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First Term Scheme of Work

WEEK	TOPIC
1	SET 1
2	SET 2
3	INDICES
4	LOGARITHM
5	CAT/PROJECT/MID TERM BREAK
6	
7	NUMBER BASED SYSTEM
8	MODULAR ARITHMETIC
9	EVERYDAY ARITHMETIC
10	SIMPLE EQUATION AND CHANGE OF SUBJECT OF FORMULA 1

PROJECT TOPIC: SET
EXPECTATIONS: GEM/ GROUP BASED PROJECT

TOPIC: REAL LIFE APPLICATION OF SET.

Rationale of the objective: To help students apply the knowledge of set in real life situation using statistics.

SDG: Focus on Sustainability: **Deadline:** 4:00pm on Friday, 21st October, 2024.

Choice of Subject: Design a survey to investigate preferred subjects by your friends from any three subjects. Collect data from your classmates and using the information:

EXPECTATIONS

- Represent the collected data neatly using Venn diagram
- Calculate the total number of students involved in the research i.e. the universal set $n(U)$
- Discuss your findings deduced from the project.

SKILLS TO ACQUIRE: team work, problem solving skills, sense initiative

STUDENTS' REFERENCE BOOK

NEW GENERAL MATHEMATICS FOR SENIOR SECONDARY SCHOOL 1, BY M.F. MACRAE ETAL

WEEKS 1 & 2

TOPIC: SETS I

Learning Objectives: Students should be able to:

- Define sets
- Use set notations
- Identify types of sets (empty, infinite, universal)
- Operate on sets (union, intersection, complement)
- Use set-builder notation

Definition of Set

A set is a well defined collection of objects or elements having some common characteristic or properties. A set can be described by

- Listing of its elements
- Giving a property that clearly defines its element

Notations used in set theory

- Elements of a set: the members of a set are called elements e.g. list the elements of set

$$A = \left\{ \text{even numbers less than 10} \right\}$$

- $n(A)$ means number of elements contained in a set
- \in means 'is an element of' or 'belongs to' e.g. $6 \in A$
- \notin means 'is not an element of' or 'did not belong to' e.g. $5 \notin A$ defined in number 1 above
- $(:)$ means such that e.g. $B = \{X : 3 \leq X \leq 10\}$ means X is a member of B such that X is a number from 3 to 10
- Equal set: two sets are equal if they contain the same elements e.g. If $S = \{a, d, c, b\}$ and $P = \{b, a, d, c, a, b\}$, then $S = P$ repeated elements are counted once
- Φ or $\{ \}$ means empty set or null set i.e. A set which has no element e.g. $\{\text{secondary school student with age 3}\}$

8. \subset means subset. B is a subset of A if all the elements of B are contained in A e.g If $A = \{1,2,3,4\}$ and $B = \{1,2,3\}$ then B is a subset of A i.e $B \subset A$
9. \cup means union: all elements belonging to two or more given sets. $A \cup B$ means list all elements in A and B e.g. If $A = \{2,4,6,8,10\}$ and $B = \{1,3,5,7,9\}$ then $A \cup B = \{1,2,3,4,5,6,7,8,9,10\}$
10. \cap means intersection i.e elements common to 2 or more sets e.g $A = \{1,2,3,4,5,6\}$ and $B = \{1,3,5,7,9\}$ then $A \cap B = \{1,3,5\}$
11. U and E means universal set i.e a large set containing all the original given set i.e A set containing all elements in a given problem or situations under consideration
12. Complement of a set i.e A^c . A^c means 'A complement' and it is the set which contains elements that are not elements of set A but are in the universal set under consideration. E.g If $E = \{\text{shoes and sock}\}$ and $A = \{\text{socks}\}$, then $A^c = \{\text{shoes}\}$

EVALUATION

1. State the elements in the given set below: $Y = \{Y: Y \in \text{integer } -4 \leq Y \leq 3\}$
2. Let $E = \{x: 10 < x < 20\}$ $P = \{\text{prime numbers}\}$ $Q = \{\text{odd numbers}\}$
Where P and Q are subsets of E
 - a) List all elements of set P (b) What is $n(P)$? (c) List all elements of set Q (d) List the elements of P^c
3. Make each of the following statements true by writing \bar{E} or E in place of *
 - a) $17 * 1,2,3,\dots,7, 8,9 \{ \quad \}$
 - b) $11 * 1,3,5,7,\dots, 19 \{ \quad \}$

TYPES OF SETS

1. Universal set: A larger set containing all other sets under consideration i.e a set of students in a school
2. Finite set: is a set which contains a fixed number of elements. This means that a finite set has an end. E.g $B = \{1,2,3,4,5\}$
3. Infinite set: is a set which has unending number of elements or which has an infinite number of elements. An infinite set has no end of its elements. E.g $D = \{5,10,15,20,\dots\}$
4. Subset: B is a subset of A if all elements of B are contained in A i.e it is a smaller set contained in a larger or bigger set. E.g if $A = \{1,2,3,4,5,6\}$ and $B = \{2,3,6\}$ then B is a subset of A i.e $B \subset A$
5. Empty set Φ or $\{ \}$. An empty set or null set contains no element
6. Disjoint set: if two sets have no elements in common, then they are said to be disjoint e.g If $P = \{2,5,7\}$ and $Q = \{3,6,8\}$ then P and Q are disjoint.

OPERATIONS OF SET

1. Intersection \cap : the intersection of two sets A and B is the set containing the elements common to A and B e.g if $A = \{a,b,c,d,e\}$ and $B = \{b,c,e,f\}$, then $A \cap B = \{b,c,e\}$
2. Union \cup : the union of A and B, $A \cup B$ is a set which includes all elements of A and B e.g if $A = \{1,3\}$ and $B = \{1,2,3,4,6\}$, then $A \cup B = \{1,2,3,4,6\}$
3. Complement of a set: the complement of a set P, P^c are elements of the universal set that are not in P e.g if $U = \{1,2,3,4,5,6\}$ $P = \{2,4,5,6\}$, then $P^c = \{1,3\}$

Examples

Given that $U = \{a,b,c,d,e,f\}$, $P = \{b,d,e\}$ $Q = \{b,c,e,f\}$

List the elements of

- a) $P \cap Q$ (b) $P \cup Q$ (c) $(P \cap Q)^c$
- (d) $(P \cup Q)^c$ (e) $P^c \cup Q$ (f) $Q^c \cap P^c$

Solution

- a) $P \cap Q = \{b,e\}$

- b) $P \cup Q = \{b, c, d, e, f\}$
 c) Since $(P \cap Q) = \{b, e\}$
 Then $(P \cap Q)^c = \{a, c, d, f\}$
 d) Since $(P \cup Q) = \{b, c, d, e, f\}$, then $(P \cup Q)^c = \{a\}$
 e) $P \cap Q$
 $P = \{a, c, f\}$
 $Q = \{b, c, e, f\}$
 Therefore $P \cap Q = \{a, b, c, e, f\}$
 f) $Q^c = \{a, d\}$
 $P^c = \{b, d, e\}$ $P \cap Q^c = \{d\}$

