(Linear) Metric Learning

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(slides based on tutorial given by Kilian Weinberger)

What is similarity?

Who is most similar?

Similar gender?

Similar age?

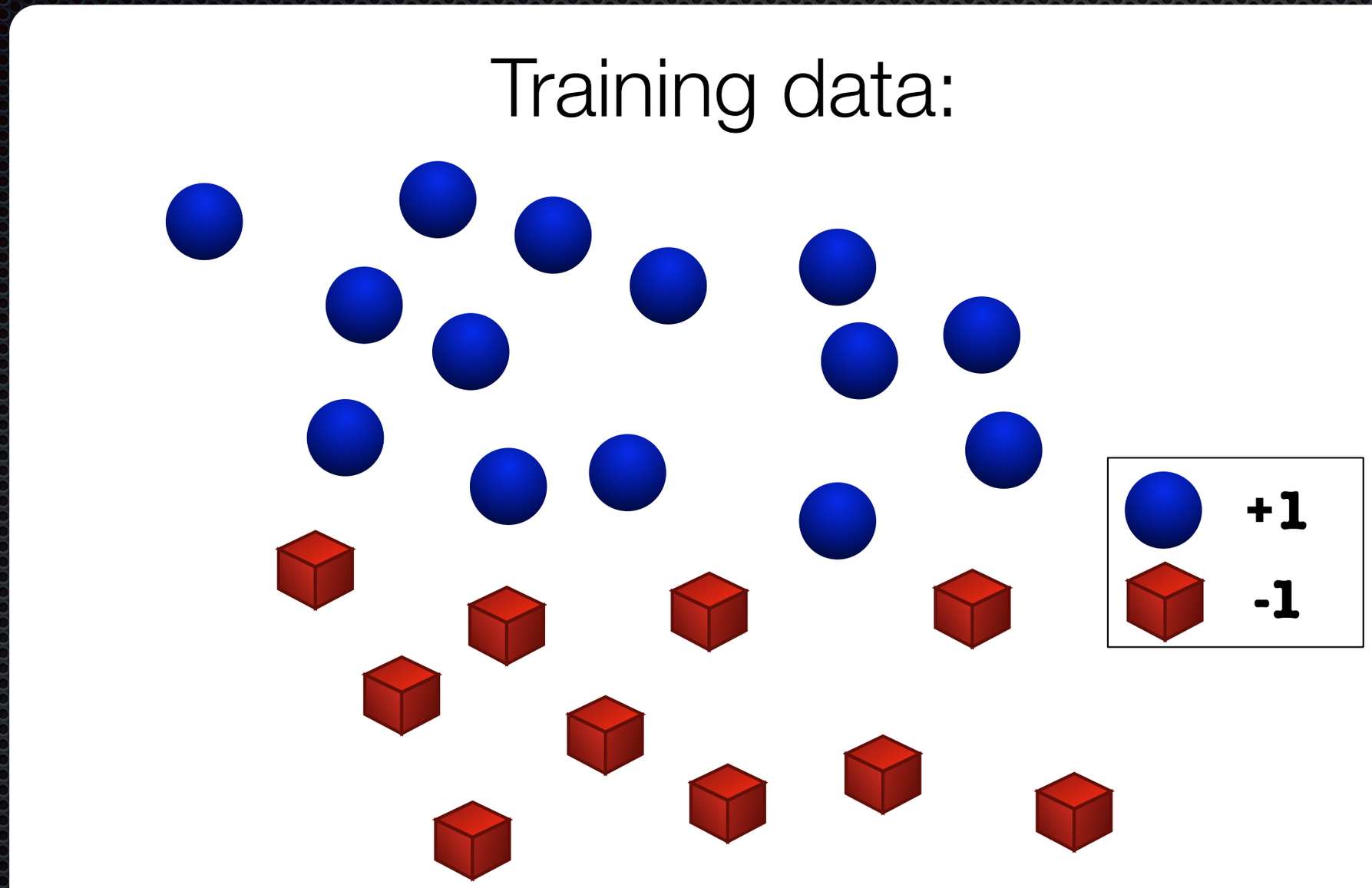
Similar hair cut?

Similarity
depends on
the context!

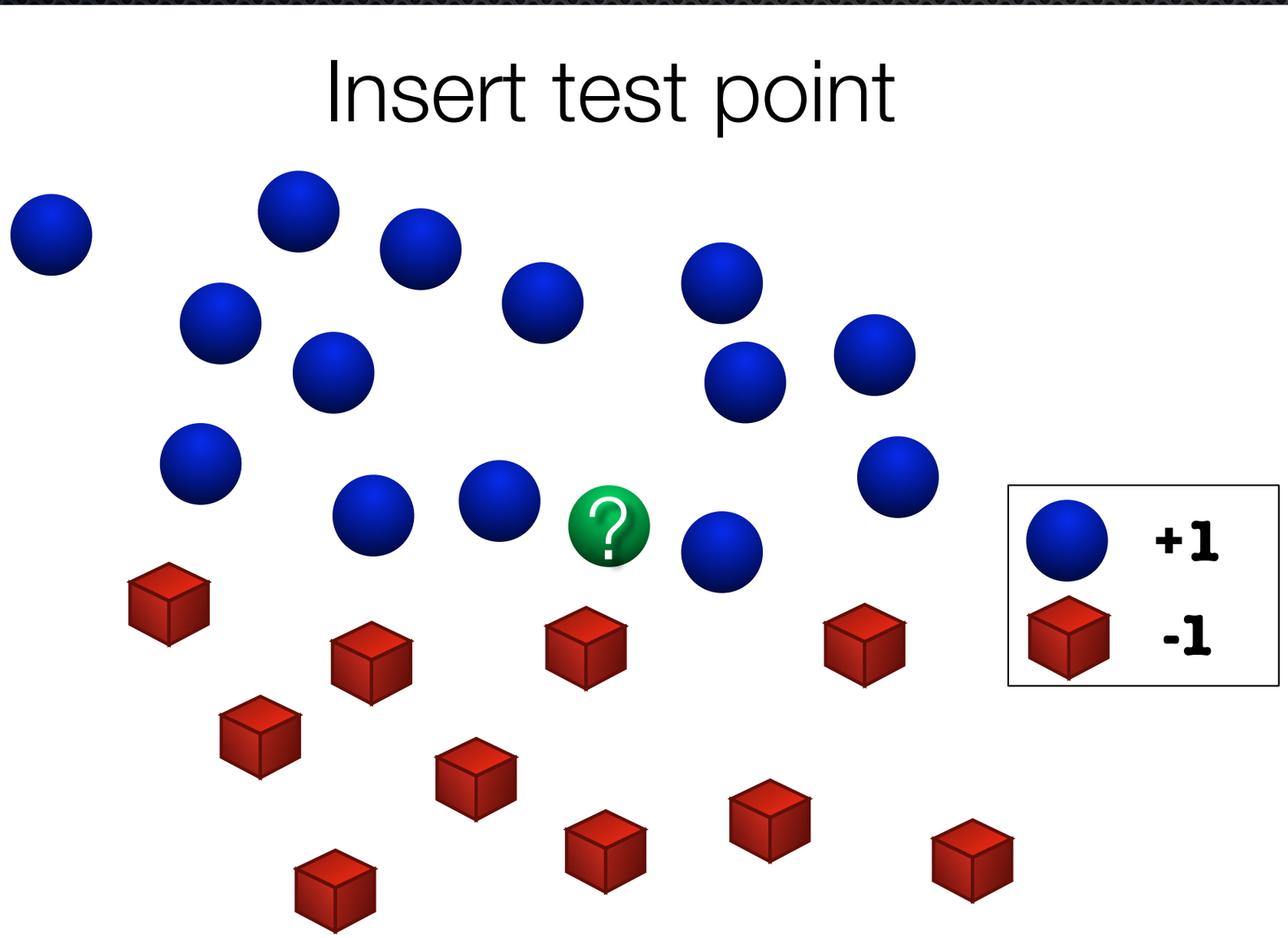
Motivation

- Many machine learning algorithms require a measure of (dis-) similarity (e.g. k-means, k-NN, SVMs, ...)
- Typically uninformed norms (e.g. **Euclidean distance**) are used as a measure of dissimilarity
- But we could incorporate our **knowledge** about the **task** and the **data**...
- Ideally we want a specific similarity measure for each task that is **semantically meaningful**.

