A Unified Model of Particle Physics and Cosmology: Origin of Inflation, Dark Energy, Dark Matter, Baryon Asymmetry and Neutrino Mass

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Abstract: I propose a unified model of particle physic and cosmology based on both a new extension of the standard particle model and the fundamental principle of the standard cosmology. It can completely and coherently describe the universe evolution from the primordial inflation to the followed reheating, to the baryogenesis, to the early hot expansion, to the later CDM condensation into the current dark energy, namely, it can simultaneously account for the common origin of inflation, reheating, baryon asymmetry, dark matter, dark energy, and neutrino mass, in particular, it establishes the internal relations between these processes and particle physics. For the evolution of each phase, I give its complete dynamical system of equations and solve them by some special techniques, the numerical results clearly show how each process is successfully implemented, moreover, illustrate that the dark energy genesis is essentially a reverse process of the slow-roll inflation. By use of fewer input parameters, the unified model not only perfectly reproduces the measured inflationary data and the current energy density budget, but also finely predicts many important quantities such as the tensor-to-scalar ratio $r_{0.05} \approx 1.86 \times 10^{-7}$, the inflaton mass $M_\Phi \approx 8.88 \times 10^{10}$ GeV, the reheating temperature $T_{re} \approx 2.2 \times 10^{11}$ GeV, the CDM mass $M_S \approx 256$ GeV, $\eta_B \approx 6.14 \times 10^{-10}$ and $h \approx 0.73$, and also it clarifies and eliminates the "Hubble tension". Finally, we expect the ongoing and future experiments to test the model.

Keywords: beyond standard model; inflation; dark energy; dark matter; baryogenesis; neutrino mass

I. Introduction

The standard model of particle physics (SM) and the ΛCDM model of cosmology (the cosmologic constant energy + the cold dark matter) together have successfully accounted for a great deal of the cosmic observations from the BBN era to the present day [1], but they can not address the origin of the hot big bang of the universe [2], namely what happened before the standard hot expansion, and also can not answer the origins of the current dark energy [3], cold dark matter (CDM) [4], and baryon asymmetry [5], in addition, the generation of the sub-eV neutrino mass is yet a puzzle [6]. At present the theoretical and experimental investigations have clearly indicated that the very early universe certainly underwent the inflation phase and the followed reheating one [7]. These two processes not only provide the initial conditions of the hot expansion, but also are related to the universe matter genesis [8], nevertheless the relations between them and particle physics are unknown, so their evolution dynamics have been unestablished as yet. To solve all of the above problems, we have to seek an underlying theory beyond the SM and ΛCDM, therefore this becomes the most challenging research for particle physics and cosmology. This aspect is currently attracting more and more attentions of theoretical and experimental physicists [9].

In fact, there have been numerous theories about the explanations of the inflation, dark energy, dark matter, baryon asymmetry and neutrino mass, which include some unified particle models [10], some paradigms of the inflation and reheating [11], some special dark energy models [12], even some models based on the non-standard gravity [13], many CDM candidates [14], the modified Newtonian dynamics [15], many mechanisms of leptogenesis and baryogenesis [16], and many models of neutrino mass [17]. However, a wide variety of these proposals have a common shortcoming, namely they are only aiming at one or two specific aspects of the above-mentioned universe phenomena rather than considering internal connections among them, in other words, these phenomena are dealt in isolation without regard to their integration in the universe evolution, this is obviously unnatural and inadvisable because the uniqueness of the universe origin and evolution destines that there are surely some internal relations among these universe ingredients. Today, the vast majority of these models have been ruled out by the recent data and analyses [18].

At the present day, by means of the analyses for the power spectra of the anisotropic and polarized temperature of the cosmic microwave background (CMB) [19], we have obtained the following inflationary data, the tensor-to-scalar ratio, the scalar spectral index, the running of the spectral index, and the scalar power spectra. On the other hand, from the global analyses of cosmology which includes CMB, BBN, structure formation, gravitational lenses, particle physic experiments, etc. [20], we have extracted the following universe data, the dark energy density, the CDM density, the ratio of the baryon number density to the photon one, and the neutrino mass sum. The present optimum values of these cosmological data are given as follows [1],

$$\begin{aligned}
&r_{0.05} < 0.036\,, \quad n_s \approx 0.965\,, \quad \frac{dn_s}{dlnk} \approx -0.004\,, \quad ln(10^{10}\Delta_R^2) \approx 3.04\,,\\
&\Omega_{DE} \approx 1 - \frac{0.143}{h^2}\,, \quad \Omega_{CDM} \approx \frac{0.12}{h^2}\,, \quad \eta_B \approx 6.14\times 10^{-10}, \quad \sum_i m_{\nu_i} \sim 0.1\,\mathrm{eV},
\end{aligned} \tag{1}$$

where $h \approx 0.73$ or $h \approx 0.674$ is the current scaling factor for Hubble expansion rate, namely $H_0 = 100h\ \mathrm{kms^{-1}Mpc^{-1}}$, the former value is directly measured by the distance ladder approach at the low red-shift, whereas the latter value is derived from the Planck CMB data (which are created at the high redshift) by assuming the ΛCDM model, this inconsistency is called as the "Hubble tension" [21]. However, these data undoubtedly contain the key information of the universe origin and evolution, any one successful theory of particle physics and cosmology has to confront them unavoidably, therefore Eq. (1) severely constrains new model builds [22].

Based on the universe concordance and the nature unification, I attempt to build a unified model of particle physics and cosmology, it can naturally relate the above-mentioned universe ingredients together and really establish connections among them in the universe evolution, of course, this is also fitting to Occam's Razor. Firstly, I put forward to a new extension of the SM, which covers the SM particles and the dark particles beyond the SM. Secondly, on the basis of the new particle model as well as the fundamental principle of the standard cosmology, I in detail research the dynamical evolutions of the inflation, the reheating and the current era, in particular, I introduce some new ideas and techniques to solve all of the above-mentioned issues elegantly and completely. The idea framework of the unified model will be described in the next Section and shown by Fig. 3. Lastly, the model numerical results will clearly show the evolution of each phase, they not only perfectly fit all of the observed data in Eq. (1), especially eliminate the "Hubble tension", but also give many interesting predictions. In a word, this model can successfully account for the origin and evolution of the universe in a unified and integrated way.

The remainder of this paper is organized as follows. In Section II, I outline the new extension of the SM, and then discuss the neutrino mass, leptogenesis mechanism and dark matter annihilation. I give a complete solution of the slow-roll inflation in Section III. I discuss the reheating evolution and the baryogenesis in Section IV. I discusses the current CDM condensation and dark energy genesis in Section V, includes eliminating the "Hubble tension". Section VI is a summary of the numerical results of the unified model. Section VII is devoted to conclusions.

II. Particle Model

The unified theory is based on the following particle model. I assume that below the GUT scale of $\sim 10^{16}$ GeV, the particle contents and symmetries in the universe are showed by Table 1 (where the irrelevant quarks and gauge bosons of the SM are all omitted), all kinds of the notations are explained by the caption. The SM particles are all in the visible sector, while the particles beyond SM (BSM) are all inhabiting in the dark sector. The dark symmetries include two discrete symmetries of $Z_2 \otimes Z_2'$. N^0 is a super-heavy Dirac fermion without any charge, it is purely neutral so that it becomes a mediator between the SM sector and the dark one. The heavy E^- is a dark charged lepton. The doublet scalar Φ is the inflation field whose mass is $\sim 10^{11}$ GeV, its dynamical evolution leads to the inflation and hot big bang of the primordial universe. The neutral S and ϕ are two real scalars, the former has -1 parity under Z_2', while the latter has -1 parity under Z_2. The electroweak breaking is implemented by $\langle H\rangle \approx 174$ GeV as usual, while the dark Z_2 is spontaneously broken by $\langle\phi\rangle \sim 1$ TeV, but Z_2' is always unbroken due to $\langle S\rangle = 0$. After the model symmetry breakings, the E^- mass is generated by $\langle\phi\rangle$, while ν_L^0 and ν_R^0 are combined into the Dirac neutrino with a sub-eV mass. Finally, the stable S whose mass is about 250 GeV will gradually cool into the current CDM after its annihilation is decoupled, in the more later stage the CDM becomes super-cool so that it eventually condenses into the current dark energy. Note that the global $B-L$ number is always conserved in the model, in addition, the model symmetry and the fermion arrangement guarantee that the model is as free-anomaly as the SM. In a word, this extension of the SM has fully and perfectly accommodated all of the ingredients required by the universe evolution.

	SM (visible sector)				BSM (dark sector)			
Fields	H	l_α	$e^-_{\beta R}$	$N^0_L\,, N^0_R$	$E^-_L\,, E^-_R$	$\nu^0_{\beta R}$	Φ	$S\,,\,\phi$
$SU_L(2)\otimes U_Y(1)$	$(2,1)$	$(2,-1)$	$(1,-2)$	$(1,0)$	$(1,-2)$	$(1,0)$	$(2,1)$	$(1,0)$
Dark Z_2 parity	1			1	$1\,,\,-1$	-1	-1	$1\,,\,-1$
Dark Z_2' parity	1			1	$-1\,,\,-1$	1	1	$-1\,,\,1$

Table 1: The particle contents and symmetries of the unified model. The notation explanations are as follows, $H=(H^+,H^0)^T$, $l_\alpha=(\nu^0_{\alpha L},e^-_{\alpha L})^T$, $(\alpha,\beta=1,2,3)$ are the fermion family indices. N^0 is purely a neutral Dirac-fermion with a super-heavy mass, E^- is a dark charged lepton with a TeV-scale mass, $\nu^0_{\alpha L}$ and $\nu^0_{\beta R}$ will be combined into the Sub-eV Dirac neutrino. $\Phi=(\Phi^+,\Phi^0)^T$ is the super-heavy inflation field, the neutral S and ϕ are two real scalars. The third row is the quantum numbers under $SU_L(2)\otimes U_Y(1)$, the last two rows are respectively the dark parities under Z_2 and Z_2'. $SU_L(2)\otimes U_Y(1)$ and Z_2 are respectively broken by $\langle H\rangle$ and $\langle\phi\rangle$, but Z_2' is unbroken due to $\langle S\rangle=0$, so the stable S whose mass is about 250 GeV will become the cold dark matter, and eventually condense into the dark energy. Note that the global $B-L$ number is always conserved in the model.

Based on the particle contents and symmetries in Table 1, the full invariant Lagrangian of the model are

$$
\begin{aligned}
\mathscr{L} =\ & \overline{E_L}i\gamma^\mu D_\mu E_L+\overline{E_R}i\gamma^\mu D_\mu E_R+\overline{\nu_R}i\gamma^\mu\partial_\mu\nu_R+(D^\mu\Phi)^\dagger D_\mu\Phi+\frac{1}{2}\partial^\mu S\partial_\mu S+\frac{1}{2}\partial^\mu\phi\partial_\mu\phi\\
&+[\,Y^e_{\alpha\beta}\overline{l_\alpha}\,e_{\beta R}H+Y^\nu_{\alpha\beta}\overline{l_\alpha}\,\nu_{\beta R}(i\tau_2\Phi^*)+y^l_\alpha\overline{l_\alpha}\,N_R(i\tau_2H^*)-\overline{N_L}M_N N_R+y^\nu_\beta\overline{N_L}\,\nu_{\beta R}\phi\\
&\quad+y^E\overline{E_L}\,E_R\phi+y^e_\beta\overline{E_L}\,e_{\beta R}S+h.c.]\\
&-V_H-V_\Phi-V_S-V_\phi+\mu_0[\,\phi\,\Phi^\dagger H+h.c.]\\
&-|H|^2(\lambda_1\phi^2+\lambda_2S^2+2\lambda_3|\Phi|^2)-\phi^2(\frac{\lambda_4}{2}S^2+\lambda_5|\Phi|^2)-\lambda_6S^2|\Phi|^2,\\
& V_H=-\mu_H^2|H|^2+\lambda_H|H|^4,\qquad V_\phi=-\frac{\mu_\phi^2}{2}\phi^2+\frac{\lambda_\phi}{4}\phi^4,\\
& V_\Phi=\mu_\Phi^2|\Phi|^2+\lambda_\Phi|\Phi|^4+\cdots\,,\qquad V_S=\frac{\mu_S^2}{2}S^2+\frac{\lambda_S}{4}S^4+\cdots\,,
\end{aligned}
\tag{2}
$$

where D_μ is the gauge covariant derivative, τ_2 is the second Pauli matrix, and the irrelevant parts of the SM Lagrangian are all omitted. Note that any Majorana-type mass or couplings are all prohibited by the global $B-L$ number conservation. The N^0 fermion has an inherent mass of $M_N\sim10^9$ GeV. $[Y^e_{\alpha\beta},Y^\nu_{\alpha\beta},y^l_\alpha,y^\nu_\beta,\cdots]$ are all Yukawa coupling parameters, the repeated family indices are summed by default. We can individually rotate the flavor spaces of l,e_R,ν_R so as to make real diagonal Y^e (which is namely the mass eigenstate basis of the charged lepton) and real y^ν, then the irremovable complex phases in Y^ν,y^l,y^e will become CP-violating sources in the lepton sector. V_H and V_ϕ have usual self-interacting potentials, but V_Φ and V_S have unusual potential forms, later we will in detail give the inflationary potential in Eq. (29), here I only write the quadratic and quartic terms of their series expansions since the Φ and S masses are only related to these terms, the higher order terms with the dimension being $\geqslant 6$ are all suppressed by the power of the squared Planck mass. Note that the special potential form of V_Φ will lead that Φ has a distinctive dynamic evolution, similar, V_S will lead to the distinctive dynamic evolution of S. The triple scalar couplings with $\mu_0 \sim 10^9$ GeV is very important, it is a key knot linking all kinds of the following vacua. Since both M_N and μ_0 are $\sim 10^9$ GeV, then this indicates that they may all result from a special symmetry breaking at this super-high scale. Finally, I assume $[\lambda_1, \lambda_2, \lambda_3, \cdots] \ll 1$, namely these couplings between two different scalars are all very weak and negligible. In conclusion, Eq. (2) completely describes all kinds of the interactions among the model particles from the primordial inflation to the present universe.

The model symmetries are spontaneously broken by the following vacuum structures of the scalar fields,

$$\begin{aligned}
&\phi \rightarrow \phi^0 + v_\phi\,, \quad H \rightarrow \begin{bmatrix} 0 \\ \frac{h^0+v_H}{\sqrt{2}} \end{bmatrix}, \quad \Phi \rightarrow \begin{bmatrix} \Phi^+ \\ \Phi^0 + \frac{v_\Phi}{\sqrt{2}} \end{bmatrix}, \quad S \rightarrow S, \\
&v_\phi \sim 1\ \text{TeV}, \quad v_H \approx 246\ \text{GeV}, \quad v_\Phi \sim 10\ \text{eV}, \quad \langle S \rangle = 0\,,
\end{aligned} \tag{3}$$

where these vacuum expectation values are hierarchical and they indicate the sequence of the symmetry breakings. After the vacuum breakings, ϕ^0 and h^0 become two massive real scalars with neutral charges, Φ still keeps its original structure due to $v_\Phi \ll M_\Phi$, and S is always unbroken due to $\langle S \rangle = 0$. From the total potential minimum in Eq. (2), we can derive that the above three vacuum expectation values are completely determined by the following system of equations,

$$\begin{aligned}
&-\mu_\phi^2 + \lambda_\phi v_\phi^2 + \lambda_1 v_H^2 + \lambda_5 v_\Phi^2 = \frac{\mu_0 v_\Phi v_H}{v_\phi}\,, \quad -\mu_H^2 + \lambda_H v_H^2 + \lambda_1 v_\phi^2 + \lambda_3 v_\Phi^2 = \frac{\mu_0 v_\phi v_\Phi}{v_H}\,, \\
&\mu_\Phi^2 + \lambda_\Phi v_\Phi^2 + \lambda_3 v_H^2 + \lambda_5 v_\phi^2 = \frac{\mu_0 v_\phi v_H}{v_\Phi}\,,
\end{aligned} \tag{4}$$

where all kinds of the parameters are chosen such as $[\lambda_\phi, \lambda_H, \lambda_\Phi] \sim 0.1$, $[\lambda_1, \lambda_2, \lambda_3, \cdots] \ll 1$, $\mu_\phi \sim v_\phi$, $\mu_H \sim v_H$, $\mu_\Phi \sim 10^{11}$ GeV and $\mu_0 \sim 10^9$ GeV, thus Eq. (4) can guarantee the vacuum stability.

