

Homework 4

Due Date: April. 23, 2019

(1) **(10 pts)** Given two strings s and t , write a function to determine if t is an anagram of s in at worst case $O(n \log n)$

(2) **(10 pts)** There are N students in a class. Some of them are friends, while some are not. Their friendship is transitive in nature. For example, if A is a **direct** friend of B, and B is a *direct* friend of C, then A is an **indirect** friend of C. And we defined a friend circle is a group of students who are direct or indirect friends.

Given a $N \times N$ matrix M representing the friend relationship between students in the class. If $M[i][j] = 1$, then the i^{th} and j^{th} students are *direct* friends with each other, otherwise not. And you have to output the number of friend circles among all the students.

So for example:

Input:

$[[1, 1, 0], [1, 1, 0], [0, 0, 1]]$

Output: 2

Explanation: The 0th and 1st students are direct friends, so they are in a friend circle. The 2nd student himself is in a friend circle. So return 2.

Write pseudo-code to solve this problem.

(3) **(10 pts)** In a town, there are N people labelled from 1 to N . There is a rumor that one of these people is secretly the town judge.

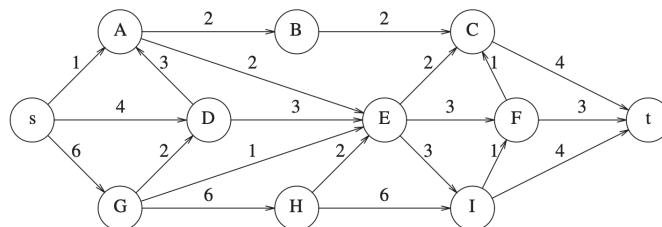
If the town judge exists, then:

1. The town judge trust nobody.
2. Everybody (except the town judge) trusts the town judge.
3. There is exactly one person that satisfies properties 1 and 2.

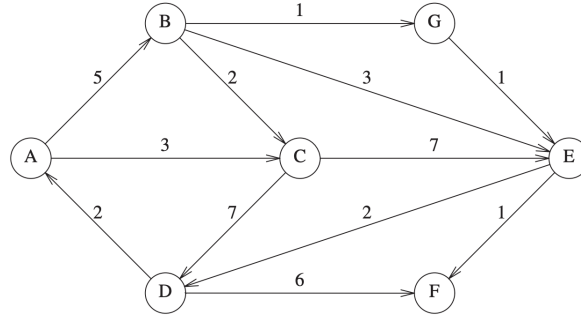
You are given trust, an array of pairs $\text{trust}[i] = [a, b]$ representing that the person labeled a trusts the person labeled b . If the town judge can be identified, return the label of the town judge, otherwise -1.

Describe *in english* how you would solve this problem.

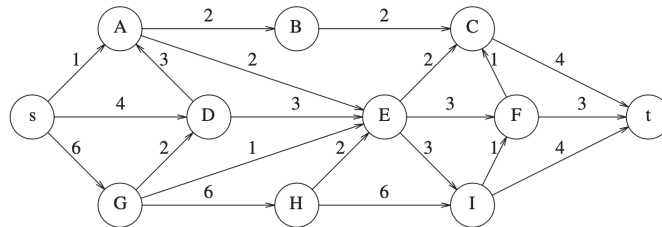
(4) **(10 pts)** Give the topological sort of the following graph starting at s .



(5) (10 pts) Show the table at each step in dijkstra's algorithm for the following graph if s =



(6) (10 pts) Get the maximum flow for the following graph show the residual and flow graph at each step. Then prove the maximum flow is correct by finding the min-cut.

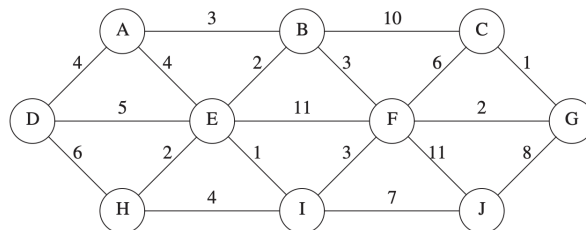


(7) (10 pts) Perform quicksort using the median of three method to find the pivot on the following array. [64, 12, 68, 23, 97, 38, 81, 76, 55, 32, 48, 29, 46]. Show each step. If the partition size is ≤ 3 put the partition in sorted order.

(8) (10 pts) Using a set from 0 to 5, perform the following unions using the union-by-height. Show the result of each union. When heights are the same make the second tree a child of the first tree

union(4,5)
union(2,3)
union(0,1)
union(5,3)
union(1,5)

(9) (10 pts) Find the minimum spanning tree using prim's algorithm for the following graph. Show each step.



(10) (10 pts) Determine if the following graph has a euler circuit.