k-nearest neighbor classification

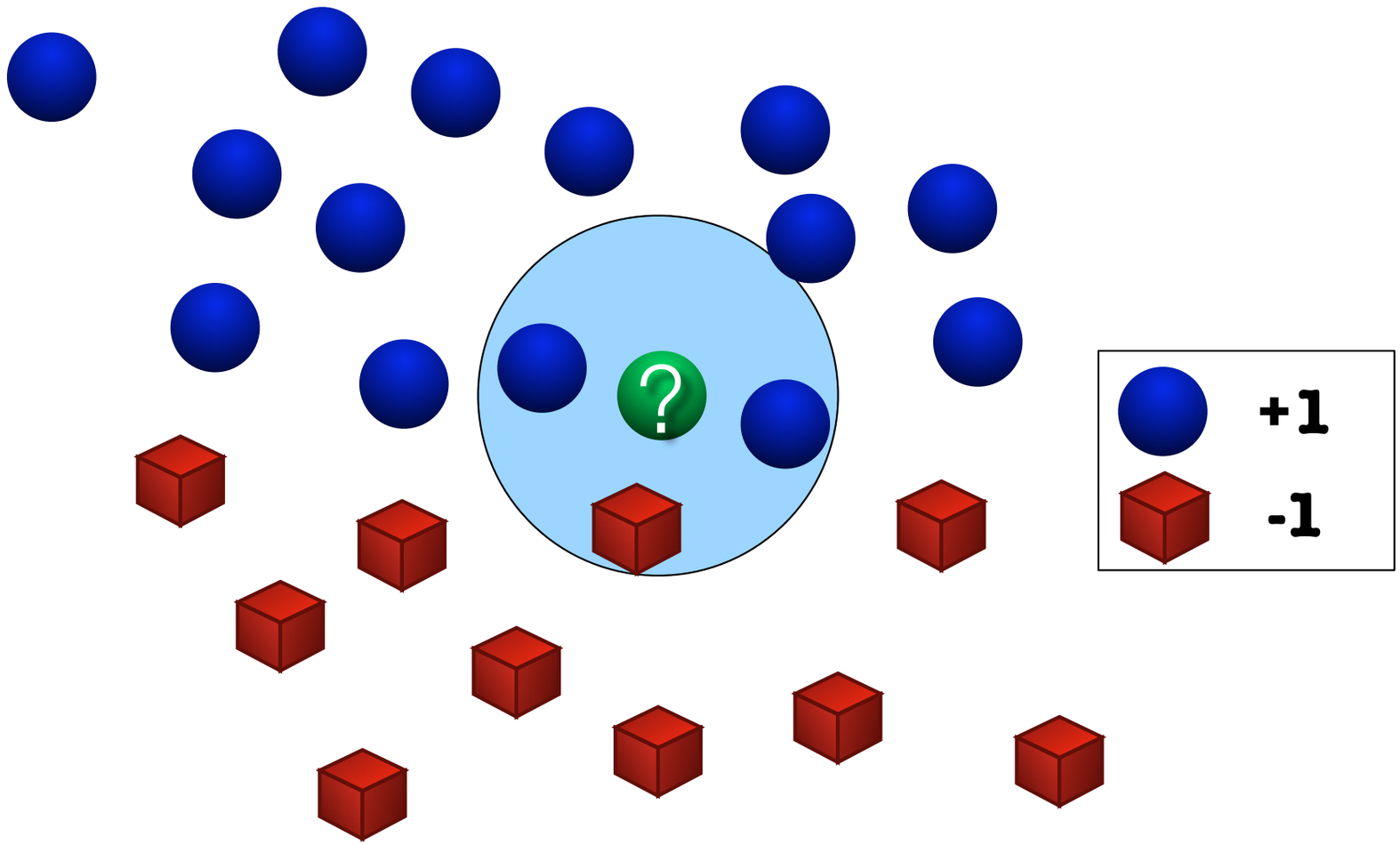


k-nearest neighbor classification



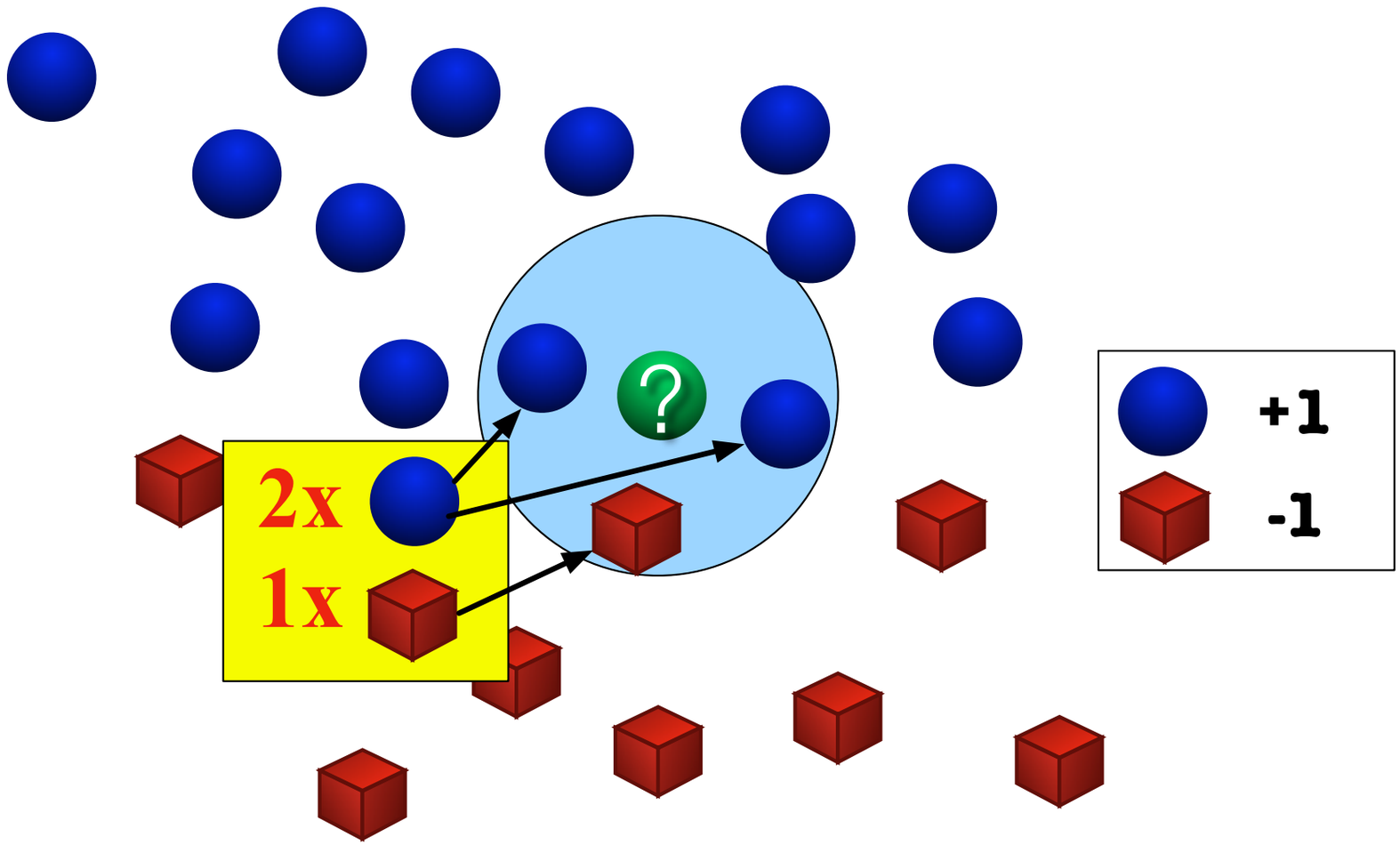
k-nearest neighbor classification

Find k nearest neighbors



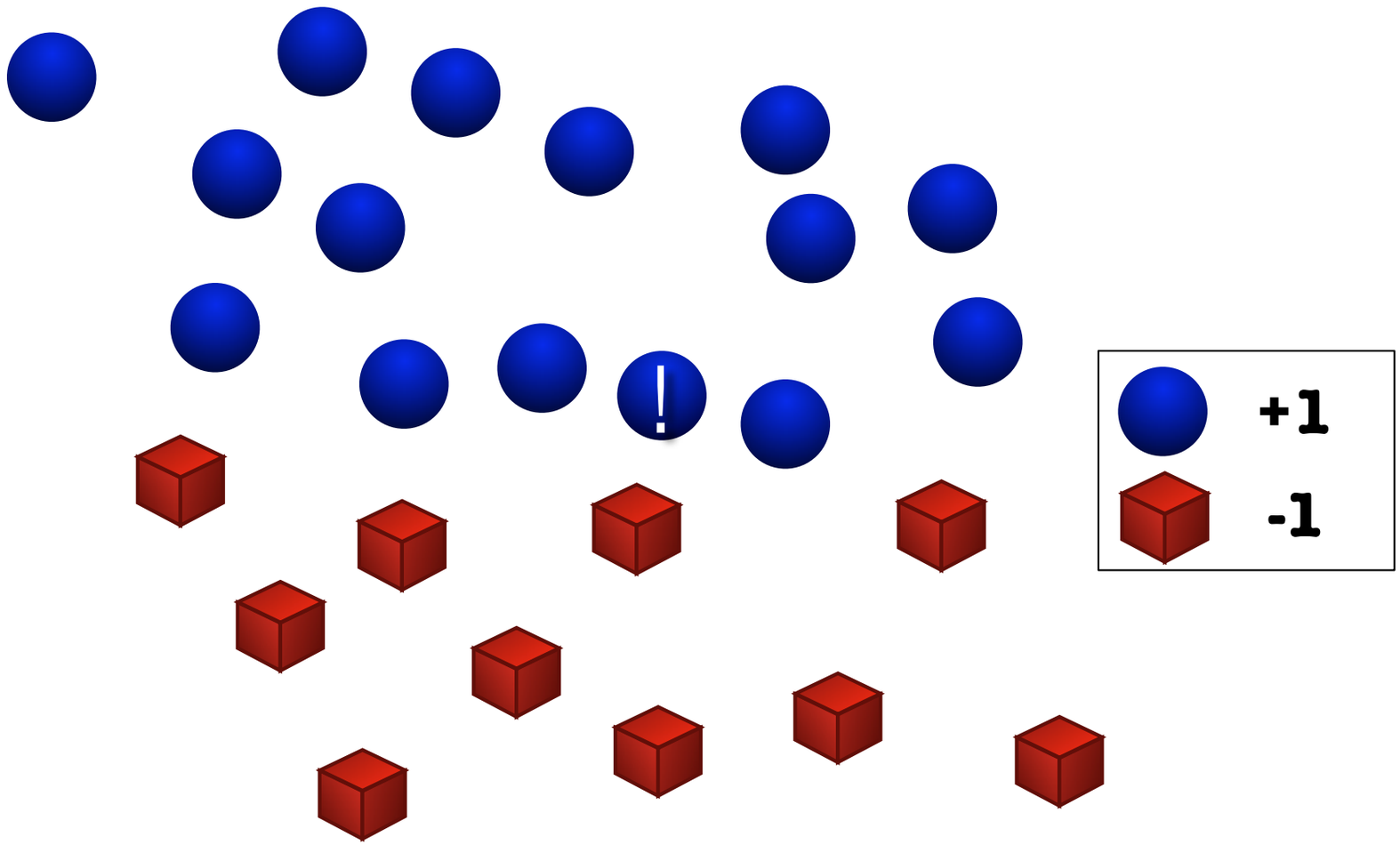
k-nearest neighbor classification

Let neighbors vote



k-nearest neighbor classification

