Flowline Model User Guide

Moving-grid, width- & depth-integrated ice flow model

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ABSTRACT

The flowline_model_demo.m script, supporting scripts, and other supporting files include the basic code for a moving-grid, width- and depth-integrated numerical ice flow model. The files are set-up to run a 100-year model simulation for a simple model glacier but can be freely modified to simulate fast-flowing tidewater glacier systems.

The proper citation for the code is as follows: E. M. Enderlin, I. M. Howat, and A. Vieli, High Sensitivity of tidewater glacier dynamics to shape, *The Cryosphere Discuss*., **7**, 551-572, 2013. doi: 10.5194/tcd-7-551-2013

Table of Contents

Introduction

Model purpose

Flowline models, or width- and depth-integrated models of ice flow, have been used to model the behavior of glaciers in a wide variety of geographic settings including Greenland [Nick et al., 2009, 2012; Vieli and Nick, 2011], Alaska [Nick et al., 2007a; Colgan et al., 2012], Svalbard [Vieli et al., 2001,2002], Iceland [Nick et al., 2007b], and Antarctica [Jamieson et al., 2012]. The use of a width-integrated lateral resistive stress term means that similar type models cannot be used to model ice flow in regions where there is strong flow convergence, such as ice streams. The implicit use of the Shallow Ice Approximation (i.e., the model is depth-integrated) further restricts the use of similar type models to fast-flowing tidewater glaciers where plug-like ice flow dominates.

Although higher-order models have become more readily available in the past several years, flowline models maintain several advantages over high-order ice flow models. The width- and depth-integration of the models greatly reduces their computational requirements, allowing the user to run a large number of simulations over relatively short time periods. The use of a physically-based calving law, unique to flowline models, allows the calving front position to freely adjust with changes in the glacier stress balance and environmental forcing. Given the potential importance of backstress provided by floating ice, the use of a physically-based calving law is requisite for realistically modeling changes in tidewater glacier dynamics. Flowline models can also be easily implemented using finite difference methods in Matlab, making them more user friendly than most higher-order models.

The flowline model associated with this user guide (flowline_model_demo.m) is a moving-grid, width- and depth-integrated ice flow model with a physically-based calving law [Benn et al., 2007; Nick et al., 2010]. The default model is set-up to execute a 100-year model simulation for an idealized glacier in Matlab. The model can be modified to reproduce observed ice flow for real glacier systems by tuning the geometry, stress, and environmental forcing parameterizations to match observations. Please see **Considerations for real glacier systems** (pp. 11-13) prior to modeling real glacier systems.

Published model results obtained from the modified code should be accompanied by a citation to E. M. Enderlin, I. M. Howat, and A. Vieli, High Sensitivity of tidewater glacier dynamics to shape, *The Cryosphere Discuss*., **7**, 551-572, 2013. doi: 10.5194/tcd-7-551-2013

Notation

The parameters used throughout the model and their units are listed in the table below. The addition of 'm' or 'n' after any of the symbols included below indicates that they are the values on the staggered grid (staggered grid values = mean of neighboring grid cells) or that they are the values obtained for the new iteration, respectively. For example, the *Hm* vector contains thickness values on the staggered grid and the *Hn* vector contains thickness values for the next time step.

Symbol	Description	Units
L	domain length	m
dx	grid spacing (dx0 = desired grid spacing)	m
x	distance vector (0 = ice divide)	m
rho_i, rho_sw, rho_fw	density of ice, seawater, & fresh water	kg m^{-3}
g	gravitational acceleration	m s^{-2}
m	basal sliding exponent	unitless
beta	basal roughness factor	$s^{1/m}$ $m^{-1/m}$
n	flow law exponent	unitless
A	rate factor	Pa^{-n} s^{-1}
E	enhancement factor	unitless
ELA	equilibrium line altitude	m a.s.l.
fwd	fresh water depth in crevasses	m
H	ice thickness	m
H_max	maximum thickness before model breaks	m
U	ice speed	m s^{-1}
dUdx	strain rate (gradient in speed)	s^{-1}
h	ice surface elevation	m a.s.l.
dhdx	ice surface slope (gradient in elevation)	unitless
hb	bed elevation	m a.s.l.
W	glacier width	m
dt	time step	s
*year**	current decimal model year	yr
Hf	thickness required to remain grounded	m
gl	grounding line grid cell (*x*(*gl*) = grounding line position in m from the divide)	unitless
c	calving front grid cell (*x*(*c*) = calving front position in m from the divide)	unitless
Rxx	longitudinal resistive stress	Pa
crev	crevasse tip depth	m
ice_end	last ice-covered grid cell (see **Maintaining stability**, p. 16)	unitless
sl	first grid cell with *hb* < 0	unitless
N	effective pressure	Pa
F	ice flux	m^3 s^{-1}
dHdt	rate of ice thickness change from ice flow	m s^{-1}
SMB	surface mass balance	m s^{-1}
submelt	submarine melt rate beneath floating ice	m s^{-1}
xf	distance from the divide where the ice begins floating	m
xl	number of desired grid spaces to *xf*	unitless
gamma	lateral resistance linearization term	$(m\ s^{-1})^{(1-n)/n}$
eta	basal resistance linearization term	$(m\ s^{-1})^{(1-m)/m}$
v	effective viscosity	Pa s
G_minus, G, G_plus	coefficients for U_{k-1},U_k, U_{k+1} (k is the position index)	Pa
T	gravitational driving stress	Pa
M	sparse tridiagonal U-coefficient matrix	Pa

*The previous model year, which is used for plotting annual profiles, as well as the starting and ending model years are indicated by '_previous','_start', and '_end' suffixes, respectively.

