

Quantum Neural Network for Continuous Reinforcement Learning

Erik Sorensen

Data Science Honors Project Proposal

Introduction

Quantum computing and machine learning are two exciting fields. Quantum computing brings a new perspective on computers by using properties of quantum mechanics to do computations, which can be much faster than the classical computers we know today. Not only are they faster, but some tasks that we thought were impossible with the computers we know today, can be accomplished using quantum computers. (CITATION) Machine learning learns complex problems from large amounts of data that are too difficult for humans to manually solve. It has been used in facial recognition (CITATION) and identifying weeds in precision agriculture (CITATION). A more recent sub-field of machine learning is *reinforcement learning*. Reinforcement learning uses a positive or negative reinforcement signal from an environment, known as the *reward*, to provide feedback to the learning system so that it may learn and improve to maximize this reward. Some areas that use reinforcement learning are in economics and artificial intelligence in games. (CITATION) Reinforcement learning is an attractive tool to use. While other machine learning techniques often times require a well tuned, large dataset to learn, reinforcement learning only requires a reward from an environment and time to train. Even further, using a quantum computer to do reinforcement learning has the potential to solve problems that would be too computationally complex for classical computers to solve. This increases the scope of problems we can solve exponentially.

Reinforcement learning

Reinforcement learning may be applied to many fields. As an example, I will use a vacuuming robot to show a specific use case of how reinforcement learning can be applied in the world. Lets imagine if this robot used reinforcement learning techniques. The robots main goal is to vacuum an *environment*, while avoiding furniture and other obstacles.

With reinforcement learning, the robot can self-teach itself by exploring the room. The only feedback the robot gets is *observations* about its surroundings via sensors, and a *reward* signal that indicates how well the robot is doing at its job. Every action the robot takes it will receive either a positive reward signal for good actions or a negative reward signal for bad actions. The robots goal then is to maximize this positive reward signal so that it knows it is doing a good job. The part of the robot that decides the next *action* it should take, whether it be going left or right or going back to charge, we will call the *agent*.

How then can the robot learn to efficiently do this task with these tools it has at its disposal? This problem is called the reinforcement problem [1], which gives the instructions for most reinforcement learning algorithms. It is stated here:

For an agent in a certain state at a certain time, what action should be taken that will maximize the overall reward, now and in the future?

To summarize, the goal of reinforcement learning is to find a *policy* which maximizes the *reward*. A *policy* (π) is a probability distribution that describes the probability the agent will take a certain action given it is in a certain state. The *optimal policy* is a policy that on average receives the greatest total reward. This is useful, especially in terms of the vacuuming robot who does not know anything about the environment it has been placed in. By defining the rewards the robot will search toward, it will learn itself how to best maximize this reward in the environment. Since we defined the robots reward as avoiding furniture in the environment, it

will learn the best paths it can take to avoid these obstacles. All methods of reinforcement learning have this same goal, which is to find the optimal policy to maximize reward.

Value Function and Action-Value Function

One method for discovering the optimal policy is to estimate the value of reward the agent receives in each state. Once this value is known, the best action an agent can take is the one that will accumulate the highest value across all future states. This method is called value iteration which returns a prediction of the expected accumulative, discounted, future reward, measuring how good each state is [2]. The expected total reward is called the return. The value function is described by

$$V(s) = E\{R_t | s_t = s\}$$

where E is expected value operator, R_t is the total return that an agent receives given state s_t at time t , and s is the current state that the agent is in. Another solution to finding the optimal policy is called Q-learning. Q-learning expands on the value function where instead of determining the value of being in a certain state, it maximizes the reward over infinitely many successive steps, starting from the current state. The action-value function is given by

$$Q(s, a) = E\{r_{t+1} + \gamma V(s_{t+1}) | s_t = s, a_t = a\}$$

where $r_t + 1$ is the reward from moving into state $s_t + 1$ from s_t by taking action a [3]. The action-value function has many benefits for learning, one of which is that it integrates well with *online learning*. Online learning algorithms are executed on data acquired in sequence [2]. The agent will use the current estimate of the optimal policy while exploring the environment in search of a better estimate of the optimal policy.

Q-learning is a famous reinforcement learning algorithm that uses the action-value function to converge on the optimal policy iteratively, meaning that after each step it obtains a better estimate of the action-value and will eventually converge on the true optimal policy values for each step [3]. The formula for Q-learning is given by

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_t + \gamma \max_a Q(s_{t+1}, a) - Q(s_t, a_t)]$$

[4], where $Q(s_t, a_t)$ is the *action value*, s_{t+1} is the state moved to out of s_t by taking action a_t , r_t is the reward from this transition, α is the learning rate, and γ is the discount rate which describes the discount rate of future rewards.

