42.2 \times 10^6 - 6.37 \times 10^6 = 35.8 \times 10^6 \text{ m} = 35800$ km.

Ex. An artificial satellite is revolving at a height of 500 km above the earth's surface in a circular orbit, completing one revolution in 98 minutes. Calculate the mass of the earth. Given G = 6.67 × 10^{-11} Nm^2 kg^{-2}, radius of the earth = 6.37 × 10^6 m.

Sol. The gravitational force of attraction exerted by the earth on the satellite is the necessary centripetal force. Therefore, if h be the height of the satellite above the earth's surface, then

$$\frac{GM_e m}{(R_e + h)^2} = \frac{mv_0^2}{(R_e + h)} \qquad ....(1)$$

where m is the mass of the satellite, (Re + h) is the distance of the satellite from the centre of the earth and v_o is the orbital velocity of the satellite. If the period of revolution of the satellites be T, then

$$T = \frac{2\pi(R_e + h)}{v_v}$$

Substituting from this the value of v_o in eq. (i), we get

$$\frac{GM_e m}{(R_e + h)^2} = \frac{4\pi^2 m(R_e + h)}{T^2}$$

$$\therefore \quad M_e = \frac{4\pi^2(R_e + h)^3}{GT^2}$$

Here, orbital radius (Re + h) = (6.37 × 10^6 m) + (0.5 × 10^6 m) = 6.87 × 10^6 m and period of revolution T = 98 × 60 = 5.88 × 10^3 s.

$$\therefore \quad M_e = \frac{4 \times (3.14)^2 \times (6.87 \times 10^6)^3}{(6.67 \times 10^{-11}) \times (5.88 \times 10^3)^2} = 5.54 \times 10^{24} \text{ kg.}$$

Ex. If the period of revolution of an artificial satellite just above the earth be T and the density of earth the ρ, then prove that ρ T^2 is a universal constant. Also calculate the value of this constant.G = 6.67 × 10^{-11} m^3 $kg^{-1}s^{-2}$.

Sol. If the period of revolution of a satellite about the earth be T, then

$$T^2 = \frac{4\pi^2(R_e + h)^3}{GM_e},$$

where h is the height of the satellite from earth's surface.

$$\therefore \quad M_e = \frac{4\pi^2(R_e + h)^3}{GT^2}$$

The satellite is revolving just above the earth, hence h is negligible compared to R_e.

$$\therefore \quad M_e = \frac{4\pi^2 R_e^3}{GT^2}$$

But $M_e = \frac{4}{3}\pi R_e^3 \rho$, where ρ is the density of the earth. Thus

$$\frac{4}{3}\pi R_e^3 \rho = \frac{4\pi^2 R_e^3}{GT^2}$$

$$\therefore \quad \rho T^2 = 3\pi/G$$

Which is a universal constant. To determine its value,

$$\rho T^2 = \frac{3\pi}{G} = \frac{3 \times 3.14}{6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}} = 1.41 \times 10^{11} \text{ kg } m^{-3}s^2$$

Ex. An artificial satellite of mass 200 kg revolves around the earth in an orbit of average radius 6670 km. Calculate its orbital kinetic energy, the gravitational potential energy and the total energy in the orbit. (Mass of earth $= 6.0 \times 10^{24}$ kg, $G = 6.67 \times 10^{-11}$ Nm^2kg^{-2})

Sol. The kinetic energy of a satellite (mass m) revolving in an orbit of radius r around the earth (mass M_e) is

$$K = \frac{GM_e m}{2r}$$

$$= \frac{(6.67\times10^{-11}Nm^2kg^{-2})\times(6.0\times10^{24}kg)\times200kg}{2\times(6670\times10^3 m)} = 6.0 \times 10^9 \text{ J}.$$

The gravitational potential energy is

$$U = -\frac{GM_e m}{r} = -12.0\times10^9 \text{ J}$$

The total energy is $E = K + U = 6.0 \times 10^9 + (-12.0 \times 10^9) = -6.0 \times 10^9$ J.