The symmetry breakings directly give rise to the mass terms of the scalar bosons, after their mass matrices are diagonalized, the mass eigenvalues are given as follows,

$$\begin{aligned}
&M_{\phi^0}^2 \approx 2\lambda_\phi v_\phi^2\,, \quad M_{h^0}^2 \approx 2\lambda_H v_H^2\,, \\
&M_\Phi^2 \approx \frac{\mu_0 v_\phi v_H}{v_\Phi}\,, \quad M_S^2 = \mu_S^2 + \lambda_2 v_H^2 + \lambda_4 v_\phi^2 + \lambda_6 v_\Phi^2\,,
\end{aligned} \tag{5}$$

$M_{h^0} \approx 125$ GeV is exactly the measured mass of the SM Higgs boson, ϕ^0 is the dark neutral boson, the mixing angle between ϕ^0 and h^0 is $\sim \frac{\lambda_1 v_H}{\lambda_\phi v_\phi} + \frac{\mu_0 v_\Phi}{M_\phi^2} \ll 1$. Eq. (5) indicates $M_{\phi^0} \sim 500$ GeV, $M_\Phi \approx \mu_\Phi \sim 10^{11}$ GeV, and $M_S \approx \mu_S \sim 250$ GeV is a suitable parameter. We will actually obtain $M_\Phi \approx 8.88 \times 10^{10}$ GeV from the inflation solution in Section III. Later one will see that these mass values have important implications for phenomena of particle physics and cosmology.

The E^- mass and the e_α^- ones directly result from $\langle \phi \rangle$ and $\langle H \rangle$, respectively, note that there is no mixing between E^- and e_α^- since S is unbroken (namely $\langle S \rangle = 0$). However, the ν_α masses are generated by the following effective couplings at the low energy, which is derived from Eq.

(2) by integrating out the super-heavy Φ and N, namely there are

$$\begin{aligned}
&\mathscr{L}^{eff}_{neutrino}=\bar{l}_{\alpha}\left[Y^{\nu}_{\alpha\beta}\frac{\mu_0\phi}{M^2_{\Phi}}+y^l_{\alpha}y^{\nu}_{\beta}\frac{\phi}{M_N}\right]\nu_{\beta R}(i\tau_2H^*)\,,\\
\Longrightarrow\, &M_{\nu}=M^a_{\nu}+M^b_{\nu}=-Y^{\nu}_{\alpha\beta}\frac{v_{\Phi}}{\sqrt{2}}-y^l_{\alpha}y^{\nu}_{\beta}\frac{v_{\phi}v_H}{\sqrt{2}M_N}\;\Longrightarrow\sum_i m_{\nu_i}=\mathrm{Tr}[U^{\nu}_L M_{\nu}U^{\nu\dagger}_R]\,,\\
&M_e=-Y^e_{\alpha\beta}\frac{v_H}{\sqrt{2}}\,,\qquad M_E=-y^E v_{\phi}\,,
\end{aligned}\tag{6}$$

where M_{ν} is diagonalized by these two unitary matrices of U^{ν}_L and U^{ν}_R which respectively rotate $\nu_{\alpha L}$ and $\nu_{\beta R}$. Obviously, this mechanism of generating neutrino mass is a Dirac-type seesaw, which is different from the usual Majorana-type seesaw [23]. All kinds of the Yukawa parameters are chosen such as $y^E\sim 0.5$, $Y^e\sim 10^{-2}$, $Y^{\nu}\sim 10^{-3}$, $y^l y^{\nu}\sim 10^{-7}$, and $M_N\sim 10^9$ GeV, then we naturally obtain $M_E\sim 0.5$ TeV, $M_e\sim 1$ GeV, $M^a_{\nu}\sim M^b_{\nu}\sim 10^{-2}$ eV. Note that M^a_{ν} has three eigenvalues, but M^b_{ν} has only one eigenvalue. Provided $M^a_{\nu}\sim 0.01$ eV and $M^b_{\nu}\sim 0.05$ eV, then this may lead to such mass spectrum as $m_{\nu_1}\approx 0.005<m_{\nu_2}\approx 0.01<m_{\nu_3}\approx 0.05$ (eV as unit), thereby we naturally explain that $\triangle m^2_{32}\approx 2.4\times 10^{-3}$ eV^2 is much larger than $\triangle m^2_{21}\approx 7.5\times 10^{-5}$ eV^2, which is exactly required by the experimental data of the neutrino oscillation. Under the flavor basis of real diagonal Y^e and real y^{ν}, then U^{ν}_L is identified as the lepton mixing matrix U_{PMNS}, thus the complex phases in Y^{ν} and y^l are transferred into the CP-violating phase in U_{PMNS}. We can further fit the neutrino mixing angles by choosing a suitable texture of M_{ν}, but here we do not go into it. Based on both the neutrino oscillation experiments and the astrophysics investigations [1], I will take suitable $\sum m_{\nu}\approx 0.065$ eV as an input parameter of the unified model, see the following Table 2.

The dark sector of this model has very important phenomena and implications for cosmology. Firstly, the Φ field slow-roll causes the primordial inflation, see Section III. Secondly, after the inflation the Φ decay brings about the universe reheating and the hot big bang, see Section IV, at the same time, it also leads to the matter-antimatter asymmetry by the following leptogenesis mechanism. In the light of Eq. (2), the Φ decay modes have $\Phi\rightarrow l^c+\nu_R$ and $\Phi\rightarrow H+\phi$, furthermore, $\Phi\rightarrow l^c+\nu_R$ and $\Phi^*\rightarrow l+\nu^c_R$ have CP asymmetric decay widths through the interference between the tree diagram amplitude and the one-loop diagram one, as shown in Fig. 1, the relevant decay width and CP asymmetry are calculated as follows,

$$\begin{aligned}
&\Gamma(\Phi\rightarrow l^c+\nu_R)=\frac{M_{\Phi}}{16\pi}\mathrm{Tr}[Y^{\nu\dagger}Y^{\nu}]\ll\Gamma(\Phi\rightarrow H+\phi)=\frac{M_{\Phi}}{16\pi}(\frac{\mu_0}{M_{\Phi}})^2,\\
&A_{CP}=\frac{\Gamma(\Phi\rightarrow l^c+\nu_R)-\Gamma(\Phi^*\rightarrow l+\nu^c_R)}{\Gamma_{\Phi}}\\
&\quad\approx[\frac{1}{2\pi}\frac{M_N}{\mu_0}ln\frac{M_N}{M_{\Phi}}]\,\mathrm{Im}[\sum_{\alpha,\beta}Y^{\nu\dagger}_{\beta\alpha}y^l_{\alpha}y^{\nu}_{\beta}]=[\frac{1}{\pi}\frac{M^2_N}{\mu^2_0}ln\frac{M_N}{M_{\Phi}}]\frac{M^2_{\Phi}\mathrm{Im}\mathrm{Tr}[M^{a\dagger}_{\nu}M^b_{\nu}]}{(v_{\phi}v_H)^2}\,,
\end{aligned}\tag{7}$$

where $Y^{\nu}\sim 10^{-3}<\frac{\mu_0}{M_{\Phi}}\sim 10^{-2}$, so the total width Γ_{Φ} is approximately equal to $\Gamma(\Phi\rightarrow H+\phi)$. In Eq. (7), the CP-violating sources are purely from the irremovable complex phases in Y^{ν} and/or y^l, accordingly they are contained in M^a_{ν} and M^b_{ν}, therefore the CP violation in the leptogenesis is closely related to the CP violation in the neutrino experiments [24]. Since $\frac{M_N}{\mu_0}\sim 1$ and $\frac{M_N}{M_{\Phi}}\sim 10^{-2}$, the factor terms of the two square brackets are all ~ 1. Provided $Y^{\nu}\sim 10^{-3}$ and $y^l y^{\nu}\sim 10^{-7}$ as before, then we can naturally obtain $A_{CP}\sim 10^{-10}$, in other terms, since

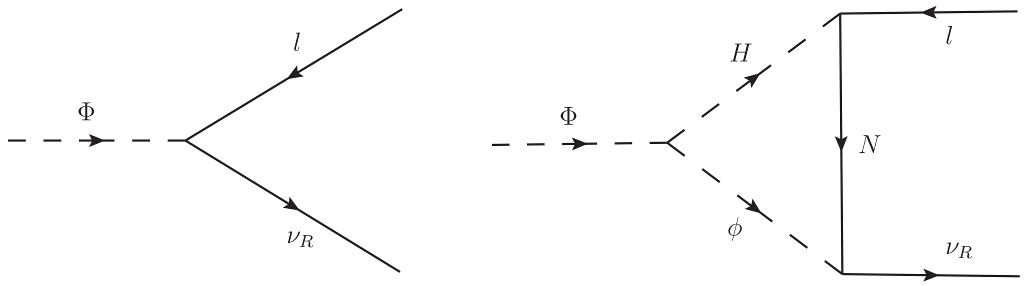

Figure 1: The tree and one-loop diagrams of $\Phi \to l^c + \nu_R$. The CP asymmetry of this decay can equally generates the asymmetric anti-lepton and the asymmetric ν_R although the net lepton number is conserved as zero, the latter is forever frozen in the dark sector, whereas the former is partly converted into the baryon asymmetry through the SM sphaleron transition.

there are $M_\Phi \sim 10^{11}$ GeV, $M_\nu \sim 10^{-11}$ GeV and $v_\phi v_H \sim 10^5$ GeV^2, then we certainly obtain $A_{CP} \sim 10^{-10}$. This value is very vital for the baryon asymmetry, see the following Eq. (37). In addition, by a simple calculation one can prove that $\Gamma(\Phi \to l^c + \nu_R)$ is smaller than the universe expansion rate at the temperature of $T = M_\Phi \approx 8.88 \times 10^{10}$ GeV, so the decay in Fig. 1 is really an out-of-equilibrium process. Note that the dilute process of $l^c + \nu_R \to \phi + H$ via the t-channel N mediation is invalid at any temperature because $\frac{|y^l y^\nu|^2}{M_N} \ll \frac{1}{M_{Pl}}$ (M_{Pl} is the Planck mass) guarantees that its reaction rate is always severely out-of-equilibrium. As a result, the CP asymmetry in Eq. (7) can equally generate the asymmetric anti-lepton and the asymmetric ν_R although the net lepton number is conserved as zero, the latter is forever frozen in the dark sector, whereas the former will be partly converted into the baryon asymmetry through the SM sphaleron transition [25], see Section IV. Finally, I have to stress that the amount of the matter-antimatter asymmetry is closely related to these fundamental quantities of M_Φ, M_ν, v_ϕ and v_H by Eq. (7), this is rightly a characteristic of the unified model.

In the hot evolution of the dark sector, all kinds of the dark particles are thermally produced by the interactions in Eq. (2), nevertheless, these heavy dark particles of N^0, E^-, ϕ^0 are early depleted by their decays. Because S is unbroken (namely $\langle S \rangle = 0$), the dark Z_2' is always conserved, in addition, provided $M_S < M_E$, this thus guarantees that S is a stable dark particle without any decay, so it will become the CDM. A pair of S can however annihilate into $S + S \to e_{\alpha R} + e^c_{\beta R}$ via the dark E^- mediation, as shown Fig. 2. Obviously, this process can be searched as the direct and indirect detections for the CDM. As the universe temperature cools, the annihilation process is terminated and the S particle are decoupled from the $e_{\alpha R}$ leptons, thus the residual S gradually cool into the current CDM, furthermore, the supercool CDM can eventually condensate into the current dark energy. The thermally averaged annihilation cross-

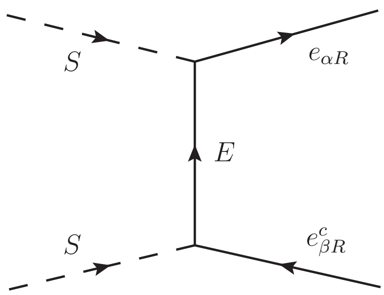

Figure 2: The annihilation of $S + S \to e_{\alpha R} + e^c_{\beta R}$ via the dark E^- mediation, which leads that the residual S will become the current CDM and eventually condensate into the dark energy. This process can be searched as the direct and indirect detections for the CDM.

section and the freeze-out temperature are simply calculated by the following relations,

$$\begin{aligned}
&\langle \sigma v_r \rangle_{T_f} = c_1 + c_2 \langle v_r^2 \rangle_{T_f} + c_3 \langle v_r^4 \rangle_{T_f} + \cdots \approx c_1 + c_2 \frac{6T_f}{M_S}\,, \quad c_1 = 0\,, \quad c_2 = \frac{M_S^2}{48\pi M_E^4} \sum_{\alpha,\beta} |y^e_\alpha y^e_\beta|^2\,, \\
&\langle \sigma v_r \rangle_{T_f} n_S(T_f) = H(T_f) = \frac{1.66\sqrt{g_*(T_f)}\,T_f^2}{M_{Pl}}\,, \quad n_S(T_f) = T_f^3 [\frac{M_S}{2\pi T_f}]^{\frac{3}{2}} e^{-\frac{M_S}{T_f}}\,, \\
&\Longrightarrow \frac{M_S}{T_f} \approx 20 + \frac{1}{2} ln \frac{M_S}{g_*(T_f) T_f} + ln \frac{M_S \langle \sigma v_r \rangle_{T_f}}{10^{-9}\,\mathrm{GeV}^{-1}}\,, \qquad (8)
\end{aligned}$$

where $v_r = 2\sqrt{1 - \frac{4M_S^2}{s}}$ is the relative velocity (where s is the squared center-of-mass energy), M_{Pl} is the Planck mass, $g_*(T_f) = 91.5$ is the effective number of relativistic degrees of freedom at $T_f \approx 10.1$ GeV. Provided $\sum\limits_{\alpha,\beta} |y^e_\alpha y^e_\beta|^2 \approx 1$, $M_E \approx 0.5$ TeV and $M_S \approx 256$ GeV, then we can solve out $\langle \sigma v_r \rangle_{T_f} \approx 1.64 \times 10^{-9}$ GeV^{-2} and $\frac{M_S}{T_f} \approx 25.4$, so there is $T_f \approx 10.1$ GeV. These values are very vital for the current density budget of the CDM and dark energy, which will be discussed in Section V.