Governing equations

The governing equations, and the linearization and discretization procedures required to solve the equations using finite difference methods, are described in detail in Enderlin et al. [2013]. Briefly, you must assume parallel ice flow so that the temporal change in ice thickness can be determined using conservation of mass, such that

$$\frac{\partial H}{\partial t} = -\frac{1}{W}\frac{\partial (UHW)}{\partial x} + B, \tag{1}$$

where H is ice thickness, U is the along-flow ice velocity, W is width, B is surface mass balance (including submarine melting), t is time, and x is distance from the ice divide.

The governing force balance equation determined through conservation of momentum is

$$2\frac{\partial}{\partial x}\left(Hv\frac{\partial U}{\partial x}\right) - \beta N U^{1/m} - \frac{2H}{W}\left(\frac{5U}{EAW}\right)^{1/n} = \rho_i g H \frac{\partial h}{\partial x}, \tag{2}$$

where β is the basal roughness factor, A is the rate factor, E is the enhancement factor, ρ_i is the density of ice, g is gravitational acceleration, h is the ice surface elevation, and v is the depth-averaged effective viscosity, which is defined as

$$v = (EA)^{-1/3}\left|\frac{\partial U}{\partial x}\right|^{-2/3}. \tag{3}$$

The RHS of Eqn. 2 is the gravitational driving stress, which is balanced by gradients in longitudinal stress (1st term LHS), basal resistance (2nd term LHS), and lateral resistance (3rd term LHS). The basal roughness factor, which is used to tune the basal resistance term, describes the roughness/slipperiness of the bed from the presence of bedrock asperities, till, water, etc. The rate and enhancement factors control the speed at which ice deforms (i.e., strain rate) under a given stress. The rate factor is controlled by the ice temperature, pressure, water content, and grain size of the ice [see Cuffey and Paterson, 2010, pp. 64-72]. The enhancement factor is a non-dimensional scalar that is used to account for additional ice deformation that cannot be explained by the rate factor alone, likely due to the presence of impurities or anisotropic fabric development within the ice.

Equation 2 is discretized, linearized, and translated into matrix-vector form so that the LHS is the cross-product of a sparse tri-diagonal velocity coefficient matrix (***M***) and the velocity vector (*U*), and the RHS is the gravitational driving stress vector (*T*). The Matlab backslash operator is used to calculate the velocity at the next time step (*U*=***M****T*). The new velocity vector is then used to solve Eqn. 1. This process is repeated iteratively in order to solve for temporal changes in the ice thickness and velocity of the glacier.

Running the Model

Matlab search path

The flowline_model folder contains all the Matlab scripts and supporting files needed to run ice flow simulations using the flowline model. The folder should be saved within a current Matlab search path. To check the current Matlab search paths, open Matlab and type *path* into the command window.

If your folder is outside of the set search paths, you can add the folder by typing *path('flowline_model_path',path)* into the command line, replacing *flowline_model_path* with the location of your flowline_model folder. For example, if the folder is saved under the user Sally's main directory, *'flowline_model_path'* should be substituted with '*/users/sally/flowline_model'* in order to move this folder to the top of the Matlab search path. ***Be sure to include the single quotes in the command line where indicated.***

Running the demonstration

Before running the demonstration, you will need to open the flowline_model_demo.m, flowline_model_demo_geometry.m, and flowline_model_demo_SMB.m scripts to change the directory referenced within the scripts. Find each line in the script that starts with *cd* and change the referenced directory to the path containing the flowline_model folder. Following the same example from **Matlab search path**, each of these lines should now be *cd /users/sally/flowline_model_demo*. After saving the changes made to each script, you can run the flowline_model_demo.m script.

Overview

The flowline model is set-up to complete a 100-year simulation of ice flow starting from an idealized geometry. The initial, idealized ice thickness and elevation, bed topography and width, and submarine melting profiles can be found in the flowline_model_demo_initialization.mat, flowline_model_demo_ascii_geometry.txt, and flowline_model_demo_ascii_submelt.txt files, respectively. The annual elevation and speed profiles are displayed in Figures 1 & 2. The inset in Figure 1 also contains a plot of the elevation profile near the terminus. The grounding line position and speed are plotted annually in two time series subplots in the remaining figure. At the end of the simulation, the profile data are automatically saved as flowline_model_demo_original.mat and the grounding line position, thickness, and speed timeseries data are saved as flowline_model_demo_gltimeseries_original.mat.

The demonstration simulation should be run several times using slightly modified parameters before inputting more complex observational data into the model. The initial values for the demonstration parameters are selected to maintain a relatively constant grounding line position throughout the 100-year simulation. Changing any of the key parameters will influence ice flow, as described in **Testing model sensitivity using the demonstration** (pp. 10-11). If you wish to compare the influence of parameter variations on ice flow, the file name under which you are saving the data must be changed.

Detailed explanation of the model code

A concise, nearly line-by-line explanation of the model code can be found within the comments in the Matlab script. The thorough explanation of critical code details provided below is a supplement to the comments in the script.

