IBM ILOG CPLEX优化软件

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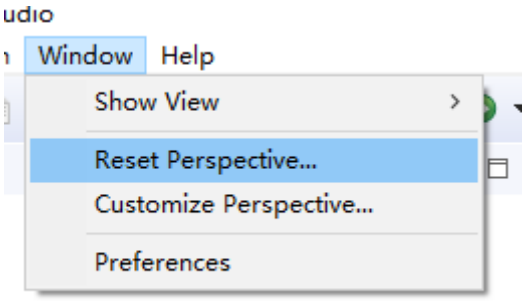
使用IBM ILOG OPL求解规划问题

入门

准备工作

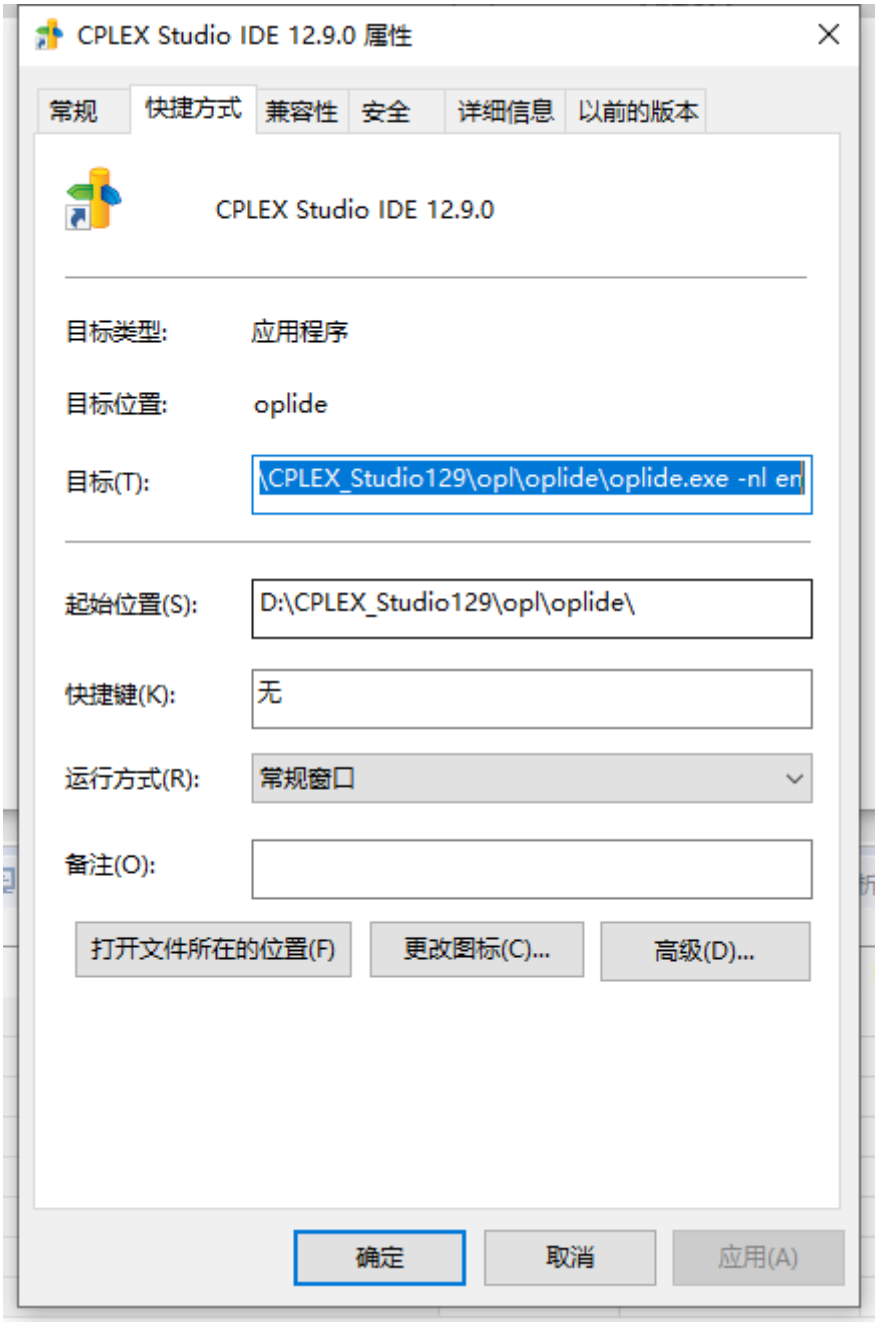


新建项目后，先复位OPL透视图将所有界面恢复到最初的配置

报错

描述	资源	路径	位置	类型
错误（1 项）				
ÔËÐÐÅäÖá°配置 1¡±²»´æÔÚ£	assignment		未知	OPL 问题标记

这个主要是汉化导致的错误，退出程序，对软件启动文件右键属性修改目标属性，加上 `-nl en` 以后软件将以英文启动，并且不会出现中文相关的报错了

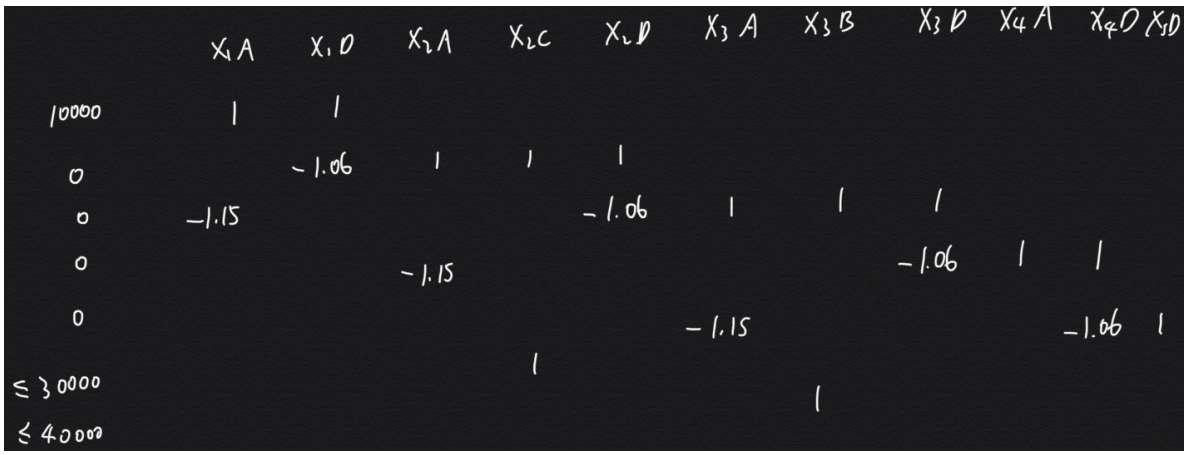


无法正常显示报错信息

在cmd中通过 `oplrun -p + 地址` 得到正常显示的报错信息

第一题——例2-13连续投资问题

抽象出结构



分离数据，完成代码，debug后运行

运行结果（最优解和最优目标）

问题 Scripting log Solutions Conflicts Relax

```
// solution (optimal) with objective 143750
// Quality There are no bound infeasibilities.
// There are no reduced-cost infeasibilities.
// Maximum Ax-b residual              = 3.63798e-12
// Maximum c-B'pi residual            = 0
// Maximum |x|                       = 71698.1
// Maximum |slack|                   = 100000
// Maximum |pi|                      = 1.40185
// Maximum |red-cost|                = 0.03036
// Condition number of unscaled basis = 1.7e+01
//

Product = [71698
         28302 0 30000 0 42453 40000 0 0 0 48821];
```

Engine Log记录

问题 Scripting log Solutions Conflicts Relaxations Engine log

```
Tried aggregator 1 time.
LP Presolve eliminated 11 rows and 0 columns.
Aggregator did 1 substitutions.
Reduced LP has 4 rows, 10 columns, and 16 nonzeros.
Presolve time = 0.00 sec. (0.01 ticks)
Initializing dual steep norms . . .

Iteration log . . .
Iteration:     1   Dual objective     =        145898.113208
```

第二题——习题2-11 工厂生产问题

数学模型

$$
\begin{aligned}
\max z=&(1.25-0.25)\times(x_1+x_2)+(2.00-0.35)\times(x_6+x_7)\\
&+(2.80-0.50)\times x_9-\frac{300}{6\ 000}\times(5x_1+10x_6)\\
