

Project Outline and Plan

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| School of Computing  Faculty of Engineering AND PHYSICAL SCIENCES |

Creating Learning Resources for Solving PDEs using the Finite Element Method

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**BSc Computer Science with Mathematics**

**COMP3931 Individual Project**

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# 1. Introduction

Partial Differential Equations (PDEs) are equations that relate functions of multiple variables, and their rates of change. The word ‘partial’ refers to the fact the rates of change referred to may not be with respect to all the parameters of a given function. PDEs are very well suited to describing complicated systems with many variables, and often arise in physical contexts. Therefore, it is incredibly important and useful to be able to interpret and solve these equations in order to gain greater insight into the systems they describe. The problem is, PDEs are at best difficult, and at worst mathematically impossible to solve analytically. We want a consistent approach which allows us to reliably solve all PDEs with as little error as possible.

The Finite Element Method (FEM), is a numerical method that attempts to meet such requirements. It works by assuming the solution must be a finite concatenation of “elements” which are primitive functions like lines or triangles. You can then obtain a series of linear equations where the variables are the coefficients of our elements. Solving the system allows us to obtain an approximate solution. The more elements you use, the more accurate your solution will be.

## 2 Aims, Objectives and Deliverables

## 2.1 Aims

The primary aim of my project is to create a set of learning materials that can teach students about FEM, with the ultimate goal of producing enough content to kick-start the creation of a new module here at the University of Leeds. The main method of delivering the information will be through a series of Jupyter Notebooks, but there will also be supplementary materials such as set-up guides for the required software.

## 2.2 Objectives

There are 4 objectives I aim to have met by the end of the project. They are as follows:

* To have produced a set of 4-6 interactive Jupyter Notebooks containing all the information needed for students to gain a good understanding of FEM.
* To have gained evidence from students that the Notebooks are effective, interesting, and engaging.
* To have created the notebooks using an agile development structure, hinging on rapid production, testing and iteration.
* To have written a well informed literature review around the subject of FEM.

## 2.3 Deliverables

The deliverables for my project are the 4-6 Jupyter Notebooks as previously discussed. The topics I would like to cover in these notebooks start with understanding the different vector spaces that are needed to solve the PDEs, then how the equations are manipulated to get them to a state where a computer can solve them. Along the way many examples will be used, and exercises will be written to cement understanding. Then we will explore more complicated problems such as material elasticity and fluid flow. Different kinds of IVP/BVPs will also be covered, in addition to visualisation of the solutions obtained for each example.

The report will also be a large deliverable for me, as in it I will discuss how effective the notebooks were, and how well I managed to use the agile development methodology.

## 3. Project Plan

The first stage was the initial research phase. Here I explored a few sources, different textbooks as well as online resources. This is where I gained the knowledge I will need to create the Notebooks. The information is split into three sections. The first and largest of the sections is the mathematical theory. This relies on knowledge of vector space algebra and calculus. Then there is the practical side where I learnt to use the Python library FENICSx to solve problems. Finally there is the visualisation, where I am exploring ways to visualise the solutions obtained from FENICSx.

The next stage is the development and testing phase. This will follow an agile procedure, where a notebook is created in a week, then tested the following week. While the notebook is being tested, the development next notebook can begin. If potential changes are highlighted during the testing phase, they can be implemented and a re-evalutation can begin. The testers will read each notebook and complete associated exercises. Then, they will fill out an online questionnaire where they will be asked questions relating to the readability and pacing of the content, how well they understood the material and the effectiveness of the exercises in aiding understanding. Any other feedback will also have the chance to be submitted.

The final stage is the report writing phase. Here I will evaluate how effective the notebooks were, why I made the decisions I did when creating the notebooks how well I used the agile development methodology.

## 3.1 Timeline

Since I am using an agile development system, the plan for creating the notebooks is subject to change as development progresses. However, I am still providing a basic outline. The actual dates are likely to diverge from this, but it is a good baseline to work towards. All of the time before Christmas was spent researching FEM and how it is implemented.

Timeline

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## 4. Risk Mitigation

One ‘risk’ I need to account for is if one of the testers is unable to load the dependencies. To account for this I will be including a document that explains the setup procedure for Windows and Mac, that will allow the student to follow and run the notebooks on their own machines.

Another ‘risk’ is that my testers either withdraw from the project or do not have enough time to rigorously test the notebooks. At the minute I have 3 testers in mind, if it seems to be too much work to test everything between them, I can attempt to find more and split the notebooks into subsections for them to test. Also I could ask them to test the notebooks in a less detailed way and compare their accounts their accounts accordingly. This is not the I would ideally go with but it is a backup option.

I have faith that the testers will not decide to withdraw from the project, as I am picking people who I feel are reliable and trustworthy. If they are thinking about it I will try my best to lessen the workload for them to encourage them to stay on. Otherwise, I will hopefully be able to find replacements through people I know, or my supervisor.

## 5. Ethics

Since my project uses user testing to a large extent, I will have to make some ethical considerations when collecting the students’ feedback. I will give each tester a Project Information Sheet, which will outline to them what the project is about, what will be required of them and what data I want to collect from them. Then I will get them to sign a user consent form, getting their consent to participation in writing. I will also give them the opportunity to remain anonymous in the final report.

# List of References

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