Ex. With what velocity must a body be thrown upward from the surface of the earth so that it reaches a height of 10 R_e ? Earth's mass $M_e = 6 \times 10^{24}$ kg, radius $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Sol. Let m be the mass of the body. The gravitational potential energy of the body at the surface of the earth is

$$U = -\frac{GM_e m}{R_e}$$

The potential energy at a height 10 R_e above the surface of the earth will be

$$U' = -\frac{GM_e m}{(R_e + 10R_e)}$$

$\therefore$ increase in potential energy is

$$U' - U = -\frac{GM_e m}{11R_e} - \left(-\frac{GM_e m}{R_e}\right) = \frac{10}{11}\frac{GM_e m}{R_e}$$

This increase will be obtained from the initial kinetic energy given to the body. Hence if the body be thrown with a velocity v, then

$$\frac{1}{2}mv^2 = \frac{10}{11}\frac{GM_e m}{R_e}$$

or

$$v = \sqrt{\frac{20GM_e}{11R_e}}$$

Substituting the given values, we get

$$v = \sqrt{\frac{20\times(6.67\times10^{-11})\times(6\times10^{24})}{11\times(6.4\times10^6)}} = 1.07\times10^4 ms^{-1}$$

Ex. A rocket is launched vertically from the surface of the earth with an initial velocity of 10 kms^{-1}. How far above the surface of the earth would it go ? Mass of the earth $= 6.0 \times 10^{24}$kg, radius = 6400 km and $G = 6.67 \times 10^{-11}$ Nm^2 kg^{-2}.

Sol. Suppose the rocket (mass m) goes to a height h above the earth's surface. Solving like the last example, the increase in gravitational potential energy of the rocket is

$$U' - U = -\frac{GM_e m}{(R_e + h)} - \left(-\frac{GM_e m}{R_e}\right) = \frac{GM_e mh}{R_e(R_e + h)}$$

This increase will be obtained from the initial kinetic energy given to the rocket. If the rocket be launched with a velocity v, then

$$\frac{1}{2}mv^2 = \frac{GM_e mh}{R_e(R_e + h)}$$

or $$R_e(R_e + h)\, v^2 = 2\, G\, M_e\, h$$

$$\therefore \quad h = \frac{R_e^2 v^2}{2GM_e - R_e v^2}$$

Given : R_e = 6400 km = 6.4×10^6m, v = 10kms^{-1} = 10^4 ms^{-1}, $M_e = 6.0 \times 10^{24}$kg and $G = 6.67 \times 10^{-11}$ Nm^{2}kg^{-2}

$$\therefore \quad h = \frac{(6.4\times10^6)^2 \times (10^4)^2}{\{(2\times(6.67\times10^{-11})\times(6.0\times10^{24})\} - \{(6.4\times10^6)\times(10^4)^2\}}$$

$$= \frac{6.4\times6.4\times10^{20}}{2\times6.67\times6.0\times10^{13} - 6.4\times10^{14}}$$

$$= \frac{6.4\times6.4\times10^{7}}{2\times6.67\times6.0 - 6.4\times10} = \frac{6.4\times6.4\times10^{7}}{16}$$

$= 2.56 \times 10^7$ m $= 2.56 \times 10^4$ km.

9 . GEO-STATIONARY SATELLITES

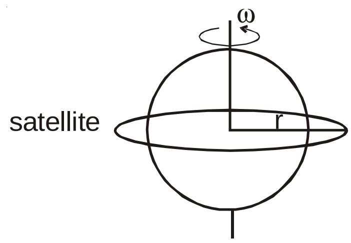
Such satellites which are stationary with respect to an observer on earth are termed as Geostationary satellites. They are also called Parking satelliltes.

The direction of rotation of geo-stationary saellites is from west to east, the time period of 24 hours and its angular velocity is same as that of axial velocity of earth, revolving around its axis.

Geo-stationary satelliltes can be launched just above the equator.

The radius of orbit of Geo-stationary satellite is r = 42000 km and it sheight above the surface of earth is h = 36000 km

Different values of satellite



(a) Angular velocity (ω) = 7.1×10^{-5} rad/sec

(b) Linear velocity (v) = 3.1 km/sec

(c) Time period (T) = 24 hours

(d) Height above the earth's surface (h) = 36000 km (approx)

At time, t the angular dispalcement of earth and Geo stationary satellite is same.

Angular momentum of satellite is conserved and it is equal to

$$J = mvr = mr\sqrt{\frac{gR_e^2}{r}} = mR_e\sqrt{gr} = m\sqrt{GM_e r}$$

Satellites behaves like freely falling bodies towards planet.

The Satellites revolves aroudn the earth in an orbit with earth as centre of a focus.

If a packet is released form the satellite, it will not fall on the earth but will remain revolving in the saem orbit with the same speed as the satellite.

No graviational force of satellite is used up for provinding necessary centripetal force. Due to gravitational force the effective value of accelertion due to gravity becomes gaff = 0, as a result effective weight becomes waif = 0, so the man sitting in the satellite enjoys weightlessens. Man experiences this weighhlessness condition only when weight of satellite is very less therfore gravitational effect of satellite is negligible.