On the basis of the above-mentioned particle model, we can describe the idea framework of the universe origin and evolution by the sketch shown as Fig. 3. In sequence, the universe went through the primordial inflation, the followed reheating, the early hot expansion, the transformation from the radiation-dominated to the matter-dominated, the supercool CDM condensation into the dark energy and the present DE-dominated universe. The primordial inflation is implemented by the Φ field slowly rolling. Φ has the two physical states or energy forms of Φ_{DE} and Φ_{DM} due to its special nature. Φ_{DE} is an inert condensed state with a negative pressure, it has no kinetic energy and can not take part in couplings to the other fields, whereas Φ_{DM} is an excited massive particle state with a vanishing pressure, it has kinetic energy and can interact with the other particles, see Eq. (10). Inappropriately, the relationship between Φ_{DE} and Φ_{DM} is analogous to ice and vapour, which are merely the two different physical states of the same material. The same physical meanings also apply to the S field, namely it has also the two physical states or energy forms of S_{DE} and S_{DM}, see Eq. (39). In brief, I give the above explanations about the unknown physical nature of the dark energy and the dark matter. In the following Sections, we will see that the physical essence of the slow-roll inflation is that the superheavy dark matter Φ_{DM} is slowly growing from the primordial dark

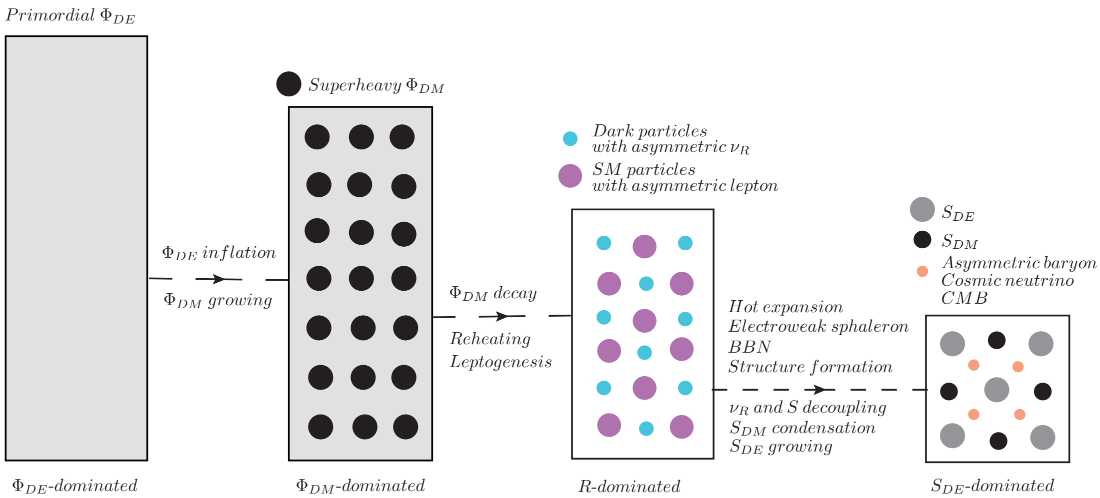

Figure 3: The sketch of the universe origin and evolution described by the unified model. The universe energy is step by step released and reduced from the primordial dark energy Φ_{DE} to the current S_{DE}-dominated energy budget, the whole evolution process is analogous to a cascade of hydropower stations, there is not the so-called "cosmological constant problem" in the model.

energy Φ_{DE}, namely a process of Φ_{DE} gradually converting into Φ_{DM}. After the inflation is terminated, the Φ_{DM} decay is responsible for the reheating and the leptogenesis. When the hot bath is formed and the radiation begins to dominate the universe, the asymmetric lepton and the asymmetric right-handed neutrino have equally been generated, but they are isolated in the visible sector and the dark sector, respectively. In the hot expansion stage, the asymmetric right-handed neutrino in the dark sector is forever frozen out, whereas the asymmetric lepton in the visible sector can be partly converted into the baryon asymmetry through the SM sphaleron transition. The followed evolution in the visible sector is well-known. In the dark sector, the stable S is decoupled from ν_R below the T_f temperature, as the universe temperature declining, S will gradually cool into the CDM denoted by S_{DM}. In the very later stage, the temperature is more and more approaching to absolute zero, the kinetic energy of S_{DM} is completely exhausted, thus the supercool S_{DM} will eventually condense into S_{DE} which is namely the current dark energy, this condensation is in essence that S_{DE} is slowly growing from S_{DM} or S_{DM} gradually converting into S_{DE}, therefore, the current condensation is essentially a reverse process of the primordial inflation. Although there is a great difference about 106 orders of magnitude between the primordial dark energy denoted by Φ_{DE} and the current dark energy denoted by S_{DE}, the universe energy is step by step released and reduced through a cascade of the above-mentioned evolutions, this is analogous to a cascade of hydropower stations at the Changjiang River, by which a huge drop of water potential is converted into electrical energy, therefore there is not naturally the so-called "cosmological constant problem" in the model. Finally, I emphasize that all of these assumptions of the unified model are moderate, reasonable and consistent, by which we can successfully and completely account for the universe origin and evolution.

III. Primordial Inflation

The dynamic evolution of the primordial inflation is described by what follows. According to the standard paradigm [26], the inflation field Φ is considered as spatially uniform distribution, but there are very small fluctuations, which will become sources of the structure formation. Under the flat FLRW metric, namely $g_{\mu\nu} = \mathrm{Diag}(1, -a^2, -a^2, -a^2)$ where $a(t)$ is the scale factor of the universe expansion, the energy density and pressure of Φ are given by its energy-momentum tensor as follows,

$$\begin{aligned}
&\mathscr{L}_\Phi = g^{\mu\nu}\partial_\mu\Phi^\dagger\partial_\nu\Phi - V_\Phi, \quad T^\mu_\nu(\Phi) = 2g^{\mu\beta}\partial_\beta\Phi^\dagger\partial_\nu\Phi - \delta^\mu_\nu\mathscr{L}_\Phi, \\
&\Longrightarrow T^0_{\ 0} = \rho_\Phi = |\dot{\Phi}|^2 + V_\Phi, \quad -\frac{1}{3}\delta^i_j T^j_{\ i} = P_\Phi = |\dot{\Phi}|^2 - V_\Phi,
\end{aligned} \tag{9}$$

where $\mathscr{L}_\Phi$ is the Lagrangian of pure Φ and $V_\Phi = V(\Phi^\dagger\Phi) = V(|\Phi|^2)$ is its self-interacting potential energy, $\dot{\Phi} = \frac{d\Phi}{dt}$ and $|\dot{\Phi}|^2 = \dot{\Phi}^\dagger\dot{\Phi} = \dot{\Phi}^+\dot{\Phi}^- + \dot{\Phi}^{0*}\dot{\Phi}^0$ is the kinetic energy of Φ. Obviously, the potential energy and the kinetic energy together determine ρ_Φ and P_Φ, and vice versa. ρ_Φ and P_Φ are however two super-high values in the inflation period.

I now introduce the dark energy Φ_{DE} and the dark matter Φ_{DM}, they are merely two energy forms or physical states of the same Φ field, each of them has own density and pressure, which are determined by the following relations,

$$\begin{aligned}
&P_{\Phi_{DE}} = -\rho_{\Phi_{DE}}, \quad P_{\Phi_{DM}} = 0\,, \quad \rho_{\Phi_{DE}} + \rho_{\Phi_{DM}} = \rho_\Phi, \quad P_{\Phi_{DE}} + P_{\Phi_{DM}} = P_\Phi = w_\Phi\rho_\Phi, \\
&\Longrightarrow \rho_{\Phi_{DE}} = -w_\Phi\rho_\Phi = \frac{-2w_\Phi}{1-w_\Phi}V_\Phi, \quad \rho_{\Phi_{DM}} = (1+w_\Phi)\rho_\Phi = |\dot{\Phi}|^2 + \frac{1+w_\Phi}{1-w_\Phi}V_\Phi = 2|\dot{\Phi}|^2,
\end{aligned} \tag{10}$$

where I employ Eq. (9). w_Φ is a parameter-of-state varying with the time, which relates the total pressure to the total energy density, there is generally $-1 \leqslant w_\Phi \leqslant 0$, Φ is purely Φ_{DE} when $w_\Phi = -1$, while Φ entirely becomes Φ_{DM} when $w_\Phi = 0$. Φ_{DE} is an inert condensed state with a negative pressure, it has only potential energy without kinetic energy, so $\rho_{\Phi_{DE}}$ contributes a part of the total V_Φ, in contrast, Φ_{DM} is an excited massive particle state with a vanishing pressure, so it carries both kinetic energy and potential energy (which is the rest of the total V_Φ), and both are always equal to each other. In short, Eq. (10) explicitly shows the inherent relations among all kinds of the energy forms of the Φ field, the physical implications of Φ_{DE} and Φ_{DM} will be further clear in the following context.

At the beginning of the inflation, the Φ field is purely in the Φ_{DE} form (or state), then Φ_{DM} is slowly growing from Φ_{DE}, Φ_{DM} is more and more generated and Φ_{DE} is more and more depleted, thus Φ_{DE} gradually converts into Φ_{DM}, this process is namely so-called slow-roll inflation. The dynamics of the inflationary evolution are collectively determined by the Friedmann equation, the Φ continuity equation and the Φ_{DM} growth equation, which are respectively

$$\begin{aligned}
&\rho_\Phi = \rho_{\Phi_{DE}} + \rho_{\Phi_{DM}} = 3\tilde{M}_p^2 H^2, \\
&\dot{\rho}_\Phi + 3H\rho_\Phi(1+w_\Phi) = 0 \Longrightarrow -\dot{\rho}_{\Phi_{DE}} = \dot{\rho}_{\Phi_{DM}} + 3H\rho_{\Phi_{DM}}, \\
&\dot{\rho}_{\Phi_{DM}} = -2\eta(t)H\rho_{\Phi_{DM}},
\end{aligned} \tag{11}$$

where I employ Eq. (10). $\tilde{M}_p = \frac{1}{\sqrt{8\pi G}} \approx 2.43\times10^{18}$ GeV is the reduced Planck mass, $H(t) = \frac{\dot{a}(t)}{a(t)}$ is the universe expansion rate, the proportional parameter $-\eta(t) > 0$ controls the Φ_{DM} growth rate, in fact η is namely one of the slow-roll parameters defined below. Once the evolution of $\eta(t)$ is specified, Eq. (11) is then a closed system of equations, from which we can solve all the evolutions of $\rho_{\Phi_{DM}}$, $\rho_{\Phi_{DE}}$, ρ_Φ and H. The above continuity equation indicates that the $\rho_{\Phi_{DM}}$

growth in the comoving volume is entirely from the $\rho_{\Phi_{DE}}$ reduction, therefore the primordial inflation is rightly the process of Φ_{DM} growing from Φ_{DE}. The Φ field will entirely become the pure Φ_{DM} form (or state) at the end of the inflation.

From Eqs. (10) and (11), we can easily derive

$$\eta(t) = -\frac{dln\rho_{\Phi_{DM}}}{2Hdt} = -\frac{dln|\dot{\Phi}|}{Hdt}, \quad \epsilon(t) = -\frac{dln\rho_{\Phi}}{2Hdt} = -\frac{\dot{H}}{H^2} = \frac{3(1+w_{\Phi})}{2}, \tag{12}$$

$$-1 = w_{\Phi}(0) \leqslant w_{\Phi}(t) \leqslant w_{\Phi}(t_{inf}) = 0, \quad 0 = \epsilon(0) \leqslant \epsilon(t) \leqslant \epsilon(t_{inf}) = \frac{3}{2}, \tag{13}$$

where η and ϵ are two slow-roll parameters defined as usual, they respectively characterize the $\rho_{\Phi_{DM}}$ varying rate and the ρ_{Φ} one. Note that η and ϵ themselves also vary with the inflationary time, or else the inflation will continue on without termination. Eq. (13) gives the inflationary boundary condition, hereinafter we take $t = 0$ as the time of inflation begin and use the "inf" subscript to indicate the time of inflation finish. In addition, we can obtain the expansion acceleration equation,

$$\frac{\ddot{a}}{a} = (1-\epsilon)H^2 = -\frac{1+3w_{\Phi}}{2}H^2. \tag{14}$$

Eq. (14) shows that the accelerating or decelerating expansion only depends on the value of ϵ or w_{Φ}, there are $\ddot{a}(t) \geqslant 0$ when $0 \leqslant \epsilon \leqslant 1$ and $\ddot{a}(t) < 0$ when $1 < \epsilon \leqslant \frac{3}{2}$, the former is in the Φ_{DE}-dominated universe, whereas the latter is in the Φ_{DM}-dominated universe. $\dot{H} \leqslant 0$ also indicates that the expansion rate and the total energy density are always decreased in the inflation period.

Put Eq. (10) and Eq. (12) together, we can use ρ_{Φ} and ϵ to express all kinds of the energy forms as follows,

$$\rho_{\Phi_{DE}} = (1-\frac{2\epsilon}{3})\rho_{\Phi}, \quad \rho_{\Phi_{DM}} = \frac{2\epsilon}{3}\rho_{\Phi}, \quad |\dot{\Phi}|^2 = \frac{\epsilon}{3}\rho_{\Phi}, \quad V_{\Phi} = (1-\frac{\epsilon}{3})\rho_{\Phi}. \tag{15}$$

Eqs. (11), (12) and (15) together make up the fundamental equations of the inflationary evolutions, while Eq. (13) is the boundary condition. If we can provide the evolution of any one of the nine inflationary quantities, H, ρ_{Φ}, $\rho_{\Phi_{DE}}$, $\rho_{\Phi_{DM}}$, $|\dot{\Phi}|^2$, V_{Φ}, ϵ, η, w_{Φ}, in principle, then the solutions of the other inflationary quantities will completely be determined by this system of equations. In what follows, we will find the solution for the inflation problem by a special technique.

One of the inflationary features is that the universe size or the scale factor expands about 10^{25} times in an extremely short duration, therefore we use the e-fold number to characterize the inflationary time span instead of the scale factor, it is defined as follows,

$$N(t) = ln\frac{a(t_{inf})}{a(t)} = \int_t^{t_{inf}} H(t')dt' \Longrightarrow \dot{N}(t) = -H(t), \tag{16}$$
$$0 = a(0) \leqslant a(t) \leqslant a(t_{inf}), \quad +\infty = N(0) \geqslant N(t) \geqslant N(t_{inf}) = 0,$$

where the starting point of the inflation is set as $a(0) = 0$ and $N(0) = +\infty$. Eq. (16) now acts as the role of $\frac{\dot{a}}{a} = H$ since $N(t)$ replaces $a(t)$ as the time scale, it will frequently be used in the following formula derivations.

By use of Eqs. (11), (12), (15) and (16), we can order by order give the slow-roll parameters by the total energy density ρ_Φ and its derivative as follows,

$$
\begin{aligned}
&\rho'_\Phi = \frac{d\rho_\Phi}{dN} = 3\rho_{\Phi_{DM}}, \quad \rho''_\Phi = \frac{d^2\rho_\Phi}{dN^2} = 3\rho'_{\Phi_{DM}}, \quad \rho'''_\Phi = \frac{d^3\rho_\Phi}{dN^3} = 3\rho''_{\Phi_{DM}}, \ \ldots \\
&\epsilon = \frac{dln\rho_\Phi}{2dN}, \quad \eta = \frac{dln\rho'_\Phi}{2dN} = \frac{dln\rho_{\Phi_{DM}}}{2dN}, \quad \theta = \frac{dln(-\rho''_\Phi)}{2dN} = \frac{dln(-\rho'_{\Phi_{DM}})}{2dN}, \quad \delta = \frac{dln|\rho'''_\Phi|}{2dN}, \ \ldots
\end{aligned} \tag{17}
$$

$$
\Longrightarrow \frac{dln\epsilon}{2dN} = \eta - \epsilon\,, \quad \frac{dln(-\eta)}{2dN} = \theta - \eta\,, \quad \frac{dln|\theta|}{2dN} = \delta - \theta\,, \ \ldots \tag{18}
$$

where hereinafter the "$'$" superscript denotes a derivative with regard to N. These slow-roll parameters in Eq. (17) are closely related to the observable quantities of the inflation, ϵ and η are relevant to the tensor-to-scalar ratio and the scalar spectral index, θ is involved in the running of the spectral index, see the following Eq. (27), therefore finding the correct solutions of these slow-roll parameters is a key of solving the inflation problem.

From the previous fundamental equations and Eq. (16), we can also derive the equation of motion of Φ and its formal solution as follows,

$$
\ddot{\Phi} + 3H\dot{\Phi} + \Phi\frac{dV_\Phi}{d|\Phi|^2} = 0 \Longrightarrow \Phi'' - 3\Phi' + (3-\epsilon)\Phi\tilde{M}_p^2\frac{dln|V_\Phi|}{d|\Phi|^2} = 0\,, \tag{19}
$$

$$
\begin{aligned}
&\Longrightarrow \Phi_1(N) = \Phi_2(N) = \Phi_3(N) = \Phi_4(N) \Longrightarrow \frac{d|\Phi|}{dN} = \sqrt{2}\,|\Phi'_i| = |\Phi'| = \tilde{M}_p\sqrt{\epsilon}\,, \\
&d\varphi = \sqrt{\sum_i (d\Phi_i)^2} = \sqrt{2d\Phi^\dagger d\Phi} \Longrightarrow \frac{d\varphi}{dN} = \sqrt{2}\,|\Phi'|\,, \\
&\Longrightarrow \frac{\varphi(N)}{\sqrt{2}\tilde{M}_p} = \frac{|\Phi(N)| - |\Phi(0)|}{\tilde{M}_p} = \int_0^N \sqrt{\epsilon(N')}dN',
\end{aligned} \tag{20}
$$

where $\Phi' = \frac{d\Phi}{dN}$, $|\Phi|^2 = \frac{1}{2}\sum\limits_i \Phi_i^2$, Φ_i (i=1,2,3,4) are four real degree of freedoms of Φ, obviously the solutions of Φ_i are degenerate since $V(|\Phi|^2)$ is fully symmetric with regard to them. In Eq. (20), I introduce an auxiliary field $\varphi(N)$ in order to get rid of the multiple-component difficulty, and we can freely fix $\varphi(0) = 0$. We can immediately calculate the φ and $|\Phi|$ evolution once $\epsilon(N)$ is provided. $\varphi' > 0$ (namely $\dot{\varphi} < 0$) indicates that φ and $|\Phi|$ are gradually reduced with the time.