&-\frac{321}{10\ 000}\times(7x_2+9x_7+12x_9)-\frac{250}{4\ 000}\times(6x_3+8x_8)\\
&-\frac{783}{7\ 000}\times(4x_4+11x_9)-\frac{200}{4\ 000}\times 7x_5\\
\text{s. t.}\ &\begin{cases}
5x_1+10x_6\leqslant 6\ 000\\
7x_2+9x_7+12x_9\leqslant 10\ 000\\
6x_3+8x_8\leqslant 4\ 000\\
4x_4+11x_9\leqslant 7\ 000\\
7x_5\leqslant 4\ 000\\
x_1+x_2=x_3+x_4+x_5\\
x_6+x_7=x_8\\
x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9\geqslant 0
\end{cases}
\end{aligned}
$$

分离数据，完成代码，debug后运行

运行结果（最优解和最优目标）

问题 Scripting log Solutions Conflicts Relaxations Engine log

```
// solution (optimal) with objective 1146.6002955665
// Quality There are no bound infeasibilities.
// There are no reduced-cost infeasibilities.
// Max. unscaled (scaled) Ax-b resid.          = 4.54747e-13 (1.13687e-13)
// Max. unscaled (scaled) c-B'pi resid.        = 1.11022e-16 (1.11022e-16)
// Max. unscaled (scaled) |x|                  = 1200 (1200)
// Max. unscaled (scaled) |slack|              = 0 (0)
// Max. unscaled (scaled) |pi|                 = 1.09132 (1.09132)
// Max. unscaled (scaled) |red-cost|           = 0.310379 (0.310379)
// Condition number of scaled basis            = 7.2e+01
//

Production = [1200
        230.05 0 858.62 571.43 0 500 500 324.14];
```

Engine Log记录

问题 Scripting log Solutions Conflicts Relaxations Engine log

```
Tried aggregator 1 time.
LP Presolve eliminated 1 rows and 0 columns.
Aggregator did 1 substitutions.
Reduced LP has 5 rows, 8 columns, and 15 nonzeros.
Presolve time = 0.00 sec. (0.01 ticks)

Iteration log . . .
Iteration:     1   Scaled dual infeas =             1.399998
Iteration:     3   Dual objective     =          1730.111111
```