EVALUATION

Given that $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{2, 4, 6, 8\}$ $B = \{1, 2, 5, 9\}$ and $C = \{2, 3, 9, 10\}$

Find: a) $A \cap B \cap C$ (b) $C^c \cap (A \cap B)$ (c) $C \cap (A \cap B)^c$ (d) $C \cup (A \cap B)$

GENERAL EVALUATION

- Given that $U = \{1, 2, 3, \dots, 19, 20\}$ and $A = \{1, 2, 4, 9, 19, 20\}$ $B = \{\text{perfect square}\}$ $C = \{\text{factors of 24}\}$.
 Where A, B, and C are subsets of universal set U
 - List all the elements of all the given sets
 - Find (i) $n(A \cup B)$ (ii) $n(A \cup B \cup C)$ (iii) $n(A \cup B \cap C)$
 - Find (i) $A \cap B \cap C$ (ii) $A \cup (B \cap C)$ (iii) $(A \cap B) \cup C$
- List all the subsets of the following sets
 - $A = \{\text{Knife, Fork}\}$
 - $P = \{a, e, i\}$

STUDY ASSIGNMENT

NGM SSS1 pages 71-72, exercise 5b and 5c.

WEEKEND ASSIGNMENT

- If $A = \{a, b, c\}$ $B = \{a, b, c, e\}$ and $C = \{a, b, c, d, e, f\}$ find $A \cap B(A \cup C)$ A. $\{a, b, c, d\}$ B. $\{a, b, c, d, e\}$
 C. $\{a, b, d, d, e\}$ D. $\{a, b, c\}$
- If $Q = \{0 < x < 30, x \text{ is a perfect square}\}$, $P = \{x: 1 \leq x \leq 10, x \text{ is an odd number}\}$ find $Q \cap P$ A. $\{1, 3, 9\}$ B. $\{1, 9, 4\}$
 C. $\{1, 9\}$ D. $\{19, 16, 25\}$

Use the following information to answer questions 3 – 5

A, B and C are subsets of universal set U such that $U = \{0, 1, 2, 3, \dots, 11, 12\}$, $A = \{x: 0 < x < 7\}$, $B = \{4, 6, 8, 10\}$, $C = \{1 < x < 8\}$

- Find $(A \cup C)^c$ A. $\{0, 1, 9\}$ B. $\{2, 3, 4, 5\}$ C. $\{2, 3, 5, 7\}$ D. $\{0, 1, 2, 9\}$
- Find $A \cap B \cap C$
- $A \cup B \cap C$ A. $\{1, 2, 3, 4, 5, 6, 7\}$ B. $\{2, 3, 5, 7\}$ C. $\{6, 8, 10, 12\}$ D. $\{4, 5, 7, 9, 11\}$

THEORY

- The universal set U is the set of integers: A, B and C are subsets of U defined as follows
 $A = \{\dots, -6, -4, -2, 0, 2, 4, 6, \dots\}$
 $B = \{X: 0 < x < 9\}$
 $C = \{X: -4 < x < 0\}$
 - Write down the set A^c , where A^c is the complement of A with respect to U
 - Find $B \cap C$
 - Find the members of set $B \cup C$, $A \cap B$, and hence show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- The universal set U is the set of all integers and the subsets P, Q, R of U are given by
 $P = \{X: X < 0\}$, $Q = \{\dots, -5, -3, -1, 1, 3, 5, \dots\}$, $R = \{X: -2 < X < 7\}$

- Find $Q \cap R$
- Find R^c where R^c is the complement of R with respect to U
- Find $P^c \cap R^c$
- List the members of $(P \cap Q)$

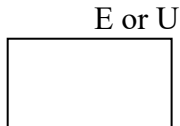
TOPIC: SETS II

Learning Objectives: Students should be able to use Venn diagram to solve problem.

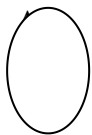
THE VENN DIAGRAM

The venn diagram is a geometric representation of sets using diagrams which shows different relationship between sets

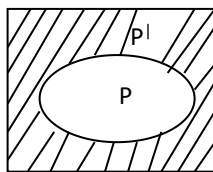
Venn diagram representation



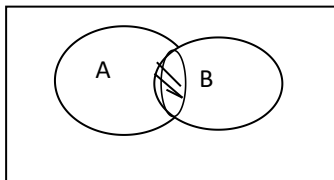
The rectangle represents the universal set i.e E or U



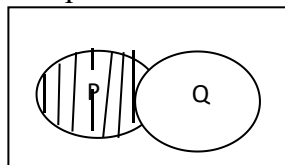
The oval shape represents the subset A.



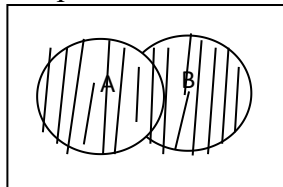
The shaded portion represents the complement of set P i.e P^c or P^c



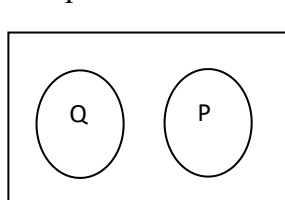
The shaded portion shows the elements common to A and B i.e $A \cap B$ or A intersection B.



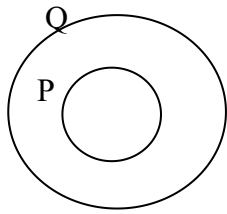
The shaded portion shows P intersection Q^c i.e $P \cap Q^c$



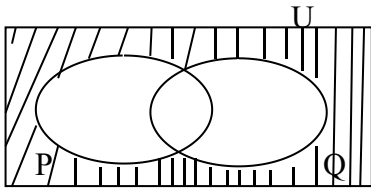
The shaded portion shows $A \cup B$ i.e A union B



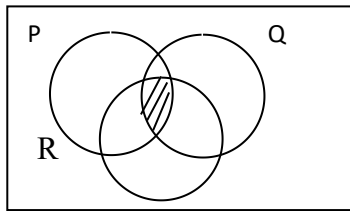
This shows that P and Q have no common element. i.e P and Q are disjoint sets i.e $P \cap Q = \Phi$



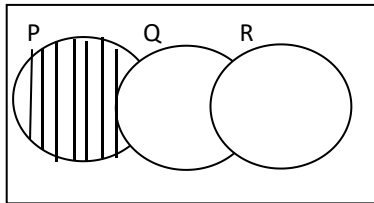
P is a subset of Q i.e $P \subset Q$



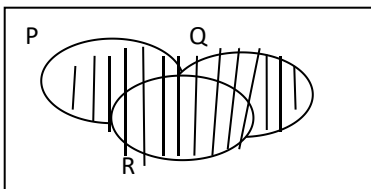
$P^c \cap Q^c$ or $(P \cup Q)^c$. This shows elements that are neither in P nor Q but are represented in the universal set.



