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CS 302 - 1002 : Data Structures
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CS 302 ASSIGNMENT 3

1(a) 1 digit : ⌴ } O(1)
- ⌴ }

10 digits: |||||||||| } O(1) n times = O(n)
- |||||||||| }

↓↓ O(1), n times

1(b) Compute a^n where a is a 3-digit number.

||| 3 digits
× ||| 3 digits
||||| 3^2 multiplications
+ ||||| X each multiplication = O(1)
||||| XX 3^n multiplications
|||||| Therefore $O(3^n)$

1c) Divide n-digit number with a 1-digit number > 1

|||||
|||||||
↑ 1 digit # > 1

each digit in quotient must try out digits 0-9 and is O(1) in runtime occurring n number of times

Therefore O(n)

2) Show for any constant $K > 0$, $O(\log^K n) = O(n)$

$$\lim_{n\to\infty} \frac{n}{\ln^K(n)} = \lim_{n\to\infty} \frac{1}{\frac{K \cdot \ln^{K-1}(n)}{n}}$$

$$= \lim_{n\to\infty} \frac{n}{K \cdot \ln^{K-1}(n)}$$

$$= \frac{1}{K} \lim_{n\to\infty} \frac{n}{\ln^{K-1}(n)} \quad \frac{\infty}{\text{undefined}} = \infty.$$

$f(x) = n$ is faster in growth rate.

3) Prove or disprove $2^n = \Theta(3^n)$

$$\lim_{n\to\infty} \frac{2^n}{3^n} = \lim_{n\to\infty} \frac{2^n \ln(2)}{3^n \ln(3)}$$

$$= \frac{\ln(2)}{\ln(3)} \cdot \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$$

2^n and 3^n do not have the same asymptotic growth rate. 3^n has a faster growth rate.

4) $\log_{10} n = 4 \log_2 n < \frac{n}{2^{64}} = 4000n + 22 < n \log_2 n = 2n^{1.5} - n \log n$
$< \frac{n^2}{2} + 20n - 4 < 1.0001^n = 2^{64} = n\sqrt{n}$

5) INSERTION SORT 5, 3, 10, 2, 9, 1

3 5

~~5~~ ~~3~~ 10 2 9 1

i ← 1

key ← 3

j ← ~~0~~ -1

↓ (next iteration)

3 5 10 2 9 1

i ← 2

key ← 10

j ← 1 (next iteration)

2 3 5 10

~~3~~ ~~5~~ ~~10~~ 2 9 1

i ← 3

key ← 2

j ← ~~2~~ ~~1~~ 0 -1 → (next line / iteration)

9 10

2 3 5 ~~10~~ ~~9~~ 1

i ← 4

key ← 9

j ← ~~3~~ 2 (next iteration)