第三题——统一花费的集合覆盖问题

构建思路

这个问题就类似于`dancing line`问题，难点在于如何生成正确的矩阵

构建输入矩阵

通过递归求出`k`元组和`k+1`元组的所有元素，并保存在两个数组中，并逐一比对是否符合配对关系

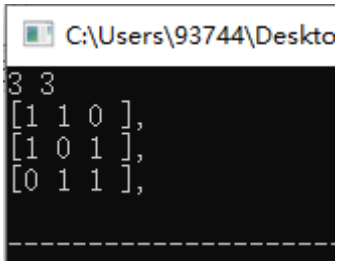
```
#include<iostream>
using namespace std;
int main()
{
    int k=5;    //题目要求的k值
    int a=1,b=1;
    for(int i=1;i<=k;i++){
        a=a*(k*2+2-i)/i;
    }//k元组个数
    for(int i=1;i<=k+1;i++){
        b=a*(k*2+2-i)/i;
    }//k+1元组个数
    cout<<a<<" "<<b<<endl;//输出矩阵行和列
    int bi1[a][k],bi2[b][k+1],count=0,t[k+2],flag=0;
    for(int i=0;i<k+2;i++){
        t[i]=i;
    }//t数组辅助递归的实现，先初始化
    while(t[1]<=k+2){
        for(int i=0;i<k;i++){
            bi1[count][i]=t[i+1];
        }//记录当前递归的情况
        ++t[k];
        if(t[k]>k*2+1){
            flag=1;
            while(flag!=0 && flag<k){
                t[k-flag+1]--;
                t[k-flag]++;
                if(t[k-flag]<t[k-flag+1]){
                    while(flag!=0){
                        t[k-flag+1]=t[k-flag]+1;
                        flag--;
                    }
                }
                else
                    flag++;
            }//递归实现
        }
        count++;
    }
    /*for(int i=0;i<a;i++)
    {
        for(int j=0;j<k;j++)
        {
            cout<<bi1[i][j]<<" ";
        }
        cout<<endl;
    }*/          //输出k元组，用于debug
    cout<<endl;
    count=0;flag=-1;
    for(int i=0;i<k+2;i++){
        t[i]=i;
    }
    while(t[1]<=k+1){   //递归原理相同
```

```
        for(int i=0;i<k+1;i++){
            bi2[count][i]=t[i+1];
        }
        ++t[k+1];
        if(t[k+1]>k*2+1){
            flag=0;
            while(flag!=-1 && flag<k){
                t[k-flag+1]--;
                t[k-flag]++;
                if(t[k-flag]<t[k-flag+1]){
                    while(flag!=-1){
                        t[k-flag+1]=t[k-flag]+1;
                        flag--;
                    }
                }
                else
                    flag++;
            }
        }
        count++;
    }
    /*for(int i=0;i<b;i++){
        for(int j=0;j<k+1;j++)
        {
            cout<<bi2[i][j]<<" ";
        }
        cout<<endl;
    }*/         //输出k+1元组，用于debug
    cout<<endl;
    int axis[a][b],temp;
    for(int i=0;i<a;i++){
        for(int j=0;j<b;j++){
            axis[i][j]=0;
        }
    }
    for(int i=0;i<a;i++){           //枚举k元组比对是否符合约定条件
        int z;
        temp=0;
        flag=0;
        for(int j=0;j<b;j++){       //枚举k+1元组
            for(z=0;z<k+1;z++){     //逐个元素比对
                if(bi2[j][z] == bi1[i][temp]){
                    temp++;
                    if(temp==k)
                        break;
                }
            }
            if(temp==k){            //满足约定条件，将矩阵对应元素设置为1
                flag=1;
                axis[i][j]=1;
            }
            temp=0;
        }
    }
    for(int i=0;i<a;i++){           //按照格式输出矩阵
        cout<<"[";
        for(int j=0;j<b;j++){
            cout<<axis[i][j]<<" ";
```

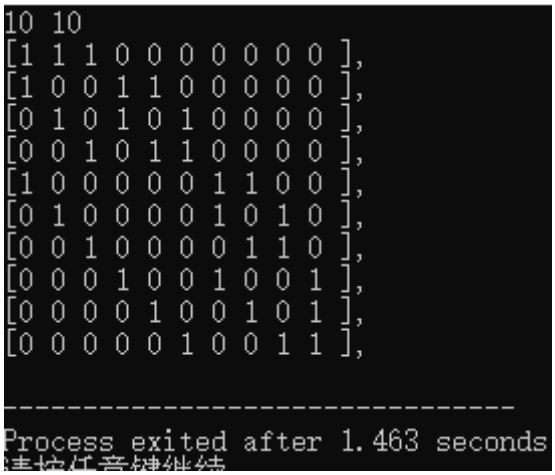
```
        }
        cout<<"],"<<endl;
    }
}
```