Using the action-value function has two main advantages over the value function for learning. Firstly, it is easier to learn the action-value function to determine the optimal policy. Second, it is a simple task to evaluate policies with the action-value function because the next action of the agent is always determined by the best action that maximizes the action-value function (i.e. $a = \operatorname{argmax}_a' Q(s, a')$).

Deep Reinforcement learning

Neural Networks

Often, the action-value function $Q(s, a)$ is represented as a table, where each state-action pair s, a maps to a particular reward value in the table. Generally, this table can get quite large with big environment spaces, sometimes they can be nearly *continuous* in size. In these scenarios, Q-learning becomes impractical because the table becomes much too large to converge on the optimal policy in a reasonable amount of time. Instead of this approach, we can try to approximate a function $f(s, a)$ of $Q(s, a)$ so that instead of directly learning each value in the table, we can learn the *parameters* of this function instead.

To do this, we can use *function approximation*. Function approximation methods expect to receive examples of the desired input-output behavior of the function they are trying to approximate. We use these methods for

value prediction simply by passing to them the s, a of each update as a training example. We then interpret the approximate function they produce as an estimated value or action-value function [5].

Once we frame the problem as a function approximation problem, we can use *neural networks* to learn the approximation of these functions. Neural networks are a computer learning algorithm that is modeled after a simplified version of biological neurons that are grouped into *layers*, where each layer is the result of previous layers multiplied by a *weight*. Neural networks must be trained by a technique called *supervised learning*, which is a machine learning task of learning a function that maps an input to an output based on example input-output pairs collected from an environment. Usually, supervised learning is done for classification problems, but neural networks may be applied to other problems by carefully selecting an *activation function*. An activation function is a function that describes the output of a neuron. Usually, activation functions compresses the outputs of neurons into a sigmoidal shape within a certain range, similarly to how biological neurons are activated.

Q-Networks

When Neural Networks are applied to Q-learning they are called Q-Networks [6]. We can rewrite the update formula for Q-learning written previously to frame it like a supervised learning problem for Q-Networks. If we let $y_t = r_t + \gamma \max_a Q(s_{t+1}, a)$, then

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[y_t - Q(s_t, a_t)].$$

The last thing we need is instructions for how the Q-Network can learn. The quantity $\nabla_Q L = -[y_t - Q(s_t, a_t)]$ can be obtained by taking the gradient of the quadratic loss function L with respect to $Q(s_t, a_t)$:

$$L \equiv \frac{1}{2}[y_t - Q(s_t, a_t)]^2.$$

The update formula now adjusts $Q(s, a)$ to minimize the value of L . This is called *gradient descent*. A Q-Network is represented by the function $Q(s, a; \theta)$, where θ represents the parameters of the neural network being used to estimate the true action-value. Using θ in our loss function, we can now obtain a gradient with respect to the parameters θ ,

$$\nabla_\theta L = -[y_t - Q(s_t, a_t; \theta)] \nabla_\theta Q(s_t, a_t; \theta).$$

Like we did before, we can use gradient descent to learn our network parameters θ ,

$$\theta \leftarrow \theta + \alpha[r + \gamma \max_a Q(s_t, a; \theta) - Q(s_t, a_t; \theta)] \nabla_\theta Q(s_t, a_t; \theta).$$

Having a reinforcement learning algorithm expressed in this way has many benefits. Lets think back to our vacuuming robot example. Say the robot is tasked with vacuuming the living room. The living room has specific characteristics that define it, say a dining room table and several chairs. The robot can now identify this room based on its characteristics and can now effectively identify and vacuum the room. Not only that, but the robot must recognize the dining room, and then remember this several states later.

Stochastic Policy Gradient Theorem and Actor Critic Algorithms

Modern computers can handle the computation of *deep neural networks*, or neural networks with many layers, much better than Q-learning because of recent techniques in parallelization and GPU matrix multiplication [7]. Furthermore, the continuous nature of neural networks and Q-Networks plays well with *continuous variable quantum computing*.

Continuous Variable Quantum Computing

Strawberry fields

Proposed Study

References

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- [6]: V. Mnih, K. Kavukcuoglu, D. Silver, A. Graves, I. Antonoglou, D. Wierstra, and M. Riedmiller, ArXiv Preprint ArXiv:1312.5602 (2013).
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