Lines 12-45

The time- and space-independent parameters are defined at the beginning of the model because they are constant and uniform, and thus, they do not need to be redefined throughout the model simulation.

Distance: The length of the model domain (*L*) and desired grid spacing (*dx0*) must be defined first in order to set-up the ice thickness and speed profiles for the simulation. The domain length must be long enough to ensure that the ice thickness reaches zero prior to the end of the domain to maintain model stability. The desired grid spacing is used to constrain the size of the moving-grid used to continuously track the grounding line position.

Stress parameters: The glacier stress balance equation contains several parameters that influence glacier behavior (see **Governing Equations**, p. 4, Eqn. 2). The influence of each parameter on ice flow is described in the **Model Details** (pp. 14-17) section of this guide. The basal sliding exponent (*m*), basal roughness factor (*beta* = β in Eqn. 2), flow law exponent (*n*), rate factor (*A*), and enhancement factor (*E*) are defined as constant and uniform in the demonstration in order to keep the model as simple as possible.

Environmental forcing parameters: The equilibrium line altitude (*ELA*) is defined in order to calculate surface accumulation and ablation independently. For the physically-based calving law included in the model, the depth of fresh water in the crevasses (*fwd*) must also be defined (see **Calving law**, p. 16).

Time step: The time step is set as a constant for the demonstration. To optimize model performance for simulations of real glacier systems, the time step should be allowed to freely adapt. See **Maintaining stability** (pp. 16-17) for more details.

Lines 46-76

Initialization: The initialization mat-file contains initial idealized thickness (*H*) and speed (*U*) profiles for a distance vector (renamed *x_initial*) and the starting year (*year* = 0) for the simulation. The initialization data are obtained at all points in the desired distance vector (*x*) using the Matlab interp1 function. Missing data are extrapolated for the remaining *x* using the 'extrap' property in the interp1 function. The initial strain rates (*dUdx*) are calculated as the spatial derivative of speed using the Matlab gradient function.

Geometry: The bed elevation (*hb*) and width (*W*) profiles are obtained at all points in *x* within the flowline_model_demo_geometry.m supporting script.

Plot set-up: Three empty plots will appear at the beginning of the simulation that will be re-positioned and filled-in after the first simulation year has elapsed (see **Plotting**, pp. 18-19 for details on the model plots).

Lines 78-234 (Mass Conservation)

Flotation Thickness: The thickness that the ice must exceed in order to remain grounded (*Hf*) depends on the density contrast between the ice and seawater (ρ_i = 917 kg m^{-3} & ρ_{sw} = 1028 kg m^{-3}) as well as the elevation of the bed.

Grounding line & ice domain end: The grounding line is located at the last grid cell where $H \geq Hf$. The end of the ice-covered domain (*ice_end*) is also defined. For a calving glacier, $ice\_end \neq c$ (see **Maintaining stability**, pp. 16-17 for an explanation).

Surface elevation: For grounded ice, the surface elevation (*h*) is calculated as the sum of the bed elevation and ice thickness. Where the ice is floating, the surface elevation calculation takes into account the difference between the density of ice and seawater. The surface slope is calculated as the spatial derivative of the surface elevation using the Matlab gradient function.

Rate factor: The rate factor (*A*) is set to linearly increase with distance from the ice divide in order to simulate the increase in ice "softness" (i.e., deformability) as the ice warms from deformational and frictional heating along flow.

Calving: The softness of the ice influences the longitudinal resistive stress that promotes crevasse opening (*Rxx*). Seaward of the grounding line, the crevasse penetration depth (*crev*) is calculated using stresses that promote crevasses opening as well as stresses that promote crevasses closure (see **Calving law**, p. 16). The calving front is located at the inland-most grid cell where the surface crevasse depth equals or exceeds the ice surface elevation. If the crevasses never penetrate to sea level, the calving front is set to a maximum position corresponding to an arbitrary minimum ice thickness. If the calving front is located inland of the grounding line, it is set to the grounding line position.

Effective Pressure: Where the bed is above sea level (hb>0), the effective pressure is equal to the ice overburden pressure. Where the bed is below sea level ($hb \leq 0$), the effective pressure is the difference between the ice overburden pressure and the basal water pressure at the bed. An easy and open connection between the ocean and the bed is assumed such that the basal water pressure is a function of the depth of the bed below sea level.

Stress Balance: see below

Change in thickness assuming continuity: The change in ice thickness from ice flow convergence/divergence between grid cells is calculated as the width-averaged difference in ice flux entering and exiting each grid cell, where the ice flux is the product of the ice speed, thickness, and width. The difference in ice flux is obtained using the Matlab gradient function.

Change in thickness from SMB & submarine melting: See **Surface Mass Balance (SMB) & Submarine melting** (p. 18) for details on the parameterizations. The change in ice thickness from environmental forcing during each time step is added to the change in thickness from ice flow convergence/divergence.

Moving-grid: The precise grounding line position (*xf*) is identified as the point where the new ice thickness equals the flotation thickness using the Matlab interp1 function. The precise grounding line position is then divided by the desired grid spacing and rounded to the nearest integer to determine the approximate number of grid spaces of a size ≅ dx0 between its position and the ice divide (*xl*). The adjusted grid spacing is obtained by dividing *xf* by *xl.* The use of the desired grid spacing in this calculation ensures that the grid will remain ≅ dx0 throughout the simulation. Using the adjusted grid spacing, a new distance vector is created and parameters are obtained for each grid cell in the new distance vector using the interp1 function. The new grounding line, last ice-covered grid cell, and calving front positions are also calculated.