输出样例

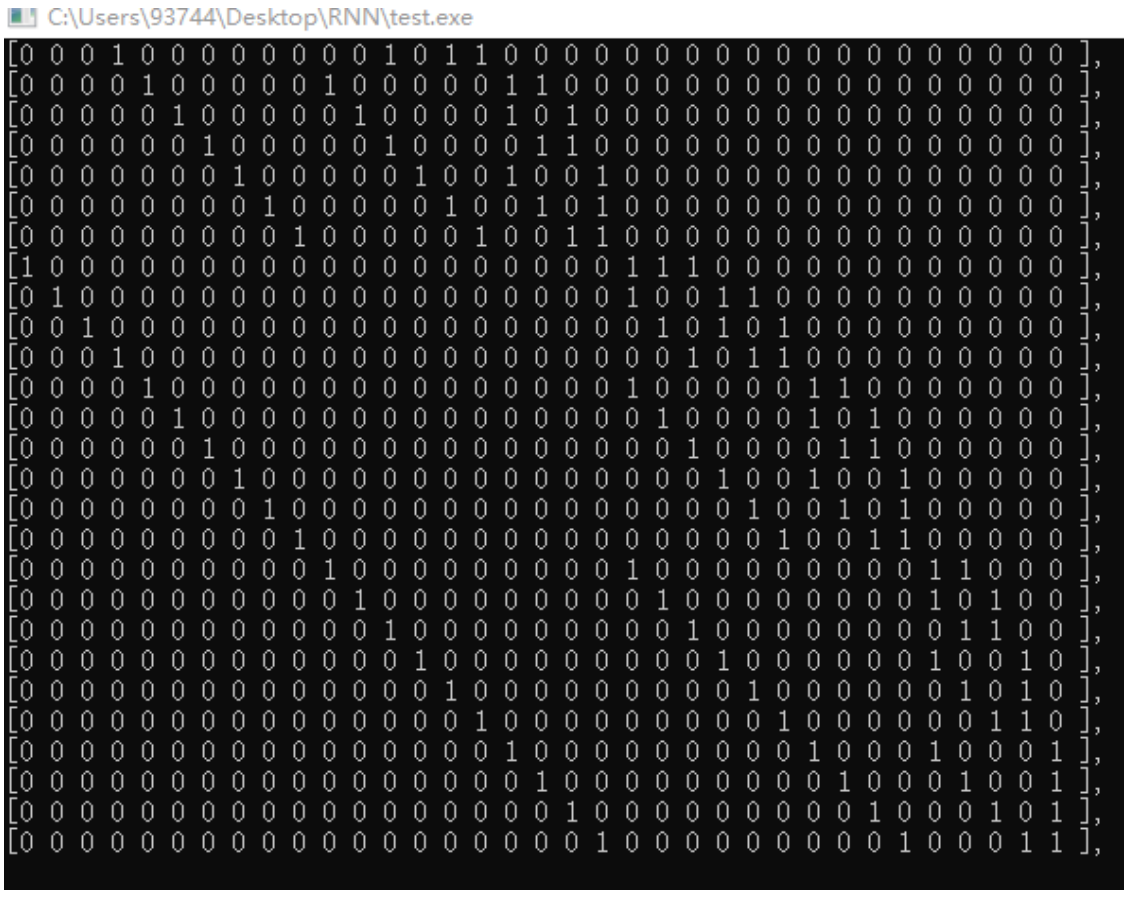
- k=1



- k=2



- k=3



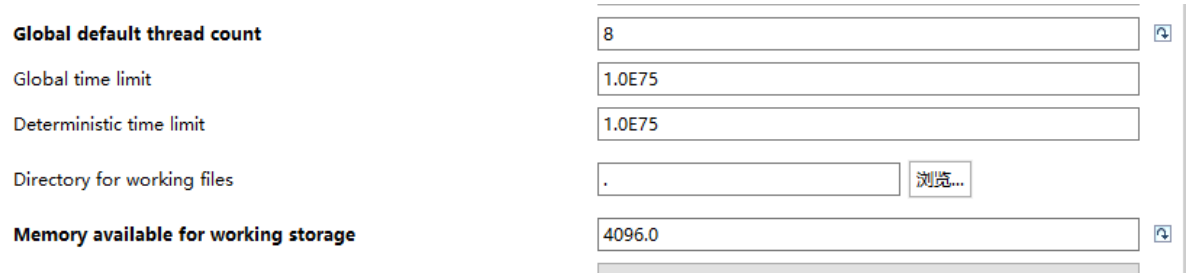
- k=4 k=5矩阵过大，就不截图了

ILOG程序

在解决统一花费的集合覆盖问题时，只需要把抽象的集合问题数学模型中，花费函数 `cj` 统一定义为1，即每一个 `k+1` 元组被选取的权重一致，并且此时的 `xj` 为 `bool` 类型变量

