习题五

陈政宏
1736011
智科1班

1. $\nabla f(x) = [2x_1, 50x_2]^T$ $\varepsilon = 0.01$

$A = \begin{bmatrix} 2 & 0 \\ 0 & 50 \end{bmatrix}$

$g_0 = \nabla f(x_0) = [4, 100]^T$, $\|g_0\| = 100.0799$

$x_1 = x_0 - \frac{g_0^T g_0}{g_0^T A g_0} g_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - 0.02003 \begin{bmatrix} 4 \\ 100 \end{bmatrix} = \begin{bmatrix} 1.91983 \\ -0.03 \end{bmatrix}$

$x_2 = x_1 - \frac{g_1^T g_1}{g_1^T A g_1} g_1 = \begin{bmatrix} 1.91988 \\ -0.003 \end{bmatrix} - 0.48089 \begin{bmatrix} 3.8376 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 0.07348 \\ 0.06913 \end{bmatrix}$

$f(x_2) = 0.124872$ $\|g_2\| > \varepsilon$

…… $x = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ 最优解 $x^* = [0, 0]^T$ $f(x^*) = 0$

2. $g(x) = \nabla f(x) = [-10 + 2x_1 - x_2, -4 + 2x_2 - x_1]^T$

$G(x) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ $G(x)^{-1} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$x_1 = x_0 - G(x)^{-1} g(x_0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix} \begin{bmatrix} -10 \\ 4 \end{bmatrix} = [8, 6]^T$

$\nabla f(x_1) = [0, 0]^T$ $\|\nabla f(x_1)\| = 0 < 0.01$

最优解 $x^* = [8, 6]^T$ $f(x^*) = 8$

3. $g(x) = \nabla f(x) = [8x_1 + 9, 4x - 3]^T$ $x_1 = x_0 + t_0 p_0 = \begin{bmatrix} -\frac{9}{8} t_0 \\ \frac{3}{4} t_0 \end{bmatrix}$

$G(x) = \begin{bmatrix} 8 & 0 \\ 0 & 4 \end{bmatrix}$ $G(x)^{-1} = \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{4} \end{bmatrix}$ $g(x_0) = [9, -3]^T$

$p_0 = -G(x)^{-1} g(x_0) = \begin{bmatrix} \frac{1}{8} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 9 \\ -3 \end{bmatrix} = [-\frac{9}{8}, \frac{3}{4}]^T$ $x_1 = x_0 + t_0 p_0 = \begin{bmatrix} -\frac{9}{8} t_0 \\ \frac{3}{4} t_0 \end{bmatrix}$

$f(x_1) = \frac{99}{16} t^2 - \frac{99}{8} t + 16$ $\therefore t = 1$ $f(x)$ 最小.

$x_1 = [-\frac{9}{8}, \frac{3}{4}]$ $\nabla f(x_1) = [0, 0]^T$ $\|\nabla f(x_1)\| = 0$

$\therefore$ 最优解 $x^* = [-\frac{9}{8}, \frac{3}{4}]^T$ $f(x^*) = \frac{151}{16}$

5. $\nabla f(X) = [4x_1 - x_2,\ 2x_2 - x_1]^T$ 初始点假设 $X_0 = [0,1]^T$

$\therefore P_0 = -\nabla f(X_0) = [-1,-2]^T \quad X_1 = X_0 + t_0 P_0 = \begin{bmatrix}0\\1\end{bmatrix} + t_0\begin{bmatrix}-1\\-2\end{bmatrix} = \begin{bmatrix}t_0\\1-2t_0\end{bmatrix}^T$

$\nabla f(X)\Big|_{t=t_0}^{t} = 16t_0 - 5 = 0 \quad \therefore t_0 = 0.125$

$X_1 = [0.3125, -0.375]^T \quad \nabla f(X_1) = [0.875, 0.4375]^T$

$\lambda_0 = \dfrac{\|\nabla f(X_1)\|^2}{\|\nabla f(X_0)\|^2} = 0.191406$

$\therefore P_1 = -\nabla f(X_1) + \lambda_0 P_0 = [-0.683594, -0.820312]^T$

$X_2 = X_1 + t_1 P_1 = \begin{bmatrix}-0.3125 - 0.683594\,t_1\\ 0.375 - 0.820312\,t_1\end{bmatrix}$