This shows the elements common to set P, Q and R i.e the intersection of three sets P, Q and R i.e $P \cap Q \cap R$



This shows the elements in P only, but not in Q and R i.e $P \cap Q^c \cap R^c$



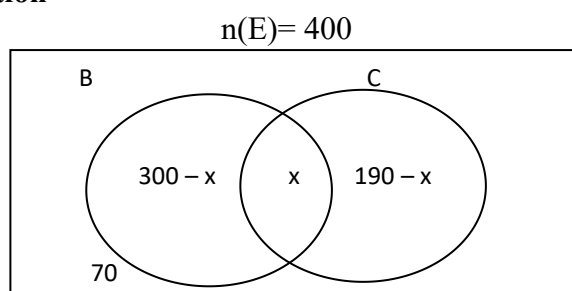
This shaded region shows the union of the two sets i.e $P \cup Q$

USING THE VENN DIAGRAM TO SOLVE PROBLEMS INVOLVING TWO SETS

Examples:

1. Out of 400 final year students in a secondary school, 300 are offering Biology and 190 are offering Chemistry. If only 70 students are offering neither Biology nor Chemistry. How many students are offering (i) both Biology and Chemistry? (ii) At least one of Biology or Chemistry?

Solution



Let the number of students who offered both Biology and Chemistry be x i.e $(B \cap C) = x$. From the information given in the question

$$\begin{aligned}n(E) &= 400 \\n(B) &= 300 \\n(C) &= 190 \\n(B \cap C)^c &= 70\end{aligned}$$

Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then:

$$\begin{aligned}300 - x + x + 190 - x + 70 &= 400 \\560 - x &= 400 \\-x &= 400 - 560 \\x &= 160\end{aligned}$$

Number of students offering both Biology and Chemistry = 160

(ii) Number of students offering at least one of Biology and Chemistry from the Venn diagram includes those who offered biology only, chemistry only and those who offered both i.e

$$\begin{aligned}300 - x + 190 - x + x &= 490 - x \\490 - 160 \text{ (from (i) above)} &= 330\end{aligned}$$

2. In a youth club with 94 members, 60 likes modern music and 50 likes traditional music. The number of them who like both traditional and modern music are three times those who do not like any type of music. How many members like only one type of music

Solution

Let the members who do not like any type of music = x

Then,

$$n(T \cap M) = 3x$$

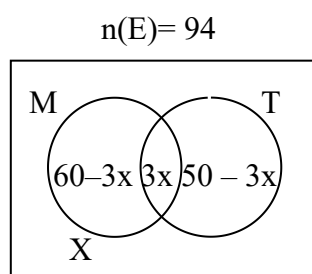
Also,

$$n(E) = 94$$

$$n(M) = 60$$

$$n(T) = 50$$

$$n(M \cup T)^c = x$$



Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then

$$\begin{aligned}60 - 3x + 3x + 50 - 3x + x &= 94 \\110 - 2x &= 94 \\16 &= 2x\end{aligned}$$

Divide both sides by 2

$$\frac{16}{2} = \frac{2x}{2}$$

$$x = 8$$

Therefore number of members who like only one type of music are those who like modern music only + those who like traditional music only.

$$\begin{aligned}60 - 3x + 50 - 3x \\110 - 6x\end{aligned}$$

$$= 110 - 6(8) = 110 - 48$$

$$= 62$$

EVALUATION

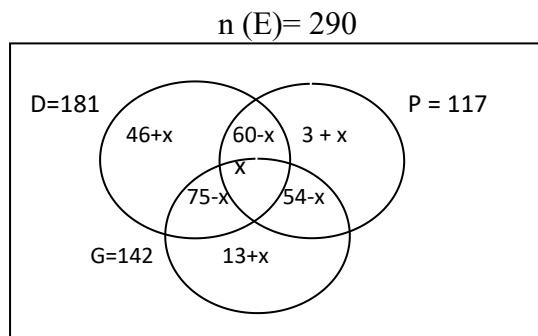
- Two questions A and B were given to 50 students as class work. 23 of them could answer question A but not B. 15 of them could answer B but not A. If $2x$ of them could answer none of the two questions and 2 could answer both questions.
 - Represent the information in a Venn diagram.
 - Find the value of x
- In a class of 50 pupils, 24 like oranges, 23 like apples and 7 like the two fruits.
 - How many do not like oranges and apples
 - What percentage of the class like apples only

USING VENN DIAGRAM TO SOLVE PROBLEMS INVOLVING THREE SETS

Examples:

- In a survey of 290 newspaper readers, 181 of them read the Daily Times, 142 read the Guardian, 117 read the Punch and each read at least one of the papers, If 75 read the Daily Times and the Guardian, 60 read the Daily Times and Punch and 54 read the Guardian and the Punch.
 - Draw a Venn diagram to illustrate the information
 - How many read:
 - all the three papers.
 - exactly two of the papers.
 - exactly one of the papers.
 - the Guardian only.

Solution



$$n(P) = 117$$

$$n(E) = 290$$

$$n(D) = 181$$

$$n(G) = 142$$

$$n(D \cap G) = 75$$

$$n(D \cap P) = 60$$

$$n(G \cap P) = 54$$

From the Venn diagram, readers who read Daily Times only

$$= 181 - (60 - x + 75 - x + x) = 181 - (135 - x) = 46 + x$$

$$\text{Punch readers only} = 117 - (60 - x + 54 - x + x) = 117 - (114 - x) = 117 - 114 + x = 3 + x$$

Guardian readers only

$$= 142 - (75 - x + 54 - x + x)$$

$$= 142 - (129 - x)$$

$$= 142 - 129 + x$$

$$= 13 + x$$

Where:

x is the number of readers who read all the three papers

Since the sum of the number of elements in all regions is equal to the total number of elements in the universal set, then:

$$46 + x + 75 - x + 13 + x + 60 - x + x + 54 - x + 3 + x = 290$$

$$251 + x = 290$$

$$x = 290 - 251$$

$$x = 39$$

b(i): number of people who read all the three papers = 39

(ii) from the Venn diagram, number of people who read exactly two papers

$$= 60 - x + 75 - x + 54 - x$$

$$= 189 - 3x = 189 - 3(39) \text{ from the above}$$

$$= 189 - 117 = 72$$

(iii) also, from the Venn diagram, number of people who read exactly only one of the papers

$$= 46 + x + 13 + x + 3 + x$$

$$= 62 + 3x = 62 + 3(39)$$

$$= 62 + 117 = 179$$

(iv) number of Guardian reader only

$$= 13 + x$$

$$= 13 + 39 = 52$$

2. A group of students were asked whether they like History, Science or Geography. Their responses are as follows:

Subject liked	Number of students
All three subjects	7
History and Geography	11
Geography and Science	09
History and Science	10
History only	20
Geography only	18
Science only	16
None of the three subjects	03