Plotting annual data: See **Plotting** (pp. 18-19) for details regarding each plot. The original file will plot the elevation and speed profiles for the entire model domain, a zoomed inset of the elevation profile, and time series of the grounding line position and speed after each model year (if *year* - *year_previous* ≥ 1).

Displaying data: The Matlab disp function can be used to display data after each model year (if *year* - *year_previous* ≥ 1).

Breaking the loop & saving data: The while loop breaks and the data are saved using the specified file names if the current model year > *year_end*. Otherwise, the loop continues to the next iteration ($i = i+1$).

Lines 238-377 (Stress Balance)

Linearize the lateral resistance term: For a flow law exponent (n) > 1, the lateral resistance term in the stress balance equation (**Governing equations**, p. 4, Eqn. 2, 3rd term on the LHS) must be linearized before creating the velocity coefficient matrix that is used to calculate the new velocity vector. The linearization term (*gamma*) is set as $U^{(1-n)/n}$ such that $gamma \times U = U^{1/n}$. The maximum value for *gamma* ensures that the linearization term does not approach infinity as the ice speed approaches zero. The maximum value must change with the choice of n.

Calculate the longitudinal stress term on the staggered grid: The longitudinal stress term (**Governing equations**, p. 4, Eqn. 2, 1st term LHS) must be solved on the staggered grid for proper convergence. If the grid spacing is small (~100s of meters), the ice thickness can be calculated on the staggered grid by taking the arithmetic mean of the neighboring grid cells. As the grid spacing increases, however, the staggered grid values should be obtained using cubic interpolation methods because the arithmetic mean will not be able to account for the curvature between the neighboring grid cells. The effective viscosity (v), which is a measure of the ice "softness" or deformability, should be calculated on the staggered grid using Eqn. 3 (see **Governing equations**, p. 4). In order to obtain correct values for *vm* (i.e., v on the staggered grid), the rate factor, enhancement factor, and strain rates must be calculated on the staggered grid and then input into Eqn. 3. The maximum value for the effective viscosity ensures that it does not approach infinity as the strain rate approaches zero. The maximum value must change with the choice of *n*.

Linearize the basal resistance term (if necessary): For a basal sliding exponent (*m*) > 1, the basal resistance term in the stress balance equation (**Governing equations**, p. 4, Eqn. 2, 2rd term on LHS) must be linearized before creating the velocity coefficient matrix used for calculating the new velocity vector. The linearization term (*eta*) is set as $U^{(1-m)/m}$ such that *eta*×*U* = $U^{1/m}$. The maximum value for *eta* ensures that the linearization term does not approach infinity as the ice speed approaches zero.

Set-up the stress balance equation in matrix-vector form: At the kth grid cell, the discretized form of Eqn. 2 can be described by the equation

$$G\_minus \times U_{k-1} + G \times U_k + G\_plus \times U_{k+1} = T_k, \quad (4)$$

where $G\_minus = G_{k-1}$, $G = G_k$, $G\_plus = G_{k+1}$ and *T* is the gravitational driving stress. The velocity coefficients (*G_minus*, *G*, *G_plus*) are defined in the model code as well as in the supplementary material in Enderlin et al. [2013]. It is important to note that the values for the velocity-coefficients and *T* are defined by the hydrostatic equilibrium boundary condition from the calving front to the end of the ice covered domain (i.e., from *c*:*ice_end*) as described in detail in **Ice seaward of the calving front** (p. 17). In matrix-vector form, the coefficients are contained in a sparse tridiagonal matrix (***M***).

Obtain new velocities & strain rates: The new velocity vector is obtained by either cross-multiplying the inverted coefficient matrix and the *T* vector ($U = \boldsymbol{M}^{-1}T$) or using the Matlab backslash operator (*U*=***M****T*). The use of the backslash operator is preferable over the matrix inversion because of its decreased computation time and increased numerical stability. For the ice-free cells in the model domain, the velocity is set to zero so that the velocity vector is the same length as the thickness and width vectors in the ice flux calculation. The strain rates are calculated for the entire model domain using the Matlab gradient function.

Checking for convergence & looping: The values obtained for the new velocity vector depend on the velocity vector that is input into the stress balance equations. Therefore, at each time step, the velocity vector must be solved iteratively until the difference in velocity between consecutive iterations satisfies a threshold convergence criterion. As the default, the velocity vector is considered to have sufficiently converged if the sum of the difference between the velocity vectors obtained from consecutive iterations (*U* & *Un*) is less than 10% of the sum of the *U* vector. If this criterion is satisfied, the loop breaks and the new velocity and strain rate vectors are input into the Conservation of Mass loop in order to solve for the ice flux. If the criterion is not satisfied, the loop continues and the new velocity and strain rate vectors are used to solve the stress balance equation. If the velocity vector does not converge after a large number of iterations (default = 50), the loop breaks and the last velocity and strain rate vectors are input into the Conservation of Mass loop.