$\nabla f(X)\Big|_{t=t_1}^{t_1} = 0 \quad \therefore t_1 = 0.456912$

$X_2 = [0,0]^T \quad \nabla f(X_2) = [0,0]^T \quad \|\nabla f(X_2)\| = 0$

最优解 $X^* = [0,0]^T \quad f(X^*) = 0$

6. $g(X) = \nabla f(X) = [8x_1 - 40,\ 2x_2 - 12]^T \quad A = \begin{bmatrix}8&0\\0&2\end{bmatrix} \quad H_0 = I$

$\therefore X_0 = [8,19]^T \quad g_0 = [24,6]^T$

$X_1 = [4.86154, 8.21538]^T \quad g_1 = [-1.10768, 4.43076]^T$

$\therefore S_0 = X_1 - X_0 = [-3.13846, -0.78462]^T$

$\therefore y_0 = g_1 - g_0 = [-25.10768, -1.56924]^T$

$H_1 = H_0 + \dfrac{S_0 S_0^T}{S_0^T y_0} - \dfrac{H_0 y_0 y_0^T H_0}{y_0^T H_0 y_0} = \begin{bmatrix}0.12697 & -0.03149\\ -0.03149 & 1.0038\end{bmatrix}$

$P_1 = -H_1 g_1 = [-0.28017, 4.48248]^T$

$\therefore X_1 = X_1 + tP_1 = [4.86154 - 0.28017t,\ 8.21538 + 4.48248t]^T$

$f(X_1 + tP_1)\big|_t = 0 \quad t = -0.48674$

$X_2 = X_1 + tP_1 = [5,6]^T \quad g(X_2) = [0,0]^T$

$\therefore X^* = [5,6]^T \quad f(X^*) = 0$

7. 沿方向 $e_1=[1,0]^T$ $e_2=[0,1]^T$ $f(X_0)=60$

$f(X_0+te_1)=t^2-10t+60$ $\frac{df}{dt}=0$ $t=5$

$X_1=X_0+te_1=[5,0]^T$ $f(X_1)=35$

沿方向 e_2 $f(X_1+te_2)=t^2-9t+35$ $\frac{df}{dt}=0$ $t=\frac{9}{2}$

$X_2=X_1+te_2=[5,\frac{9}{2}]^T$ $f(X_2)=14.75$

…… e_1 $t=2.0352$ $X_9=[7.9883,5.9766]^T$ $f(X_9)=8.00004$

e_2 $t=0.0176$ $X_{10}=[7.9883,5.9941]^T$ $f(X_{10})=8.0001$

最优解 $X^*=[7.9883,5.9941]^T$ $f(X^*)=8.0001$

8. $f(X_0)=5$ $f(X_2)=0.133387$ $f(X_3)=-1.826469$

$X_4=\frac{1}{3}(\sum X_i-X_{max})=[0.612,0.612]^T$

$X_5=2X_4-X_1=[1.224,1.224]^T$ $f(X_5)=-5.19347$

$X_6=X_4+\alpha(X_5-X_4)=[1.2852,1.2852]^T$ $f(X_6)=-5.46718$

X_6 代替 X_1

…… 第五次迭代 $f(X_6)=-5.467182$ $f(X_{10})=-6.650356$ $f(X_{13})=-6.591614$

$X_{15}=2X_{14}-X_6=[2.6316,2.6316]^T$

$X_{16}=X_{14}+\beta(X_{15}-X_{14})=[2.295,2.295]^T$

$f(X_{16})=-6.738925$ $|f(X_2-f(X_4)|=0.058<\varepsilon$

∴ 最优解 $X^*=[2.295,2.295]^T$ $f(X^*)=-6.738925$