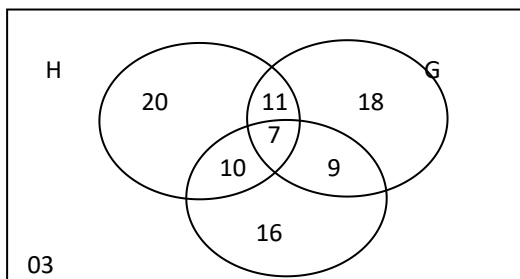
a) Represent the information in a Venn diagram

b) How many students were in the group?

c) How many students like exactly two subjects

Solution

a) $n(E) = ?$



b) Number of students in the group = sum of the elements in all the regions i.e

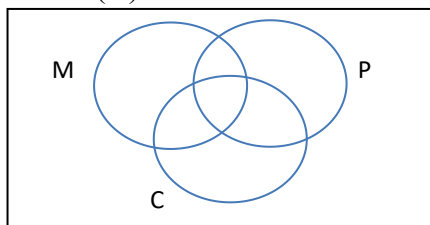
$$\text{Number of students in the group} = 20 + 18 + 16 + 11 + 9 + 10 + 7 + 3 = 94$$

c) Number of students who like exactly two subjects = $11 + 9 + 10 = 30$

Evaluation

- In a community of 160 people, 70 have cars, 82 have motorcycles, and 88 have bicycles: 20 have both cars and motorcycles, 25 have both cars and bicycles, while 42 have both motorcycles and bicycles. Each person rode on at least any of the vehicles
 - Draw a Venn diagram to illustrate the information.
 - Find the number of people that has both cars and bicycles.
 - How many people have either one of the three vehicles?

2. $N(U)$



The score of 144 candidates who registered for Mathematics, Physics and Chemistry in an examination in a town are represented in the Venn diagram above.

- How many candidate register for both Mathematics and Physics
- How many candidate register for both Mathematics and Physics only

GENERAL EVALUATION

- In a senior secondary school, 80 students play hockey or football. The numbers that play football is 5 more than twice the number that play hockey. If 5 students play both games and every students in the school plays at least one of the games. Find:
 - The number of students that play football
 - The number of students that play football but not hockey
 - The number of students that play hockey but not football
- A, B and C are subsets of the universal set U such that

$$U = \{0, 1, 2, 3, 4, \dots, 12\}$$

$$A = \{x: 0 \leq x < 7\} \quad B = \{4, 6, 8, 10, 12\} \quad C = \{1 < y < 8\} \text{ where } Y \text{ is a prime number.}$$

- Draw a venn diagram to illustrate the information
- Find (i) $B \cup C$ (ii) $A \cap B \cap C$

STUDY ASSIGNMENT

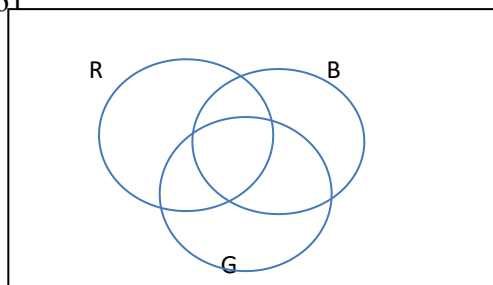
NGM SSS1, page 106, exercise 8d, numbers 11-17.

WEEKEND ASSIGNMENT

- In a class of 50 pupils, 24 like oranges, 23 like apples and 7 like the two fruits. How many students do not like oranges and apples? (a) 7 (b) 6 (c) 10 (d) 15
- In a survey of 55 pupils in a certain private school, 34 like biscuits, 26 like sweets and 5 of them like none. How many pupils like both biscuits and sweet? (a) 5 (b) 7 (c) 9 (d) 10
- In a class of 40 students, 25 speaks Hausa, 16 speaks Igbo, 21 speaks Yoruba and each of the students speaks at least one of the three languages. If 8 speaks Hausa and Igbo, 11 speaks Hausa and Yoruba, 6 speaks Igbo and Yoruba. How many students speak the three languages? (a) 3 (b) 4 (c) 5 (d) 6

Use the information to answer question 4 and 5

$N(U) = 61$



The Venn diagram above shows the food items purchased by 85 people that visited a store in one week. Food items purchased from the store were rice, beans and garri.

4. How many of them purchased garri only? (a)8 (b)10 (c) 14 (d)12
5. How many of them purchased the three food items? (a) 5 (b)7 (c) 9 (d)11

THEORY

1. In a certain class, 22 pupils take one or more of Chemistry, Economics and Government. 12 take Economics (E), 8 take Government (G) and 7 take Chemistry (C). nobody takes Economics and Chemistry and 4 pupils take Economics and Government
 - a) Using set notation and the letters indicated above, write down the two statements in the last sentence.
 - b) Draw the Venn diagram to illustrate the information
2. How many pupils take
 - a) Both Chemistry and Government?
 - b) Government only?

WEEK 3 TOPIC: INDICES

Learning Objectives: Students should be able to:

- i. solve problems on standard form.
- ii. use the standard notation of indices appropriately.
- iii. identify indices as a shorthand notation of the standard form.
- iv. solve problems of indices equations applying the laws of indices.

INDICES: are numbers expressed in powers on ten i.e. 2^5 . The analysis and simplification of indices depends on the basic interpretation and rules of indices as enumerated below.

LAWS OF INDICES

1. $a^M \times a^N = a^{M+N}$
2. $a^M \div a^N = a^{\frac{M}{N}}$
3. $a^0 = 1$
4. $a^{\frac{M}{N}} = (\sqrt[N]{a})^M$
5. $(a^M)^N = a^{M \times N} = a^{MN}$
6. $a^{-M} = \frac{1}{a^M}$

EXAMPLES:

Write down the values of the following in index form:

- (i) $7^6 \times 7^2$ (ii) $5X^2 \times 4X^0 \times 2X^{-6}$ (iii) $16r^{11} \div 4r^7$ (iv) $(\frac{8}{27})^{-\frac{2}{3}}$

Solution

(I) $7^6 \times 7^2 = 7^{6+2} = 7^8$

(II) $5X^2 \times 4X^0 \times 2X^{-6} = (5 \times 4 \times 2) X^{2+0+(-6)} = 40X^{-4} = \frac{40}{X^4}$

(III) $16r^{11} \div 4r^7 = (16 \div 4)r^{11-7} = 4r^4$

(IV) $(2^3)^5 = 2^{3 \times 5} = 2^{15}$

(V) $(\frac{8}{27})^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{\frac{8}{27}})^2} = \frac{1}{(\frac{2}{3})^2} = 1 \div \frac{4}{9} = 1 \times \frac{9}{4} = \frac{9}{4} = 2.25 \text{ or } 2\frac{1}{4}$

Simplify the following:

(a) $\sqrt{\frac{25X^4}{16}} \times (\frac{X}{2})^{-3}$ (b) $3X^{\frac{1}{3}}Y^{\frac{1}{2}} \div 6X^{\frac{1}{6}}Y^{\frac{3}{4}}$

Solution

$$(a) \sqrt{\frac{25X^4}{16}} \times \frac{1}{\left(\frac{X}{2}\right)^3} = \frac{5X^2}{4} \times \frac{8}{X^3} = \frac{10}{X}$$

$$(b) (3 \div 6) X^{\frac{1}{3} + \frac{1}{6}} Y^{\frac{1}{2} + \frac{3}{4}} = \left(\frac{1}{2}\right) X^{\frac{2-1}{6}} Y^{\frac{2-3}{4}} = \frac{X^{\frac{1}{6}}}{2Y^{\frac{-1}{4}}}$$

Evaluation

Simplify the following questions:

$$(1) (27)^{-\frac{2}{3}} (2)^3 \sqrt[3]{27a^{-9}} (3). 27^{m+2} \times 4^{m \div 6} 2^{2m} \quad (4) -10a^2b^3 \div (-5a^{\frac{3}{2}}b^{\frac{-1}{2}}) \quad (5) 125^{-\frac{2}{3}} \times 64^{-\frac{1}{4}} \times 25^0 \quad (6) \sqrt[3]{\frac{27Y^6}{(36)^{\frac{1}{2}}}}$$

ASSIGNMENT: See New General Mathematics SS 1

Exercise 1d numbers 11-13

Exercise 1e numbers 4-5

WEEK 4

LOGARITHMS OF WHOLE NUMBERS

Learning Objectives: Students should be able to:

- define logarithm
- deduce a relationship between indices and logarithm
- use the graph of $y = 10^x$ for multiplication and division
- find the logarithms and antilogarithm of numbers greater than one.
- use logarithm tables in calculation.
- use logarithm table to solve problems relating to capital market and other real life problems.

The logarithms of any number N to any base M is the index or power to which the base must be raised, to equal the number N.

The logarithms of any given number consist of two parts called the characteristics and the mantissa. The characteristic is a whole number which can either be positive, zero or negative integers, While the Mantissa is the decimal (fractional) part of the integers always from the table values.

EXAMPLE

$$399 = 2.6010. \quad 2 \text{ Is the characteristics of the number and } 6010 \text{ from table is the Mantissa or } 3.99 \times 10^{2.6010}$$

1. Find the Logarithms of the following numbers:

$$(a) 8615 \quad (b) 690460 \quad (c) 1.607$$

Solution

$$(a) 8615 = 8.615 \times 10^{3.9353} : \text{ in mathematics table, check logarithm of 86 under 1 difference } 5 = 9350 + 3$$

$$(b) 690460 = 6.90460 \times 10^{5.8391}$$

$$(c) 1.607 = 1.607 \times 10^{0.2059}$$

ANTILOGARITHM: Is the opposite of logarithm.

2. Find the original number of the following logarithms numbers:

$$(a) 10^{0.27} \quad (b) 10^{3.568} \quad (c) 6.3892$$

Solution

$$(a) 10^{0.27} = 1.862, \text{ from antilogarithm table check 27 under zero since there is no third value and the zero before the point (characteristics) determines where the point occupies in the number. Add onto every positive characteristics to determine your value}$$

$$(b) 10^{3.568} = 3698.0 \text{ or } 3698$$

$$(c) 6.3892 = 2450000.0$$

MULTIPLICATION OF NUMBERS

When multiplying numbers in logarithms, their table values are been added before checking antilogarithms for its solutions.

EXAMPLE

Evaluate the following using table:

(a) 143.8×23.46 (b) 8234×70000

Solution

(a) $143.8 \times 23.46 =$	NO	LOG
	143.8	$10^{2.1577}$
	23.46	+ $10^{1.3703}$
		$10^{3.5280}$

Antilogarithm of 5280 = 3374

$143.8 \times 23.46 = 3374.0$

(b) $8234 \times 70000 =$	NO	LOG
	8234	$10^{3.9158}$
	70000	+ $10^{4.8451}$
		$10^{8.7609}$

Antilog of 7609 = 5766 characteristics is 8+1=9 numbers before point

$8234 \times 70000 = 576600000$

DIVISION OF NUMBERS IN LOGARITHMS: When dividing numbers in logarithms we subtract their values

EXAMPLE

Evaluate the following numbers using table:

(a) $912.4 \div 30.42$ (b) $36.75 \times 284.7 \div 26.45$

SOLUTION

(a) $912.4 \div 30.42 =$	NO	LOG
	912.4	$10^{2.9602}$
	30.42	- $10^{1.4832}$
		$10^{2.9602-1.4832} = 10^{1.4770}$

Antilog of 4770 = 2999

$912.4 \div 30.42 = 29.99$.

(b) $36.75 \times 284.7 \div 26.45 =$

	NO	LOG
	36.75	$10^{1.5653}$
	284.7	+ $10^{2.4544}$
	$10^{1.5653+2.4544} =$	$10^{4.0197}$
	26.45	- $10^{1.4224}$
	$10^{4.0197-1.4224} =$	$10^{2.5973}$

Antilog of 5973 = 3957

$36.75 \times 284.7 \div 26.45 = 395.7$

ASSESSMENT: Using table evaluate the following numbers:

1 (a) 497.2×8.789 (b) $89 \times 34.56 \times 2.094$ (c) $8050 \div 20.15$ (d) $45.08 \div 5.462$

2 (a) $98.45 \times 56 \div 30.8$ (b) $\frac{35.26 \times 106.4}{786.5}$ (c) $\frac{78.34 \times 5.60104}{8.236}$

3. Find the antilogarithms of the following numbers:

(a) $10^{2.3900}$ (b) $10^{4.3096}$ (c) 0.5971 (d) 7.8903 (e) 2.0079

WEEK 7 NUMBER BASES

Learning Objectives: Students should be able to:

- convert numbers from other bases to base 10 and vice versa
- convert decimal fraction from other bases to base 10 and vice versa
- Convert from one base to another base
- perform some basic operations on number bases.
- apply number base system to computer programming.

Base number is the basis of which each place value column in a number system or the classification of numbers to which one or more other numbers are appended or added.

TYPES OF BASE NUMBERS.

OCTAL BASE; Octal base are numbers express in base eight. E.g. 257_8

DENARY/DECIMAL BASE: These are numbers express in base ten. E.g. 189_{ten}

BINARY: These are numbers express in base two. E.g. 11001_{two}

BICIMAL: This is the fractional binary number or fraction in base two. E.g. $(\frac{11}{10})$, $0.10101\dots$ fractions in base two.

DUODECIMAL BASE: This is the number system that is expressed in base 12.

HEXADECIMAL: Is system of numbers which is express in base 16. I.e base 2,3,4,5,6,7,8,9,A,B,C,D,E,F.