Testing model sensitivity using the demonstration

Basal resistance: The basal resistance parameterization can be modified by changing either the basal sliding exponent (m) or the basal roughness factor (β). If the basal sliding exponent is lowered (default m=3), the magnitude of the basal resistance for a particular ice flow speed increases (see **Governing equations**, p. 4, Eqn. 2, 2nd term on LHS). ***Therefore, if you change the basal sliding exponent, you must also change the basal roughness factor to prevent run-away ice flow.*** If the basal sliding exponent is fixed and only the basal roughness factor changes (default $\beta = 2\times10^4$ $s^{1/m}$ $m^{-1/m}$), the ice flow speed can be gradually adjusted. Larger values for the roughness factor promote slower speeds (more basal resistance) and smaller values promote faster speeds (less basal resistance).

Suggested sensitivity tests include:
1) m=3 and β=1.5×10^4 $s^{1/m}$ $m^{-1/m}$ (glacier will advance & thin in the interior)
2) m=2 and β=3×10^5 $s^{1/m}$ $m^{-1/m}$ (glacier will retreat & thicken in the interior)

Rate & Enhancement factors: The effective viscosity of the ice is controlled by the rate and enhancement factors (see **Governing equations**, p. 4, Eqn. 3). The rate factor, A, controls the speed at which ice deforms under a given stress (i.e., strain rate), with higher values corresponding to lower effective viscosities and higher deformation rates. The default rate factor increases linearly from the minimum value (A_min = 3.5×10^{-25} Pa^{-n} s^{-1}) at the ice divide to the maximum value (A_max = 9.3×10^{-25} Pa^{-n} s^{-1}) at the end of the model domain. These values are obtained for ice temperatures of -10°C and -5°C, respectively. The enhancement factor, E, is a non-dimensional scalar that is used to account for additional ice deformation caused by the presence of impurities or anisotropic fabric development within the ice. The default value is E=1 (no enhancement). An increase in either of these terms causes the effective viscosity to decrease and flow speeds to increase.

Suggested sensitivity tests include:
1) Decrease A_max to 3.5×10^{-25} Pa^{-n} s^{-1} (slightly thicker, slower)
2) Increase the enhancement factor to E=3 (slightly thinner, faster)

Environmental Forcing: Relatively small changes in the surface mass balance (*SMB*), submarine melt rates (*submelt*), and water depth in the crevasses (*fwd*) can trigger substantial changes in ice flow (see **Surface Mass Balance (SMB) & Submarine melting**, p. 18 for details on environmental parameterizations).

Changes in the *ELA*, accumulation rate, and ablation rate influence the *SMB* and can all be modified independently. The *ELA* can be changed at the beginning of the model (see the description of Lines 12-45 above, default *ELA*=800 m). The accumulation and ablation rates vary with the elevation of the glacier's surface with respect to the *ELA* and can be modified in the flowline_model_demo_SMB.m script. Along-flow changes in *SMB* are parameterized with a piecewise linear function (i.e., the slope of the function differs in the accumulation and ablation zone).

Changes in the submarine melt rate (*submelt*) and fresh water depth in crevasses (*fwd*) influence the thickness and length of the floating ice tongue, when floating ice is present. Higher melt rate or crevasses water depth values promote the development of thinner or shorter floating tongues, respectively.

Suggested sensitivity tests include:
1) Lower the *ELA* from 800 m to 700 m (glacier will advance and thicken)
2) Lower fwd from 4 m to 2 m (acceleration near the terminus)

Considerations for real glacier systems

In order to use the model for simulations of real glacier systems, numerous parameters must be adjusted within the model using available observational data. ***Observational data and their respective spatial coordinates should be saved as tab delimited ascii text files.*** These files can easily be input into the model using the Matlab dlmread function (see the flowline_model_demo_geometry.m script for a dlmread example). If the spatial coordinates are input as distance from the ice divide, the data can be interpolated at all grid points in the model domain using the Matlab interp1 function. The following parameters should be modified using observational data (if available):

Ice thickness & speed (*H*, *U*): These data can be used to either initialize model simulations or to establish a threshold criterion used to terminate a model run, or both. If the model is initialized with ice thickness and speed observations, you will be able to quickly identify whether appropriate values have been chosen for the stress balance (m, β, n, A, E) and environmental forcing (*ELA*, *SMB* slopes, *submelt*, *fwd*) parameters because the simulated profiles will immediately deviate from the initialization profiles if any parameter value is incorrect.

The observational data can also be used to establish a threshold criterion that breaks the model simulation when it is satisfied. If the model is used to reproduce steady-state behavior (with steady-state H & U observations), the threshold criterion should consider the offset between the observed and simulated profiles and the change in the profiles between consecutive model iterations. If the model is used to reproduce transient behavior, the threshold criterion should consider the offset between the observed and simulated profiles at each time step with available observational data.

Geometry (bed elevation (hb) and width (W)): Geometry data must be provided for the entire model domain in order to run a model simulation. If observational data are not available for the entire length of the glacier system, data gaps should be filled with interpolated values or with estimated values obtained from conservation of mass calculations. If high-resolution bed data are available, they may need to be smoothed in order to maintain numerical stability. ***Additionally, if across-flow bed elevations are available, the width-averaged bed elevations should be used in the model rather than the centerline observations.***

Stress parameters (*m*, β, *n*, *A*, *E*): Observations of basal sliding are limited to very few glaciers, so the basal sliding parameters (m, β) are typically used as free parameters tuned to produce simulated *H* & *U* profiles that reasonably match the available observational data. Typical values for *m* range from 1 to 3. Values for the basal roughness factor (β (beta in the model)) vary widely between glacier systems and should be adjusted freely with the choice of *m*. ***Although the basal roughness factor can be tuned to reproduce observational data for nearly any choice of the basal sliding exponent, it is important to consider that the choice of the basal sliding exponent will strongly influence the dynamic response of the simulated glacier to a perturbation.***

The flow law exponent (*n*) is normally set to 3 but may vary for different glacier systems. If $n \neq 3$, the maximum value for the linearization term for lateral resistance (*gamma*) and the rate factor (*A*) must be changed.

