The vector between a point and the closest point on a ray running through is given by

In other words:

The squared magnitude of this vector is given by

Simplifying yields

Without loss of generality, we can choose to be a unit vector, so that and . To simplify notation, we will let and . So, replace with and with to get

However, we want to minimize the sum of many of these distances:

However, for sake of simplicity, we will not write the sigma summation symbol or the subscripts. Anyways, simplifying using and moving terms around yields

Taking the derivative with respect to and setting it equal to 0 yields

Dividing by two gives us

Expanding yields

Combing like terms and eliminating opposing terms yields

Rewriting gives us

Moving and combining terms yields

Reintroducing summation notation gives us

Bringing out the ’s (which are “constant” gives us

Some algebra yields

Now, let be the left-hand side:

Squaring both sides yields

Multiply both sides by the denominator

and expand

and consolidate:

Substitute for :

Lo and behold: it is a quadratic. Let’s solve it:

Simplifying gives us

Recall, that

Moreover, , and , which (given that is a unit vector) implies . A similar truth holds for and , so we have

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