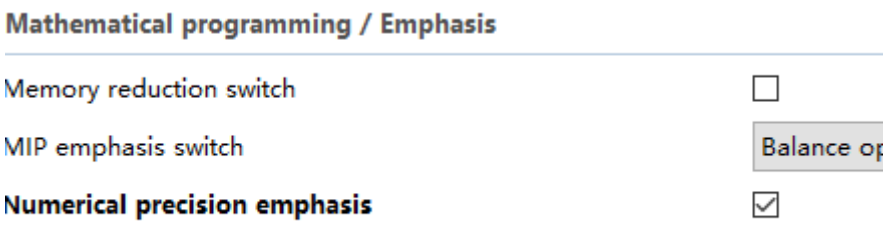
OPL ops配置文件设置

在k=1..4时，ILOG都可以很快的计算出问题的最优解，但是当k=5时，数据规模过大，ILOG很长时间无法拟合出最优解，所以需要子在ops中调整一些参数

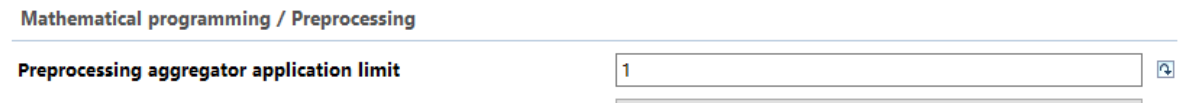


首先最直观的加速拟合过程的方法就是提高并行计算，将全局线程设置更大，此处考虑到CPU承载能力设置为8线程

随着拟合过程的进行，可行基选取的树结构会越来越复杂，从一开始的几百MB无限增长，所以需要提前设置可用内存，考虑电脑运行内存为8GB，此处设置了4GB



配置文件默认为平衡效率和稳定性，但是为了更快拟合，需要勾选强调计算精度的选项，可以加快solution拟合



配置文件中默认为-1自动选择前处理聚合应用程序，此处设置为1对MIP模型变量替换无限制，以改善拟合效率

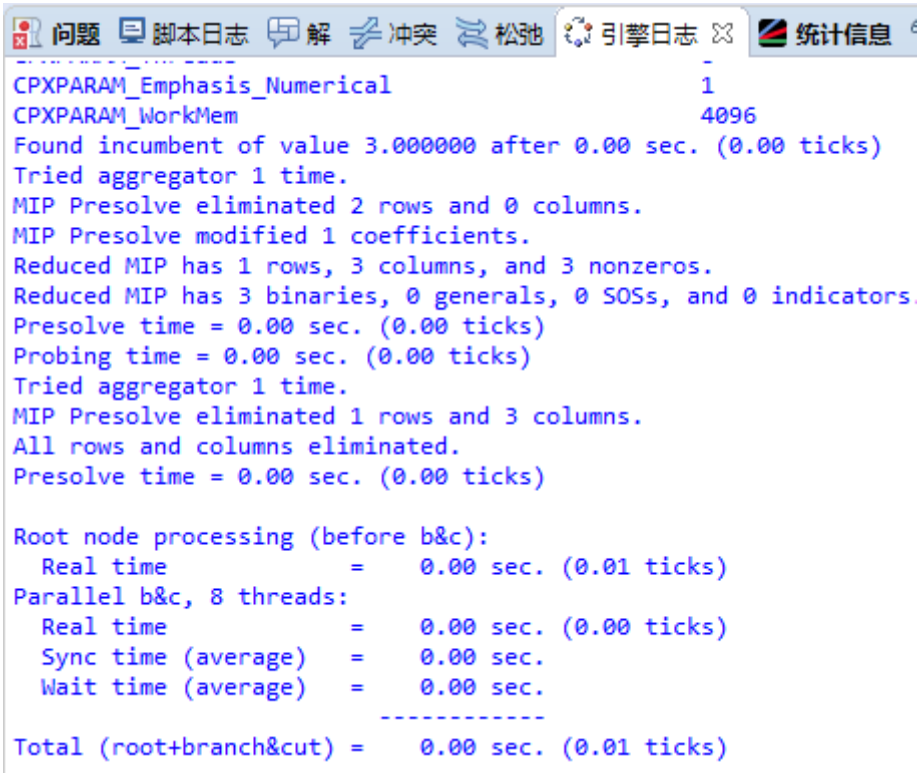
运行结果（最优解、最优目标和Engine Log）（k= 1..4）

- k=1

  最优解和最优目标

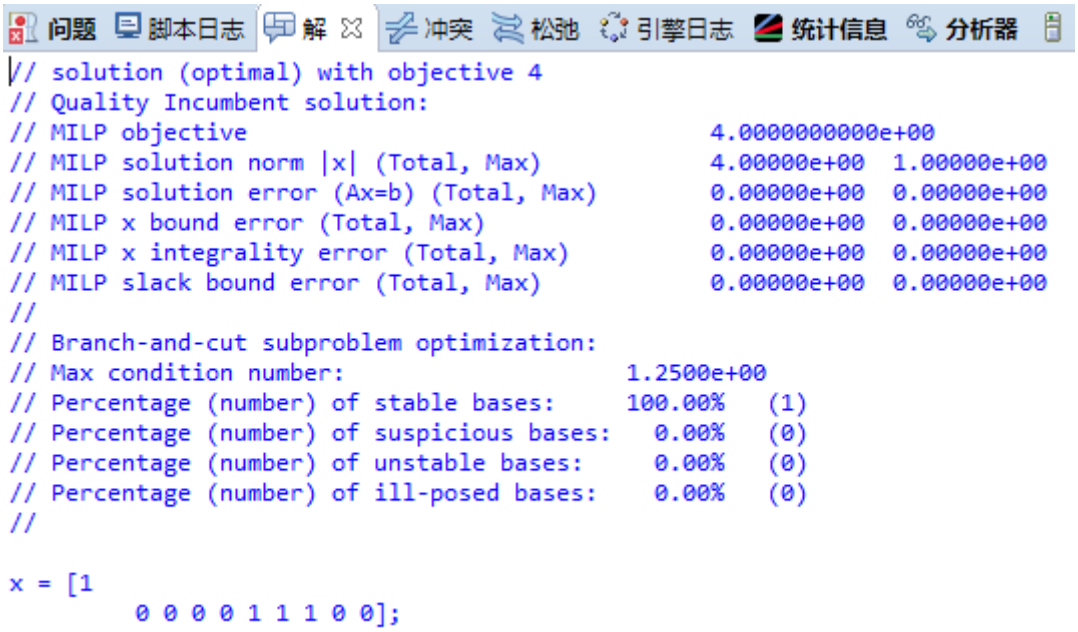
```
// solution (optimal) with objective 2
// Quality Incumbent solution:
// MILP objective                                2.0000000000e+00
// MILP solution norm |x| (Total, Max)           2.00000e+00  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//

x = [1
        1 0];
```