HINT: No number must be equal or greater than the base number in operation. If you are working in base two, the highest digit will be 1 and the lowest number is 0

EXPRESSION NUMBERS IN BASE TEN.

$450 = 4 \times 10^2 + 5 \times 10^1 + 0 \times 10^0$ in base ten.

CONVERSION OF NUMBERS TO BASE TEN

EXAMPLE 1:

Convert the following numbers to denary base:

a. 1011111_2 b. 432_5 c. 431_x .

Solution

$$\begin{aligned} \text{a. } 1011111_2 &= 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 64 + 0 \times 32 + 1 \times 16 + 1 \times 8 + 1 \times 4 + 1 \times 2 + 1 \times 1 \\ &= 64 + 0 + 16 + 8 + 4 + 2 + 1 \\ &= 95_{\text{ten}} \end{aligned}$$

$$\begin{aligned} \text{b. } 432_5 &= 4 \times 5^2 + 3 \times 5^1 + 2 \times 5^0 \\ &= 4 \times 25 + 3 \times 5 + 2 \times 1 \\ &= 100 + 15 + 2 \\ &= 117_{10}. \end{aligned}$$

$$\begin{aligned} \text{c. } 431_x &= 4 \times x^2 + 3 \times x^1 + 1 \times x^0 \\ &= 4x^2 + 3x + 1 \text{ (solve using any quadratic formula)} \end{aligned}$$

CONVERSION OF BASE NUMBERS FROM BASE TEN TO ANOTHER BASE.

1. Express the following base ten numbers to each base giving:

- 1007 to i. octal base ii. Binary base.
- 761 to (i). Base 12 (ii). Base 16

CONVERSION FROM ONE BASE TO ANOTHER

HINT: First express the number to base ten and then convert from base ten to the required base.

EXAMPLE 2:

Express 313_6 to octal base

Solution

$$\begin{aligned}
 313_6 &= 3 \times 6^2 + 1 \times 6^1 + 3 \times 6^0 \\
 &= 3 \times 36 + 1 \times 6 + 3 \times 1 \\
 &= 108 + 6 + 3 = 117_{10} \\
 117 \text{ base ten to Octal base } 8
 \end{aligned}$$

	117
8	14 r 5
8	1 r 6
8	0 r 1

Therefore 313_6 to octal base = 165_8

FRACTIONAL BASE NUMBER

EXAMPLE 3: Convert 1011.01_{two} to denary base.

Solution

$$\begin{aligned}
 1011.01_2 &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 0 \times 2^{-1} + 1 \times 2^{-2} \\
 &= 1 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1 + 1 \times \frac{1}{2} + 1 \times \frac{1}{2^2} \\
 &= 8 + 0 + 2 + 1 + \frac{1}{2} + \frac{1}{4} \\
 &= 11 \frac{3}{4}
 \end{aligned}$$

EXAMPLE:

Express $\frac{2}{3}$ as bicimal number.

Solution

$$\left(\frac{2}{3}\right)_{\text{ten}} = \left(\frac{10}{11}\right)_{\text{two}} = 0.10101010\dots$$

ASSESSMENT: Students should work the following questions

- Express the following base numbers to base ten.
 - 312.21_4
 - 1051.11_8
 - 23411_6
- Convert the following base ten numbers to bicimals:
 - $\frac{2}{5}$
 - $\frac{9}{20}$
 - $\frac{3}{4}$
 - $\frac{6}{7}$
 - $\frac{5}{6}$
- Convert the following to base; I. Base 5 ii. Base 12. iii. Base 15
 - 56_8
 - 124_7
 - 10001_2
 - 121103_4

RULES OF BASE NUMBER

- Numbers must not be equal to or greater than the base number under consideration.
- Base numbers of the same base can be added, subtracted, multiplied and divided otherwise it must first be converted to base ten or equal base before the required operation is done.
- When subtracting base numbers, the number carried from nearby to support the other becomes the base in operation added to the original number in that position.

BASIC OPERATIONS OF BASE NUMBER.

EXAMPLE 1

- (A) Find the sum of the octal numbers 174 and 233. (B) Simplify $2311_4 - 213_4$.
 (C) find the product of 214 and 23 both in base five (D). if $104_x = 68$, find the value of x?

Solution

$$\begin{array}{r}
 A \quad 1 \ 7 \ 4 \\
 + \ 2 \ 3 \ 3_8 \\
 \hline
 4 \ 2 \ 7_8
 \end{array}$$

$$\begin{array}{r}
 B. \quad 2 \ 3 \ 1 \ 1 \\
 - \quad 2 \ 1 \ 3 \\
 \hline
 2 \ 0 \ 3 \ 3_4
 \end{array}$$

$$\begin{array}{r}
 C. \quad 2 \ 1 \ 4 \\
 \times \ 2 \ 3_5 \\
 \hline
 1 \ 2 \ 0 \ 2 \\
 + \ 4 \ 3 \ 3 \\
 \hline
 1 \ 1 \ 0 \ 3 \ 2_5
 \end{array}$$

$$D. 104_x = 68$$

$$1 \times x^2 + 0 \times x^1 + 4 \times x^0 = 68$$

$$x^2 + 0 + 4 = 68$$

$$x^2 = 68 - 4 \quad : \quad x^2 = 64$$

$$x = \pm\sqrt{64} \quad : \quad x = \pm 8$$

APPLICATION OF BASE NUMBER TO COMPUTER PROGRAMMING

In computer programming the punched cards uses the binary numbers instead of the letters.

A = 1. B = 2. C = 3. D = 4. E = 5. F = 6. P = 16. U = 21. Z = 26. The binary equivalent of the number code of letters in binary, such as:

A = 00001, B = 00010, C = 00011, p = 10000, Z = 11010.

Yes = 1 and No = 0

EVALUATION: The students are to do the following questions:

1. If $4103_{\text{six}} = 2112_{\text{six}} + x_{\text{six}}$. Find x?
2. Simplify the following number bases:
- i. $11011_{\text{two}} \times 101_{\text{two}}$ ii. $614_8 - 506_8$ iii. If $123_y = 83$, find y?
3. Represent I LOVE MATHEMATICS in binary code.

ASSIGNMENT: 1. Simplify $(4001_5 - 2304_5) + 234_5$. 2. If $452_{\text{six}} - 114_{\text{six}} = Z_{\text{three}}$, what is the value of Z?

WEEK 8

MODULAR ARITHMETIC

LEARNING OBJECTIVES: Students should be able to:

- i. define the term modular arithmetic.
- ii. perform some basic arithmetic operations of addition, subtraction, multiplication, and division.
- iii. determine the residues and equivalent classes mod n.
- iv. simplify and solve simple modular algebraic expressions and equations.
- v. apply modular arithmetic in everyday life.

