人工神经网络作业

陈政培 17363011

智科1班

1.(a) ∵ 二阶为 G

∴ $\nabla f(x) = Gx + b$

$f(x) = \frac{1}{2}x^T G x + b^T x + c$

在 x^* 处 $Gx^* + b = 0$

$Gs = \lambda s$

$g^{(0)} = G(x^* + \mu s) + b$

$= Gx^* + b + G\mu s$

$= G\mu s = \frac{\lambda s}{s}\mu s = \mu\lambda s$

沿最速下降

搜索方向 $d_k = -\nabla f(x) = -(Gx+b)$

步长 $\alpha_k = \arg\min f(x_k - \alpha\nabla f(x_k))$

一阶必要 $\varphi'(\alpha)|_{\alpha=\alpha_k} = 0$

$d_k^T \nabla f(x_k + \alpha_k d_k)$

$= (-g_k^T)(b + G(x_k - \alpha_k g_k))$

$= g_k^T(-b - Gx_k + \alpha_k G g_k)$

$= g_k^T(-g_k + \alpha_k G g_k)$

∴ $\alpha_k = \frac{g_k^T g_k}{g_k^T G g_k}$

$\alpha_k = \frac{\cancel{\mu^2\lambda^2} s^T s}{\cancel{\mu^2\lambda^2} s^T G s} = \frac{s^T s}{s^T \frac{\lambda s}{s} s} = \frac{1}{\lambda}$

∴ $x^{(1)} = x^{(0)} - \frac{1}{\lambda}g^{(0)} = x^{(0)} - \mu s$

∴ $x^{(1)} = x^* = x^{(0)} - \mu s$ 得证

(b) $G = aI$ ∴ $\lambda = a$ $Gs = as$ $\alpha_k = \frac{1}{a}$ $g^{(0)} = as = Gs$

$x^{(0)} = x^* + \mu s$ ∴ $\mu = 1$ ∴ $x^{(1)} = x^* = x^{(0)} - \mu s$

2. (a) 一阶必要条件

(b) $G = [\ 10 \quad -1\ ]$

8:58 6月14日周日 17%

备忘录

2. (a) 一阶必要条件

$$\nabla q(x)=\begin{bmatrix} 10x_1-x_2-11 \\ 10x_2-x_1+11 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore \begin{cases} 10x_1-x_2-11=0 \\ 10x_2-x_1+11=0 \end{cases} \quad \therefore \begin{cases} x_1=1 \\ x_2=-1 \end{cases}$$

$$\therefore x^*=\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

(b) $G=\begin{bmatrix} 10 & -1 \\ -1 & 10 \end{bmatrix}$

$10>0$

$\begin{bmatrix} 10 & -1 \\ -1 & 10 \end{bmatrix}=101>0$

$\therefore$ G正定 $\therefore q(x)$是凸函数

$\therefore x^*$是全局极小点.

(c) G的特征值为 9、11 $\therefore$ 收敛因子 $a=\left(\dfrac{\lambda_1-\lambda_2}{\lambda_1+\lambda_2}\right)^2=0.01$

3. (a) $f(x)=\frac{1}{2}x^TGx+b^Tx$

$\nabla f(x)=Gx+b$

$x_1=x_0+t_0(-\nabla f(x))=t_0Gx_0+t_0b$

$=t_0b$

$\varphi(t)=\frac{1}{2}t_0^2b^TGb+t_0b^Tb$

$\varphi'(t)=t_0b^TGb+b^Tb=0$

$t_0=\frac{10}{11}$

......

$x^*=(0.4472, 1.8944, 3.3416, 2.7889)^T$

$g^{(0)}=Gx^0+b=(-1,0,2,\sqrt{5})^T$

$Gg^{(0)}=(-2,-1,4-\sqrt{5},-2+2\sqrt{5})^T$

$G^2g^{(0)}=(-3,-4+\sqrt{5},11-4\sqrt{5},-8+5\sqrt{5})^T$

$G^2g^{(0)}=\lambda_1g^{(0)}+\lambda_2Gg^{(0)}$

$\begin{cases} \lambda_1=-5+2\sqrt{5} \\ \lambda_2=4-\sqrt{5} \end{cases}$ $\therefore$ 只有两个独立向量

(b) $g^{(0)}, Gg^{(0)}, G^2g^{(0)}, G^3g^{(0)}$ 只有两个独立变量.

$\therefore$ 经两次迭代后终止

4. (a) $g(x) = 10x_1^2 + x_2^2$

$$\nabla g(x) = \begin{bmatrix} 20x_1 \\ 2x_2 \end{bmatrix}$$

$$G = \begin{bmatrix} 20 & 0 \\ 0 & 2 \end{bmatrix}$$

第一步 $x_0 = (0.1, 1)^T$

$g_0 = Gx_0 = (2,2)^T$

$p_0 = -H_0 g_0 = -(2,2)^T$

$\alpha_0 = -\frac{g_0^T p_0}{p_0^T G p_0} = \frac{1}{11}$

$x_1 = x_0 + \alpha_0 p_0 = (-\frac{9}{110}, \frac{9}{11})^T$

$g_1 = Gx_1 = (-\frac{18}{11}, \frac{18}{11})^T$

$s_0 = x_1 - x_0 = -(\frac{2}{11}, \frac{2}{11})^T$

$y^0 = g_1 - g_0 = -(\frac{40}{11}, \frac{4}{11})^T$

$$H_1 = H_0 - \frac{H_0 y^0 y_0^T H_0}{y_0^T H_0 y_0} + \frac{s_0 s_0^T}{y_0^T s_0} = \frac{1}{2222}\begin{bmatrix} 123 & -119 \\ -119 & 2301 \end{bmatrix}$$

第二步

$p_1 = \frac{1}{101}(18, -180)^T$

$\alpha_1 = \frac{101}{220}$

$x_2 = (0,0)^T$

$g_2 = (0,0)^T$

$s_1 = (\frac{9}{110}, -\frac{9}{11})^T$

$y_1 = (\frac{18}{11}, -\frac{18}{11})^T$

$$H_2 = \begin{bmatrix} \frac{1}{20} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

∵ $g_2 = 0$

∴ 停止

$H_2 = G^{-1}$

(b) 共轭法

$p_0 = -(2,2)^T$

$\alpha_0 = \frac{1}{11}$

$x_1 = (-\frac{9}{110}, \frac{9}{11})^T$

$g_1 = (-\frac{18}{11}, \frac{18}{11})^T$

$\beta_0 = \frac{g_1^T g_1}{g_0^T g_0} = \frac{81}{121}$

$p_1 = (\frac{36}{121}, -\frac{360}{121})^T$

第二步 $\alpha_1 = \frac{11}{40}$

$x_2 = (0,0)^T$

$g_2 = (0,0)^T$

∵ $g_2 = 0$

∴ 同样的二次终止性