Engine Log

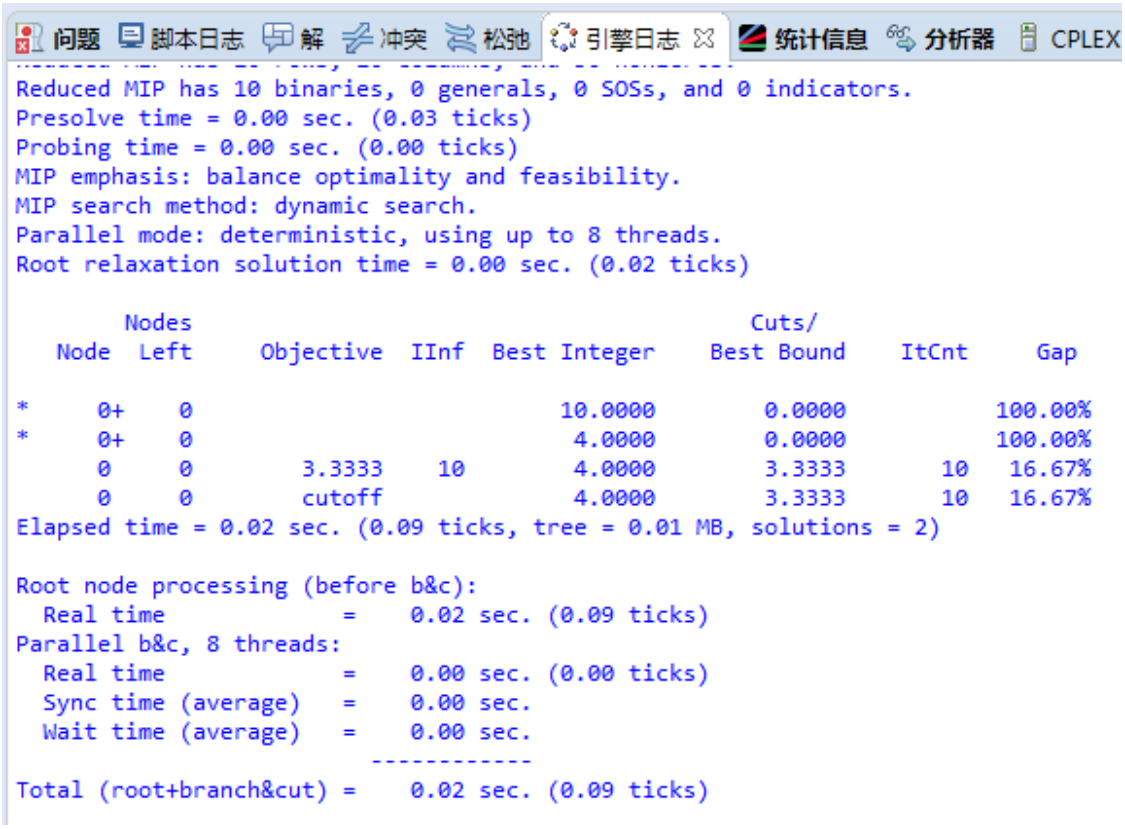


- k=2

  最优解和最优目标

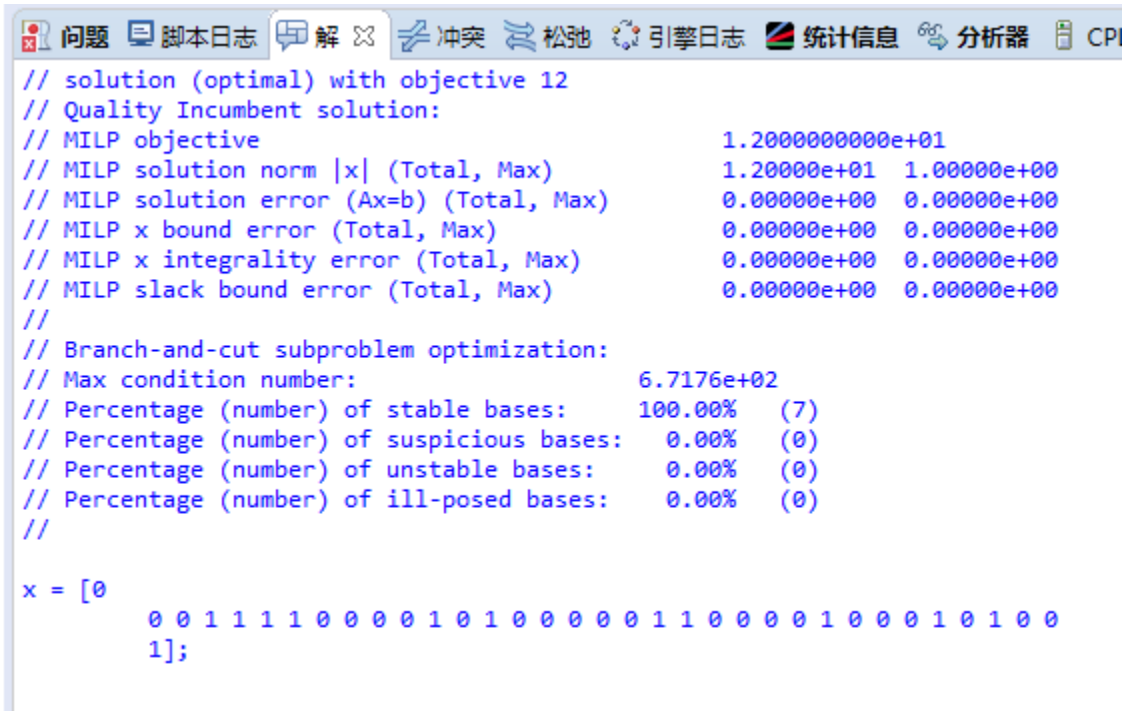


Engine Log



- k=3

  最优解和最优目标



  Engine Log

问题 脚本日志 解 冲突 松弛 引擎日志 统计信息 分析器 CPLEX S

```
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                          35.0000        0.0000           100.00%
*     0+    0                          14.0000        0.0000           100.00%
*     0+    0                          13.0000        0.0000           100.00%
      0     0        8.7500    35       13.0000        8.7500       36   32.69%
      0     0        9.5000    29       13.0000       Cuts: 8       42   26.92%
      0     0        9.7500    34       13.0000      Cuts: 12       57   25.00%
      0     0       10.1250    32       13.0000       Cuts: 6       68   22.12%
*     0+    0                          12.0000       10.1250           15.62%
      0     0       10.2500    32       12.0000   ZeroHalf: 6       79   14.58%
      0     0       10.2778    32       12.0000   ZeroHalf: 3       92   14.35%
      0     0        cutoff             12.0000                     119    0.00%
Elapsed time = 0.44 sec. (4.95 ticks, tree = 0.01 MB, solutions = 4)

Zero-half cuts applied:  22

Root node processing (before b&c):
  Real time             =    0.44 sec. (4.96 ticks)
Parallel b&c, 8 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          ------------
Total (root+branch&cut) =    0.44 sec. (4.96 ticks)
```