Although moon is also a satellite of the earth, but a person on moon does not feel weightlessness. Thereason is that the moon has a large mass and exerts a graviational force on the person (and this is the weigth of the person on the moon). On the other hand, the artifical satellite having a smaller mass does not exert gravitational force on the space-man.

9.1 Relation between velocity of projection and shape of orbit

Shape of the satellite's orbit depends on its velocity.

$V_0 = \sqrt{\dfrac{GM_e}{R_e + h}}$; orbitial velocity of the satellite's

CASES :

If $V < V_0$; In this case satellite will leave its circular orbit and finally fall to earth following spiral path.

If $V = V_0$; In this case satellite will rotate in circular path.

If $V_0 < V < \sqrt{2}\ V_0$; In this case satellite will revolve around the earth in elliptical orbit.

If $V = \sqrt{2}\ V_0$; In this case satellite will leave the graviational field of earth and escape wasy following a parabolic path.

If $V > \sqrt{2}\ V_0$; In this case the satellite will escape, following a hyperbolical path.

10. ESCAPE VELOCITY

Escape velocity is the minimum velocity that should be given to the body to enable it to escape away form the graviational field of earth.

The energy given to the body to project it with the escape velocity is called the 'Escape Energy' or 'Bindind energy'.

Total energy of a body is reduced to zero to enable it to escape away from the graviational field or earth.

The graviational potential energy of a particle at the surface of earth = $\dfrac{-GM_em}{R_e}$

$\therefore$ Escape Energy of Binding energy = + $\dfrac{GM_eM}{R_e}$ if thrown with the velocity v_e,

then $\dfrac{1}{2}\ mv_e^2 = \dfrac{GM_em}{2}$

Escape velocity for earth $v_e = \sqrt{\dfrac{2GM_e}{R_e}}$ = 11.2 km/sec

Example based on Escape velocity

Ex. A space-ship is launched into a circular orbit close to the earth's surface. What additional velocity has now to be imparted to the space-ship in the orbit to overcome the gravitational pull.

(Radius of the earth = 6400 km, g = 9.8 ms^{-2})

Sol. The speed of the space-ship in a circular orbit close to the earth's surface is given by

$$v_0 = \sqrt{gR_e}$$

and escape velocity is given by $v_e = \sqrt{2gR_e} = 1.414\sqrt{gR_e}$

Additional velocity required to escape is

$$v_e - v_0 = 1.414\sqrt{g\,R_e} - \sqrt{g\,R_e} = 0.414\sqrt{gR_e}$$

$$= 0.414 \times \sqrt{(9.8ms^{-2}) \times (6400 \times 10^3 m)} = 3.278 \times 10^3\ ms^{-1}$$

Ex. The radius of a planet is double that of the earth but their average densities are the same. If the escape velocities at the planet and at the earth are v_P and v_e respectively, then prove that $v_P = 2\ v_E$.

Sol. Escape velocity on the earth is

$$v_E = \sqrt{\frac{2GM_E}{R_E}},$$

where M_E is the mass of the earth & R_E is the radius.If the average density of earth (& also of the planet) be ρ, then $M_E = 4/3\ \pi R_E^3 \rho$.

$$\therefore \quad v_E = \sqrt{\frac{2G}{R_E}\left(\frac{4}{3}\pi R_E^3 \rho\right)} = R_E\sqrt{\frac{8}{3}G\pi\rho}$$

Similarly, the escape velocity on the planet is

$$v_P = R_P\sqrt{\frac{8}{3}G\pi\rho}$$

$$\therefore \quad \frac{v_P}{v_E} = \frac{R_P}{R_E}$$

But $R_P = 2\ R_E$

$$\therefore \quad v_P = 2\ v_E$$

Ex. The escape velocity of a body from earth is 11.2 kms^{-1}. If the radius of a planet be half the radius of the earth and its mass be one-fourth that of earth, then what will be the escape velocity from the planet ?

Sol. Escape velocity of a body from the earth's surface is

$$v_e = \sqrt{\frac{2GM_e}{R_e}},$$

where M_e is the mass of earth and R_e is the radius of earth. If the mass of a planet be M_P and radius R_P, then the escape velocity from it will be

$$v'_e = \sqrt{\frac{2GM_p}{R_p}}$$

$$\therefore \quad \frac{v'_e}{v_e} = \sqrt{\frac{M_p}{M_e} \times \frac{R_e}{R_p}}$$

Given : $M_p / M_e = 1/4$ and $R_p / R_e = 1/2$

$$\therefore \quad \frac{v'_e}{v_e} = \sqrt{\frac{1}{4} \times \frac{2}{1}} = \frac{1}{\sqrt{2}}$$

or $$v'_e = \frac{v_e}{\sqrt{2}} = \frac{11.2 km/s}{1.414} = 7.92\ km\ s^{-1}$$

Ex. A body is at a height equal to the radius of the earth from the surface of the earth. With what velocity be it thrown so that it goes out of the gravitational field of the earth ? Given : $M_e = 6.0 \times 10^{24}$ kg, $R_e = 6.4 \times 10^6$ m and $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$.

