Ops	$AB^{\intercal} \ (A \in \mathbb{R}^{a \times b}, B \in \mathbb{R}^{c \times b})$			$A^{\intercal}B\;(A\in\mathbb{R}^{a imes c},B\in\mathbb{R}^{a imes b})$			
Ops	(Jin et al.	ColMajor	DiagABT	(Jin et al.)	RowMajor	DiagATB	
CMult	0	$c(\lceil \tfrac{a}{s_0} \rceil \lceil \tfrac{b}{s_1} \rceil + \lceil \tfrac{a}{s_0} \rceil)$	c	0	$c\lceil \tfrac{a}{s_0}\rceil\lceil \tfrac{b}{s_1}\rceil + c$	$\frac{1 + \frac{c}{2}(\lceil \frac{a}{s_0} \rceil + \lceil \frac{b}{s_1} \rceil) - \lceil \frac{a}{s_0} \rceil}{1 + \frac{c}{2}(\lceil \frac{a}{s_0} \rceil \lceil \frac{b}{s_1} \rceil + \lceil \frac{b}{s_1} \rceil) - \lceil \frac{a}{s_0} \rceil}$	
Mult	bc	$c\lceil \frac{a}{s_0} \rceil \lceil \frac{b}{s_1} \rceil$	$\frac{c}{2} \left\lceil \frac{a}{s_0} \right\rceil \left\lceil \frac{b}{s_1} \right\rceil$	bc	$c \lceil \frac{a}{s_0} \rceil \lceil \frac{b}{s_1} \rceil$	$\frac{c}{2} \left\lceil \frac{a}{s_0} \right\rceil \left\lceil \frac{b}{s_1} \right\rceil$	
Rot	0	$c(\lceil \frac{b}{s_1} \rceil \log s_0 + \lceil \frac{a}{s_0} \rceil (\log s_1 + 1)) - \lceil \frac{a}{s_0} \rceil$	$\frac{c}{2}(1+2\lceil \frac{a}{s_0}\rceil \log s_1)$	$bc \log s$	$ c(\lceil \frac{b}{s_1} \rceil \log s_0 + \lceil \frac{a}{s_0} \rceil (\log s_1 + 1)) - \lceil \frac{a}{s_0} \rceil $	$\frac{\left 2+\frac{c}{2}(2\lceil\frac{a}{s_0}\rceil+\log s_0\lceil\frac{b}{s_1}\rceil)-2\lceil\frac{a}{s_0}\rceil\right }{2+\frac{c}{2}(\lceil\frac{a}{s_0}\rceil+\lceil\frac{a}{s_0}\rceil\lceil\frac{b}{s_1}\rceil+\log s_0\lceil\frac{b}{s_1}\rceil)-\lceil\frac{a}{s_0}\rceil-\lceil\frac{a}{s_0}\rceil\lceil\frac{b}{s_1}\rceil}$	

Table 6: Exact number of operations in terms of  $a,b,c,s_0,s_1$ . Here  $s=s_0s_1$ . For  $\mathsf{DiagATB}$ , the first row (resp. second row) of  $\mathsf{CMult}$  and  $\mathsf{Rot}$  represent the case when  $\mathsf{level}(A) \geq \mathsf{level}(B)$  (resp.  $\mathsf{level}(A) < \mathsf{level}(B)$ ).

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(a,b,c)	$AB^{\intercal}$ (A	$\in \mathbb{R}^{a \times b}, B \in$	$\in \mathbb{R}^{c \times b}$ )	$A^{T}B\ (A \in \mathbb{R}^{a \times c}, B \in \mathbb{R}^{a \times b})$		
(a, b, c)	(Jin et al.)	ColMajor	DiagABT	(Jin et al.)	RowMajor	DiagATB
	0	8	4	0	8	4
						4
(128, 128, 4)	512	4	2	512	4	2
	0	63	34	7680	63	18
						18
	0	24	8	0	24	12
						15
(256, 256, 8)	2048	16	8	2048	16	8
	0	191	64	30720	191	72
						75
	0	56	4	0	56	28
	0					40
(512, 769, 4)	3076	52	26	3076	52	26
	0	495	50	46140	495	238
						250
	0	208	8	0	208	104
						176
(1024, 769, 8)	6152	200	100	6152	200	100
	0	2047	140	92280	2047	1008
						1080
	0	800	16	0	800	400
						736
(2048, 769, 16)	12304	784	392	12304	784	392
	0	8703	456	184560	8703	4328
						4664

Table 7: The number of constant multiplications (CMult, first rows), multiplications (Mult, second rows), and rotations (Rot, third rows). For DiagATB, the first row (resp. second row) of CMult and Rot represent the case when  $level(A) \ge level(B)$  (resp. level(A) < level(B)).