By convention, if the inflationary potential V_Φ is provided, then the conventional slow-roll parameters are given by V_Φ as follows,

$$
\begin{aligned}
\epsilon_V &= \frac{\tilde{M}_p^2}{2}[\frac{dV_\Phi}{V_\Phi d\varphi}]^2 = \epsilon[\frac{3-\eta}{3-\epsilon}]^2 \xrightarrow{\epsilon,\,\eta \ll 1} \epsilon\,, \\
\eta_V &= \tilde{M}_p^2[\frac{d^2V_\Phi}{V_\Phi d\varphi^2}] = \frac{(\epsilon+\eta)(3-\eta)-\eta'}{3-\epsilon} \xrightarrow{\epsilon,\,\eta,\,\theta \ll 1} \epsilon + \eta - \frac{\eta'}{3}\,, \\
\xi_V^2 &= \tilde{M}_p^4[\frac{dV_\Phi}{V_\Phi d\varphi}][\frac{d^3V_\Phi}{V_\Phi d\varphi^3}] = [\frac{3-\eta}{3-\epsilon}]^2[4\epsilon\eta + \frac{\eta'(3-3\epsilon+2\eta-2\theta)-2\eta\theta'}{3-\eta}] \\
&\qquad \xrightarrow{\epsilon,\,\eta,\,\theta,\,\delta \ll 1} 4\epsilon\eta + \eta' - \frac{2\eta\theta'}{3}\,,
\end{aligned} \tag{21}
$$

where $\eta' = \frac{d\eta}{dN}$ and $\theta' = \frac{d\theta}{dN}$, and I employ the foregoing equations to derive the relations in Eq. (21), which relate these two sets of slow-roll parameters to each other. However, it should be stressed that the above approximations are held only when $\epsilon, \eta, \theta, \delta \ll 1$, this case is only in the early and middle phases of the inflation, when the inflation is close to its end, some slow-roll parameters actually become ~ 1, thus these approximations are invalid.

When the inflationary potential is characterized by $V_\Phi(\varphi)$ whose argument is φ, we can make Taylor expansion of $V_\Phi(\varphi)$ around $\varphi = 0$ and obtain the following results,

$$V_\Phi(\varphi) = V_\Phi(\varphi)|_{\varphi=0} + \frac{dV_\Phi(\varphi)}{d\varphi}|_{\varphi=0}\,\varphi + \frac{d^2V_\Phi(\varphi)}{d\varphi^2}|_{\varphi=0}\frac{\varphi^2}{2} + \cdots \Longrightarrow$$

$$M_\Phi^2 = \frac{d^2V_\Phi(\varphi)}{d\varphi^2}|_{\varphi=0} = [(3-\epsilon)\eta_V H^2]_{N=0} \Longrightarrow M_\Phi = [\sqrt{(3-\epsilon)\eta_V}\,H\,]_{t_{inf}}, \tag{22}$$

$$V_{\Phi min} = M_\Phi^2\,|\Phi(t_{inf})|^2 = V_\Phi(t_{inf}) = [(1-\frac{\epsilon}{3})\rho_\Phi]_{t_{inf}} \Longrightarrow \frac{|\Phi(t_{inf})|}{\tilde{M}_p} = \frac{1}{\sqrt{\eta_V(t_{inf})}}, \tag{23}$$

where I employ Eqs. (21) and (15). M_Φ is about the same size as H_{inf}, but M_Φ can be identified as the mass meanings of Φ only when η_V becomes positive, in the following Fig. 4, we will see how M_Φ is gradually generated from nothing as η_V evolving from negative to positive, namely M_Φ is generated by the special mechanism of the inflation evolution. Later we will work out $\eta_V(t_{inf}) \approx 2.61$, then there is $|\Phi(t_{inf})|/\tilde{M}_p \approx 0.62$, this is a very reasonable value.

A traditional and usual technique of solving the inflation problem is as the following procedure. Firstly, one has to design or guess a function form of $V(|\Phi|^2)$ or $V(\varphi)$. Secondly, one puts V_Φ into Eq. (19), and ignores the ϵ factor since there is $\epsilon \ll 1$ in the most of the inflation duration, then one can solve the Φ differential equation to find a solution of $\Phi(N)$. Thirdly, one can obtain $\epsilon(N)$ by making a derivative of $\Phi(N)$, and further one can calculate $\eta(N)$ and $\theta(N)$ by Eq. (18). Lastly, one can obtain ρ_Φ, $\rho_{\Phi_{DE}}$, $\rho_{\Phi_{DM}}$ and $|\dot{\Phi}|^2$ by Eq. (15) since $V_\Phi(N)$ and $\epsilon(N)$ are given, up to this point, the inflationary evolutions are completely solved out. Nevertheless, this procedure has two serious shortcomings. i) In the later phase of the inflation ϵ is actually ~ 1 rather than $\ll 1$, one neglecting ϵ in Eq. (19) will therefore lead to a non-rigorous and incomplete inflationary solution, in particular, this has great effect on the inflation termination and the followed reheating. ii) it is very difficult to fit precisely all of the inflationary data in Eq. (1) by this technique, in fact, a desirable inflationary potential is not be found as yet although the countless endeavours have been made. Therefore, to solve reliably and completely the inflation problem in the standard gravity framework, we have to find a new approach.

In the system of equations of the inflationary evolution, all kinds of the unknown inflationary quantities have an equal status at least in mathematical sense, thereby we can flexibly choose the η parameter as the starting point instead of the V_Φ potential. In principle, we can employ the following procedure. Firstly, we can design or guess an evolution function of $\eta(N)$ as the following Eq. (24), this amounts to specifying directly the law of the Φ_{DM} growth in Eq. (11). Secondly, Eq. (11) has now become a closed the system of equations, thus we can solve them to obtain $\rho_{\Phi_{DM}}$, $\rho_{\Phi_{DE}}$, ρ_Φ and H. Lastly, we can figure out $|\dot{\Phi}|^2$, V_Φ, ϵ and w_Φ by Eqs. (15) and (12), up to now, all of the inflationary evolutions are completely solved out. By means of this procedure, the inflationary potential is reversely worked out rather than provided. Obviously, this technique is both simple and reliable, and it can overcome the shortcomings of the traditional technique. Whatever technical means is employed, the only criterion is that it is able to fit all of the inflationary data correctly and completely.

After careful analysis and calculation, I find a suitable function form of $\eta(N)$ as follows,

$$\eta(N) = \eta(0) e^{-\alpha(\frac{N}{N_*})^2} = -\frac{e^{\alpha(1-\frac{N^2}{N_*^2})}}{N_* + 6}\,, \tag{24}$$
$$\Longrightarrow \eta' = -\eta\,\frac{2\alpha N}{N_*^2}\,, \qquad \theta = \eta - \frac{\alpha N}{N_*^2}\,,$$

where there are two independent parameters N_* and α, and $\eta(0)$ is parameterized by them. In fact, N_* is exactly corresponding to the inflationary e-fold number when the pivot scale of $k_* = 0.05$ Mpc^{-1} exits from the horizon, the model will calculate out $N_* \approx 51.1$ by the following Eq. (28). α is one of two input parameters in the inflation sector, the other one is H_{inf} in Eq. (27), we can determine $\alpha \approx 2.92$ by fitting the inflationary data, all kinds of the input parameters of the unified model are later summarized in Table 2 in VI Section. From Eq. (24), we can directly obtain η' and θ (employ Eq. (18)), obviously, there are $\eta(0) = \theta(0) = \frac{-e^{\alpha}}{N_*+6}$ and $\eta'(0) = 0$ at the end of the inflation.

Starting from Eq. (24), we can easily solve the inflationary evolutions as follows. Firstly, by use of the definition formula of η in Eq. (17) we can solve out $\rho_{\Phi_{DM}}(N)$ by integrating $\eta(N)$. Secondly, we can further integrate $\rho_{\Phi_{DM}}(N)$ to obtain $\rho_{\Phi}(N)$ by the first formula in Eq. (17). Thirdly, $\epsilon(N)$ is obtained by the second equality in Eq. (15). The derived results are,

$$\frac{\rho_{\Phi_{DM}}(N)}{\rho_{\Phi}(0)} = e^{2\int_0^N \eta(N')dN'}, \quad \frac{\rho_{\Phi}(N)}{\rho_{\Phi}(0)} = 1 + 3\int_0^N \frac{\rho_{\Phi_{DM}}(N')}{\rho_{\Phi}(0)} dN', \quad \epsilon(N) = \frac{3}{2}\,\frac{\rho_{\Phi_{DM}}(N)}{\rho_{\Phi}(N)}\,, \tag{25}$$

where there is $\rho_{\Phi_{DM}}(0) = \rho_{\Phi}(0) = 3\tilde{M}_p^2 H_{inf}^2$ due to $\epsilon(0) = \frac{3}{2}$ at the end of the inflation. Lastly, by use of the relevant relations we can also calculate $|\dot{\Phi}|^2$, V_{Φ}, η_V, w_{Φ}, etc. Note that all kinds of the energy forms are normalized to $\rho_{\Phi}(0)$.

Now we show the numerical results of this inflation model. Fig. 4 shows the inflationary evolutions of the relevant energy forms of the Φ field with N as time scale. In the early and middle phases of the inflation process, these three curves of $\rho_{\Phi}, \rho_{\Phi_{DE}}, V_{\Phi}$ almost coincide with each other, moreover, they nearly keep a constant value, the reason for this is that the growths of Φ_{DM} and $T_{\Phi} = |\dot{\Phi}|^2$ are very slow at this stages, thus there is $\dot{\Phi} \approx 0$, this is namely so-called slow-roll inflation. In the last phase of the inflation proceeding, the $\rho_{\Phi_{DM}}$ and T_{Φ} growths are more and more fast, thus ρ_{Φ}, $\rho_{\Phi_{DE}}$ and V_{Φ} turn into fall sharply and their curves are significantly separated each other. Eventually, $\rho_{\Phi_{DM}}$ exceeds $\rho_{\Phi_{DE}}$, the Φ_{DE}-dominated universe is transformed into the Φ_{DM}-dominated one, at the same time, the accelerating expansion is also changed into the decelerating one, thus the inflation is naturally over. At the time of the inflation finish, namely when $N = 0$, there are $\rho_{\Phi_{DE}} = 0$ and $\rho_{\Phi_{DM}} = \rho_{\Phi} = 2T_{\Phi} = 2V_{\Phi}$. Note that the green curve indicates $\rho_{\Phi}(\infty)/\rho_{\Phi}(0) \approx 5.65$, this means that the ρ_{Φ} amount only varies about 5.65 times from the inflation beginning to its end. In short, Fig. 4 clearly shows the full evolution of all kinds of the energy forms in the inflation process, which contain the slow-roll features, the transformations among these energy forms, and the inflation terminations.

Fig. 5 shows the inflationary evolutions of the slow-roll parameters and the parameter-of-state with N as time scale. One can see three remarkable features. i) In the most of the inflation duration, there are $\epsilon \ll 1$ and $w_{\Phi} \approx -1$, while the other curves are slowly varying. Only when $N(t) \to N(t_{inf}) = 0$, these three parameters of ϵ, η_V and w_{Φ} sharply rise, this thus leads to the inflation termination. ii) In the early and middle phases of the inflation, η is coinciding with η_V

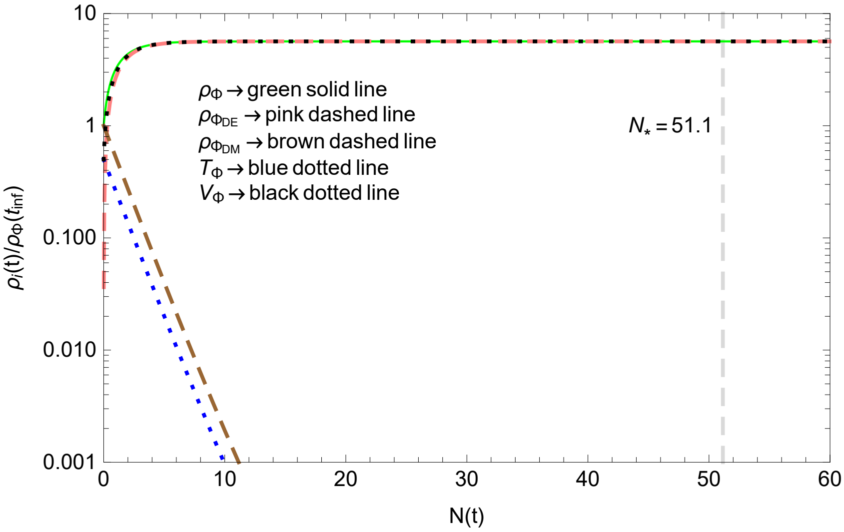

Figure 4: The inflationary evolutions of the relevant energy forms of the Φ field with the e-fold number as time scale, $N_* \approx 51.1$ is corresponding to the time of $k_* = 0.05$ Mpc^{-1} exiting from horizon.

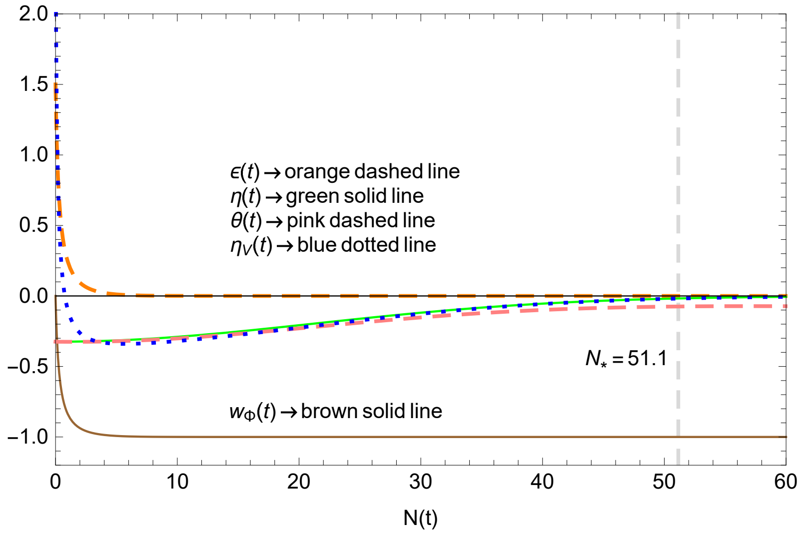

Figure 5: The inflationary evolutions of the slow-roll parameters and the parameter-of-state with the e-fold number as time scale, $N_* \approx 51.1$ is corresponding to the time of $k_* = 0.05$ Mpc^{-1} exiting from horizon.

due to $\eta \approx \eta_V$, whereas in the last phase of the inflation, η is coinciding with θ due to $\eta \approx \theta$. iii) ϵ is always positive, while η and θ are always negative, but η_V can however change from negative to positive when $N \to 0$. In view of Eq. (22), M_Φ^2 is proportional to η_V, this means that M_Φ is gradually generated from nothing as η_V evolving, certainly, this is closely related to Φ_{DM} growing, therefore M_Φ purely results from the inflationary dynamical evolution. This mass generation mechanism of the inflationary field is very different from that of the SM particles, which simply arises from the vacuum spontaneous breaking. In short, these numerical results of Fig. 4 and Fig. 5 can excellently explain the physical pictures of the primordial inflation.