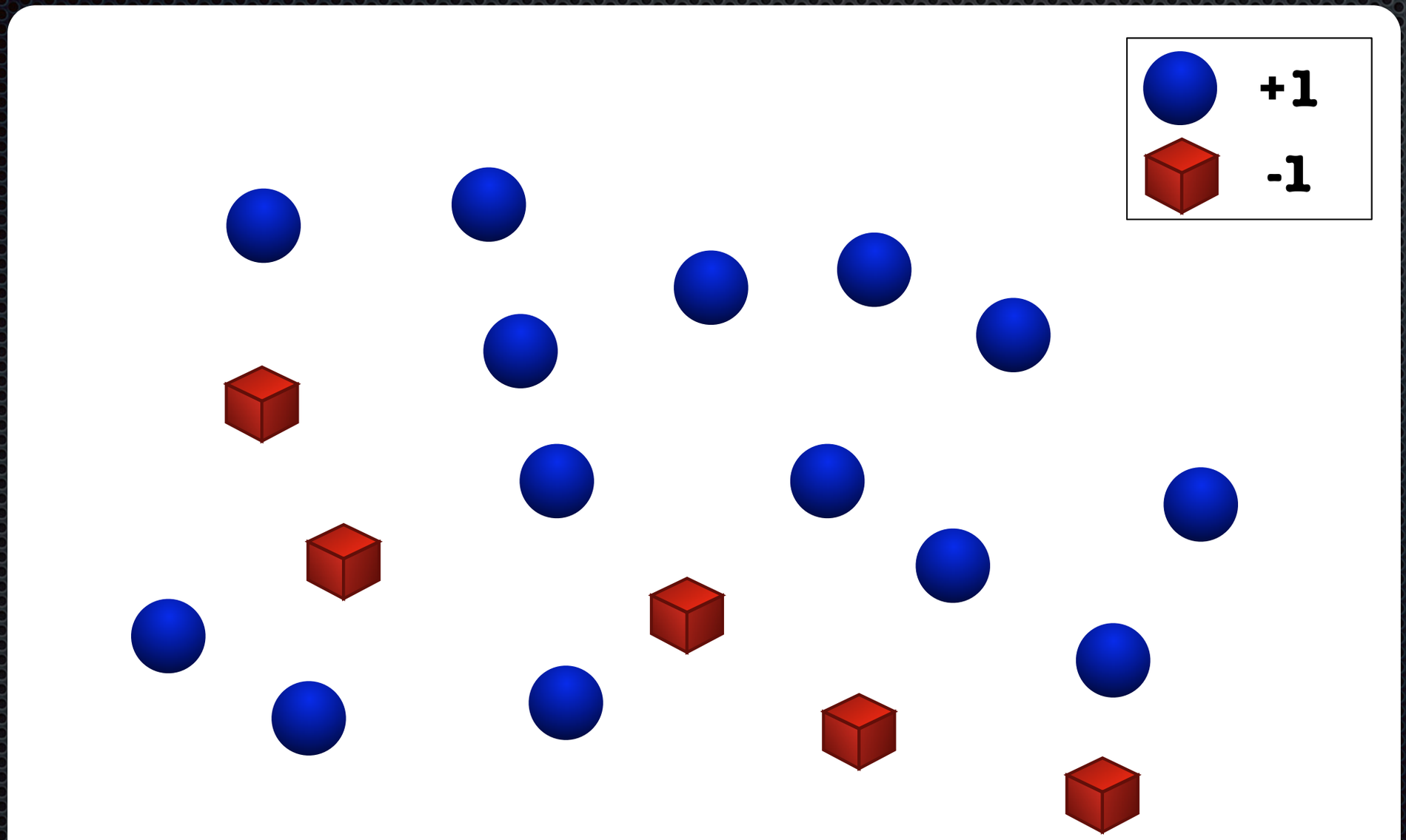
Assign most common label



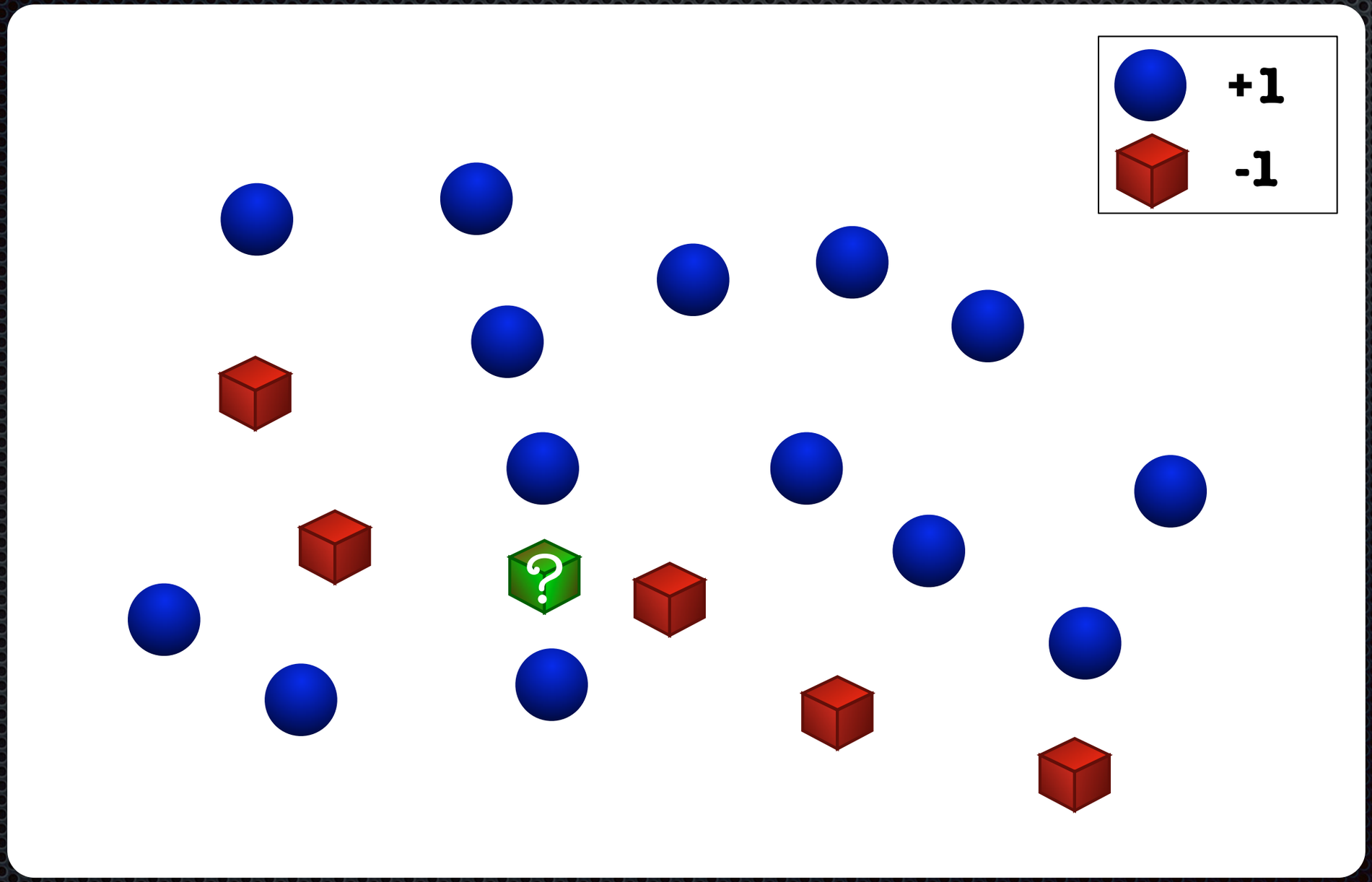
Strengths of k-NN classification

- straightforward to implement
- nonlinear decision boundaries
- complexity independent of # of classes
- asymptotic guarantees [Covert and Hart 1961]
- **relies entirely on underlying metric**

