Aula 06

Mecânica Celeste

Momento Angular

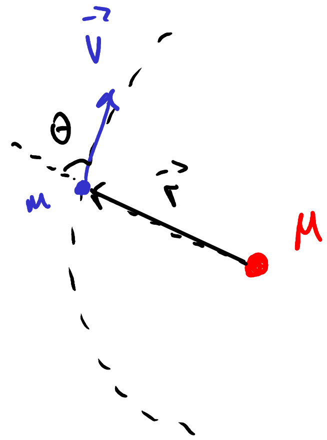
O momento angular é uma grandeza vetorial associada ao movimento de rotação.

$$\vec{L} = \vec{r} \times \vec{p}$$

Para órbitas:

v em módulo

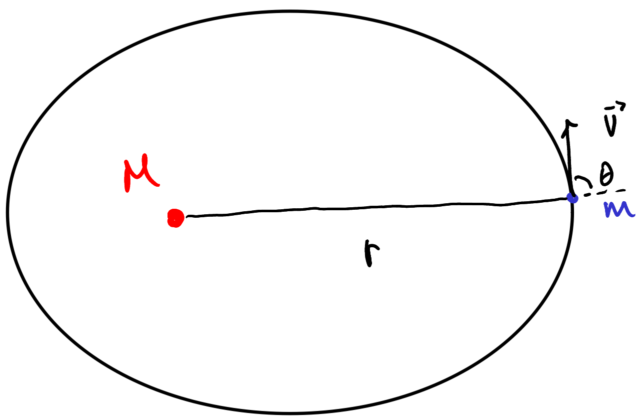
$$\boxed{L = mVr \sin\theta}$$



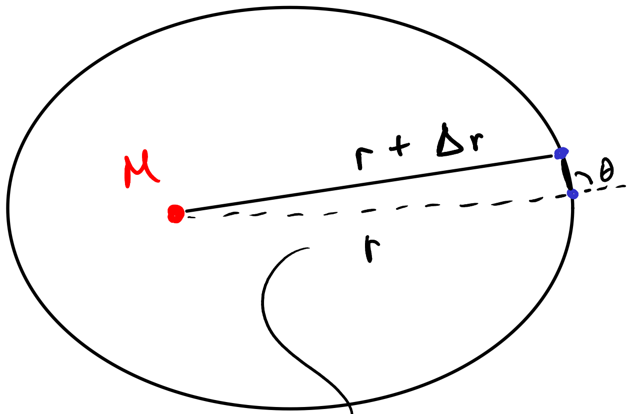
Em órbitas, o momento angular se conserva

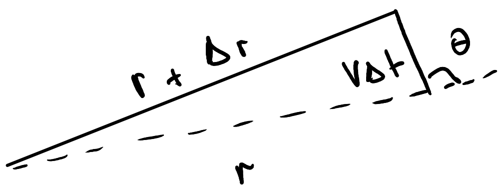
2ª Lei de Kepler: Demonstração

Partindo desse fato podemos uma demonstração para a 2ª Lei de Kepler:



Para uma passagem de tempo muito pequena





A área do triângulo é:

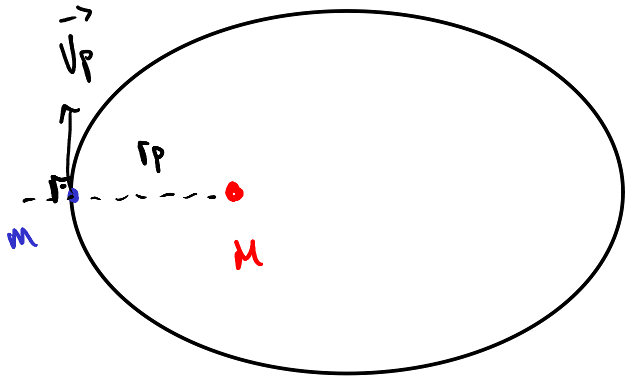
$$\Delta A = \frac{r \cdot V\Delta t \ \sin\theta}{2} \quad \therefore \quad \frac{\Delta A}{\Delta t} = \frac{rV\sin\theta}{2}$$

$$\frac{\Delta A}{\Delta t} = \frac{mrV\sin\theta}{2m}$$

constante

$$L = mrV\sin\theta \quad \therefore \quad \boxed{\frac{\Delta A}{\Delta t} = \frac{L}{2m}}$$

Momento Angular de uma órbita Elíptica



$$L = m V_p r_p \sin\theta_p$$

$$\sin\theta_p = 1$$

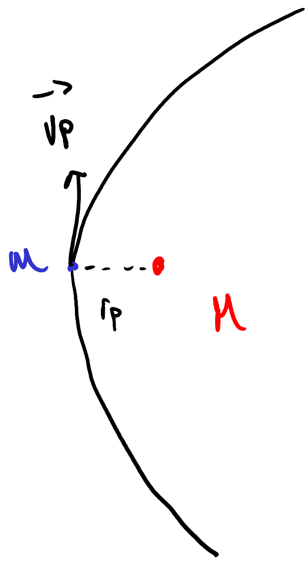
$$\therefore L = m V_p r_p$$

$$L = m \sqrt{\frac{GM}{a} \frac{1+e}{1-e}} \; a(1-e)$$

$$L = m \sqrt{\frac{GM}{a} \frac{1+e}{1-e} a^2 (1-e)^2}$$

$$L = m \sqrt{GMa(1+e)(1-e)} \quad \therefore \boxed{L = m\sqrt{GMa(1-e^2)}}$$

Momento Angular de uma órbita Hiperbólica



$$r_p = a(e-1)$$

$$V_p = \sqrt{GM\left(\frac{2}{r_p} - \frac{1}{a}\right)}$$

$$\therefore V_p = \sqrt{\frac{GM}{a} \frac{e+1}{e-1}}$$

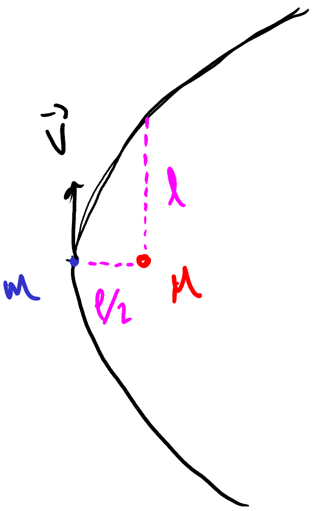
$$\therefore L = m V_p r_p \cancel{\sin\theta}$$

$$L = m \sqrt{\frac{GM}{a} \frac{e+1}{e-1}} \; a(e-1)$$

$$L = m\sqrt{\frac{GM}{a}\,\frac{e+1}{e-1}\,a^2(e-1)^2} \quad \therefore \quad L = m\sqrt{GMa(e+1)(e-1)}$$

$$\boxed{L = m\sqrt{GMa(e^2-1)}}$$

Momento Angular de uma Órbita Parabólica



$$v = \sqrt{\frac{2GM}{l/2}} \quad \therefore \quad v = \sqrt{\frac{4GM}{l}}$$

$$L = m v \frac{l}{2} \sin\theta$$

$$\therefore \quad L = 2m\sqrt{\frac{GM}{l}}\;\frac{l}{2}$$

$$L = m\sqrt{\frac{GM}{l}\,l^2}$$

$$\therefore \quad \boxed{L = m\sqrt{GMl}}$$