Now we set about addressing the observable data of the inflation. In Fig. 5, at $N_* \approx 51.1$ the slow-roll parameters and the parameter-of-state are evaluated as follows,

$$\begin{aligned}
&\epsilon_{V*} \approx \epsilon_* \approx 1.17 \times 10^{-8}, \quad \eta_* = \frac{-1}{N_* + 6} \approx -0.0175, \quad \theta_* = \eta_* - \frac{\alpha}{N_*} \approx -0.0746, \\
&\eta_{V*} \approx \epsilon_* + \eta_*(1 + \frac{2\alpha}{3N_*}) \approx -0.0183, \qquad w_{\Phi *} = \frac{2\epsilon_*}{3} - 1 \approx -1,
\end{aligned} \tag{26}$$

hereinafter the "$*$" subscript specially indicates the time of $N_* \approx 51.1$. From the cosmological perturbation theory of the structure formation [27], we know that the above slow-roll parameters are directly related to the following inflationary observable quantities,

$$\begin{aligned}
\Delta_R^2(k_*) &= \frac{H^2}{8\pi^2 \tilde{M}_p^2\, \epsilon}|_{k_*} = \frac{1}{8\pi^2 \epsilon_*}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}][\frac{H_{inf}}{\tilde{M}_p}]^2, \qquad r_* = 16\epsilon_*\,, \\
n_s(k_*) - 1 &= \frac{dln\Delta_R^2}{dlnk}|_{k_*} = \frac{dlnH^2 - dln\epsilon}{(\epsilon - 1)dN}|_{k_*} = -4\epsilon_* + 2\eta_* \approx 6(\frac{\eta'_*}{9} - \epsilon_{V*}) + 2\eta_{V*}\,, \\
\frac{dn_s}{dlnk}|_{k_*} &= \frac{dn_s}{(\epsilon - 1)dN}|_{k_*} = 4\epsilon'_* - 2\eta'_* = 8\epsilon_*(\eta_* - \epsilon_*) - 4\eta_*(\theta_* - \eta_*) \\
&\approx 24\epsilon_{V*}(\frac{2\eta'_*}{9} - \epsilon_{V*}) + 16\epsilon_{V*}\eta_{V*} - 2\xi_{V*}^2\,,
\end{aligned} \tag{27}$$

where I employ Eq. (21). $k_* = 0.05$ Mpc^{-1} is the pivot scale exciting from horizon when $N_* \approx 51.1$, the relation between k_* and N_* will be given by Eq. (28). H_{inf} is the expansion rate at the time of the inflation finish, it is the second input parameter in the inflation sector, we can determine $H_{inf} \approx 4.49 \times 10^{10}$ GeV by fitting the inflationary data. $\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)} \approx 5.65$ has been calculated out by Eq. (25), also see Fig. 4. Note that the $\frac{\eta'_*}{9}$ factor in Eq. (27) is the same order of magnitude as ϵ_{V*}, so their contributions are actually negligible. We can employ either of the two sets of slow-roll parameters to calculate the inflationary data, put Eq. (26) into Eq. (27), we can perfectly reproduce all of the measured inflation data in Eq. (1), moreover, $M_\Phi \approx 8.88 \times 10^{10}$ GeV is given by Eq. (22), the detailed results are all summarized in Table 2 in VI Section.

k_* is defined and calculated as follows,

$$\begin{aligned}
k_* &= \frac{a_* H_*}{c} = \frac{H_0}{c}\frac{H_{inf}}{H_0}\frac{H_*}{H_{inf}}\frac{a_*}{a_{inf}}\frac{a_{inf}}{a_{req}}\frac{a_{req}}{a_{ref}}\frac{a_{ref}}{a_0} \\
&= [\frac{H_0}{c\,h}][\frac{g_*(T_0)}{2}]^{\frac{1}{3}}[\Omega_\gamma(T_0)h^2]^{\frac{1}{3}}[\frac{H_{inf}}{H_0/h}]^{\frac{1}{3}}[\frac{T_{re}}{T_0}]^{\frac{1}{3}}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}]^{\frac{1}{2}}e^{-N_*},
\end{aligned} \tag{28}$$

where c is the speed of light, $\frac{H_0}{ch} = \frac{100}{3\times 10^5}$ Mpc^{-1} and $H_0/h \approx 2.13 \times 10^{-42}$ GeV are two fixed constants, a detailed derivation of Eq. (28) is seen in Appendix I. At the present day, the CMB

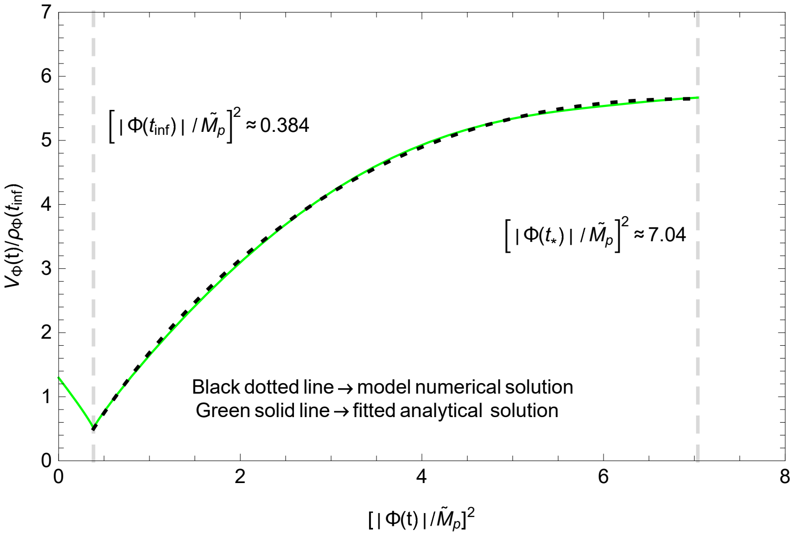

Figure 6: The inflationary potential evolution with $[\frac{|\Phi(t)|}{\tilde{M}_p}]^2$ as variable, t_{inf} and t_* are respectively corresponding to $N = 0$ and $N_* \approx 51.1$, the black dotted curve is the numerical solution of the model, while the green solid curve is the analytical solution of Eq. (29), the latter is perfectly fitting the former.

temperature is $T_0 \approx 2.7255$ K $\approx 2.35 \times 10^{-4}$ eV [1], $g_*(T_0) \approx 4.1$ is the effective number of relativistic degrees of freedom (which includes the ν_R contribution, see the following Eq. (38)), the photon energy density parameter $\Omega_\gamma(T_0)h^2 \approx 2.47 \times 10^{-5}$ will be calculated out by Eq. (49) in Section V, and the reheating temperature $T_{re} \approx 2.2 \times 10^{11}$ GeV will be work out by Eq. (36) in Section IV. Once the above quantities are input into Eq. (28), we can immediately solve out $N_* \approx 51.1$ corresponding to $k_* = 0.05$ Mpc^{-1}. It should be emphasized that Eq. (28) relates these fundamental quantities of the inflation, reheating and current universe together, this is rightly another characteristic of the unified model.

Finally, we can find out an analytical function form of the inflationary potential $V(|\Phi|^2)$ through fitting its numerical solution. Since the $\epsilon(N)$ solution has been given in Fig. 5, substitute it into Eq. (20) and make a numerical integration, then we can obtain a numerical solution of $|\Phi(N)|/\tilde{M}_p$, on the other hand, the $V_\Phi(N)$ solution has been given in Fig. 4, put these two solutions together, thus we can translate $V_\Phi(N)$ with N as variable into $V(|\Phi|^2/\tilde{M}_p^2)$ with $[|\Phi|/\tilde{M}_p]^2$ as variable, this is easily achieved by a computer, the calculated result is shown by the black dotted curve in Fig. 6, where $[|\Phi(t_{inf})|/\tilde{M}_p]^2 \approx 0.384$ and $[|\Phi(t_*)|/\tilde{M}_p]^2 \approx 7.04$ are two field values corresponding to $N = 0$ and $N_* \approx 51.1$, respectively. Apparently, the evolution of $V(|\Phi|^2)$ in Fig. 6 is more smooth and steady in comparison with the evolution of $V_\Phi(N)$ in Fig. 4, moreover, in the inflation duration $[|\Phi|/\tilde{M}_p]^2$ varying amount is much smaller than N varying one. Note that when $N \geqslant N_*$ (namely $t \leqslant t_*$), there is actually $\dot{\Phi}/\tilde{M}_p \approx 0$, namely $\Phi/\tilde{M}_p$ is approximating to the limit value of $[|\Phi|/\tilde{M}_p]^2 \to 7.04$, accordingly there is the limit value of $V_\Phi/\rho_\Phi(t_{inf}) \to 5.65$, namely the potential is approximating to the constant value. The reason for this is of course that the universe is entirely filled with the primordial dark energy in the very early phase of the inflation.

After making a great effort, I eventually find such function form of $V(|\Phi|^2)$ as

$$\frac{V(|\Phi|^2)}{\rho_\Phi(t_{inf})} = 5.75\left[1 - e^{-0.078(x-0.384)^2}\right]^{0.45} + e^{\frac{4.5}{x-7.04}}, \tag{29}$$
$$x = [\frac{|\Phi(t)|}{\tilde{M}_p}]^2, \quad [\frac{|\Phi(t_{inf})|}{\tilde{M}_p}]^2 = \frac{1}{\eta_V(t_{inf})} \approx 0.384, \quad [\frac{|\Phi(t_*)|}{\tilde{M}_p}]^2 \approx 7.04,$$

where $\rho_\Phi(t_{inf}) = 3\tilde{M}_p^2 H_{inf}^2$, the adjustable parameters include 5.75, 0.078, 0.45 and 4.5. The potential form of Eq. (29) obviously satisfies the model requirement in Eq. (2), note that it is very different from the Starobinsky-type inflationary potential [28]. In Fig. 6, we use the green solid curve to show the analytical solution of Eq. (29), it can perfectly fit the model numerical solution shown by the black dotted curve. When $x \to 0.384$, the first term of Eq. (29) is $\to 0$ and its second term is $\to \frac{1}{2}$, this is corresponding to the potential valley bottom, which is namely where the inflation ends. When $x \to 7.04$, its first term is $\to 5.65$ and its second term is $\to 0$, this is corresponding to the potential plateau platform, which is namely the early phase of the inflation. In particular note that the second term of Eq. (29) constrains there must be $x \to 7.04$ with $x < 7.04$, once $x \to 7.04$ with $x > 7.04$ the potential will become an infinity, this is certainly unacceptable, therefore Eq. (29) naturally explains the limited values of both the inflationary field and the inflationary potential. If we now start from Eq. (29) to deal with the inflation problem by use of the usual procedure, in principle, we can also derive all kinds of the foregoing results which are obtained by my ansatz, but this potential form of Eq. (29) can not at all be guessed in advance. In conclusion, Fig. 6 clearly shows the inflationary slow-roll evolution and Eq. (29) explicitly gives us deep insights into the inflationary potential. Up to now, All of the inflation problems have completely been solved under the unified model.

IV. Reheating and Baryogenesis

At the end of the inflation, Φ_{DE} is vanishing and the Φ field entirely becomes Φ_{DM}, thus the Φ_{DM}-dominated universe comes into the decelerating expansion era. Since Φ_{DM} is an excited particle state with kinetic energy, then it can interact with the other particles via those couplings in Eq. (2), by virtue of its superheavy mass, Φ_{DM} is quite unstable and can shortly decay into one SM particle and one dark particle, the phenomena of the Φ_{DM} decay has been discussed in Section II. On the other hand, the Φ_{DM} decay has very important cosmological implications, in fact, it not only directly produces the hot bath of the universe, namely the reheating universe, but also simultaneously generates the matter-antimatter asymmetry by the foregoing leptogenesis mechanism. In what follows, we will discuss the reheating evolution and the baryon asymmetry genesis.

From now on, we take off the "DM" subscript of Φ_{DM} since Φ has been pure Φ_{DM}. The Φ decay and its subsequent processes directly produce the earliest radiation of the universe, which are hot plasma consisting of the SM and dark particles. The total energy of the universe now includes the two components of ρ_Φ and ρ_R. The dynamic of the reheating evolution are collectively controlled by Friedmann equation and the continuity equations, namely

$$\rho_\Phi + \rho_R = 3\tilde{M}_p^2 H^2, \quad \dot{\rho}_\Phi + 3H\rho_\Phi = -\Gamma_\Phi \rho_\Phi, \quad \dot{\rho}_R + 4H\rho_R = \Gamma_\Phi \rho_\Phi, \tag{30}$$

where Γ_Φ is the Φ decay width given in Eq. (7). Eq. (30) physical implications are very clear, it is a closed system of equations, the evolutions of ρ_Φ and ρ_R are completely determined by the

Γ_Φ value and the initial conditions. For the reheating calculation, I take suitable $\frac{\mu_0}{M_\Phi} \approx 0.01$ in Eq. (7) as an input parameter from the particle model, which is the only parameter in the reheating sector, see Table 2.

In order to solve the system of equations of Eq. (30), I define the dimensionless energy densities and time variable as follows,

$$\tilde{\rho}_i(\tilde{t}) = \frac{\rho_i}{3\tilde{M}_p^2\Gamma_\Phi^2}\,, \quad \tilde{t} = (t - t_{inf})\Gamma_\Phi = \frac{t - t_{inf}}{\tau_\Phi}\,, \quad 0 < \tilde{t} \leqslant \tilde{t}_{ref} = \frac{t_{ref} - t_{inf}}{\tau_\Phi}, \tag{31}$$

$$\Longrightarrow \tilde{\rho}_\Phi(0) = (\frac{H_{inf}}{\Gamma_\Phi})^2, \quad \tilde{\rho}_R(0) = 0\,, \tag{32}$$

where $i = (\Phi, R)$ and τ_Φ is the Φ lifetime. The time of the reheating beginning is namely the time of the inflation finish, while the time of the reheating finish is specially indicated by the "ref" subscript. Eq. (32) is namely the initial values of the reheating evolution. By use of the numerical values in Table 2, then the model gives $\frac{H_{inf}}{\Gamma_\Phi} \approx 2.54 \times 10^5$, so the reheating is indeed a severely out-of-equilibrium process in its early phase. By use of Eq. (31), we can recast Eq. (30) as follows,

$$\tilde{\rho}_\Phi + \tilde{\rho}_R = (\frac{H}{\Gamma_\Phi})^2, \quad \frac{d\tilde{\rho}_\Phi}{d\tilde{t}} + [3(\frac{H}{\Gamma_\Phi}) + 1]\tilde{\rho}_\Phi = 0\,, \quad \frac{d\tilde{\rho}_R}{d\tilde{t}} + 4(\frac{H}{\Gamma_\Phi})\tilde{\rho}_R - \tilde{\rho}_\Phi = 0\,. \tag{33}$$

Now the evolutions of $\tilde{\rho}_\Phi(\tilde{t})$ and $\tilde{\rho}_R(\tilde{t})$ only depend on their initial values in Eq. (32), and also we can easily obtain their numerical solutions.

In fact, we are much more interested in the energy density parameters and the total parameter-of-state, which are defined by

$$\Omega_i(\tilde{t}) = \frac{\rho_i}{\rho_\Phi + \rho_R}\,, \quad w_T(\tilde{t}) = \frac{P_\Phi + P_R}{\rho_\Phi + \rho_R} = \frac{\Omega_R}{3}\,, \tag{34}$$

where $P_\Phi = 0$ and $P_R = \frac{\rho_R}{3}$. Fig. 7 numerically shows the reheating evolutions of Ω_Φ and Ω_R with $\tilde{t}$ as time scale. As the Φ decay proceeding, the Φ density parameter is continuously decreasing from the initial $\Omega_\Phi = 1$ to the final $\Omega_\Phi \approx 0$, so Φ is eventually exhausted, meanwhile, Ω_R is gradually increasing from the initial $\Omega_R = 0$ to the final $\Omega_R \approx 1$, thus the universe is entirely filled by radiation. The $w_T(\tilde{t})$ evolution is also in agreement with this, which is gradually increasing from the initial $w_T = 0$ to the final $w_T = \frac{1}{3}$, as a result, the initial Φ-dominated universe is transformed into the finial R-dominated one. In the reheating period and after it, the universe is obviously decelerating expansion and the expansion rate is continuously declining.

