The vector between a point $\langle x, y\rangle$ and the closest point on a ray running through $\langle u, v\rangle$ is given by

$$\langle x, y\rangle - (xu + yv)\langle u, v\rangle$$

In other words:

$$\langle x - (xu + yv)u, y - (xu + yv)v\rangle$$

The squared magnitude of this vector is given by

$$(x - (xu + yv)u)^2 + (y - (xu + yv)v)^2$$

Simplifying yields

$$x^2 + y^2 - 2xu(xu + yv) + u^2(xu + yv)^2 - 2yv(xu + yv) + v^2(xu + yv)^2$$

Without loss of generality, we can choose $\langle u, v\rangle$ to be a unit vector, so that $u = \cos\theta$ and $v = \sin\theta$. To simplify notation, we will let $c = \cos\theta$ and $s = \sin\theta$. So, replace u with c and v with s to get

$$x^2 - 2xc(xc + ys) + c^2(xc + ys)^2 + y^2 - 2ys(xc + ys) + s^2(xc + ys)^2$$

However, we want to minimize the sum of many of these distances:

$$\sum_{i=1}^{n} x_i^2 - 2x_ic(x_ic + y_is) + c^2(x_ic + y_is)^2 + y_i^2 - 2y_is(x_ic + y_is) + s^2(x_ic + y_is)^2$$

However, for sake of simplicity, we will not write the sigma summation symbol or the i subscripts. Anyways, simplifying using $c^2 + s^2 = 1$ and moving terms around yields

$$x^2 + y^2 - 2xc(xc + ys) - 2ys(xc + ys) + (xc + ys)^2$$

Taking the derivative with respect to θ and setting it equal to 0 yields

$$2xs(xc + ys) - 2xc(-xs + yc) - 2yc(xc + ys) - 2ys(-xs + yc) + 2(xc + ys)(-xs + yc) = 0$$

Dividing by two gives us

$$xs(xc + ys) - xc(-xs + yc) - yc(xc + ys) - ys(-xs + yc) + (xc + ys)(-xs + yc) = 0$$

Expanding yields

$$x^2sc + xys^2 + x^2cs - xyc^2 - xyc^2 - y^2cs + xys^2 - y^2cs - x^2cs + xyc^2 - xys^2 + y^2cs = 0$$

Combing like terms and eliminating opposing terms yields

$$x^2sc + xys^2 - xyc^2 - y^2cs = 0$$

Rewriting $s = \sqrt{1 - c^2}$ gives us

$$x^2c\sqrt{1-c^2}+xy(1-c^2)-xyc^2-y^2c\sqrt{1-c^2}=0$$

Moving and combining terms yields

$$xy(1-2c^2)=(y^2-x^2)c\sqrt{1-c^2}$$

Reintroducing summation notation gives us

$$\sum x_i y_i(1-2c^2)=\sum (y_i^2-x_i^2)c\sqrt{1-c^2}$$

Bringing out the c's (which are "constant" gives us

$$(1-2c^2)\sum x_i y_i = c\sqrt{1-c^2}\sum (y_i^2-x_i^2)$$

Some algebra yields

$$\frac{\sum x_i y_i}{\sum (y_i^2-x_i^2)}=\frac{c\sqrt{1-c^2}}{1-2c^2}$$

Now, let A be the left-hand side:

$$\frac{c\sqrt{1-c^2}}{1-2c^2}=A$$

Squaring both sides yields

$$\frac{-c^4+c^2}{4c^4-4c^2+1}=A^2$$

Multiply both sides by the denominator

$$-c^4+c^2=A^2(4c^4-4c^2+1)$$

and expand

$$-c^4+c^2=4A^2c^4-4A^2c^2+A^2$$

and consolidate:

$$0=(4A^2+1)c^4+(1-4A^2)c^2+(A^2)$$

Substitute q for c^2:

$$0=(4A^2+1)q^2+(1-4A^2)q+(A^2)$$

Lo and behold: it is a quadratic. Let's solve it:

$$q=\frac{4A^2-1\pm\sqrt{(1-4A^2)^2-4(4A^2+1)(A^2)}}{2(4A^2+1)}$$

Simplifying gives us

$$q=\frac{4A^2-1\pm\sqrt{1-12A^4-12A^2}}{8A^2+2}$$

Recall, that

$$A = \frac{\sum x_i y_i}{\sum (y_i^2 - x_i^2)}$$

Moreover, $q = c^2$, and $c = \cos(\theta)$, which (given that $\langle u, v \rangle$ is a unit vector) implies $u = c$. A similar truth holds for s and v, so we have

$$\boxed{\begin{gathered} A = \frac{\sum x_i y_i}{\sum (y_i^2 - x_i^2)} \\ u^2 = \frac{4A^2 - 1 \pm \sqrt{1 - 12A^4 - 12A^2}}{8A^2 + 2} \\ u^2 + v^2 = 1 \end{gathered}}$$