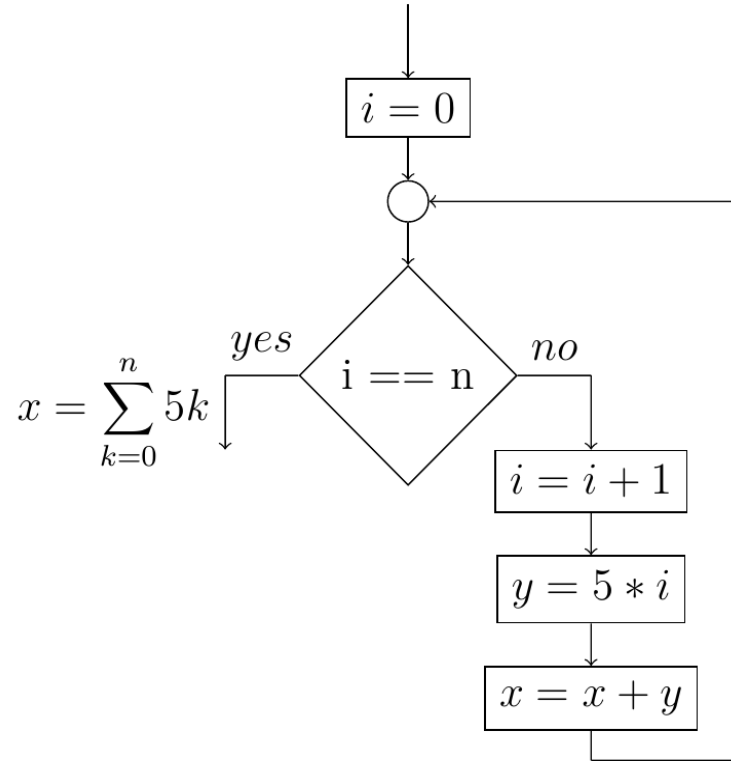
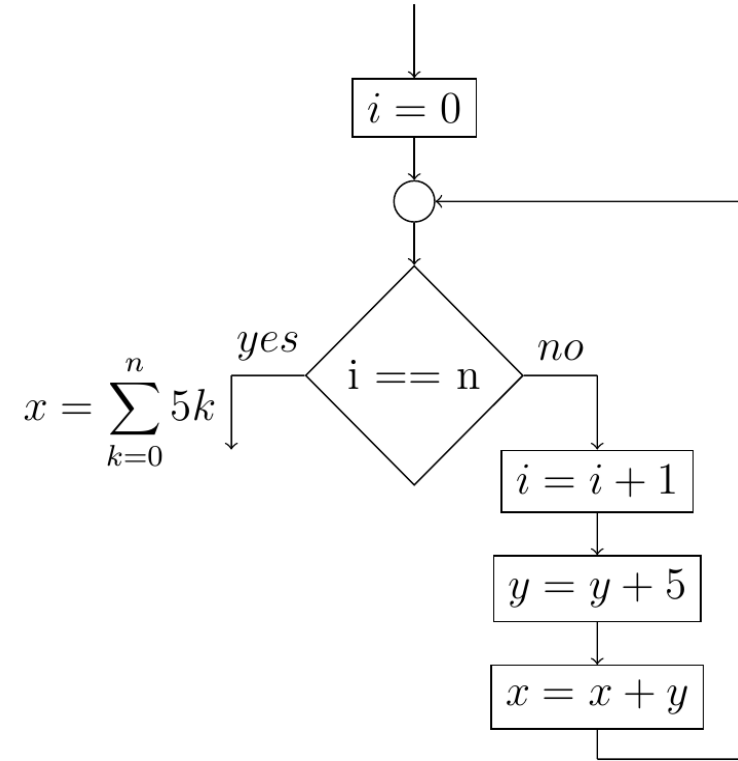


Consider these control flow graph fragments (assume x and y to be 0 initially):

1.

