

Question: We built a random number generator and it just rolls dice in the background and prints the result of the roll. We love prime numbers so a dice with (10^9+7) (a famous prime number) sides is used. To simulate random behaviour, the machine rolls the dice n number of times, where n equals the $2^{((10^9)+7)}$ (th) prime number. We want to know the probability of the largest result among all these rolls being a prime number too. You can stop the machine from rolling dices till the Heat Death of the universe by telling us the answer beforehand. Calculate the first 10 digits after the decimal place.

The flag will look like -> `csictf{first_10_digits_after_the_decimal_point}`

Solution: So basically the total probability will be the probability of $(10^9 + 7)$ rolling + probability of the next highest prime number rolling and being the highest roll + probability of the next highest prime number rolling and being the highest roll and so on. But the 2nd, 3rd Terms in this series will be negligible when looking only at first 10 decimal places. To solve the first part, i've attached an image.

$$\text{Total numbers} = 10^9 + 7$$

$$10^9 + 7 \text{ is prime}$$

$$P(\text{rolling } 10^9 + 7 \text{ in } 2^{10^9+7} \text{ rolls}) = 1 - P(\text{not rolling } 10^9 + 7 \text{ in } 2^{10^9+7} \text{ rolls})$$

$$P(\text{not rolling } 10^9 + 7 \text{ in } 2^{10^9+7} \text{ rolls}) = \left(\frac{10^9 + 6}{10^9 + 7} \right)^2$$

$$\approx 0.000 \dots \dots \dots 1$$

$$\therefore P(\text{rolling } 10^9 + 7 \text{ in } 2^{10^9+7} \text{ rolls}) = 1 - 0.000 \dots \dots \dots 1$$

$$= 0.999 \dots \dots \dots$$

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