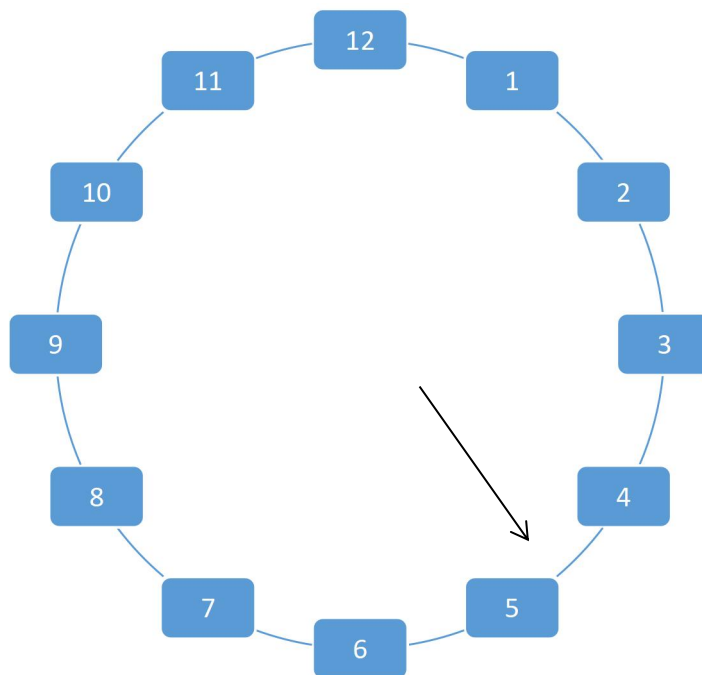
Modular arithmetic is a branch of Mathematics use to predict the outcomes of cyclic events such as days of the week, market days, months of the year, time etc.

RULES OF MODULAR ARITHMETIC

- The modulo value must be greater than the number worked upon.
- When using cyclic pattern in adding numbers, you must count clock wise direction.
- In subtraction of numbers , you must count anti -clock wise direction

EXAMPLE:

The shorter hand of a clock points to 5 on a clock face. What number does it point to after 30 hours?



Solution

➤ In subtraction of numbers , you must count anti -clock wise direction

EXAMPLE:

The shorter hand of a clock points to 5 on a clock face. What number does it point to after 30 hours?

a. 15 b. 102

Solution

$$15(\text{mod}4) = 15 \div 4$$

3 remainder 3, therefore 3 the remainder is taken as 3 mod 4

$$102 \text{mod} 4 = 102 \div 4 = 25 \text{ remainder } 2$$

$$102 (\text{mod } 4) = 2 \text{ mod } 4$$

ADDITION OF MODULO ARITHMETIC

EXAMPLE

Find the following modulo addition

a. $42 \oplus 28 (\text{mod } 8)$ b. $54 \oplus 25 (\text{mod } 5)$

Solution

a. $42 \oplus 28 = 70 \text{ mod } 8$

Solution

➤ In subtraction of numbers , you must count in anti -clock wise direction

$$70 \text{ mod } 8 = 6 \text{ mod } 8$$

b. $54 \oplus 25 = 79 \text{ mod } 5$

$$79 \text{ mod } 5 = 4 \text{ mod } 5$$

SUBTRACTION OF MODULO NUMBERS

Find the simplest form of the following in their giving moduli.

a. $-5 \text{ mod } 6$ b. $-17 \text{ mod } 10$ c. $-75 \text{ mod } 7$

Solution

a. $-5 \text{ mod } 6 = -6 \times 1 + 1 = 1 \text{ mod } 6$ the value added to the negative number to give the require result becomes your result.

b. $-17 \text{ mod } 10 = -10 \times 2 + 3 = 3 \text{ mod } 10.$

c. $-75 \text{ mod } 7 = -7 \times 11 + 2 = 2 \text{ mod } 7.$

MULTIPLICATION OF MODULO NUMBERS

Evaluate the following in their moduli.

$$16 \otimes 7 \pmod{5} \quad \text{b.} \quad 21 \otimes 65 \pmod{4}$$

Solution

A $16 \times 7 = 112 \pmod{5}$, which is $2 \pmod{5}$

$$\text{a.} \quad 21 \times 65 = 1365 \pmod{4} = 1 \pmod{4}$$

EQUATION OF MODULO

Solve the following equations in their giving moduli

$$\text{a.} \quad 3x = 5 \pmod{7} \quad \text{b.} \quad 2x + 3 = 1 \pmod{6} \quad \text{c.}$$

SOLUTION

$$\text{a.} \quad 3x = 5 + 7, \therefore 3x = 12$$

$$X = 4 \pmod{7}$$

$$\text{b.} \quad 2x + 3 = 1 + 6, \quad 2x + 3 = 7$$

$$2x = 7 - 3, \text{ then } 2x = 4$$

$$X = 2 \pmod{6}$$

Assignment: Solve the following questions;

1. Use the cycle number in modulo 6 to simplify the following.

$$\text{(i)} \quad 2 + 10 \quad \text{(ii).} \quad 5 + 5 \quad \text{(iii)} \quad 15 + 37 \quad \text{(iv)} \quad 2 - 9 \quad \text{(v)} \quad 0 - 22$$

2. A toy car starts at a point 0 and runs around a circular track of 2 meters. How far is the car from its starting point along the track when it has gone :

$$\text{(a)} \quad 6\text{m} \quad \text{(b)} \quad 15\text{m} \quad \text{(c)} \quad 21\text{m} \quad \text{(d)} \quad 87\text{m}$$

3. Find the following numbers in their simplest form in modulo 4:

$$\text{(i).} \quad 62 \quad \text{(ii).} \quad 102 \quad \text{(iii)} \quad -56 \quad \text{(iv)} \quad -78 \quad \text{(v)} \quad -202$$

4. Solve the following equations in the set of positive integers of each modular arithmetic:

$$\text{i.} \quad 3X + 4 = 7 \pmod{8} \quad \text{ii.} \quad 4X - 3 = 6 \pmod{7} \quad \text{iii.} \quad X^2 - 2X + 2 = 0 \pmod{5}$$

$$\text{iv.} \quad 3^x = 2 \text{ in (a). Mod 5 (b). Mod 6 (c). mod 9}$$

NB: Some modular arithmetic operation symbols were omitted here.

WEEK 9

EVERY DAY ARITHMETIC

LEARNING OBJECTIVE: Students will acquire skills necessary in solving problems met in Mathematics and everyday life.

RATIO (REVISION)

Examples

$$1. \text{ If } 57:95=12:x, \text{ evaluate } x.$$

Solution:

$$\text{If } 57:95=12:x, \text{ then } \frac{57}{95} = \frac{12}{x}$$

Clear fractions

$$57x = 12 \times 95$$

$$x = \frac{12 \times 95}{57} = 20$$

2. The selling price of a second hand car is originally ₦273 000.00. The dealer reduces the price in the ratio 11:13. What is the new selling price of the car?