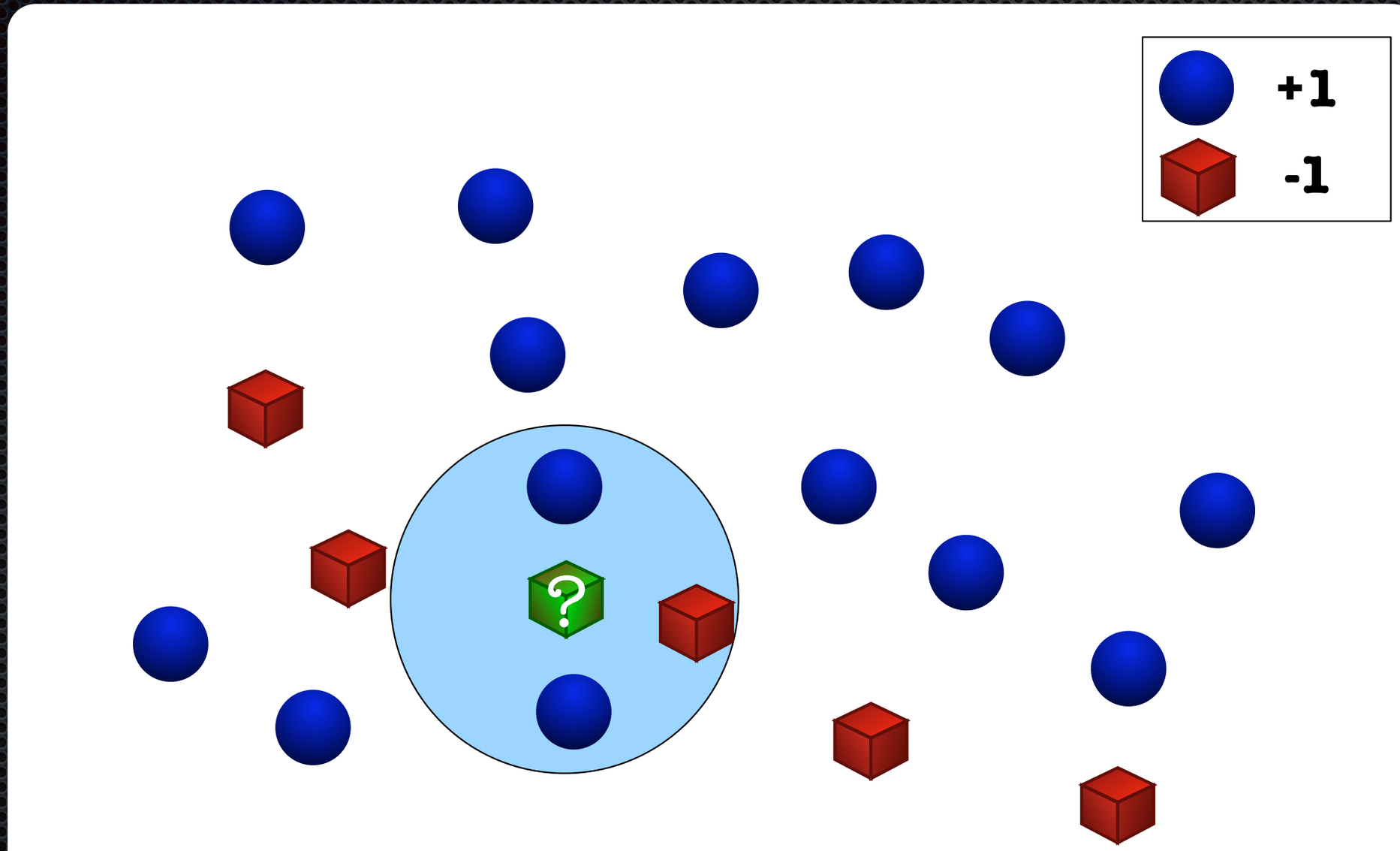
Problem with Euclidean distance



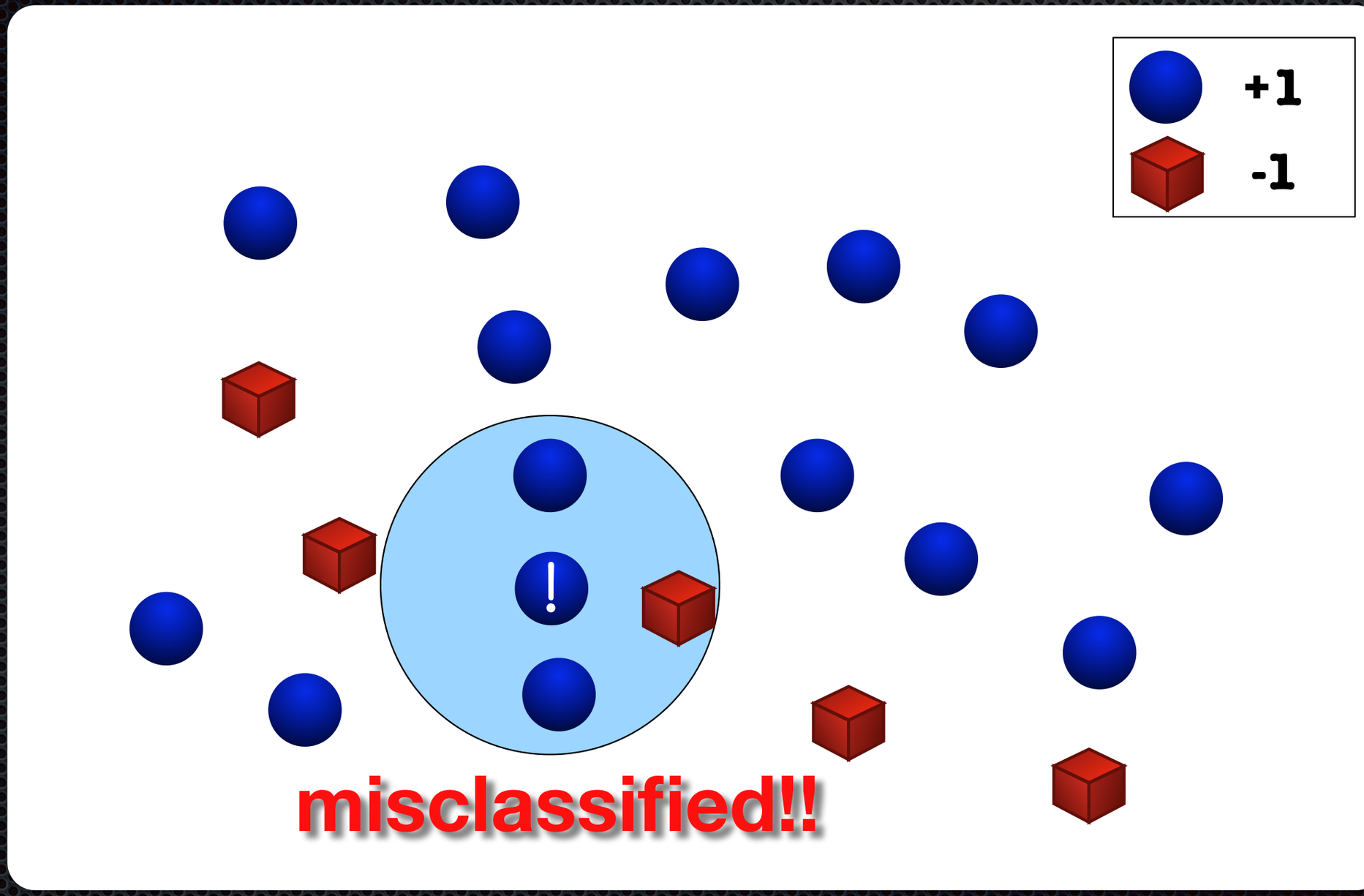
Problem with Euclidean distance



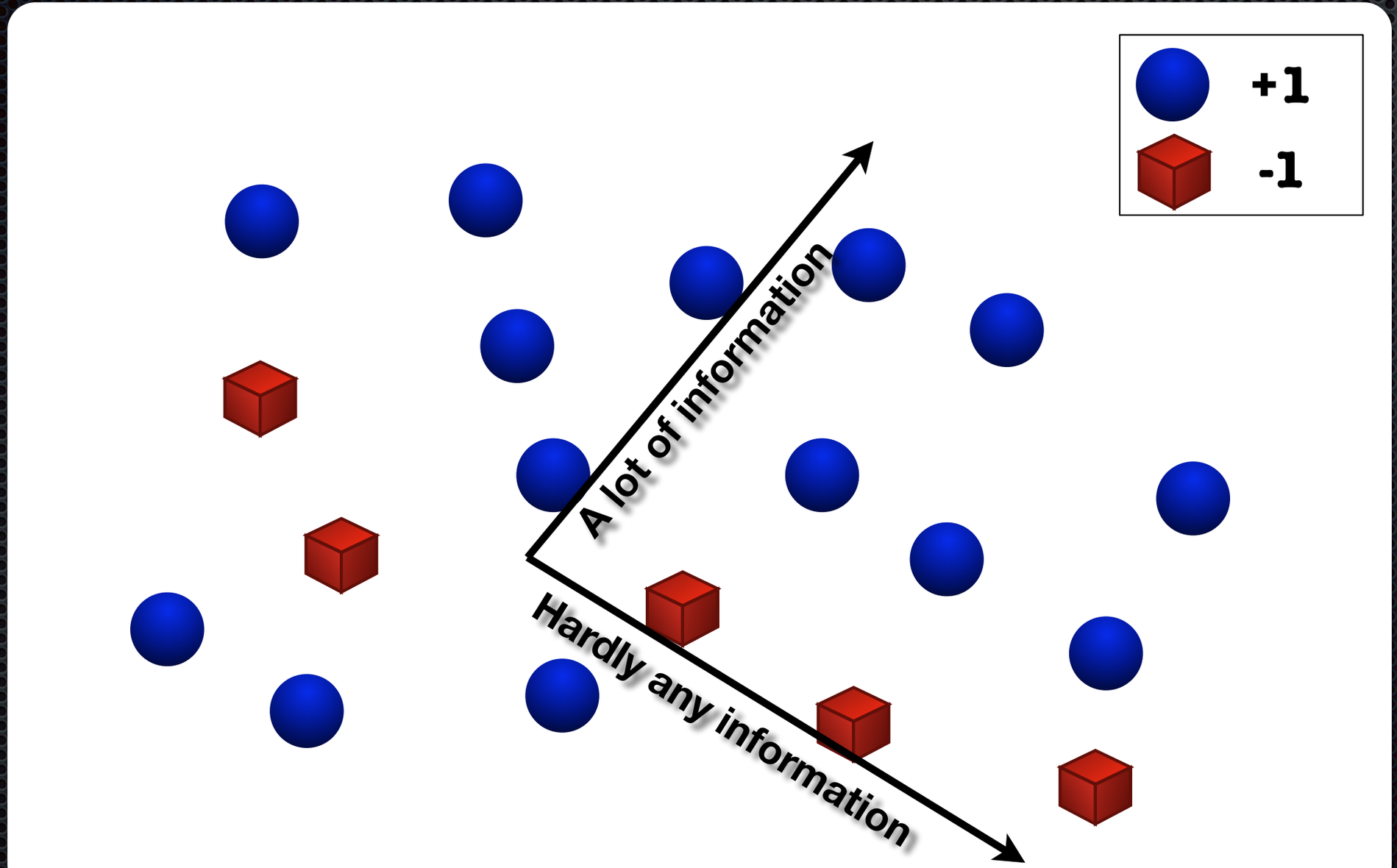
Problem with Euclidean distance



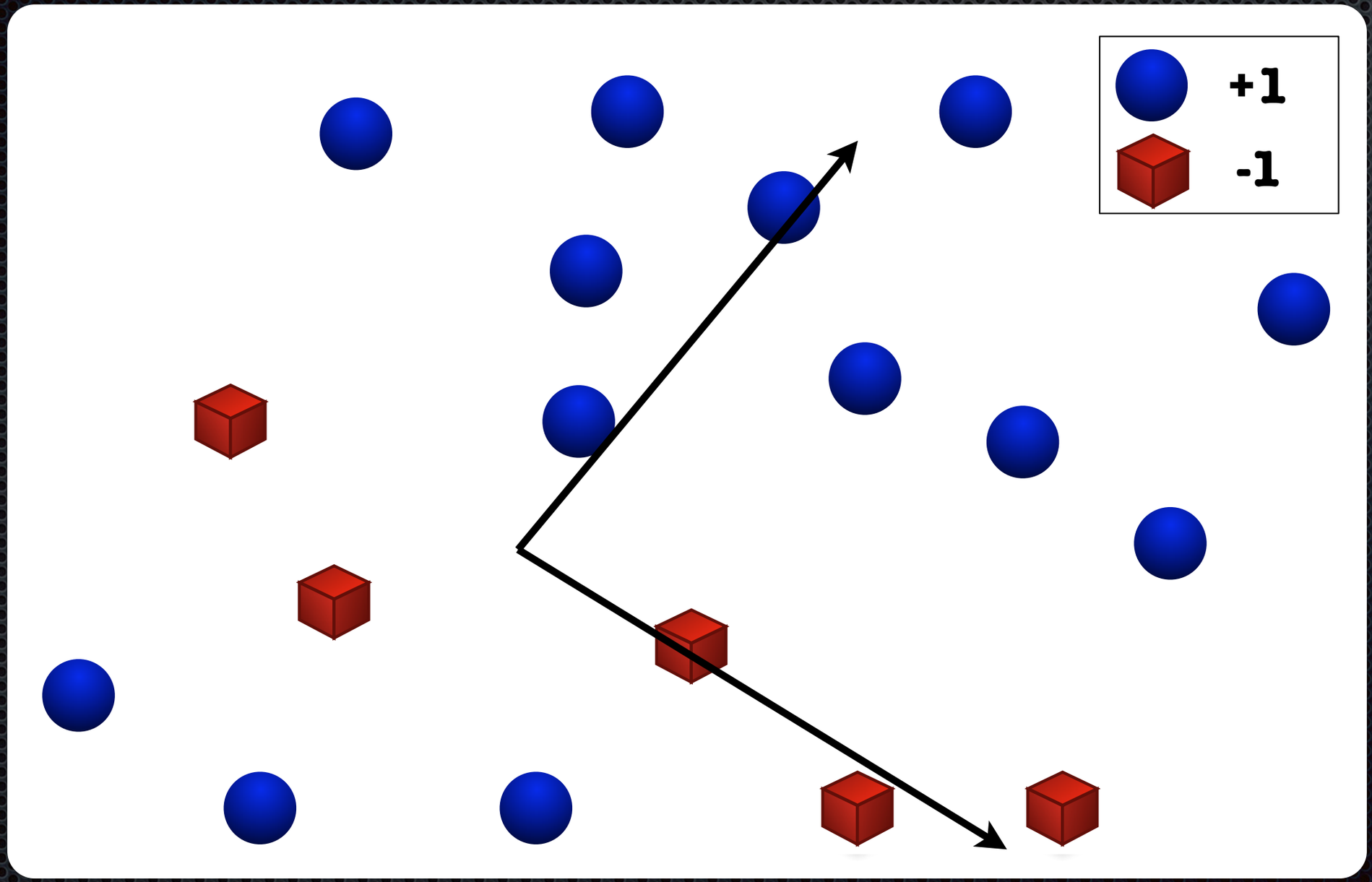
Problem with Euclidean distance



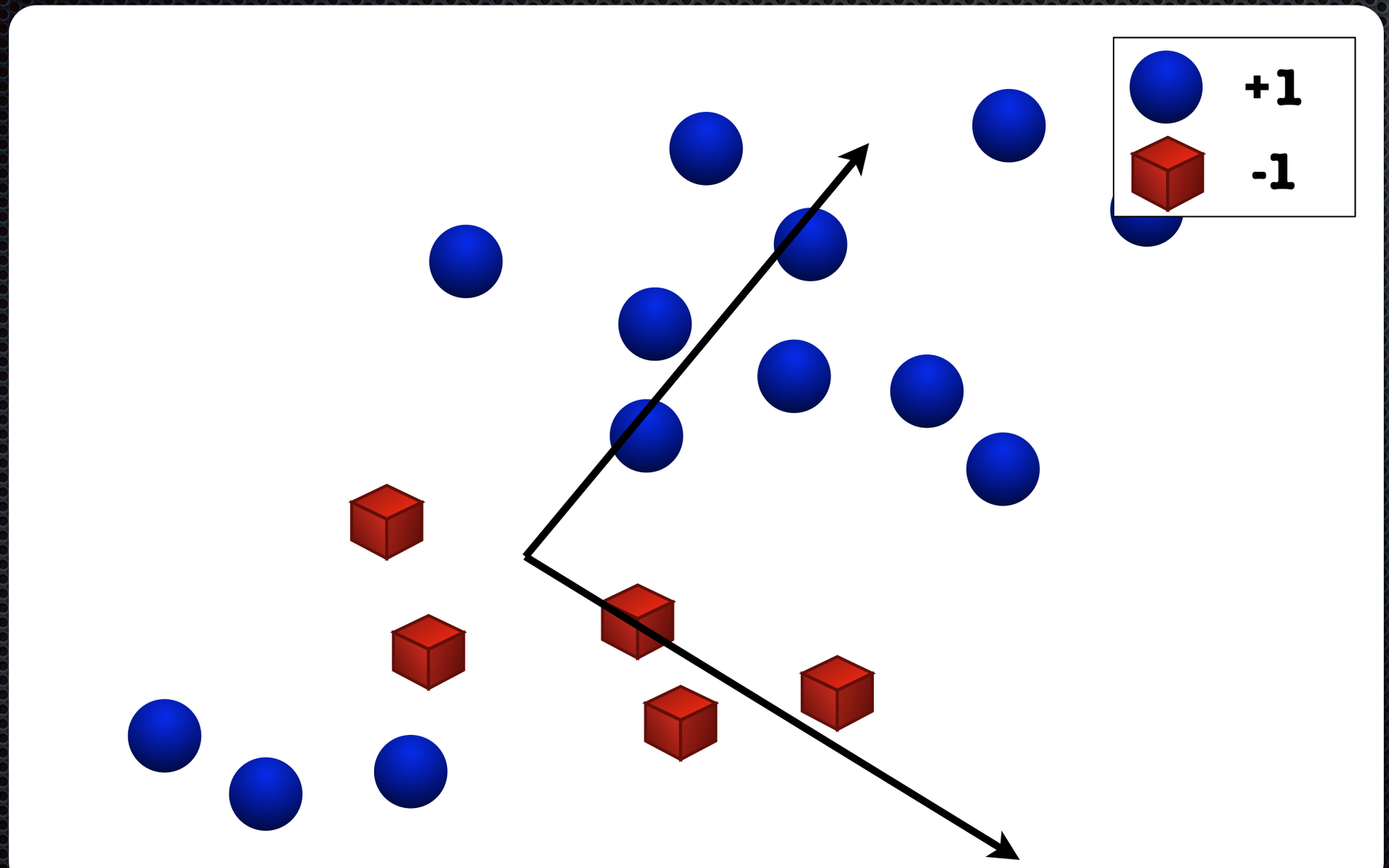
Problem with Euclidean distance