In Fig. 7, the time of the $\Phi - R$ equality, $\tilde{t}_{req} \approx 1.054$, is a key time point in the reheating process, the universe energy is Φ-dominated in $0 < \tilde{t} \leqslant \tilde{t}_{req}$ (namely $t_{inf} < t \leqslant t_{req}$), once $\tilde{t} > \tilde{t}_{req}$ ($t > t_{req}$) the universe enters into the radiation-dominated era, at $\tilde{t} \approx 10$ the reheating process is essentially over, after that the universe will start a evolution of the hot expansion driven by the radiation. In addition, $\tilde{t}_{req} \approx 1.054$ means $t_{req} - t_{inf} \approx \tau_\Phi$, namely the Φ decay mostly happens around t_{req} , thus $\rho_R(t_{req})$ essentially arrives the highest radiation energy density, accordingly the radiation temperature $T_{re}(t_{req})$ is the highest temperature in the reheating process. To sum

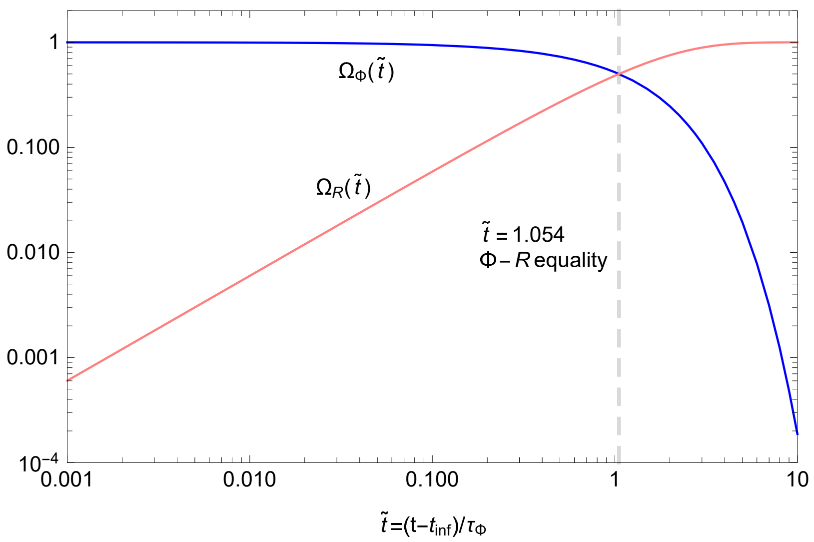

Figure 7: The reheating evolutions of the Φ and radiation energy density parameters with the dimensionless $\tilde{t}$ as time scale, $\tilde{t} \approx 1.054$ is the time of $\rho_\Phi = \rho_R$ and $\tilde{t} \approx 10$ is the time of the reheating finish.

up, $\tilde{t}_{req}$ and T_{re} are determined by the following relations,

$$\Omega_\Phi(\tilde{t}_{req}) = \Omega_R(\tilde{t}_{req}) = \frac{1}{2} \Longrightarrow \tilde{t}_{req} = \frac{t_{req} - t_{inf}}{\tau_\Phi} \approx 1.054\,, \tag{35}$$

$$H(t_{req}) = \Gamma_\Phi \sqrt{2\tilde{\rho}_R(\tilde{t}_{req})}\,, \quad T_{re} = [\tilde{M}_p \Gamma_\Phi]^{\frac{1}{2}} \left[\frac{90\tilde{\rho}_R(\tilde{t}_{req})}{\pi^2 g_*(T_{re})}\right]^{\frac{1}{4}} = [\tilde{M}_p H(t_{req})]^{\frac{1}{2}} \left[\frac{45}{\pi^2 g_*(T_{re})}\right]^{\frac{1}{4}}\,, \tag{36}$$

where $g_*(T_{re}) = 121$ is the effective number of relativistic degrees of freedom, which includes all of the model particles except Φ. The numerical calculation gives $\frac{H(t_{req})}{\Gamma_\Phi} \approx 0.58$ and $T_{re} \approx 2.2 \times 10^{11}$ GeV, this demonstrates that the Φ decay is indeed out-of-equilibrium in the period of $t_{inf} < t < t_{req}$, but the thermal equilibrium is roughly formed after $t > t_{req}$. When $t = t_{ref}$, there is $T(t_{ref}) \approx 7.81 \times 10^{10}$ GeV $< M_\Phi \approx 8.88 \times 10^{10}$ GeV, the Φ particle can not be newly produced from the hot bath, thus the reheating is naturally over. The relevant results of the reheating are all listed in Table 2 in Section VI. Finally, we stress that the reheating is closely related to the inflation and particle physics, this is another one characteristic of the unified model.

Now we discuss the baryogenesis through the foregoing leptogenesis mechanism. In the light of the discussions in Section II, $\Phi \to l^c + \nu_R$ is a relatively slow and weak decay mode, but it has the three remarkable features. i) Its decay rate has the CP asymmetry about $A_{CP} \sim 10^{-10}$, which is given by Eq. (7). ii) This decay is really out-of-equilibrium in the reheating process since $\Gamma(\Phi \to l^c + \nu_R) < H(t)$ ($t_{inf} < t \leqslant t_{req}$). iii) Although the net lepton number is conserved as zero, the CP asymmetric decay can equally generate the anti-lepton asymmetry and the ν_R one. After the reheating, these two asymmetries are respectively isolated in the SM sector and dark sector, and also they can not be erased each other. Therefore, the ν_R asymmetry is forever frozen in the dark sector, whereas the anti-lepton asymmetry is partly converted into the baryon asymmetry through the SM sphaleron process [25]. In short, although this baryogenesis mechanism does not fully fulfil the Sakharov's three conditions [29], it is indeed put into effect in the unified model.

After the reheating is completed, the hot bath has fully been formed, the universe then enters into the hot expansion era. The hot evolution in the dark sector will be discussed in Section V, while the hot evolution in the SM sector is exactly the well-known hot big bang paradigm. Because the $B-L$ number is always conserved, above the electroweak scale the sphaleron transition can partly convert the generated anti-lepton asymmetry into the baryon asymmetry [30]. The relevant relations are in detail given as follows,

$$\begin{aligned}
&[\frac{\bar{n}_l - n_l}{s}]_{T_{re}} = [\frac{n_{\nu_R} - \bar{n}_{\nu_R}}{s}]_{T_{re}} = [\frac{n_\Phi A_{CP}}{s}]_{T_{re}} = [\frac{\rho_\Phi A_{CP}}{M_\Phi s}]_{T_{re}} = [\frac{\rho_R}{s}]_{T_{re}} \frac{A_{CP}}{M_\Phi} = \frac{3T_{re}A_{CP}}{4M_\Phi}\,,\\
&\eta_B = [\frac{s}{n_\gamma}]_{T_0} Y_B(T_0) = [\frac{s}{n_\gamma}]_{T_0} Y_B(T_{ew}) = [\frac{s}{n_\gamma}]_{T_0} c_s Y_{B-L}(T_{ew}) = [\frac{s}{n_\gamma}]_{T_0} c_s Y_{B-L}(T_{re})\\
&\quad = c_s [\frac{s}{n_\gamma}]_{T_0} [\frac{\bar{n}_l - n_l}{s}]_{T_{re}},
\end{aligned} \tag{37}$$

where s is the entropy density and $Y_B(T) = [\frac{n_B - \bar{n}_B}{s}]_T$ is the yield, $c_s = \frac{28}{79}$ is the SM sphaleron coefficient, $[\frac{s}{n_\gamma}]_{T_0} \approx 7.38$ includes the ν_R contribution to $g_*(T_0) \approx 4.1$ (see Eq. (38)). Eq. (37) clearly shows that η_B is collectively determined by the inflaton mass, the reheating temperature and the CP asymmetry, this again manifests that the unified model closely relates the inflation, reheating and particle physics together. In Table 2, I take suitable $A_{CP} \approx 1.29 \times 10^{-10}$ as an input parameter from the particle model, then the unified model naturally predicts $\eta_B \approx 6.14 \times 10^{-10}$.

V. Current Dark Matter and Dark Energy

The universe hot expansion leads that the radiation energy is red-shift, accordingly the universe temperature which is denoted by the photon temperature is continuously declining. The hot evolution in the dark sector is however different from that in the SM sector. In the light of the discussion in Section II, these heavy dark particles of N^0, E^-, ϕ^0 are all depleted by their decays in the very early universe, only the stable S and ν_R can survive in the dark sector. In fact, ν_R has been decoupled from the hot bath below the temperature of $M_N \sim 10^9$ GeV, the dark sector is completely separated from the SM sector after the S annihilation is frozen out at $T_f \approx \frac{M_S}{25.4} \approx 10.1$ GeV, therefore, the relativistic ν_R eventually becomes the dark radiation background, while the non-relativistic S becomes the CDM. From the entropy conservation, we can derive the effective temperature of ν_R as follows,

$$\begin{aligned}
&\frac{a^3(M_N)}{a^3(T)} = \frac{T_{\nu_R}^3}{M_N^3} = \frac{g_*(T)T^3}{g_*(M_N)M_N^3}\,,\\
&\Longrightarrow (\frac{T_{\nu_R}}{T})^3 = \frac{2 + \frac{7}{8} \times 6[(\frac{T_{\nu_L}}{T})^3 + (\frac{T_{\nu_R}}{T})^3]}{117.5} \Longrightarrow (\frac{T_{\nu_R}}{T})^3 \approx 0.0348\,,
\end{aligned} \tag{38}$$

where there must be $T < m_e \approx 0.5$ MeV (namely after the electron-positron annihilation), $(\frac{T_{\nu_L}}{T})^3 = \frac{4}{11}$ is the well-known effective temperature of ν_L in the SM sector. Eq. (38) gives the present-day $T_{\nu_R} = 0.0348^{\frac{1}{3}} T_0 \approx 0.9$ K, in addition, we can calculate the effective number of neutrinos at the recombination as $N_{eff} = 3[1 + (T_{\nu_R}/T_{\nu_L})^4] \approx 3.13$, which is safely within the current limit from the CMB data analysis.

The dark particle S has however a special nature. The effective temperature of S (namely its kinetic temperature) scales as $T_S \propto E_S \propto p_S^2 \propto a^{-2}$ on account of the momentum redshift where E_S and p_S are respectively the S kinetic energy and momentum, by contrast $T_{\nu_R} \propto a^{-1}$, therefore T_S is much faster dropping than T_{ν_R}, at present day T_S is essentially approaching to absolute zero, in other words, the S kinetic energy is almost exhausted so that it actually becomes supercool matter, as a result, the supercool S_{DM} is eventually condensing into the dark energy S_{DE} via its special self-interacting potential, this phenomenon is exactly a cosmological effect of the Bose–Einstein condensate which generally occurs at the extremely low temperature.

When the universe temperature cools to $T_{eq} \approx 1$ eV, namely the universe age is about 5×10^4 years, the total matter density exceeds the radiation one, thus the universe is transformed from the radiation-dominated to the matter-dominated. Note that T_{eq} is below $T_{BBN} \approx 0.1$ MeV but above $T_{Recom} \approx 0.3$ eV. We can regard the time of the matter-radiation equality as the starting point at which S_{DM} begins to condense into S_{DE}. As the universe expansion and cooling, the S_{DM} effective temperature is rapidly dropping and approaching to absolute zero, accordingly the S_{DM} kinetic energy is depleted very fast, thus more and more S_{DM} become supercool matter and condense into S_{DE}, so more and more S_{DE} are grown from them. To some extent, the S condensation is essentially a reverse process of the Φ inflation discussed in III Section, namely it is actually a process of S_{DE} slowly growing from S_{DM}, or S_{DM} gradually converting into S_{DE}. It should be stressed that the S condensation is very different from baryon and electron condensing into the usual material (namely the visible world), the former is a pure boson system, whereas the latter is a pure fermion system. In a word, the special evolution of S (namely the reverse evolution of the inflation) eventually leads to the dark matter and dark energy in the current universe.

From now on, we directly use the abbreviations of "DM" and "DE" to denote S_{DM} and S_{DE} respectively, their energy density and pressure are given as follows,

$$\begin{aligned}
&P_{DM} = 0\,, \quad P_{DE} = -\rho_{DE}\,, \quad \rho_{DM} + \rho_{DE} = \rho_S, \quad P_{DM} + P_{DE} = P_S = w_S\rho_S, \\
&\Longrightarrow \rho_{DM} = (1 + w_S)\rho_S, \quad \rho_{DE} = -w_S\rho_S,
\end{aligned} \tag{39}$$

where w_S is a parameter-of-state varying with the time, which is in the range of $0 \geqslant w_S \geqslant -1$. The physical implications of Eq. (39) is the same as that of the Φ field in Eq. (10), see those explanations below Eq. (10). After $T < T_{eq} \approx 1$ eV, the universe energy includes the four components of the photon ρ_γ, the neutrino ρ_ν (which contains the ν_L and ν_R energy), the baryon ρ_B, and the dark ρ_S (which contains ρ_{DM} and ρ_{DE}), their dynamical evolutions are determined by the following system of equations,

$$\rho_\gamma + \rho_\nu + \rho_B + \rho_S = 3\tilde{M}_p^2 H^2, \tag{40}$$

$$\begin{aligned}
&\dot{\rho}_\gamma + 4H\rho_\gamma = 0\,, \quad \dot{\rho}_\nu + 3H\rho_\nu(1 + w_\nu) = 0\,, \quad \dot{\rho}_B + 3H\rho_B = 0\,, \\
&\dot{\rho}_S + 3H\rho_S(1 + w_S) = 0 \Longrightarrow \dot{\rho}_{DE} = -(\dot{\rho}_{DM} + 3H\rho_{DM}),
\end{aligned} \tag{41}$$

$$\dot{\rho}_{DE} = \kappa(T)H\rho_{DM}. \tag{42}$$

Eq. (40) is Friedmann equation, it is in charge of the expansion rate and it relates the SM sector and the dark sector together. Eqs. (41) are the continuity equations of the four energy components. The neutrino is relativistic state in the early phase, but it later turns into non-relativistic state since it has a sub-eV mass, so there is $w_\nu = \frac{1}{3}$ for the relativistic neutrino and $w_\nu = 0$ for the non-relativistic neutrino. The last equality in Eqs. (41) indicates that the S_{DE}

growth is entirely from the S_{DM} reduction in the comoving volume. Eq. (42) is namely the growth equation of S_{DE}, the parameter $\kappa(T)$ characterizes the growth rate (or one can also call it as the condensing rate), it depends on the temperature, κ will rapidly increase as T is more and more low. Once the $\kappa(T)$ function is provided, then the above system of equations is closed, thus we can solve the evolution of each energy component.

That is similar to the inflation process, we can introduce the e-fold number of the condensation process as follows,

$$\begin{aligned} &N(T) = ln\frac{a(T)}{a(T_{eq})} = ln\frac{T_{eq}}{T} \Longrightarrow \dot{N}(t) = H(t), \\ &a(T_{eq}) \leqslant a(T) \leqslant a(T_0) = a_0, \quad 0 = N(T_{eq}) \leqslant N(T) \leqslant N(T_0) = N_0, \end{aligned} \tag{43}$$

where T_{eq} is the starting temperature of the S condensation and T_0 is the present universe temperature. Note that $\dot{N}(t)$ is positive in Eq. (43), namely N is increasing with the time, this is different from the negative $\dot{N}(t)$ defined in Eq. (16), one should not confuse them.

Now we use N as the time variable of the energy evolution, and normalize all kinds of the energy densities to the initial $\rho_\gamma(N(T_{eq})) = \rho_\gamma(0)$, then we can easily derive the following initial relations,

$$\begin{aligned} &\rho_\gamma(0) + \rho_\nu(0) = \rho_B(0) + \rho_S(0), \quad \rho_{DM}(0) = \rho_S(0), \quad \rho_{DE}(0) = 0\,, \quad w_S(0) = 0\,, \\ &\frac{\rho_\nu(0)}{\rho_\gamma(0)} = \frac{21}{8}[(\frac{T_{\nu_L}}{T_{eq}})^4 + (\frac{T_{\nu_R}}{T_{eq}})^4]\,, \quad \frac{\rho_B(0)}{\rho_\gamma(0)} = \frac{n_\gamma(0)}{\rho_\gamma(0)}\frac{M_B n_B(0)}{n_\gamma(0)} = \frac{3.6\times 10^{10}\,\eta_B[\frac{M_B}{\mathrm{GeV}}]}{\pi^4\,[\frac{T_{eq}}{\mathrm{eV}}]}\,, \\ &\frac{\rho_S(0)}{\rho_\gamma(0)} = \frac{M_S n_S(0)}{\rho_\gamma(0)} = \frac{M_S}{T_f\sqrt{g_*(T_f)}}\frac{0.85\times 10^{-9}\,\mathrm{GeV}^{-2}}{\langle\sigma v_r\rangle_{T_f}\,[\frac{T_{eq}}{\mathrm{eV}}]}\,, \end{aligned} \tag{44}$$

where $\frac{T_{\nu_L}}{T_{eq}} = (\frac{4}{11})^{\frac{1}{3}}$ and $\frac{T_{\nu_R}}{T_{eq}} \approx 0.0348^{\frac{1}{3}}$ have been given by Eq. (38), $M_B \approx 0.9383$ GeV is the baryon mass, I employ $\frac{n_B(0)}{n_\gamma(0)} = \frac{n_B(N_0)}{n_\gamma(N_0)} = \eta_B$ and $\eta_B \approx 6.14\times 10^{-10}$ has been obtained by Eq. (37). A detailed derivation of the equality of $\frac{\rho_S(0)}{\rho_\gamma(0)}$ is seen in Appendix II. According to Eq. (8), provided $\sum\limits_{\alpha,\beta}|y^e_\alpha y^e_\beta|^2 \approx 1$, $M_E \approx 0.5$ TeV and $M_S \approx 256$ GeV, we have obtained $\langle\sigma v_r\rangle_{T_f} \approx 1.64\times 10^{-9}$ GeV^{-2}, $\frac{M_S}{T_f} \approx 25.4$ and $g_*(T_f) = 91.5$, now put them into Eq. (44), then we can determine $T_{eq} \approx 0.928$ eV and $\frac{\rho_S(0)}{\rho_B(0)} \approx 6.46$. Although $\rho_B(0)$ and $\rho_S(0)$ are the same order of magnitude, evidently their origins are very different in Eq. (44). Note that $\frac{\rho_{DM}(N_0)}{\rho_B(N_0)} \approx 5.36 < \frac{\rho_S(0)}{\rho_B(0)} < \frac{\rho_S(N_0)}{\rho_B(N_0)} \approx 22.8$, this means that a part of the S particles are surely condensed into the dark energy after they become supercool.