Sol. Escape velocity of a body from the earth's surface is

$$v_e = \sqrt{\frac{2GM_e}{R_e}},$$

Where R_e is the radius of the earth (distance of the body from the centre of the earth). If the body is at a height R_e from the earth's surface, then the distance of the body from the centre of the earth will be 2 R_e. Hence in this case, the escape velocity of the body will be

$$v'_e = \sqrt{\frac{2GM_e}{2R_e}}$$

Substituting the given values $v'_e = \sqrt{\frac{2\times(6.67\times10^{-11})\times(6.0\times10^{24})}{2\times(6.4\times10^6)}}$

$$= 7.9 \times 10^3 \text{ ms}^{-1} = 7.9 \text{ kms}^{-1}$$

Ex. A body of mass 100 kg falls on the earth from infinity. What will be its velocity on reaching the earth ? Energy ? Radius of the earth is 6400 km and g = 9.8 ms^{-2}. Air friction is negligible.

Sol. A body projected up with the escape velocity v_e will go to infinity. Therefore, the velocity of the body falling on the earth from infinity will be v_e. Now the escape velocity on the earth is

$$v_e = \sqrt{2gR_e}$$

$$= \sqrt{2\times(9.8\text{ms}^{-2})\times(6400\times10^3\text{m})}$$

$$= 1.12 \times 10^4 \text{ ms}^{-1} = 11.2 \text{ kms}^{-1}$$

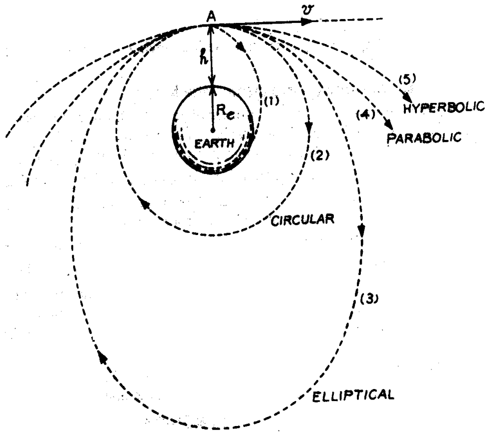
The kinetic energy acquired by the body is

$$K = 1/2mv_e^2$$

$$= \frac{1}{2}\times100\text{kg} \times (11.2 \times 10^3 \text{ ms}^{-1})^2 = 6.27 \times 10^9 \text{ J}$$

11. LAUNCHING OF AN ARTIFICIAL SATELLITE AROUND THE EARTH

Now-a-days, artificial satellites can be put into stable orbits around the earth. This is done by means of multistage rockets. The satellite is placed on the rocket which is launched from the earth. When the rocket reaches its maximum vertical height h, a special mechanism gives a thrust to the satellite at point A (see fig.), producing a horizontal velocity v. We know that the orbital velocity of a satellite orbiting in a circular orbit at a height h above the earth's surface is



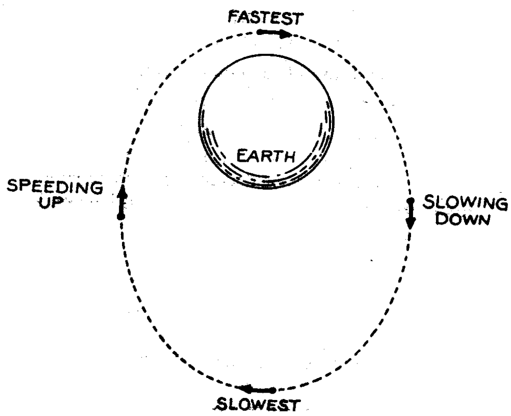
$v_0 = \sqrt{\frac{GM_e}{R_e+h}}$ and at this height the escape velocity is

$v_e = \sqrt{\frac{2GM_e}{R_e+h}}$ where M_e is mass and R_e is radius of earth.

(1) If $v < \sqrt{\frac{GM_e}{R_e+h}}$, then the satellite moving on a 'parabolic' path will come closer and closer to the earth and finally fall on the earth.