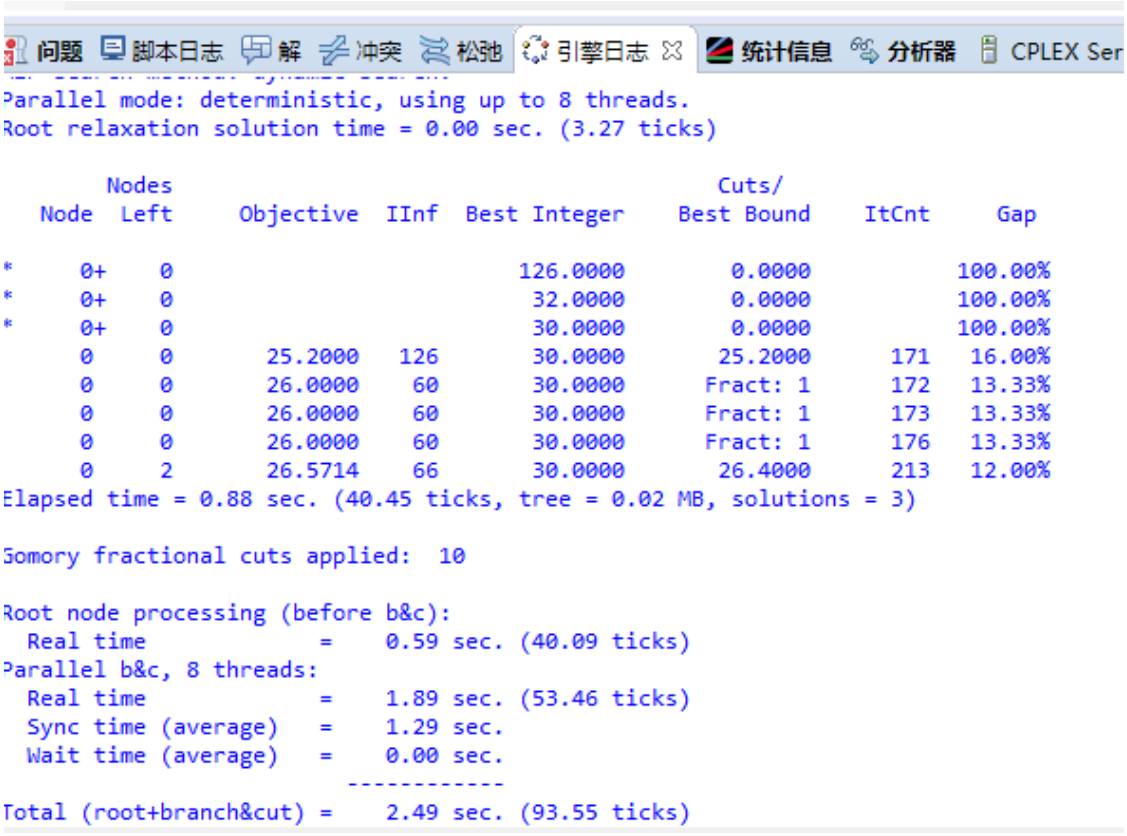
- k=4

  最优解和最优目标

问题 脚本日志 解 冲突 松弛 引擎日志 统计信息 分析器 CPL

```
// solution (optimal) with objective 30
// Quality Incumbent solution:
// MILP objective                                3.0000000000e+01
// MILP solution norm |x| (Total, Max)           3.00000e+01  1.00000e+00
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
//
// Branch-and-cut subproblem optimization:
// Max condition number:                    7.7483e+03
// Percentage (number) of stable bases:     100.00%   (69)
// Percentage (number) of suspicious bases:   0.00%   (0)
// Percentage (number) of unstable bases:     0.00%   (0)
// Percentage (number) of ill-posed bases:    0.00%   (0)
//

x = [0
        0 1 0 0 0 1 0 0 1 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 1 1 0 0 0 0 0 0
        0 1 0 0 0 0 1 1 0 0 0 0 0 0 1 1 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0
        0 0 1 0 1 1 1 1 0 0 0 0 0 1 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 1 0
        0 0 0 1 0 0 0 0 0 0 0 0 0 0 1 0 0 1 0 0 1 1 0 0 0 0];
```

Engine Log



k=5次优解分析

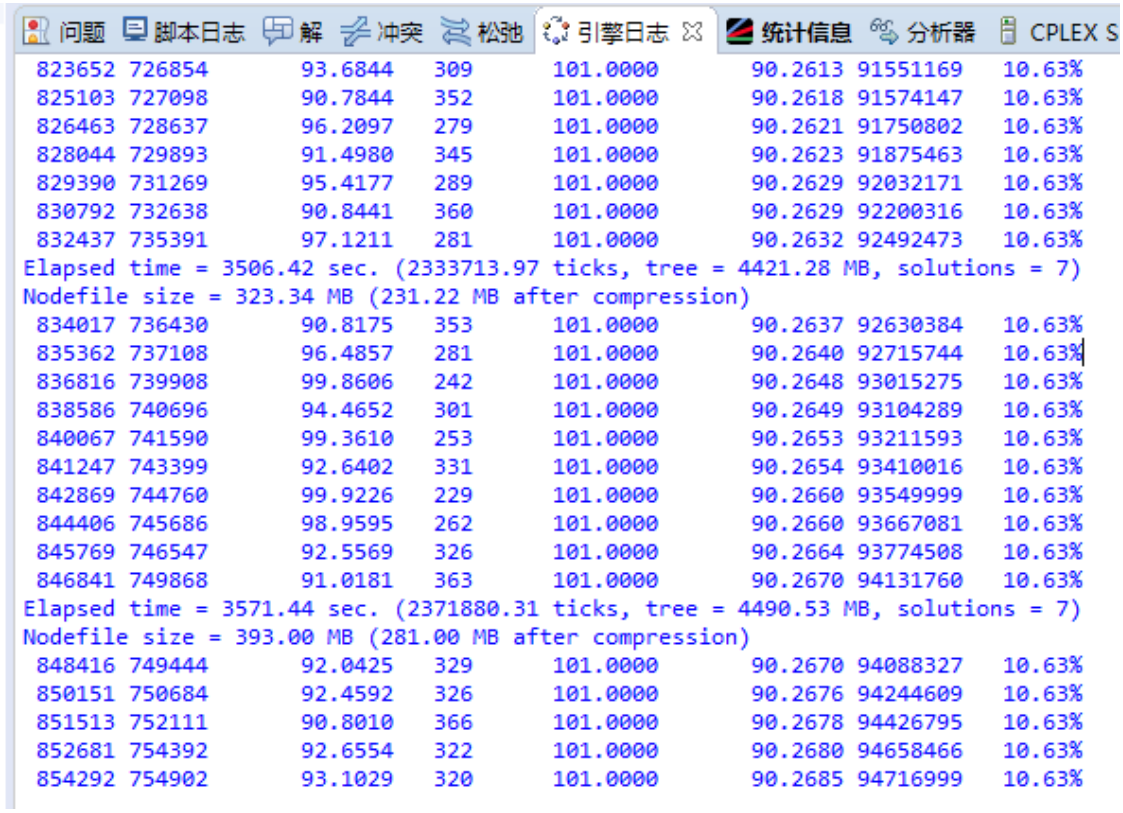
Engine Log 配置好ops后运行

问题 脚本日志 解 冲突 松弛 引擎日志 统计信息 分析器 CPLEX Se

```
Reduced MIP has 462 rows, 462 columns, and 2772 nonzeros.
Reduced MIP has 462 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.00 sec. (1.00 ticks)
Probing time = 0.00 sec. (0.16 ticks)
Tried aggregator 1 time.
Reduced MIP has 462 rows, 462 columns, and 2772 nonzeros.
Reduced MIP has 462 binaries, 0 generals, 0 SOSs, and 0 indicators.
Presolve time = 0.00 sec. (3.72 ticks)
Probing time = 0.00 sec. (0.16 ticks)
MIP emphasis: balance optimality and feasibility.
MIP search method: dynamic search.
Parallel mode: deterministic, using up to 8 threads.
Root relaxation solution time = 0.14 sec. (145.18 ticks)

        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt     Gap

*     0+    0                          462.0000        0.0000           100.00%
*     0+    0                          110.0000        0.0000           100.00%
*     0+    0                          108.0000        0.0000           100.00%
*     0+    0                          106.0000        0.0000           100.00%
*     0+    0                          104.0000        0.0000           100.00%
*     0+    0                          102.0000        0.0000           100.00%
      0     0       77.0000   462      102.0000       77.0000     1056   24.51%
      0     0       80.5852   455      102.0000     Cuts: 106     1247   20.99%
      0     0       82.3184   453      102.0000  ZeroHalf: 70     1466   19.30%
      0     0       83.4929   446      102.0000  ZeroHalf: 75     1748   18.14%
      0     0       84.6125   441      102.0000  ZeroHalf: 66     2095   17.05%
      0     0       85.5231   439      102.0000  ZeroHalf: 77     2492   16.15%
      0     0       86.8162   415      102.0000  ZeroHalf: 32     3331   14.89%
      0     0       87.1999   419      102.0000  ZeroHalf: 24     3610   14.51%
      0     0       87.4310   416      102.0000  ZeroHalf: 18     3874   14.28%
      0     0       87.5333   417      102.0000   ZeroHalf: 9     4059   14.18%
      0     0       87.5954   421      102.0000   ZeroHalf: 7     4197   14.12%
      0     0       87.6657   417      102.0000   ZeroHalf: 8     4354   14.05%
      0     0       87.7408   421      102.0000   ZeroHalf: 8     4509   13.98%
      0     0       87.7877   420      102.0000   ZeroHalf: 6     4624   13.93%
      0     0       87.8646   417      102.0000   ZeroHalf: 6     4750   13.86%
      0     0       87.9614   412      102.0000   ZeroHalf: 9     4925   13.76%
      0     0       88.0049   418      102.0000   ZeroHalf: 4     5076   13.72%
      0     0       88.1268   419      102.0000   ZeroHalf: 8     5309   13.60%
      0     0       88.2101   424      102.0000   ZeroHalf: 8     5561   13.52%
      0     0       88.2963   415      102.0000  ZeroHalf: 11     6041   13.44%
      0     0       88.3779   412      102.0000   ZeroHalf: 9     6697   13.36%
      0     0       88.5119   410      102.0000   ZeroHalf: 9     7834   13.21%
```