Solution:

Let the new selling price be ₦x then,

$$\frac{x}{273\,000} = \frac{11}{13}$$

X = ₦231 000.00 is the new price

3. Find which ratio is greater, 7:13 or 8:15

Ans: 7:13

4. Express the ratio 8:13 in the form 1:n

Ans 1:1.625

5. A plan is made of a school compound. The length of a laboratory, 15.6m is represented on the plan by a line 7.8cm long. Find the scale of the plan in the form 1:n

Ans 1: 200

Assignment: New General Mathematics SS1 Page 163 Exercise 13a,b

RATE:

Ratios compare quantities which are of the same kind. For example 4kg:7kg, 1cm:5km etc while Rates compare quantities of different kinds for example speed (distance and time) and density (mass and volume)

Further examples:

a. A worker is paid ₦3 360 for an 8 hour day. Her rate of pay is ₦420 per hour

b. A cyclist travels 28km in 2 hours. His rate is 14km per hour. In this case, the rate is called speed.

c. A 4 metre beam, of uniform cross-section and mass 120kg has a mass of 30 kg per metre. This rate gives the mass per unit length

d. A piece of metal has a volume of 20 cm^3 and a mass of 180 g. Its density is 9g/cm^3 . The density of gases, liquids and solids is the rate giving the mass per unit volume.

e. A town of 32 000 people has an area of 40 km^2 . The population density of the town is 800 people/ km^2 . Population density is a rate giving the average number of people per unit of area.

Example 1: Find in km/h, the rate at which a car travels if it goes $38\frac{1}{2}\text{ km}$ in 35 min.

Ans: 66 km/h

2. A village is roughly square in shape. Its perimeter is about 6km. If the population density of the village is 1 200 people/ km^2 , find the approximate population of the village

Ans: 2 700 people

Exercise 13 c, page 164

Proportional division:

This involves dividing a quantity into a number of equal shares then distributing the shares in a given ratio. For example: If a sum of money is to be divided in the ratio 3:8:13, then it will be divided into 24 shares (3+8+13), then distributed in 3 shares, 8 shares and 13 shares.

Example 1: Divide ₦1 170.00 between Bola and Ola, so that their shares are in the ratio 8:5

Ans: Bola receives ₦720.00 while Ola receives ₦450.00

Check ₦720.00 + ₦450.00 = ₦1 170.00

2. Divide 299 into three parts in the ratio $\frac{1}{3}:\frac{5}{6}:\frac{3}{4}$

Ans: 52:130:117

3. X,Y,Z shared ₦85.00 so that for every ₦1.00 that X gets, Y gets ₦3.00 and for every ₦2.00 that Y gets, Z gets ₦3.00. Find Y's share.

Ans: ₦30.00

4. A and B invest money in a business. A invests ₦525 000.00 for 4 months and B invests ₦900 000.00 for 3 months. How should they share the first year's profits of ₦272 000.00?

Ans: Find the investment of each

A's investment = ₦525 000.00 \times 4 months = ₦2 100 000.00

B's investment = ₦900 000.00 \times 3 months = ₦270 000.00

Their investments are in the ratio 7:9=16

Sharing the profit:

A's share: ₦119 000.00 and B's share: ₦153 000.00

Assignment: Exercise 13d Page 166 NGM SS 1

WEEKS 10 & 11

SIMPLE EQUATION AND VARIATION

LEARNING OBJECTIVES: Students should be able to:

- solve problems involving linear equations involving brackets and fractions, word problems leading to linear equations, substitute in linear equations
- Change of subject of formula involving brackets, roots and powers; subject of formula and substitution

SIMPLE EQUATION: is any algebraic equation with one unknown.

EXAMPLE

1. Solve for p in the equation $p - 7 = 24$

Solution

If $P - 7 = 24$, then add 7 to both sides of the equation

$$P - 7 + 7 = 24 + 7$$

$$P = 31$$

2. Solve the equation $5(c + 2) - 3(3c - 5) = 1$

Solution

$5c + 10 - 9c + 15 = 1$. First open the bracket, collect like terms and simplify.

$$5c - 9c + 10 + 15 = 1$$

$$-4c + 25 = 1, \text{ subtract 25 from both sides of the equation}$$

$$-4c = 1 - 25,$$

$$-4c = -24, \text{ divide -4 by both sides}$$

$$C = 6.$$

3. Solve the equation $1\frac{1}{2}y = 9$

4. Solve $a - 10\frac{1}{2} = 10\frac{1}{2} - \frac{2}{5}a$

5. Solve $8b - (3b + 4) = 11$

6. Solve $4(3x - 1) = 11x - 3(x - 4)$

7. Solve $3x - [3(1 + x) - 2x] = 3$

8. Solve $\frac{1}{5}x - \frac{1}{7}x = \frac{7}{15}$

9. Solve $\frac{4n+1}{3} - 1\frac{1}{2} = \frac{2n+5}{6}$

10. A trader buys n oranges at the rate of 5 oranges for ₦90.00. Eight of the oranges are bad. So she sells the rest at the rate of 4 oranges for ₦120.00 and makes a profit of ₦900.00. Find n.

CHANGE OF SUBJECT OF FORMULAE

A formula is an equation consisting of letters which represent quantities.

EXAMPLE

Make each of the following letters giving the subject of formula:

(a) $A = ax + b$, x (b) $T = a + (n-1)d$, a (c) $T = 2\pi\sqrt{\frac{l}{g}}$, g

Solution

(a) $A = ax + b$. make x the subject of formula

Subtract b from both sides

$A - b = ax$ divide both sides by a

$$\frac{A-b}{a} = x$$

(b) $T = a + (n-1)d$, a . subtract $(n-1)d$ from both sides

$$T - (n-1)d = a \text{ or } a = T - nd + d$$

(c) $T = 2\pi\sqrt{\frac{l}{g}}$, g Divide both sides by 2π

$$\frac{T^2}{4\pi^2} = \frac{l}{g}, \text{ cross multiply}$$

$T^2g = 4\pi^2l$, divide both sides by T^2

$$g = \frac{4\pi^2l}{T^2}$$

Evaluation:

1. Make the given letters the subject of the formula of the following equations:

(a) $\frac{x}{y} = \frac{P+3Q}{Q-3P}$, Q (b) $A = \sqrt{gd(1 + \frac{3h}{d})}$, h, d (c) $A = \pi r \sqrt{h^2 - r^2}$, r

2. Solve the following equations:

(a) $8y - 19 = 5 + 3y$ (b) $12 - 3t - 9 = 3 - 5t$ (c) $2 = 5(5w - 2) - 9(3w - 2)$ (d) $\frac{2x}{3} + \frac{x}{4} = 6$ (e) $\frac{2(5z-3)}{3} -$

$$\frac{3(5z-2)}{5} = \frac{8}{15}$$

Assignment: Exercise 6a, b, c, d, e, f, g, h New General Mathematics SS 1 Page 77

Revise for Examination