(2) If $v = \sqrt{\frac{GM_e}{R_e+h}}$ then the satellite will revolve around the earth in a 'circular' orbit of radius (R_e + h). This is the limiting (minimum) velocity of the satellite to revolve around the earth. If the velocity is less than this, the satellite will come closer and closer to the earth and finally fall on the earth.

(3) If v is greater than $\sqrt{\frac{GM_e}{R_e+h}}$, but less than $\sqrt{\frac{2GM_e}{R_e+h}}$ then the satellite will revolve in an 'elliptical' orbit with centre of the earth at one focus of the orbit. In elliptical orbit, the speed of the satellite varies. It slows down as it gets farther from earth's surface, and speeds up again as it approaches the earth (see fig.)



(4) If $v=\sqrt{\frac{2GM_e}{R_e+h}}$, that is, If the horizontal velocity given to the satellite is equal to the escape velocity, then the satellite moving on a 'parabolic' path would escape, never to return.

(5) If $v>\sqrt{\frac{2GM_e}{R_e+h}}$, then the satellite moving on a 'hyperbolic' path would escape.

SOLVED EXAMPLES

Ex. Four particles, eath of mass m, are palced at the ceorners of squar and moving along a circle of radius r under the infinuence of mutual gravitational attraction. The speed of each particle wil be -

Sol. Resultant force on particle '1'

$F_r = \sqrt{2}\ F + F'$

or $F_r = \sqrt{2}\ \frac{Gm^2}{2r^2} + \frac{Gm^2}{4r^2} = \frac{mv^2}{r}$ or $v = \sqrt{\frac{Gm}{2}\left(\frac{2\sqrt{2}+1}{4}\right)}$

Ex. Three particles of equal mass m are situated at the verticles of an equilateral triangle of side ℓ. What should be the velocity of eath particle, so that they move on a circular path without chanigng ℓ.

Sol. The resultant gravitational force on each particle provides it the necessary centripetal force

$$\therefore\ \frac{mv^2}{r} = \sqrt{F^2+F^2+2F^2\cos 60^o} = \sqrt{3}F$$

But, $r = \frac{\sqrt{3}}{2}\ell \times \frac{2}{3} = \frac{\ell}{3}$, $\quad \therefore v = \sqrt{\frac{Gm}{\ell}}$

Ex. At what height above the Earth's surface does the force of gravity decrease by 10%. Assume radius of earth to be 6370 km.

Sol. Force of gravity at surface of earth,

F_1 = Gm M/R^2 .... (1)

Force of gravity at height H is

F_2 = GmM $(R + H)^2$ ....(2)

Divinding (1) by (2) Rearranging

$H = R\left(\sqrt{\frac{F_1}{F_2}}-1\right)$ = 350 km where (F_2 = 9 F_1)

Ex. The mass and the radius of the earth and the moon are M_1, M_2 and R_1, R_2 respectively their centres are at dsitance d apart. The minimum speed with which a particle of mass m should be projected form a point midway between the two centres so as to escape to infinity will be -

Sol. The P.E. of the mass at d/2 due to the earth and moon is

$$U = -2\frac{GM_1m}{d} - 2\frac{GM_2m}{d}$$

or $U = -2\frac{Gm}{d}(M_1 + M_2)$ (Numertically)

$$\frac{1}{2}mV_e^2 = U$$

$$\Rightarrow V_e = 2\sqrt{\frac{G}{d}(M_1 + M_2)}$$

Ex. The diameter of planet is four times that of the earth. The time period of a pendulum on the planet, if it is a second pendulum on the earth will be (Take the mean density of the planet equal to that of the earth)

Sol. If R be the radius of the earth, then radius of the planet is 4R

Acceleration due to gravity on earth

$$g = \frac{GM}{R^2} = G\frac{4}{3}\frac{\pi R^3\rho}{R_e{}^2} = \frac{4\pi R\rho}{3}G$$

Acceleration due to gravity on planet,

$$g' = \frac{GM'}{(4R)^2} = \frac{G4\pi(4R)^3\rho}{3(4R)^2} = \frac{16\pi R\rho}{3}G$$

Time period of the pendulum on earth.

$$T = 2 = 2\pi\sqrt{\frac{\ell}{g}}\text{ sec}$$

T’ be the time period of the pendulum at planet, then

$$T' = 2\pi\sqrt{\frac{\ell}{g'}} = 2\pi\sqrt{\frac{\ell}{4g}} = \pi\sqrt{\frac{\ell}{g}}, \qquad \therefore\ T'/T = \frac{1}{2} \quad \text{or } T' = 2 \times \frac{1}{2} = 1 \text{ sec}$$