In order to solve Eqs. (40)-(42), we need provide the $\kappa(T)$ evolution, so I assume that $\kappa(T)$ is given by the following $F(N)$ function,

$$\begin{aligned} &\kappa(N(T)) = \frac{dF(N)}{dN} \Longleftrightarrow \int_0^N \kappa(N')dN' = F(N) = b\,e^{a(1-\frac{N_0}{N})}, \\ &\Longrightarrow F(0) = 0\,, \quad F(N_0) = b\,, \quad F(\infty) = b\,e^a, \end{aligned} \tag{45}$$

where $a \approx 17.25$ and $b \approx 0.186$ are two input parameters in the S condensation process, which are determined by fitting the current density budget of the dark matter and dark energy, see

Table 2. The role of Eq. (45) is very similar to that of Eq. (24) in the Φ inflation, the former controls S_{DE} growing from S_{DM}, whereas the latter is in charge of Φ_{DM} growing from Φ_{DE}.

Make use of Eq. (43) and Eq. (45), then the relevant energy densities in Eqs. (40)-(42) are analytically solved as follows,

$$\begin{aligned}
\frac{dln\rho_\gamma}{dN} &= -4 \Longrightarrow ln\frac{\rho_\gamma(N)}{\rho_\gamma(0)} = -4N, \\
\frac{dln\rho_\nu}{dN} &= -3(1+w_\nu) \Longrightarrow ln\frac{\rho_\nu(N)}{\rho_\gamma(0)} = -4N + ln\frac{\rho_\nu(0)}{\rho_\gamma(0)}\,(0 \leqslant N \leqslant N_\nu), \\
& \qquad\qquad ln\frac{\rho_\nu(N)}{\rho_\gamma(0)} = -3N - N_\nu + ln\frac{\rho_\nu(0)}{\rho_\gamma(0)}\,(N > N_\nu), \\
\frac{dln\rho_B}{dN} &= -3 \Longrightarrow ln\frac{\rho_B(N)}{\rho_\gamma(0)} = -3N + ln\frac{\rho_B(0)}{\rho_\gamma(0)}\,, \\
\frac{dln\rho_{DM}}{dN} &= -3-\kappa \Longrightarrow ln\frac{\rho_{DM}(N)}{\rho_\gamma(0)} = -3N - F(N) + ln\frac{\rho_S(0)}{\rho_\gamma(0)}\,, \\
\frac{d\rho_{DE}}{dN} &= \kappa\,\rho_{DM} \Longrightarrow ln\frac{\rho_{DE}(N)}{\rho_\gamma(0)} = ln\left[\int_0^N \kappa(N')e^{-3N'-F(N')}dN'\right] + ln\frac{\rho_S(0)}{\rho_\gamma(0)}\,,
\end{aligned} \tag{46}$$

where N_ν is the time point when the neutrino is transformed from relativistic state to non-relativistic state, see the following Eq. (47), there is $w_\nu = \frac{1}{3}$ when $0 \leqslant N \leqslant N_\nu$ and $w_\nu = 0$ when $N > N_\nu$. Since the initial conditions have been given by Eq. (44), from Eq. (46) we can immediately calculate the relevant energy evolution once we input the model parameters.

First of all, we can calculate out the three key time points in the universe evolution,

$$N_\nu = ln\frac{T_{eq}}{0.1553\Sigma m_\nu} \approx 4.52, \quad N_D = ln\frac{T_{eq}}{3.16\times10^{-4}\,\mathrm{eV}} \approx 7.99, \quad N_0 = ln\frac{T_{eq}}{T_0} \approx 8.28, \tag{47}$$

where $T_{eq} \approx 0.928$ eV, its corresponding universe age is about 5×10^4 years. N_ν is the time when the neutrino turns into non-relativistic state, its corresponding universe temperature is $0.1553\Sigma m_\nu$ where the 0.1553 factor is derived from the second equality in Eq. (49) and $\Sigma m_\nu \approx 0.065$ eV is an input parameter of the unified model. N_D is the time of the equality of the dark energy density and the total matter one, it occurs at $T \approx 3.16\times10^{-4}\,\mathrm{eV} \approx 3.67$ K, this is close to the present-day $T_0 \approx 2.35\times10^{-4}\,\mathrm{eV} \approx 2.7255$ K, the present universe age is 13.8 billion years, while the N_D time is about 4 billion years ago.

Fig. 8 numerically shows the evolutions of the relevant energy densities since the matter-radiation equality which is set as the starting point of the S condensation. ρ_γ (the yellow curve), ρ_ν (the blue dotted curve) and ρ_B (the brown curve) evidently carry out the normal hot evolutions in the SM sector. ρ_B always keeps the pure matter evolution because the baryon number is conserved. At $N_\nu \approx 4.52$, the neutrino turns into the non-relativistic state, so its evolution is transformed from the radiation to the matter, eventually the radiation is only left with the photon. By contrast, ρ_S (the green curve), ρ_{DM} (the pink curve) and ρ_{DE} (the black curve) implement special hot evolutions in the dark sector. As the universe temperature is more and more cool, more and more CDM become supercool and condense into DE, therefore ρ_{DM} is more and more deviating from the pure matter evolution, while ρ_{DE} is slowly growing and rising, as a result, the total ρ_S is gradually transformed from the initial pure DM to the final pure DE, these three curves clearly show this condensing evolution. At $N_D \approx 7.99$, the dark

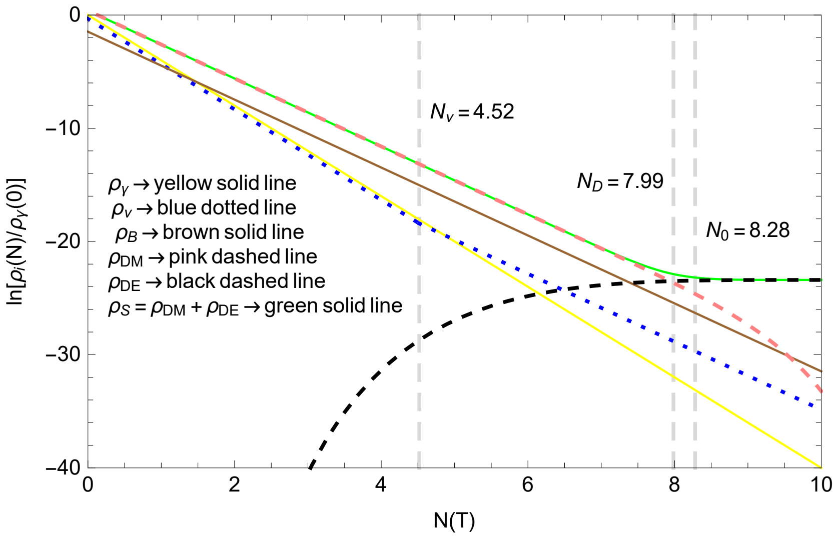

Figure 8: The evolutions of the relevant energy densities with $N(T)$ as time scale since $T_{eq} \approx 0.928$ eV. $N(T_{eq}) = 0$ is set as the starting point of the dark energy growing, and also several key time points are shown. The SM sector carries out the normal hot evolution, while the dark sector implements the S condensation, which is essentially a reverse process of the Φ inflation.

energy exceeds the total matter energy and it begins to dominate the universe, thus the universe is newly transformed from the decelerating expansion to the accelerating one. The present-day value of each energy density is evaluated at $N_0 \approx 8.28$. In the future, once all of the CDM are fully condensed into the dark energy, then the dark energy density will eventually become a constant, thus the universe will newly become a de Sitter one, which is the same state as the primordial universe of the early inflation. When the evolutions of ρ_{DM} and ρ_{DE} are compared with that of $\rho_{\Phi_{DE}}$ and $\rho_{\Phi_{DM}}$ in Fig. 4, we can see that the current S condensation is essentially a reverse process of the primordial Φ inflation, the reason for this is of course that the two fields of S and Φ have the same nature and similar dynamics. In a similar way to finding the inflationary potential of V_Φ, we can also find the condensation potential of V_S by fitting the ρ_S curve in Fig. 8, but we have to give up it in order not to increase the length of the paper.

Make use of Eq. (46), we can also calculate the density parameter of each energy component, and further obtain the total parameter-of-state and the h value, they are given by the following relations,

$$\Omega_i(N) = \frac{\rho_i(N)}{\sum\limits_i \rho_i(N)}\,, \quad w_T(N) = \frac{\sum\limits_i P_i(N)}{\sum\limits_i \rho_i(N)} = \frac{\Omega_R(N)}{3} - \Omega_{DE}(N),$$
$$\sum_i \rho_i(N_0) = 3\tilde{M}_p^2 H_0^2 \implies h \approx 0.73\,, \tag{48}$$

where the present critical energy density is $3\tilde{M}_p^2 H_0^2$ with $H_0 \approx 2.13 \times 10^{-42} h$ GeV, note that $h \approx 0.73$ is an output value of the unified model rather than an input parameter. In fact, from Eq. (46) and Eq. (48) we can analytically derive the following ratio relations among the present-day density parameters,

$$\Omega_\gamma(N_0)h^2 = \frac{1}{45}[\frac{\pi\, T_0^2}{\tilde{M}_p \frac{H_0}{h}}]^2, \qquad \frac{\Omega_\nu(N_0)}{\Omega_\gamma(N_0)} = \frac{27\Sigma m_\nu}{\pi^4 T_0}[(\frac{T_{\nu_L}}{T_0})^3 + (\frac{T_{\nu_R}}{T_0})^3],$$
$$\frac{\Omega_B(N_0)}{\Omega_\gamma(N_0)} = \frac{36\eta_B M_B}{\pi^4 T_0}, \qquad \frac{\Omega_{DM}(N_0)}{\Omega_B(N_0)} = \frac{\rho_S(0)}{\rho_B(0)}\, e^{-F(N_0)}, \tag{49}$$

where $(\frac{T_{\nu_L}}{T_0})^3 = \frac{4}{11}$ and $(\frac{T_{\nu_R}}{T_0})^3 = 0.0348$. $\Omega_\gamma(N_0)h^2$ is obviously fixed by T_0, while $\Omega_\nu(N_0)$ and $\Omega_B(N_0)$ are respectively determined by Σm_ν and η_B. Since $\frac{\rho_S(0)}{\rho_B(0)} \approx 6.46$ is obtained by Eq. (44) and $F(N_0) = b \approx 0.186$ is an input parameter, then we naturally obtain $\frac{\Omega_{DM}(N_0)}{\Omega_B(N_0)} \approx 5.36$. $h \approx 0.73$ is determined by adjusting the input parameter $a \approx 17.25$, finally Ω_{DE} is fixed by the closed relation $\sum\limits_i \Omega_i \approx 1$, the detailed results are all listed in Table 2.

Now we can simply and elegantly solve the "Hubble tension" by use of my model, and clarify which one among $h \approx 0.73$ and $h \approx 0.674$ is really correct. The Planck data analysis of the CMB is on the basis of the following relations [31],

$$r_s^* = \int_{z_*}^{\infty} \frac{c_s(z)dz}{H(z)} = \frac{1}{H(z_*)}\int_{z_*}^{\infty} \frac{c_s(z)H(z_*)dz}{H(z)}, \quad H(z_*) \approx \frac{\sqrt{\rho_B(z_*)[1+\frac{\rho_{DM}(z_*)}{\rho_B(z_*)}]}}{\sqrt{3}\tilde{M}_p},$$
$$D_A^* = \int_0^{z_*} \frac{dz}{H(z)} = \frac{1}{h[\frac{H_0}{h}]}\int_0^{z_*} \frac{H_0 dz}{H(z)}, \qquad \theta^* = \frac{r_s^*}{D_A^*} \propto \frac{h}{H(z_*)}, \tag{50}$$

where $z_* \approx 1100$ is the redshift to the recombination era at which the CMB is formed, it is corresponding to the time of $N \approx 1.2$ in Fig. 8. r_s^* is the sound horizon at z_* and D_A^* is the angular diameter distance to z_*. $c_s = [3 + \frac{\rho_B}{\rho_\gamma}]^{-\frac{1}{2}}$ is the sound speed of the baryon-photon plasma and it is independent of models, there is $\frac{1}{\sqrt{3}} \leqslant c_s(z) \lesssim \frac{1}{\sqrt{3.71}}$ for $z \geqslant z_*$. θ^* may be directly read off from the acoustic peak spacing of the CMB power spectra, so it is a fixed value. Since the integrating factor in r_s^* is dominated by $\frac{H(z_*)}{H(z)} \to 1$ when $z \to z_*$, in the same way, the integrating factor in D_A^* is dominated by $\frac{H_0}{H(z)} \to 1$ when $z \to 0$, then these two integrating factors are actually insensitive to the specific models. The Planck data analysis assumes the ΛCDM model, it employs $\frac{\rho_{DM}(z_*)}{\rho_B(z_*)} = \frac{\rho_{DM}(z)}{\rho_B(z)}|_{z=0} \approx 5.36$ to infer $h \approx 0.674$ in Eq. (50), in contrast, there is $\frac{\rho_{DM}(z_*)}{\rho_B(z_*)} \approx \frac{\rho_{DM}(z)}{\rho_B(z)}|_{z=z_{eq}} \approx 6.46$ in the unified model where $z_{eq} \approx 3400$ is the redshift at the R-M equality (which is corresponding to the time of $N = 0$ in Fig. 8), this result can be directly seen in Fig. 8, thus we can naturally infer $h \approx 0.73$ from Eq. (50). In brief, both of $\frac{h}{H(z_*)} \propto \frac{0.674}{\sqrt{1+5.36}} \propto \frac{0.73}{\sqrt{1+6.46}}$ are all fitting the Planck CMB data, but the former result is derived from the ΛCDM model, it is however inconsistent with the $h \approx 0.73$ measurements at the low redshift (namely the "Hubble tension"), whereas the latter result arises from my model, the predicted $h \approx 0.73$ is simultaneously fitting to the intersection point given by the SH_0ES data and the BAO+SNe ones, refer to FIG. 1 in [31], therefore the "Hubble tension" is naturally eliminated. In conclusion, this unified model is more successful and believable than the ΛCDM model.