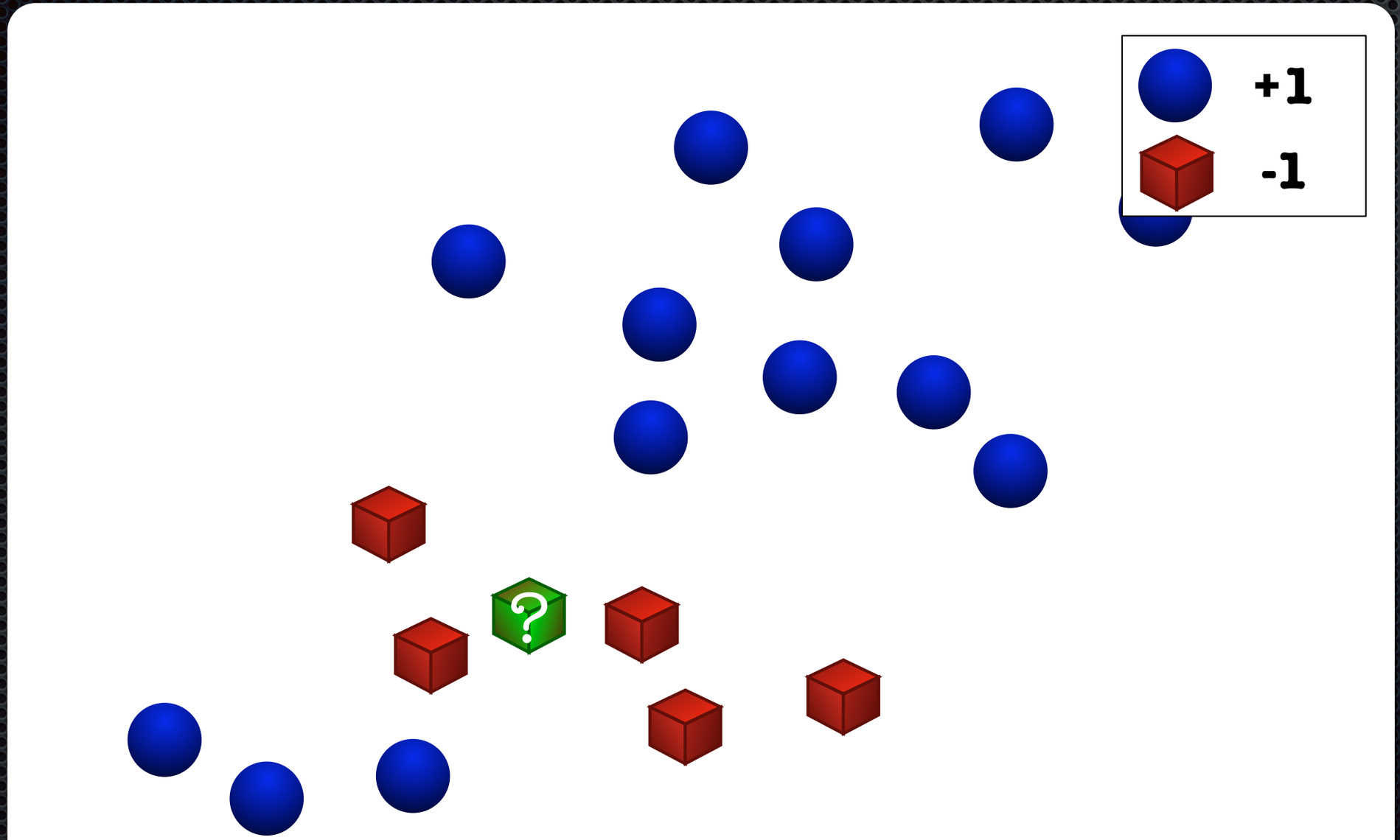
Cure: Amplify informative directions



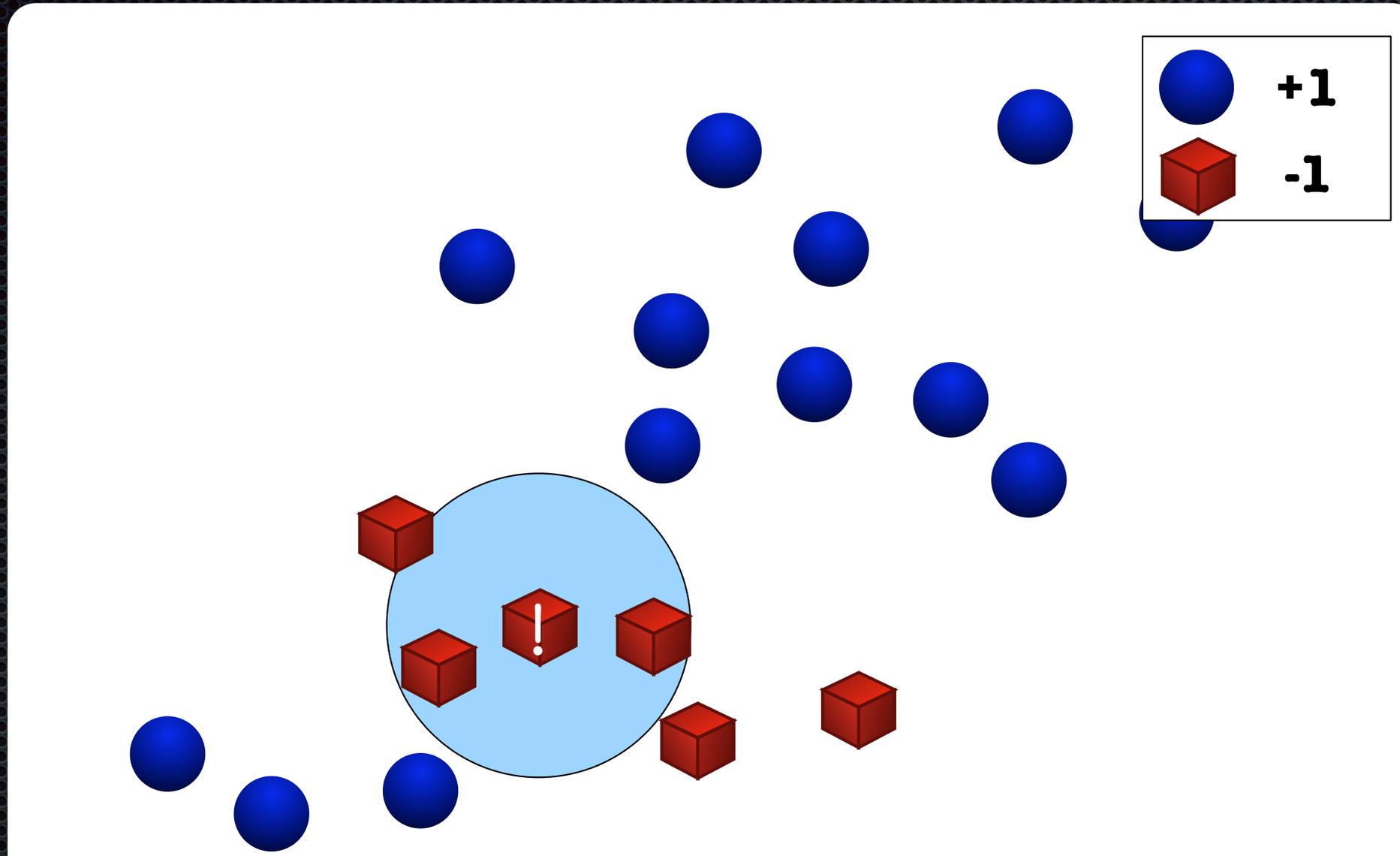
Dampen non-informative directions



Finally!!



Finally!!

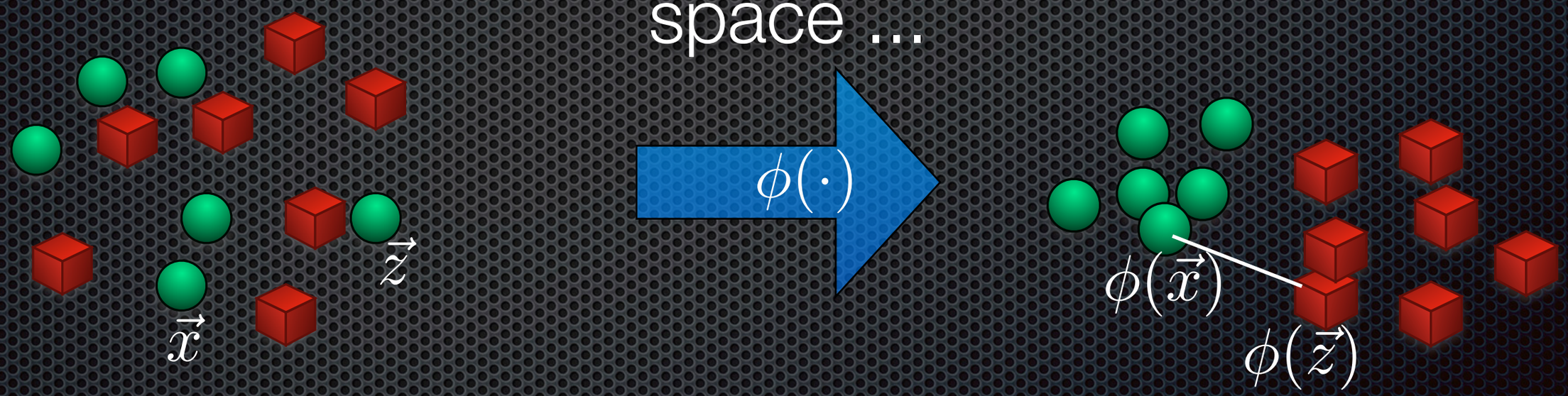


Mapping approach

Take input vectors ...

$$d(\vec{x}_i, \vec{x}_j) = \|\phi(\vec{x}_i) - \phi(\vec{x}_j)\|_2$$

... map them into some feature space ...