Values for the rate and enhancement factors (*A*, *E*) should be based on any available temperature and fabric measurements obtained for the glacier system. Data collected during ice core drilling can be used to constrain the depth-averaged values for *A* and *E* at the location of the core. It is important to consider, however, that values for these parameters can vary both along- and across-flow.

See **Stress parameterizations** (pp. 14-15) for more detailed information on the basal and lateral resistive stress parameterizations used in the model.

Environmental forcing parameters (*ELA*, *SMB* slopes, *submelt*, *fwd*): It is likely that the *SMB* profiles for most glacier systems are much more complex than the *SMB* profile parameterization in the supporting flowline_model_demo_SMB.m script. For simulations of real glacier systems, meteorological station and snowpit data can be spatially interpolated to obtain *SMB* profiles within the observational time period for the entire glacier domain. Alternatively, *SMB* profiles can be extracted from regional climate models if *in situ* data are not available. ***Note that the SMB of a glacier system can vary both spatially and temporally. In order to realistically simulate ice flow, the SMB parameterization input into the flowline model must take into account the influence of both spatial and temporal SMB variability on ice flow.***

The values chosen for the submarine melt rate (*submelt*) and the fresh water depth in surface crevasses (*fwd*) can strongly influence the shape of the floating ice tongue. Changes in the size/shape of the floating tongue can trigger changes in glacier dynamics, making the choice of appropriate parameter values important for simulations that aim to reproduce observations of dynamic change. Limited observational data have been collected for both of these parameters, however, so they must be tuned until the length (controlled by *fwd*) and freeboard (controlled by *submelt*) of the simulated floating ice tongue matches observations. The modeled *fwd* is typically on the order of meters in order to produce short (<10 km) floating tongues. Submarine melt rates range from approximately zero to ~4 m d^{-1} in Greenland [Rignot and Steffen, 2008; Motyka et al., 2011; Enderlin and Howat, 2013].

Other considerations: If the simulated glacier base is not below sea level at the grounding line (i.e., the glacier is land-terminating), the hydrostatic equilibrium boundary condition applied at the calving face must be removed from the stress balance equation (**Detailed explanation of**

the model code, pp. 6-9, Eqn. 4) so that *G_minus*, *G*, *G_plus*, and *T* are calculated from the ice divide to the grounding line. In the modified calculation, $T = \rho_i g H \frac{\partial h}{\partial x}$ for the entire ice domain and *ice_end=gl*.

The time step (*dt*) utilized for the model iterations should also be adjusted using the Courant-Friedrichs-Lewy (CFL) condition for partial differential equations in order to maximum the computational efficiency of the model. The optimal time step will decrease if the grid spacing (dx) decreases and/or the speed of the simulated glacier increases. Therefore, it is important to consider both the desired time step and grid spacing when designing simulations for real glacier systems. See **Grid spacing & time step** (p. 16) for more details.

Model Details

Boundary Conditions

The flux at the inland boundary is set to zero (i.e., $U_1 = 0$) to simulate the ice divide. At the seaward boundary, the longitudinal stress is balanced by the difference between the hydrostatic pressure of the ice and seawater. In order to impose this boundary condition, the depth-averaged stress (*Rxx*) from the calving face (*c*) to the end of the ice-covered domain (*ice_end*) is calculated as

$$R_{xx} = \frac{1}{2}\rho_i g\left[H\left(1-\frac{\rho_i}{\rho_{sw}}\right)\right], \tag{5}$$

where $H\dfrac{\rho_i}{\rho_{sw}}$ is the depth of the glacier base below sea level. After applying Glen's flow law and rearranging the terms, the seaward boundary condition becomes

$$\frac{\partial U}{\partial x} = A\left[\frac{\rho_i g}{4}H\left(1-\frac{\rho_i}{\rho_{sw}}\right)\right]^n. \tag{6}$$

Thus, from $k = c{:}ice\_end$, $G\_minus = G_{k-1} = -1$, $G = G_k = 1$, $G\_plus = G_{k+1} = 0$ such that $U_k - U_{k-1} = T_k$, and

$$T_k = A_k\left[\frac{\rho_i g}{4}H_k\left(1-\frac{\rho_i}{\rho_{sw}}\right)\right]^n dx\,, \tag{7}$$

where the subscript k is the position index.

Stress parameterizations

Basal resistive stress

The basal resistive stress, τ_{basal}, is a parameterized using a Weertman-type sliding law, such that basal resistance is a function of the effective pressure at the glacier base, N_e, basal roughness, β, and the depth-integrated ice velocity. Thus, basal resistance is calculated as

$$\tau_{basal} = \beta N_e U^{1/m}, \tag{8}$$

where m is the basal sliding exponent [Bindschadler, 1983; van der Veen and Whillans, 1996; Vieli and Payne, 2005; Nick et al., 2009, 2010].