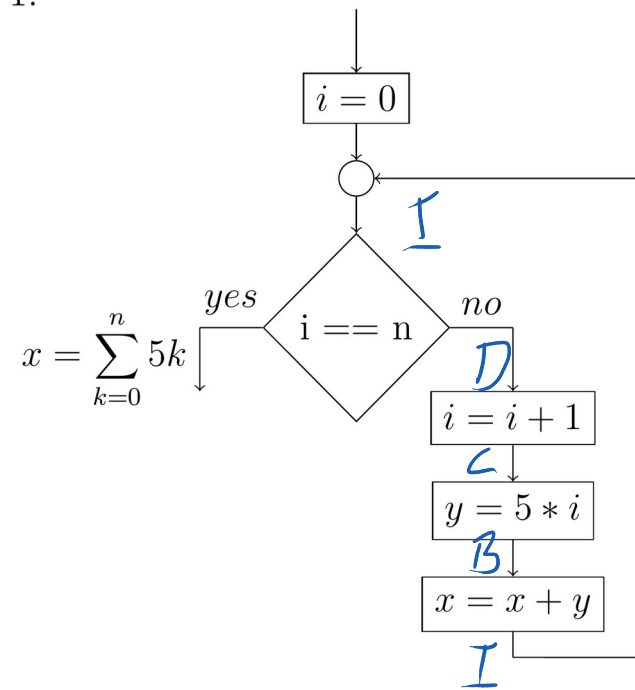


2.



Find suitable loop invariants and prove them locally consistent. Discuss, why these invariants have to be like that.

1.



$$I \equiv X = \sum_{k=0}^i 5k$$

$$B \equiv \text{wp}[x = x + y](I) = X + y = \sum_{k=0}^i 5k$$

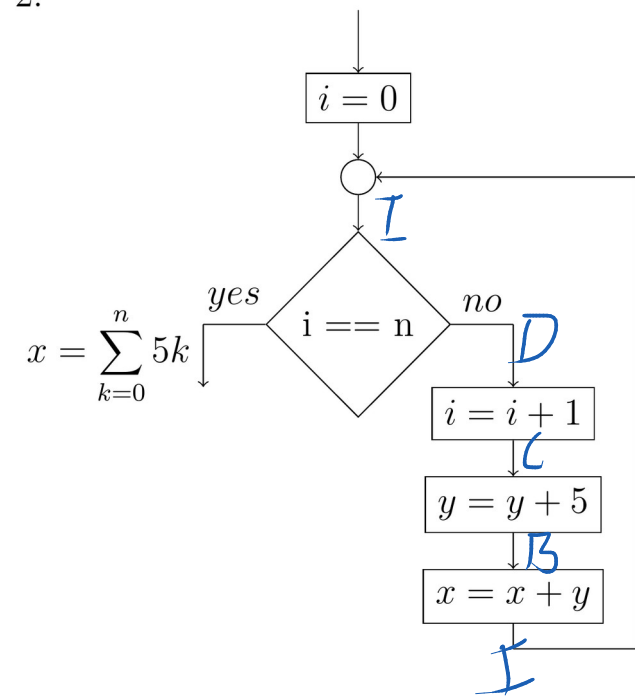
$$C \equiv \text{wp}[y = 5i](A) = X + 5i = \sum_{k=0}^i 5k$$

$$D \equiv X + 5(i+1) = \sum_{k=0}^{i+1} 5k$$

$$= \text{---} = 5(i+1) + \sum_{k=0}^i 5k$$

$$\equiv X = \sum_{k=0}^i 5k$$

2.



1. Try the same loop invariant again

2. Why didn't it work? What information are we missing?

3. How can add that information to our proof?

$$I \equiv X = \sum_{k=0}^i 5k \quad \wedge \quad y = 5i$$

$$B \equiv \text{WP}[x = x + y](I) = X + y = \sum_{k=0}^i 5k \quad \wedge \quad y = 5i$$

$$C \equiv x + y + 5 = \sum_{k=0}^i 5k$$

$$\wedge \quad y + 5 = 5i$$

$$D \equiv X + y + 5 = \sum_{k=0}^{i+1} 5k$$

$$\wedge \quad y = 5i$$

$$X = \sum_{k=0}^i 5k$$

$$\Rightarrow X + y + 5 = \sum_{k=0}^{i+1} 5k$$

$$\wedge \quad i \neq n$$

$$\wedge \quad y = 5i$$

$$\wedge \quad y = 5i$$