Fig. 9 clearly shows the evolutions of the relevant density parameters since the matter-radiation equality. Both the standard hot evolution in the visible sector and the DM condensing into the DE (or the DE growing from the DM) in the dark sector are explicitly illustrated by the

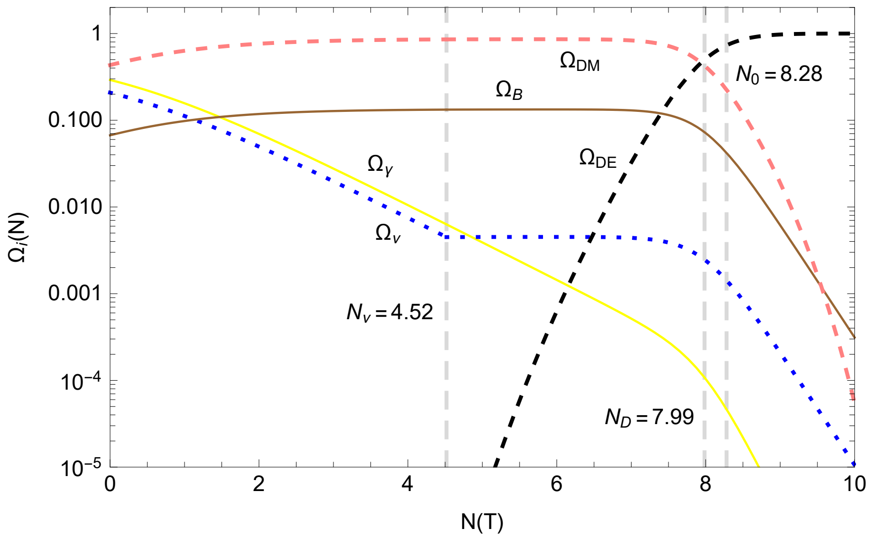

Figure 9: The evolutions of the relevant energy density parameters with $N(T)$ as time scale since $T_{eq} \approx 0.928$ eV. $N(T_{eq}) = 0$ is set as the starting point of the dark energy growing, and also several key time points are shown. The present energy density budget is evaluated at N_0, which is accurately fitting to the present data.

corresponding curves, these results further confirm our previous discussions. When $N < 0$, the universe is R-dominated. When $0 < N < N_D$, the universe is M-dominated. When $N > N_D$, the universe is DE-dominated. The present energy density budget is evaluated at N_0, which is correctly fitting to the present data. In the future, the universe will entirely be filled with the dark energy.

Fig. 10 shows the evolutions of the total parameters-of-state and the S parameters-of-state since the matter-radiation equality. When the evolution of w_S is compared with that of w_Φ in Fig. 5, the current CDM condensation into the dark energy is indeed a reverse process of the primordial slow-roll inflation. When $w_T < -\frac{1}{3}$ (namely $N > 7.78$), the universe is newly transformed from the decelerating expansion to the accelerating one. When $w_T < -\frac{1}{2}$ (namely $N > N_D$), the universe is transformed from M-dominated to DE-dominated. At the present day, ninety-five percent of the total universe energy is dark and the universe is accelerating expansion by the dark energy drive, all of these have been verified by the observations. The future fate of the universe will newly become de Sitter universe filled with pure S_{DE}, which is exactly the same state as the primordial universe filled with pure Φ_{DE}, but these two dark energy densities differ by about 106 orders of magnitude.

VI. Summary for Numerical Results

Now we summarize all kinds of the important numerical results of the unified model, and then compare them with the measured experimental data. The used physical constants are only

$$\tilde{M}_p = 2.43 \times 10^{18}\,\text{GeV}, \qquad M_B = 0.9383\,\text{GeV},$$
$$T_0 = 2.7255\,\text{K} = 2.35 \times 10^{-4}\,\text{eV}, \quad \frac{H_0}{h} = 2.13 \times 10^{-42}\,\text{GeV} = 100\,\text{km}\,\text{s}^{-1}\text{Mpc}^{-1}. \tag{51}$$

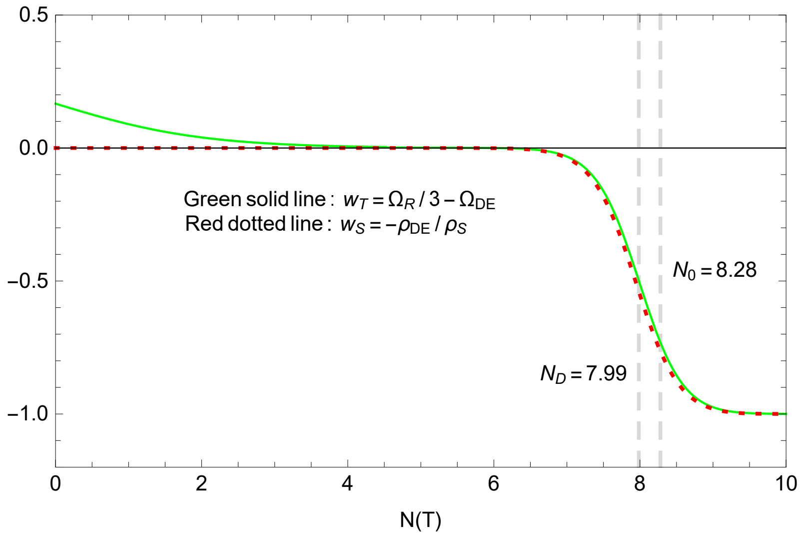

Figure 10: The evolutions of the relevant parameters-of-states with $N(T)$ as time scale since $T_{eq} \approx 0.928$ eV. $N(T_{eq}) = 0$ is set as the starting point of the dark energy growing. At $w_T = -\frac{1}{3}$ namely $N \approx 7.78$, the universe is newly transformed from the decelerating expansion to the accelerating one. At $w_T = -\frac{1}{2}$ namely N_D, the DE begins to dominate the universe. The S condensation is essentially a reverse process of the Φ inflation. The future fate of the universe will newly become de Sitter universe which is the same state as the primordial universe, but their energy densities differ by about 106 orders of magnitude.

The fundamental quantities of the model are set as the following benchmark,

$$\mu_0 \sim M_N \sim 10^9 \text{ GeV}, \quad v_\phi \sim 1 \text{ TeV}, \quad v_H \approx 246 \text{ GeV}, \quad v_\Phi \sim 10 \text{ eV}. \tag{52}$$

In addition, we fix $\sum\limits_{\alpha,\beta} |y^e_\alpha y^e_\beta|^2 \approx 1$ and $M_E \approx 0.5$ TeV so that M_S becomes an adjustable parameter. Table 2 in detail lists the relevant input parameters and all kinds of the output results in the unified model.

In the Φ inflation there are only the two input parameters of H_{inf} and α, but the inflation output results excellently fit all of the inflationary data, in particular, the model predicts $r_{0.05} \approx 1.86 \times 10^{-7}$, which is too small to be detected currently, and the inflaton mass $M_\Phi \approx 8.88 \times 10^{10}$ GeV, which purely arises from the inflationary dynamical evolution. The S condensation are controlled only by the two input parameters of a and b, their values are determined by fitting the current densities of the CDM and dark energy. The remaining input parameters are all provided by the particle model, $\frac{\mu_0}{M_\Phi}$ and A_{CP} are respectively responsible for the reheating outputs and the baryon asymmetry, M_S is in charge of the cross-section and freeze-out temperature of the S annihilation, by which the S relic density is determined, finally, $\sum m_{\nu_i}$ determines the present energy density of the cosmic neutrino. Although there are no observable data of the universe reheating as yet, the reheating output results are all very reasonable, the reheating process brings about both the primordial hot bang and the matter-antimatter asymmetry, this is also very believable. In the last panel, the current energy density budget is perfectly reproduced, moreover, $h \approx 0.73$ and $\eta_B \approx 6.14 \times 10^{-10}$ is finely predicted, in particular, the "Hubble tension" is removed. In short, all of the numerical results are consistent, reasonable and without any fine-tuning, they are very well in agreement with all of the present measured data in Eq. (1).

Relevant input parameters							
Φ inflation		S condensation		Particle model			
H_{inf}(GeV)	α	a	b	$\frac{\mu_0}{M_\Phi}$	A_{CP}	M_S(GeV)	$\sum m_{\nu_i}$(eV)
4.49×10^{10}	2.92	17.25	0.186	0.01	1.264×10^{-10}	256	0.065

Inflation output quantities							
k_*(Mpc^{-1})	N_*	$r_{0.05}$	n_s	$\frac{dn_s}{dlnk}$	$ln[10^{10}\Delta_R^2]$	$\frac{\rho_\Phi(N_*)}{\rho_\Phi(t_{inf})}$	M_Φ(GeV)
0.05	51.1	1.86×10^{-7}	0.965	-0.0040	3.042	5.65	8.88×10^{10}

Reheating output quantities					
	Ω_Φ	Ω_R	T(GeV)	H(GeV)	Γ_Φ(GeV)
$\tilde{t}_{req} = 1.054$	0.5	0.5	2.2×10^{11}	1.02×10^5	1.77×10^5
$\tilde{t}_{ref} = 10$	1.87×10^{-4}	≈ 1	7.81×10^{10}	9.15×10^3	

Current output quantities							
	Ω_γ	Ω_ν	Ω_B	Ω_{DM}	Ω_{DE}	h	η_B
	4.63×10^{-5}	0.00142	0.042	0.2251	0.7315	0.73	6.14×10^{-10}
$\Omega_i h^2$	2.47×10^{-5}	0.000754	0.02237	0.120	0.3898		

Table 2: A summary of the numerical results of the unified model.

All of the energy scales in Table 2 are below the GUT scale, so the primordial inflation really takes place after the GUT phase transition, thus the magnetic monopole problem is naturally eliminated. In the unified model, the super-high dark energy converting into the super-heavy dark matter drives the primordial inflation, while the current dark energy is grown from the CDM condensation, although there is difference about 106 orders of magnitude between these two dark energy densities, the universe energy is step by step released and reduced in the evolution processes of 13.8 billion years, the whole evolution processes are analogous to a cascade of hydropower stations, see Fig. 3, so there is not the so-called "cosmological constant problem" at all. All of these results fully demonstrate that the model is highly of self-consistence, concordance and unification. In conclusion, the unified model is indeed able to account for the universe origin and evolution elegantly and excellently, it is very successful and believable, therefore we expect it to be tested by the future experiments.

VII. Conclusions

I put forward to a unified model of particle physic and cosmology based on both a new extension of the SM and the fundamental principle of the standard cosmology. This new theory covers both the SM physics (visible sector) and the BSM one (dark sector), it can successfully account for the full process of the universe origin and evolution in a unified and integrated way, and also it elegantly explains the origins of the neutrino mass and matter-antimatter asymmetry. The universe starts from the primordial super-high dark energy Φ_{DE}, which drives the inflation and is slowly converted into the super-heavy dark matter Φ_{DM}, at the end of the inflation the Φ_{DM} decay leads to both the reheating and the leptogenesis, after that the universe enters the R-dominated era, it gradually cools by means of the hot expansion, and then the matter begins to dominate the universe, more and more S_{DM} particles become the supercool CDM so that they can eventually condensate into the dark energy S_{DE}, therefore the future universe will newly become a de Sitter one filled with the dark energy S_{DE}. In the model, Φ and S are two special scalar fields with the unusual self-interacting potentials, but they have the similar nature and dynamics. The primordial Φ inflation is implemented by Φ_{DM} slowly growing from Φ_{DE}, whereas the current S condensation is implemented by S_{DE} slowly growing from S_{DM}, the latter is essentially a reverse process of the former.

For each evolution process, I give its complete dynamical system of equations and solve them by some special techniques, in particular, I establish the internal relations between these evolution processes and particle physics, which are embodied in some key equations. The numerical results of the model are in detail shown by all kinds of the figures and Table 2. Those evolution figures clearly show how the slow-roll inflation is implemented, how the inflaton mass arises from the inflationary evolution, what the inflationary potential shape and its function form really look like, the details of reheating process, the mechanism of matter genesis, the CDM formation and condensation, and the current dark energy genesis. From the primordial super-high dark energy to the current super-low dark energy, the universe energy is step by step released and reduced in the evolution history of 13.8 billion years, which is analogous to a cascade of hydropower stations, so there is no the “cosmological constant problem”.

The unified model can excellently fit all kinds of the observation data by use of fewer input parameters. It not only perfectly reproduces the measured inflationary data and the current energy density budget, but also finely predicts some important cosmological quantities such as the tensor-to-scalar ratio $r_{0.05} \approx 1.86 \times 10^{-7}$, the inflaton mass $M_\Phi \approx 8.88 \times 10^{10}$ GeV, the reheating temperature $T_{re} \approx 2.2 \times 10^{11}$ GeV, the CDM mass $M_S \approx 256$ GeV, the baryon asymmetry $\eta_B \approx 6.14 \times 10^{-10}$, the scaling factor of expansion rate $h \approx 0.73$, in particular, it also clarifies and eliminates the “Hubble tension”, all of these results are very significant for particle physics and cosmology. In short, the model really achieves a unification of particle physics and cosmology, and also it is very successful and believable, therefore we expect that it is tested in the near future.

Acknowledgements

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Appendix I

A derivation of Eq. (28) is as follows,

$$
\begin{aligned}
k_* &= \frac{a_* H_*}{c} = \frac{H_0}{c}\frac{H_{inf}}{H_0}\frac{H_*}{H_{inf}}\frac{a_*}{a_{inf}}\frac{a_{inf}}{a_{req}}\frac{a_{req}}{a_{ref}}\frac{a_{ref}}{a_0} \\
&= \frac{H_0}{c}\frac{H_{inf}}{H_0}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}]^{\frac{1}{2}} e^{-N_*}[\frac{\rho_\Phi(t_{req})}{\rho_\Phi(t_{inf})}]^{\frac{1}{3}}[\frac{\rho_R(t_{ref})}{\rho_R(t_{req})}]^{\frac{1}{4}}[\frac{s(T_0)}{s(T_{ref})}]^{\frac{1}{3}} \\
&= \frac{H_0}{c}\frac{H_{inf}}{H_0}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}]^{\frac{1}{2}} e^{-N_*}[\frac{\rho_c(t_0)}{\rho_\Phi(t_{inf})}\frac{\rho_\gamma(t_0)}{\rho_c(t_0)}\frac{\rho_R(t_{req})}{\rho_\gamma(t_0)}]^{\frac{1}{3}}[\frac{T_{ref}}{T_{req}}][\frac{g_*^{\frac{1}{3}}(T_0)T_0}{g_*^{\frac{1}{3}}(T_{ref})T_{ref}}] \\
&= \frac{H_0}{c}\frac{H_{inf}}{H_0}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}]^{\frac{1}{2}} e^{-N_*}[\frac{H_0^2}{H_{inf}^2}]^{\frac{1}{3}}[\Omega_\gamma(T_0)]^{\frac{1}{3}}[\frac{g_*(T_{req})T_{req}^4}{2T_0^4}]^{\frac{1}{3}}[\frac{g_*^{\frac{1}{3}}(T_0)T_0}{g_*^{\frac{1}{3}}(T_{ref})T_{req}}] \\
&= [\frac{H_0}{c\,h}][\frac{g_*(T_0)}{2}]^{\frac{1}{3}}[\Omega_\gamma(T_0)h^2]^{\frac{1}{3}}[\frac{H_{inf}}{H_0/h}]^{\frac{1}{3}}[\frac{T_{req}}{T_0}]^{\frac{1}{3}}[\frac{\rho_\Phi(N_*)}{\rho_\Phi(0)}]^{\frac{1}{2}} e^{-N_*},
\end{aligned} \tag{53}
$$

where I use $a \propto \rho^{-\frac{1}{3}}$ for the Φ-dominated phase in $t_{inf} < t < t_{req}$ and $a \propto \rho^{-\frac{1}{4}}$ for the R-dominated phase in $t_{req} < t < t_{ref}$, in addition, I employ $\rho_\Phi(t_{req}) = \rho_R(t_{req})$ and $g_*(T_{req}) = g_*(T_{ref})$.

Appendix II

A derivation of the $\frac{\rho_S(0)}{\rho_\gamma(0)}$ equality in Eq. (44) is as follows,

$$
\begin{aligned}
&\frac{\rho_S(0)}{\rho_\gamma(0)} = \frac{M_S n_S(0)}{\rho_\gamma(0)} = \frac{M_S n_S(T_f)}{\rho_\gamma(T_{eq})}\frac{n_S(T_{eq})}{n_S(T_f)}, \quad \frac{n_S(T_{eq})}{n_S(T_f)} = \frac{a^3(T_f)}{a^3(T_{eq})} = \frac{g_*(T_{eq})T_{eq}^3}{g_*(T_f)T_f^3}, \\
&n_S(T_f) = \frac{H(T_f)}{\langle\sigma v_r\rangle_{T_f}} = \frac{1.66\sqrt{g_*(T_f)}\,T_f^2}{\langle\sigma v_r\rangle_{T_f} M_{Pl}}, \quad \rho_\gamma(T_{eq}) = \frac{\pi^2}{15}T_{eq}^4, \\
&\Longrightarrow \frac{\rho_S(0)}{\rho_\gamma(0)} = \frac{M_S}{T_f\sqrt{g_*(T_f)}}\frac{8.5\times10^{-10}\,\mathrm{GeV}^{-2}}{\langle\sigma v_r\rangle_{T_f}[\frac{T_{eq}}{\mathrm{eV}}]},
\end{aligned} \tag{54}
$$

where $g_*(T_{eq}) = 4.1$ and $g_*(T_f) = 91.5$.

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