The effective pressure is calculated assuming an easy and open connection between the ocean and ice-bed interface so that N_e gradually decreases to zero at the grounding line. Although short-term (i.e., hourly to monthly) variations in subglacial water pressure influence the ice flow speed throughout the melt season [Das et al., 2008; Joughin et al., 2008; Bartholomew et al., 2010; Sundal et al., 2011], inter-annual changes in ice flow speed are likely to be triggered by changes in resistive stress at/near the glacier terminus rather than changes in subglacial hydrology [Vieli and Nick, 2011]. Therefore, the simple effective pressure parameterization utilized by the model should be sufficient for studies of inter-annual changes in glacier behavior.

Values for the basal sliding exponent typically vary from m = 1-3 [Vieli and Payne, 2005; Nick et al., 2009, 2010; Vieli and Nick, 2011; Jamieson et al., 2012]. The basal roughness factor describes the roughness/slipperiness of the bed from the presence of bedrock asperities, till, water, etc. Because bed roughness cannot be constrained by surface observations, it is normally used as a free, tuning parameter in ice flow models. In regions where effective pressure is high (water pressure is low), the values selected for these parameters will strongly influence the simulated ice flow.

Lateral resistive stress

The lateral resistive stress, $\tau_{lateral}$, is parameterized by integrating the horizontal shear stress over the channel width assuming that lateral drag supports the same fraction of driving stress along a transect across the glacier [van der Veen and Whillans, 1996], such that

$$\tau_{lateral} = \frac{2H}{W}\left(\frac{5U}{EAW}\right)^{1/n}. \tag{9}$$

Although this parameterization accounts for variations in channel width along-flow, it does not account for differences in channel shape (e.g., parallel-sided, parabolic, semi-circular). A shape factor could potentially be included in Eqn. 9 to account for the influence of channel shape on the width-integrated lateral resistance.

Grounding line tracking

Flotation criterion

The grounding line position is tracked as the point where the ice thickness is equal to the thickness required for the ice to remain grounded such that the effective pressure at the glacier base equals zero (i.e., ice overburden pressure = subglacial water pressure). The physically-based contact problem used to define the grounding line position in Nowicki and Wingham [2007] is likely to identify the position of the grounding line with better precision. However, the flotation criterion has been successfully used in similar models to reproduce observed grounding line migration [e.g., Nick et al., 2009; Jamieson et al., 2012] and should sufficiently track grounding line migration over inter-annual or longer time scales.

Grid adjustment

The grounding line position is precisely and continuously tracked by stretching/contracting the coordinate system so that the grounding line coincides exactly with a grid cell. The precise grounding line position is identified at each time step using the Matlab interp1 function. Similar moving grid techniques have been shown to more accurately track the grounding line position than models that utilize fixed coordinate systems [Pattyn et al., 2012]. The specific code used to adjust the grid spacing is explained in **Detailed explanation of the model code** (pp. 6-9).

Calving law

Near tidewater glacier termini, longitudinal stretching from ice flow acceleration will create dense fields of extensional crevasses. In such settings, surface crevasses will penetrate to the depth in the ice where the net longitudinal stress becomes zero. The net longitudinal stress is calculated as the difference in stresses that promote the opening and closure of crevasses. For crevasses containing surface melt water, opening stresses include both the resistive stress of the ice and the stress of the overlying water. In contrast, the weight of the ice overburden promotes crevasse closure. Thus, the crevasse depth (*crev*) is calculated as

$$crev = \frac{R_{xx}}{\rho_i g} + \frac{\rho_{fw}}{\rho_i} fwd, \qquad (10)$$

where *Rxx* is the resistive stress. Using Glen's flow law, the resistive stress is defined as

$$R_{xx} = 2\left(A^{-1}\frac{\partial U}{\partial x}\right)^{1/n}. \qquad (11)$$

Assuming that the fracture of ice along pre-existing crevasses is a large-scale, first-order control of iceberg calving, the calving front can be identified as the inland-most ungrounded grid cell in which the surface crevasse depth equals the ice surface elevation [Benn et al., 2007; Nick et al., 2010]. The calving law can be modified to include basal crevassing using the equations described in Nick et al. [2010]. Although this crevasse penetration parameterization of calving cannot be used to model individual calving events, it provides a direct connection between changes in ice dynamics and environmental forcing that is absent from more complex ice flow models.

Maintaining stability

Grid spacing & time step

The grid spacing and time step used in the model must meet the Courant-Friedrichs-Lewy (CFL) condition for partial differential equations, described as

$$U\Delta t \le \Delta x, \qquad (12)$$

where Δt is the model time step (*dt*) and Δx is the grid spacing (*dx*) [van der Veen, 1999, p. 220]. For fast-flowing tidewater glaciers, the grid spacing should be less than or equal to one ice thickness (≤ 1 km) and speeds can reach >10 m d^{-1}. With these considerations in mind, the model time step should range from ~0.001-0.1 yr^{-1}. Although the trade-off between grid spacing and computation time makes the use of larger grid spacing more desirable, it is important to consider whether the selected grid spacing can sufficiently resolve along-flow variations in ice thickness and speed. To maximize computational efficiency, the time step should be adjusted to its maximum stable value ($\Delta t = \Delta x/U$) after each model loop.