... and use Euclidean distance after mapping

Mapping approach

$$d(\vec{x}_i, \vec{x}_j) = \|\phi(\vec{x}_i) - \phi(\vec{x}_j)\|_2$$

Resulting metric is automatically a well-defined **pseudometric**.

requirements for pseudometric:

$d(\vec{x}, \vec{z}) \geq 0$	non-negativity
$d(\vec{x}, \vec{x}) = 0$	identity
$d(\vec{x}, \vec{z}) = d(\vec{z}, \vec{x})$	symmetry
$d(\vec{x}, \vec{z}) \leq d(\vec{x}, \vec{y}) + d(\vec{y}, \vec{z})$	triangular inequality

additional requirement for metric:

$d(\vec{x}, \vec{z}) = 0 \Leftrightarrow \vec{x} = \vec{z}$	identity of indiscernibles

Mahalanobis distance metric

The simplest mapping is a linear transformation

$$\phi : \vec{x} \rightarrow \mathbf{A}\vec{x}$$

$$d^2(\vec{x}, \vec{z}) = \|\mathbf{A}(\vec{x} - \vec{z})\|_2^2$$

$$= (\vec{x} - \vec{z})^\top \mathbf{A}^\top \mathbf{A}(\vec{x} - \vec{z})$$

$$= (\vec{x} - \vec{z})^\top \mathbf{M}(\vec{x} - \vec{z})$$

positive semi-definite

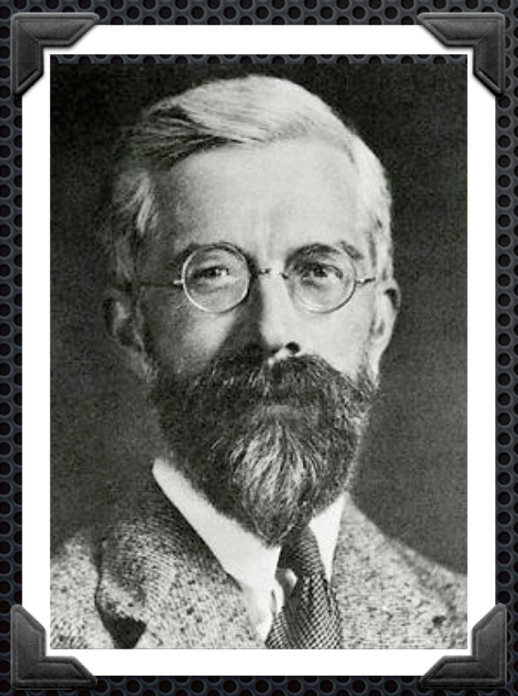
$$\mathbf{M} \succeq 0$$

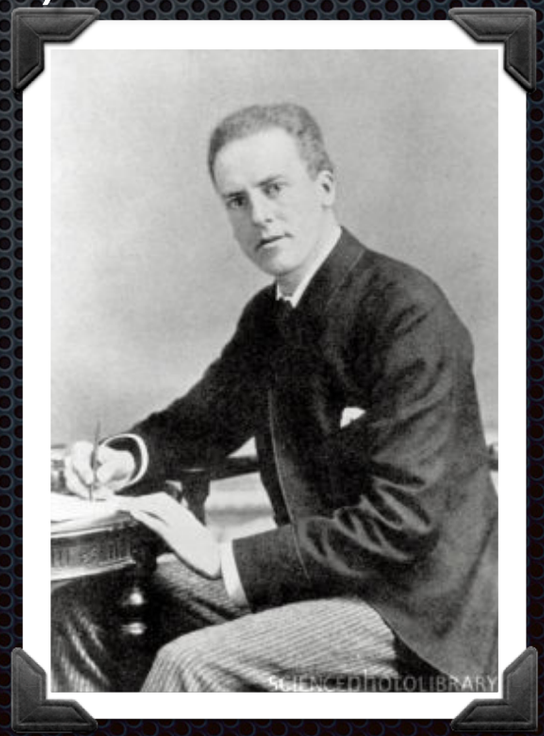
$$\mathbf{M} = \mathbf{A}^\top \mathbf{A}$$

(Generalization of Euclidean distance.)

Historic Metric Learning Algorithms

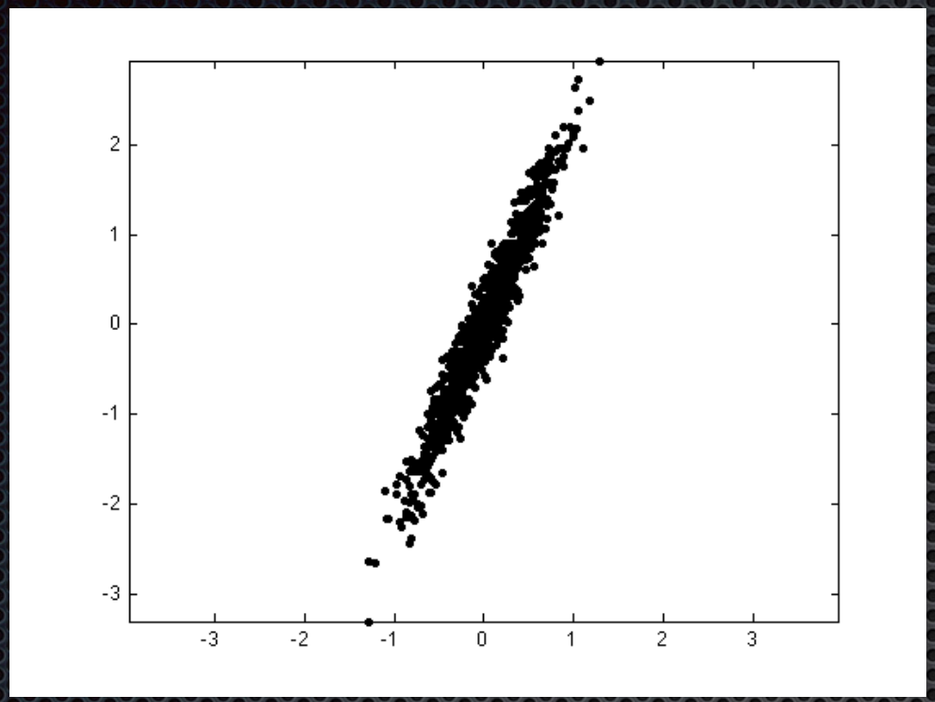
(all based on second-order statistics)

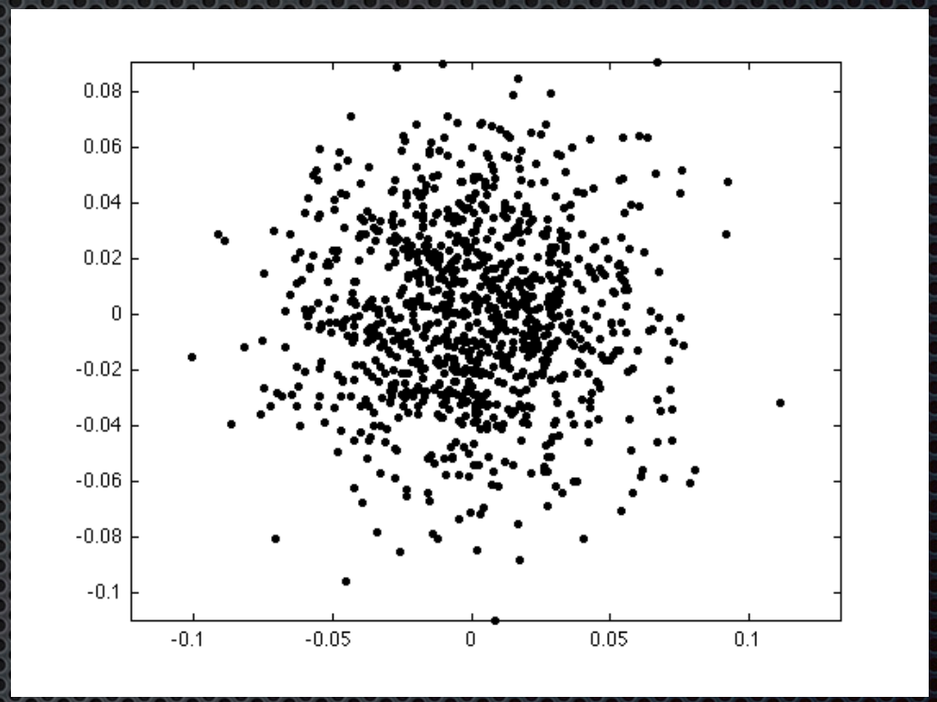




True Mahalanobis distance

(a.k.a. **whitening**)





$$M = C^{-1}$$

$$C = \sum_i (\vec{x}_i - \mu)(\vec{x}_i - \mu)^\top$$

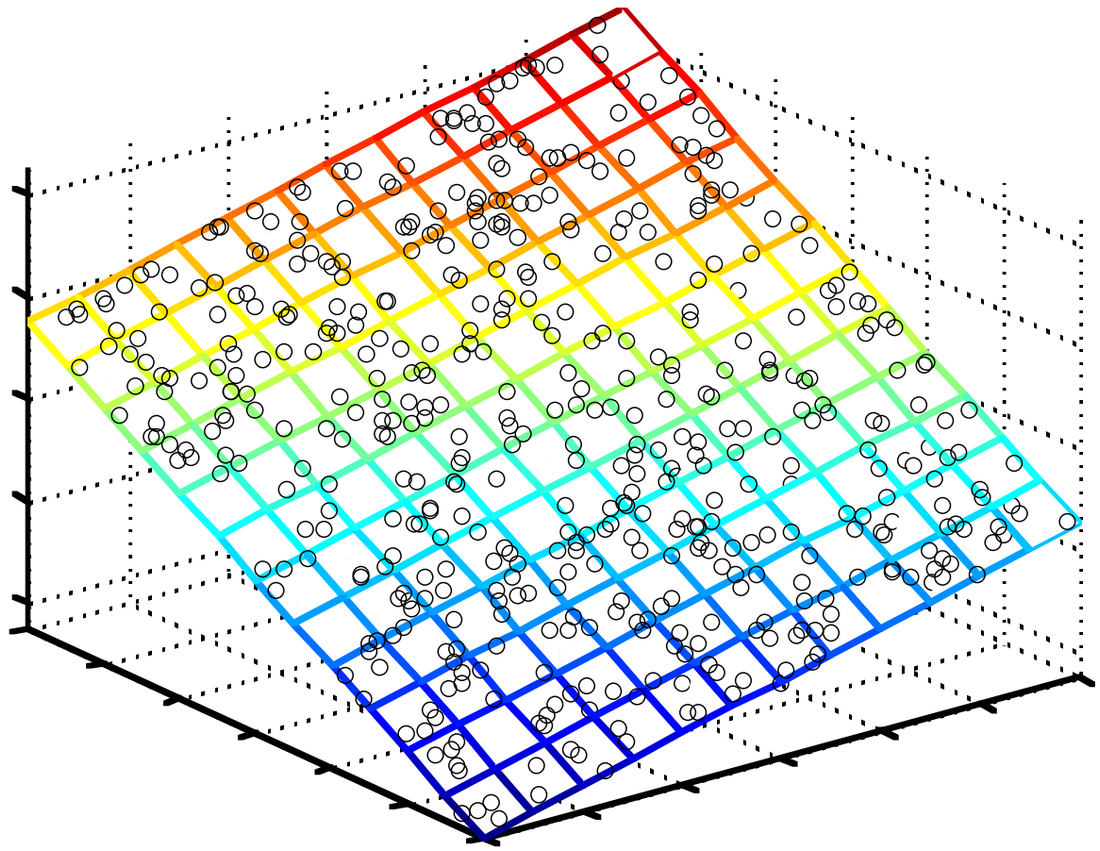
[P.C. Mahalanobis, 1936]