1 2 3 5 9 10

~~2~~ ~~3~~ ~~5~~ ~~9~~ ~~10~~ ~~1~~

i ← 5

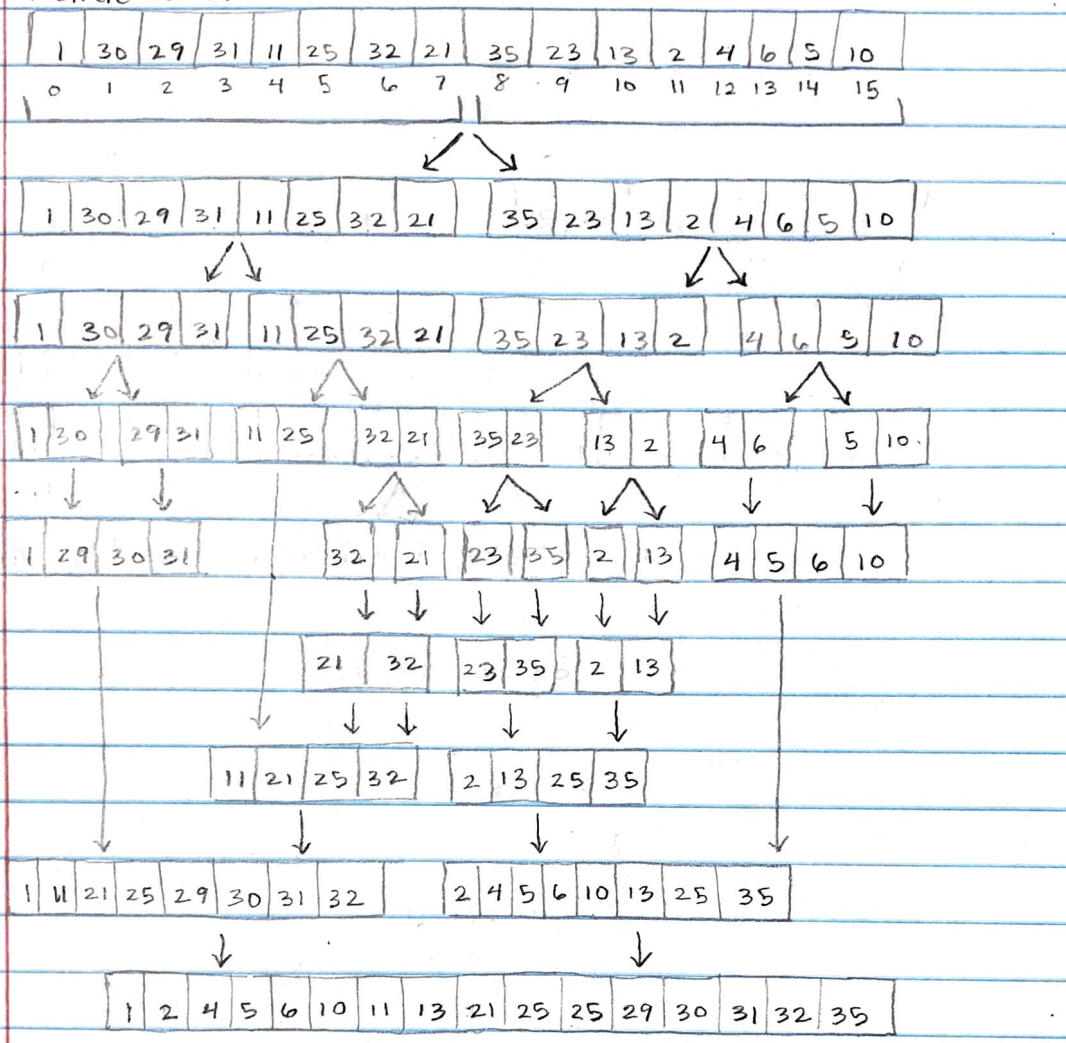
key ← 1

j ← ~~4~~ ~~3~~ ~~2~~ ~~1~~ ~~0~~ -1 → (next iteration)

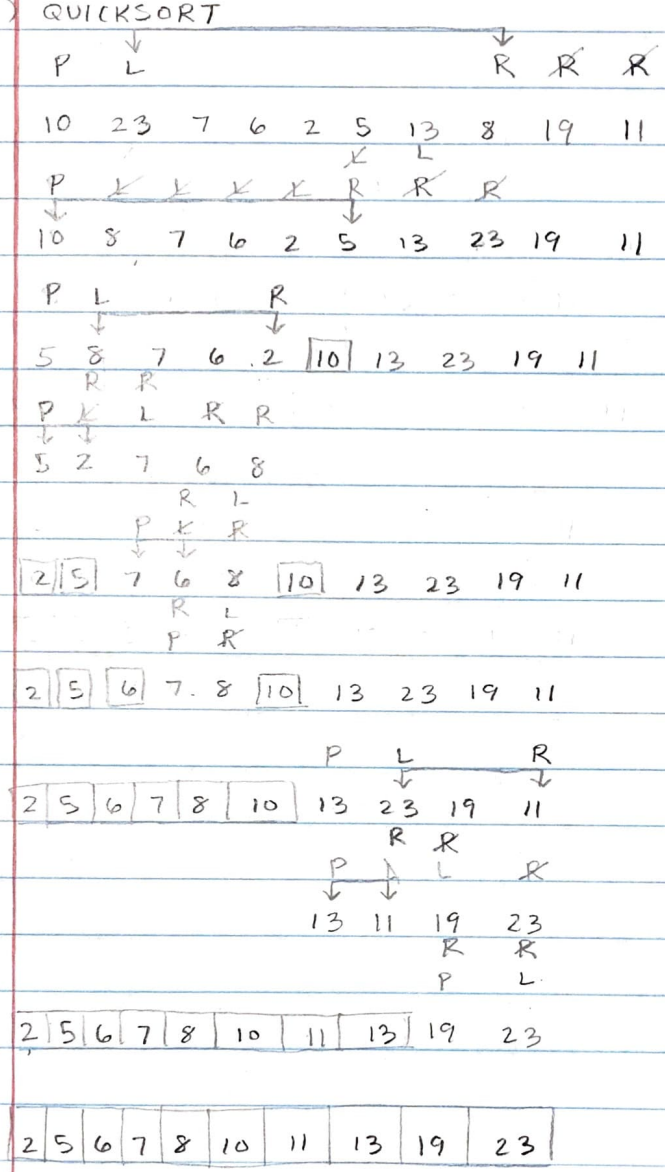
1 2 3 5 9 10

i ← 6 exit insertion sort.

6) MERGE SORT


1 30 29 31 11 25 32 21 35 23 13 2 4 6 5 10
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1 30 29 31 11 25 32 21 35 23 13 2 4 6 5 10
1 30 29 31 11 25 32 21 35 23 13 2 4 6 5 10
1 30 29 31 11 25 32 21 35 23 13 2 4 6 5 10
1 29 30 31 32 21 23 35 2 13 4 5 6 10
21 32 23 35 2 13
11 21 25 32 2 13 25 35
1 11 21 25 29 30 31 32 2 4 5 6 10 13 25 35
1 2 4 5 6 10 11 13 21 25 25 29 30 31 32 35

7) QUICKSORT


P L R R R
10 23 7 6 2 5 13 8 19 11
L L
P L L L L R R R
10 8 7 6 2 5 13 23 19 11
P L R
5 8 7 6 2 10 13 23 19 11
R R
P L L R R
5 2 7 6 8
R L
P L R
2 5 7 6 8 10 13 23 19 11
R L
P R
2 5 6 7 8 10 13 23 19 11
P L R
2 5 6 7 8 10 13 23 19 11
R R
P L R
13 11 19 23
R R
P L
2 5 6 7 8 10 11 13 19 23
2 5 6 7 8 10 11 13 19 23