Ice seaward of the calving front

At the calving front, the slope of the ice surface (*dhdx*) approaches infinity (i.e., the calving face is approximately vertical). If this surface slope is input into the stress balance equation, the driving stress (see **Governing Equations**, p. 4, Eqn. 2, RHS) will become infinitely large, resulting in model instability. To avoid this instability, the ice domain is extended to include the 'calved' ice attached to the model glacier, which gradually thins with distance from the calving front under the influence of surface and submarine melting. Thus, the surface slope never approaches infinity. The seaward boundary condition is applied from the calving front to the end of the ice domain (*ice_end*).

*NOTE: For land-terminating glaciers the ice domain will end at the grounding line (*ice_end=gl*), and the seaward boundary condition must be removed from the model code. If the model is used to simulate the retreat of a marine-terminating glacier from a body of water, the model must be modified to account for differences in the seaward boundary condition when the base of the glacier is below and above sea level at the grounding line.

Supporting script details

Geometry

The supporting flowline_model_demo_geometry.m script imports bed elevation and width profile data from a tab-delimited ascii text file using the Matlab dlmread function (`P = dlmread('flowline_model_demo_ascii_geometry.txt','\t',1,0);`). The first column in the text file contains the distance vector (`x_input = P(:,1)';`), the second column contains the bed elevation vector (`hb_input = P(:,2)';`), and the third column contains the width vector (`W_input = P(:,3)';`). The output geometry data (*hb*, *W*) are interpolated at all grid cells in the flowline model's distance vector (*x*).

Surface Mass Balance (SMB) & Submarine melting

The supporting flowline_model_demo_SMB.m script calculates the surface mass balance (*SMB*) and submarine melt rate (*submelt*) profiles for the glacier at each time step. The units for the SMB and submelt data are $m\ s^{-1}$.

SMB is parameterized as a piecewise linear function of the ice surface elevation with respect to the equilibrium line altitude (*ELA*). The slope of the function varies above and below the *ELA* (*c*1 and *c*2, respectively) so that accumulation and ablation can be modified independently. The air temperature lapse rate is incorporated in the functions as an additional scalar to ensure that the *SMB* does not get unreasonably large for *c*1 & *c*2 slopes ranging from 0-1.

The submarine melt rate parameterization is saved in a tab-delimited text file that is imported into the model using the Matlab dlmread function (`P = dlmread('flowline_model_demo_ascii_submelt.txt','\t',1,0);`). The first column in the text file contains the distance vector (`x_input = P(:,1)';`) and the second column contains the submarine melt rate vector (`submelt_input = P(:,2)';`) starting at the grounding line. The submarine melt rate is zero at the grounding line and increases to its maximum value of ~32 m yr^{-1} at a distance of 1.2 km from the grounding line. Melt rates gradually decrease to ~16 m yr^{-1} at a distance of 4km from the grounding line and remain uniform thereafter. The prescribed values are similar to melt rates from Petermann Glacier, Greenland [Rignot and Steffen, 2008]. Melt rates can, however, reach values of up to meters per day in Greenland [Enderlin and Howat, 2013].

*NOTE: Submarine melt rates must gradually increase for at least 500 m seaward of the grounding line in order to prevent an abrupt increase in the surface slope between the grounding line and the adjacent seaward grid cell. If the submarine melt rate is increased too rapidly, the model may behave unstably.

Plotting

The supporting flowline_model_demo_plots.m script plots the elevation and speed profiles as well as the grounding line position and speed after each simulated year.

The axes limits and tick marks are defined at the beginning of the model based on the range of results obtained for the sensitivity tests presented in **Testing model sensitivity using the demonstration** (pp. 10-11). The annual grounding line position, thickness, and speed are also renamed at the beginning of the script. These data are exported from the plotting script so that temporal changes in the grounding line position, thickness, and speed can be easily examined at the end of the model simulation.

Annual elevation profiles are plotted in Figure 1 (fig1). In order to correctly plot the floating ice tongue, the depth of the glacier base below sea level must be calculated using the density contrast between the ice and sea water where the ice is floating. The base of the floating ice tongue is plotted from the grounding line to the calving front. The calving face is plotted as a vertical line at the location of the calving front. The ice surface elevation is plotted from the divide to the calving front and the bed elevation is plotted for the entire model domain. If the glacier is land-terminating, only the surface and bed elevation profiles are plotted. Finally, the axes limits, tick marks, labels and plot position are set. After the last simulation year, the glacier elevation profile will be plotted with green lines but for all other years, the lines will be blue.

Annual speed profiles are plotted in Figure 2 (fig2). The speed profile is plotted from the ice divide to the calving front with units of m d^{-1} rather than m s^{-1} to simplify comparison with observational data. The axes limits, tick marks, labels and plot position are set. The current profile year is also indicated in a white box within the figure. After the last simulation year, the speed profile will be plotted with green lines but for all other years, the lines will be blue.

A small dummy plot (fig3) is also created every year in order to plot a zoomed elevation profile inset in Figure 1 using the inset.m script. The inset.m script plots the profile contained in fig3 in a small inset window within Figure 1 so that the current elevation profile is clearly shown. After the zoomed profile is plotted in Figure 1, the dummy figure is deleted and the current simulation year is added to the elevation plot.

Time series of the grounding line position and speed are plotted in the third figure window, Figure 3 (fig4). The grounding line speed changes gradually with time but the position of the grounding line may appear to move erratically. The abrupt changes in the grounding line position are the result of the small iterative adjustments in the ice speed and thickness between model loops. The sizes of the abrupt changes in the grounding line position are independent of the size of the grid spacing used in the simulation.

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