Gap收敛速度相对配置前，更快

- 次优解



得到的结果101，后续收敛非常慢，后由于内存溢出停止

以下为log分析，前期收敛较快，从13%到10.64%收敛树结构生成了 `4091.81 MB` ，后开始压缩结构存储，内存报错前 `Gap` 值最优为 `10.63%`

```
        Nodes                                         Cuts/
   Node  Left     Objective  IInf  Best Integer    Best Bound    ItCnt
Gap

*     0+    0                          462.0000        0.0000
100.00%
*     0+    0                          110.0000        0.0000
100.00%
*     0+    0                          108.0000        0.0000
100.00%
*     0+    0                          106.0000        0.0000
100.00%
*     0+    0                          104.0000        0.0000
100.00%
*     0+    0                          102.0000        0.0000
100.00%
      0     0       77.0000   462      102.0000       77.0000     1056
24.51%
      0     0       80.5852   455      102.0000     Cuts: 106     1247
20.99%
      0     0       82.3184   453      102.0000  ZeroHalf: 70     1466
19.30%
      0     0       83.4929   446      102.0000  ZeroHalf: 75     1748
18.14%
      0     0       84.6125   441      102.0000  ZeroHalf: 66     2095
17.05%
      0     0       85.5231   439      102.0000  ZeroHalf: 77     2492
16.15%
      0     0       86.8162   415      102.0000  ZeroHalf: 32     3331
14.89%
```

```
      0     0       87.1999   419      102.0000  ZeroHalf: 24     3610
14.51%
      0     0       87.4310   416      102.0000  ZeroHalf: 18     3874
14.28%
      0     0       87.5333   417      102.0000   ZeroHalf: 9     4059
14.18%
      0     0       87.5954   421      102.0000   ZeroHalf: 7     4197
14.12%
      0     0       87.6657   417      102.0000   ZeroHalf: 8     4354
14.05%
      0     0       87.7408   421      102.0000   ZeroHalf: 8     4509
13.98%
      0     0       87.7877   420      102.0000   ZeroHalf: 6     4624
13.93%
      0     0       87.8646   417      102.0000   ZeroHalf: 6     4750
13.86%
      0     0       87.9614   412      102.0000   ZeroHalf: 9     4925
13.76%
      0     0       88.0049   418      102.0000   ZeroHalf: 4     5076
13.72%
      0     0       88.1268   419      102.0000   ZeroHalf: 8     5309
13.60%
      0     0       88.2101   424      102.0000   ZeroHalf: 8     5561
13.52%
      0     0       88.2963   415      102.0000  ZeroHalf: 11     6041
13.44%
      0     0       88.3779   412      102.0000   ZeroHalf: 9     6697
13.36%
      0     0       88.5119   410      102.0000   ZeroHalf: 9     7834
13.21%
      0     0       88.7222   410      102.0000  ZeroHalf: 14     7941
13.02%
      0     0       88.7222   415      102.0000   ZeroHalf: 3     7967
13.02%
      0     2       88.7222   400      102.0000       88.7222     7967
13.02%
...
...
Elapsed time = 392.77 sec. (272300.48 ticks, tree = 384.39 MB, solutions =
7)
  72661  5807       96.1058   294      101.0000       90.0066  9310022
10.88%
Elapsed time = 1151.27 sec. (806900.98 ticks, tree = 1035.82 MB, solutions =
7)
 260167 185744       99.8571   251      101.0000       90.0224 29967702
10.87%
...
...
Elapsed time = 3178.33 sec. (2142853.00 ticks, tree = 3993.34 MB, solutions
= 7)
 773685 678698       93.4005   307      101.0000       90.2499 86090688
10.64%
Elapsed time = 3239.56 sec. (2181024.89 ticks, tree = 4091.81 MB, solutions
= 7)
 788396 692338       91.0870   341      101.0000       90.2540 87645258
10.64%
Elapsed time = 3306.80 sec. (2219194.14 ticks, tree = 4176.81 MB, solutions
= 7)
```

```
Nodefile size = 78.83 MB (56.45 MB after compression)
 802831 705947       93.6489   311      101.0000       90.2569 89199774
10.64%
Elapsed time = 3372.73 sec. (2257370.87 ticks, tree = 4249.38 MB, solutions
= 7)
Nodefile size = 151.58 MB (108.44 MB after compression)
 817920 719181       96.1040   291      101.0000       90.2597 90697048
10.63%
Elapsed time = 3439.75 sec. (2295546.96 ticks, tree = 4327.13 MB, solutions
= 7)
Nodefile size = 228.63 MB (163.49 MB after compression)
 832437 735391       97.1211   281      101.0000       90.2632 92492473
10.63%
Elapsed time = 3506.42 sec. (2333713.97 ticks, tree = 4421.28 MB, solutions
= 7)
Nodefile size = 323.34 MB (231.22 MB after compression)
 846841 749868       91.0181   363      101.0000       90.2670 94131760
10.63%
```

参考文献

1. https://wenku.baidu.com/view/7f9fe7bb48d7c1c709a145dd.html **ILOG_OPL进阶功能**