Principal Component Analysis

$$C = V \Delta V^\top$$

$$M = V V^\top$$

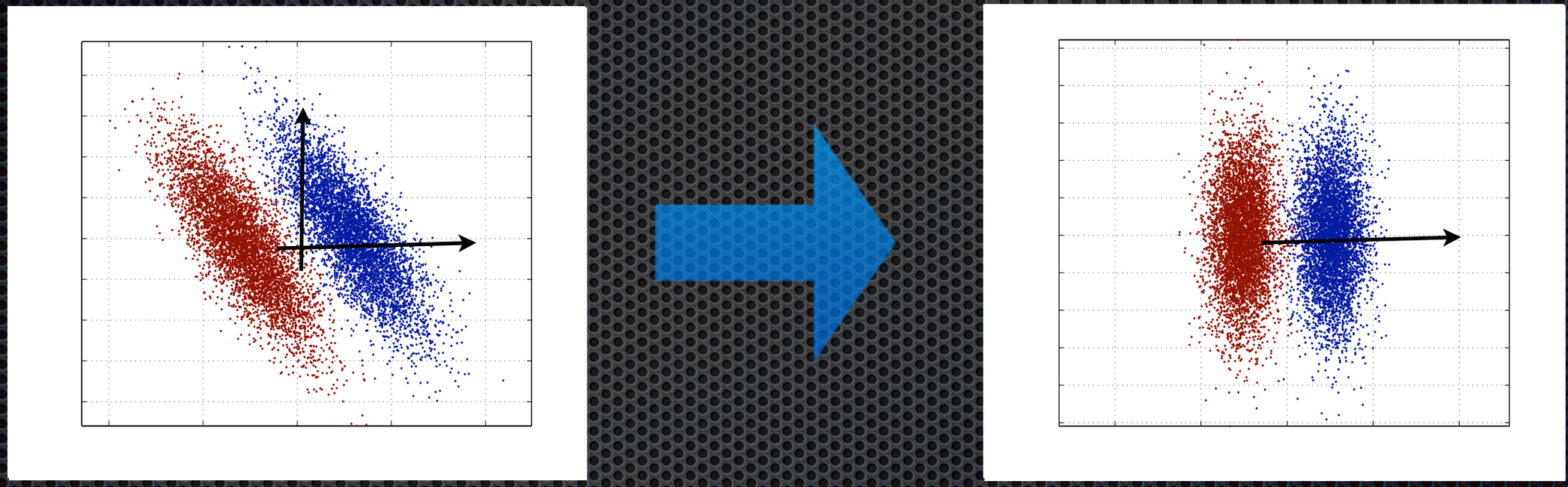
(Clip bottom k eigenvectors.)



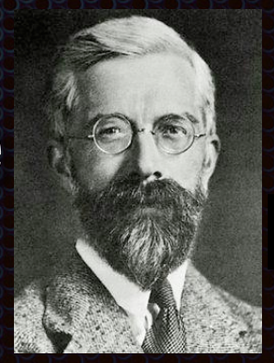
Project onto sub-space with maximum variance.

(Pearson 1901)

Liner Discriminant Analysis



Computes projection that best separates the different classes.



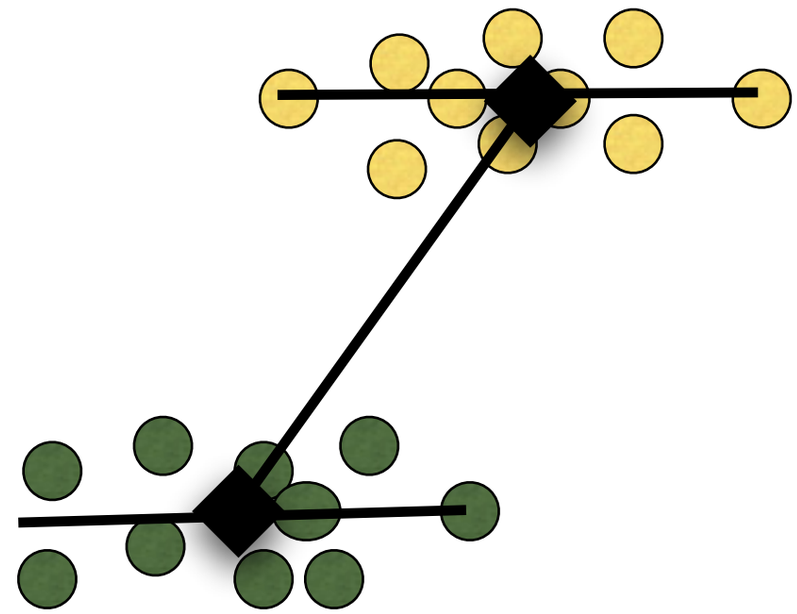
(Fisher 1936)

LDA: Optimization

between-class
covariance (weighted)

$$C_w^{-1} C_b = V \Delta V^\top$$

$$M \equiv V V^\top$$

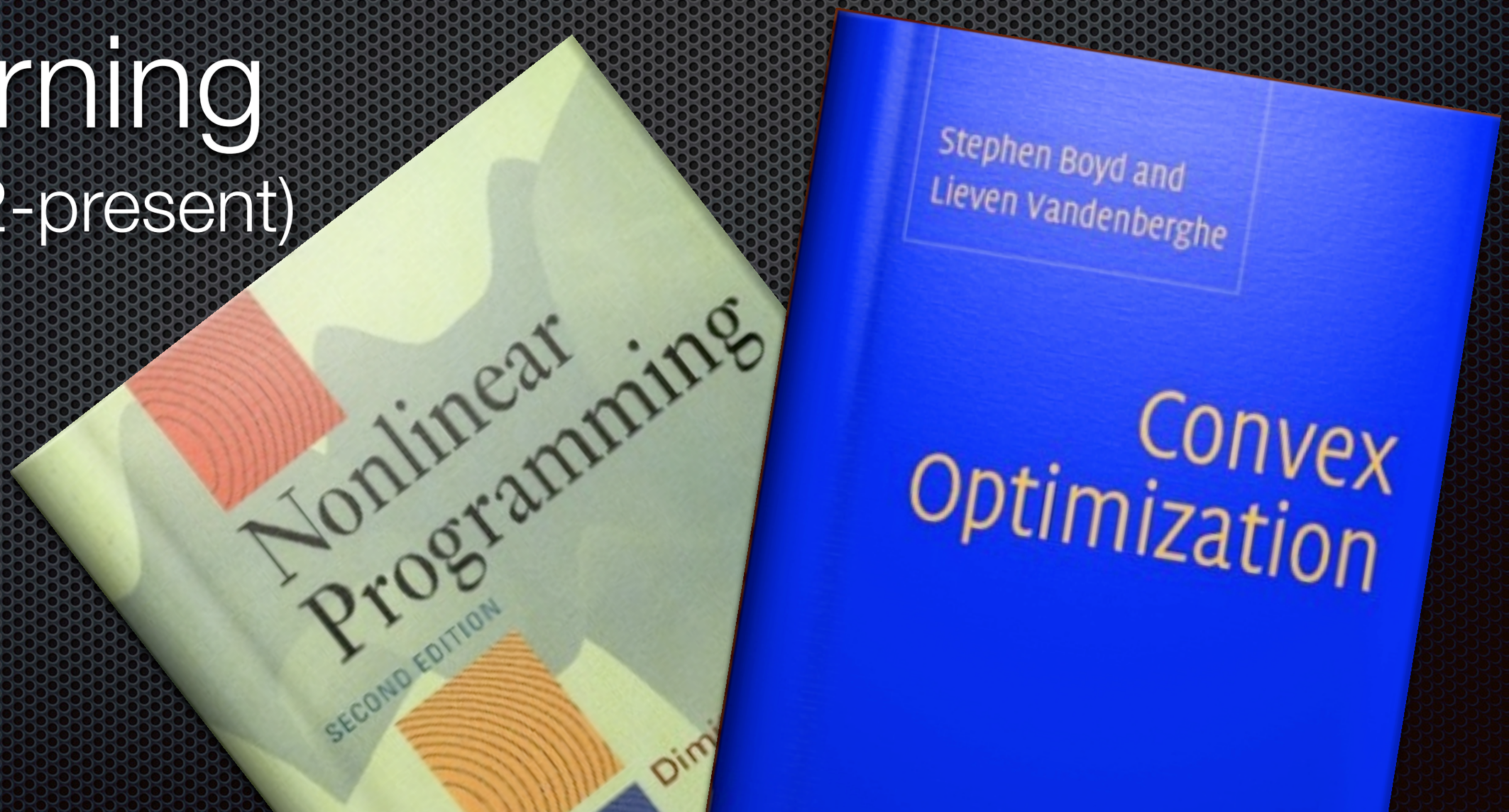


averaged within-class
covariance

PCA	Whitening	LDA
Learn “Mahalanobis” metric		
Based on second-order statistics		
Low dimensional projection		
Optimization problem		
Closed form eigenvalue solution		
unsupervised	unsupervised	supervised
not scale-free	noise sensitive	
Eigenvalue decomposition	Matrix inversion	Matrix inversion

(Clever combination of LDA + PCA + Whitening ≈ RCA [Shental et al. 2002])

Optimization based metric learning
(2002-present)



General Framework

- Constraint optimization problem
- Loss function:
  - Typically regularizer
- Constraints:
  - Enforce meaningful similarities
  - Positive semi-definiteness

Convex Optimization:

$$\min_{\mathbf{M}} \mathcal{L}(\mathbf{M})$$

$$\text{subject to:}$$

$$c_i(\mathbf{M}) \leq 0 \quad \forall c_i \in \mathcal{C}$$

$$\mathbf{M} \succeq 0$$

General Framework

- Constraint optimization problem
- Loss function:
  - Typically regularizer
- Constraints:
  - Enforce meaningful similarities
  - Positive semi-definiteness

Non-convex Optimization:

$$\min_{\mathbf{M}} \mathcal{L}(\mathbf{M})$$

$$\text{subject to:}$$

$$c_i(\mathbf{M}) \leq 0 \quad \forall c_i \in \mathcal{C}$$

$$\mathbf{M} \succeq 0$$

General Framework

- Constraint optimization problem
- Loss function:
  - Regularizer
- Approx. of kNN loss
  - Constraints:
  - Enforce meaningful similarities

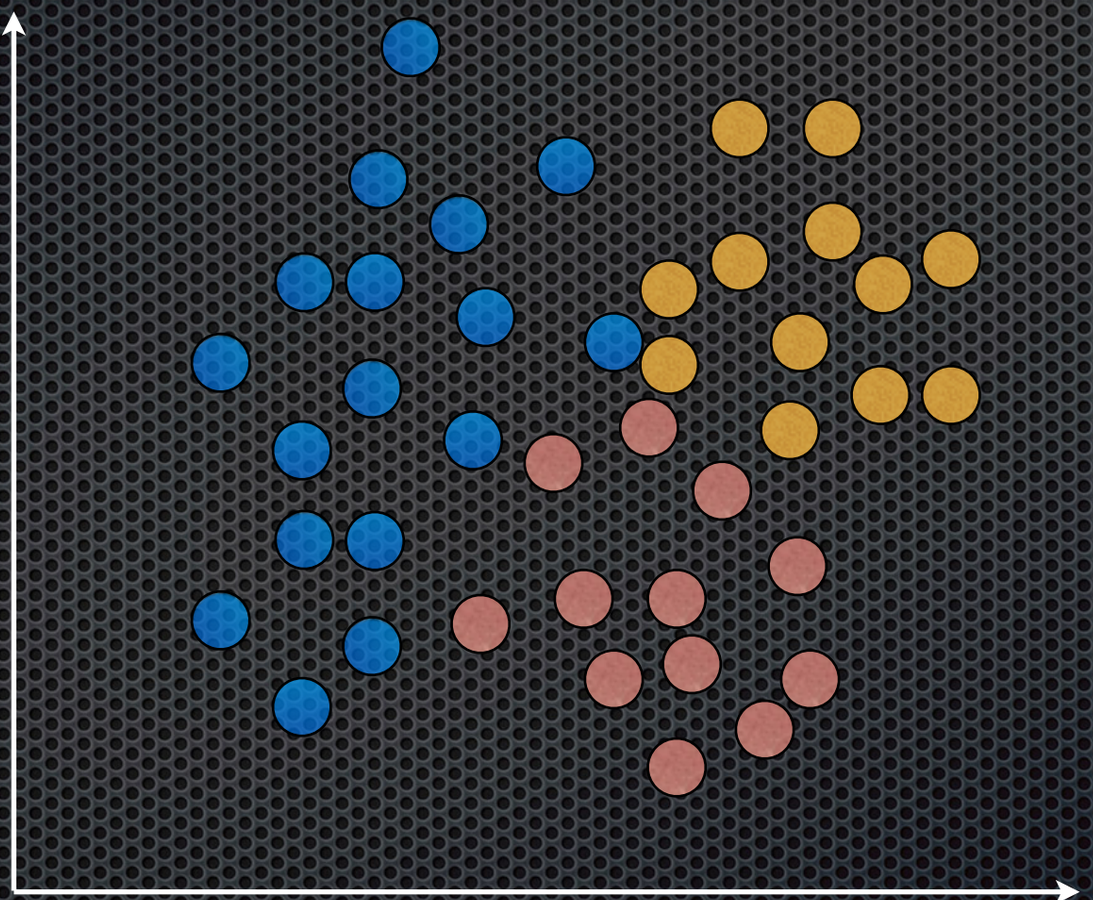
Unconstrained Optimization:

$$\min_{\mathbf{A}} \mathcal{L}(\mathbf{A}) + \sum_{c_i \in \mathcal{C}} c_i(\mathbf{A})$$

subject to:

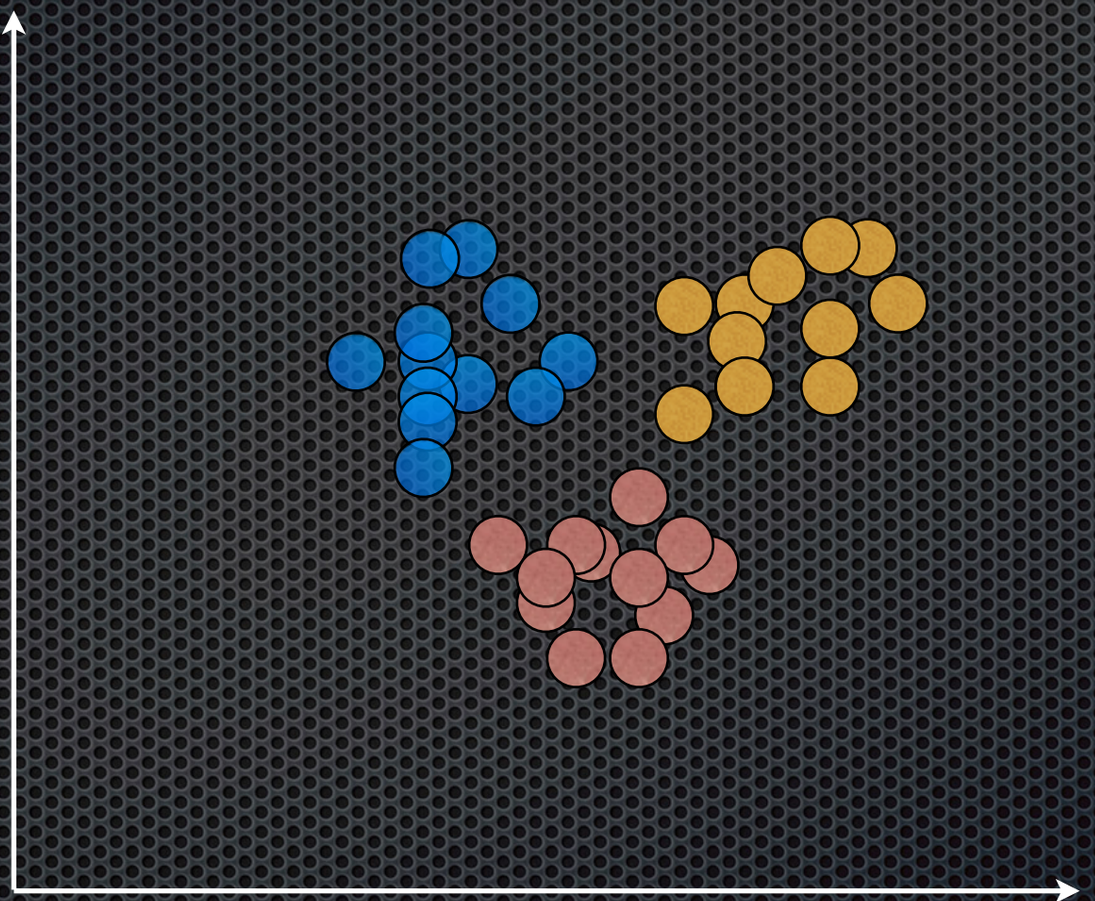
$$c_i(\mathbf{A}) \leq 0 \quad \forall c_i \in \mathcal{C}$$

Mahalanobis Metric for Clustering (MMC)



[Xing et al. (2002)]

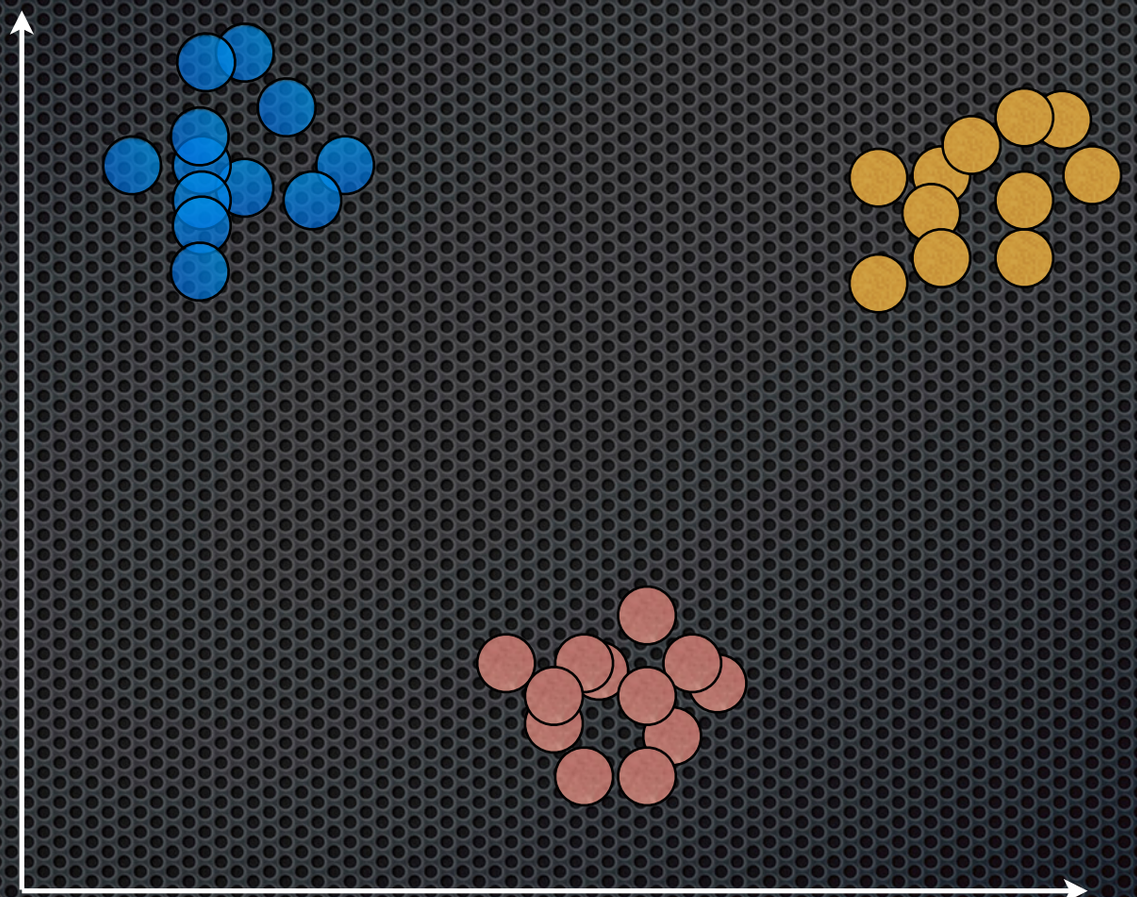
Mahalanobis Metric for Clustering (MMC)



Move **similarly** labeled inputs **together**

[Xing et al. (2002)]

Mahalanobis Metric for Clustering (MMC)



Move **different** labeled inputs **apart**

[Xing et al. (2002)]

Convex optimization problem

$$\mathcal{S} = \{(i,j) \mid \vec{x}_i \text{ and } \vec{x}_j \text{ are similarly labeled}\}$$

$$\mathcal{D} = \{(i,j) \mid \vec{x}_i \text{ and } \vec{x}_j \text{ are differently labeled}\}$$

$$\vec{x}_{ij} = (\vec{x}_i - \vec{x}_j)$$

$$\underset{\mathbf{M}}{\textbf{Maximize}} \sum_{(i,j)\in\mathcal{D}} \sqrt{\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}}$$

subject to:

$$\textbf{(1)} \sum_{(i,j)\in\mathcal{S}} \vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij} \leq 1$$

$$\textbf{(2)}\ \mathbf{M} \succeq 0.$$

Convex optimization problem

$$\underset{\mathbf{M}}{\text{Maximize}} \sum_{(i,j)\in\mathcal{D}} \sqrt{\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}}$$

pushing differently labeled inputs apart

$$\text{subject to:}$$

$$(1)\ \sum_{(i,j)\in\mathcal{S}} \vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij} \leq 1$$

$$(2)\ \mathbf{M} \succeq 0.$$

Convex optimization problem

$$\underset{\mathbf{M}}{\textbf{Maximize}} \sum_{(i,j)\in\mathcal{D}} \sqrt{\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}}$$

subject to:

$$(1)\ \sum_{(i,j)\in\mathcal{S}} \vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij} \leq 1$$

$$(2)\ \mathbf{M} \succeq 0.$$

pulling similar points together

Convex optimization problem

$$\underset{\mathbf{M}}{\textbf{Maximize}} \sum_{(i,j)\in\mathcal{D}} \sqrt{\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}}$$

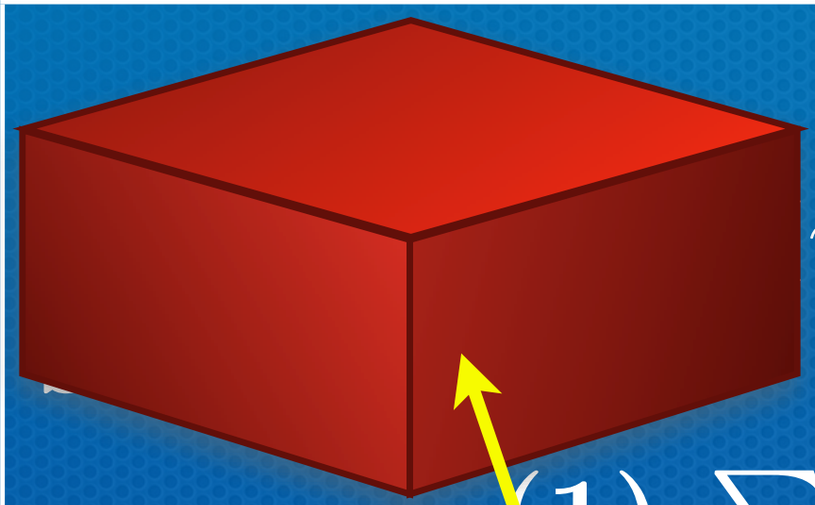
subject to:

$$(1)\ \sum_{(i,j)\in\mathcal{S}} \vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij} \leq 1$$

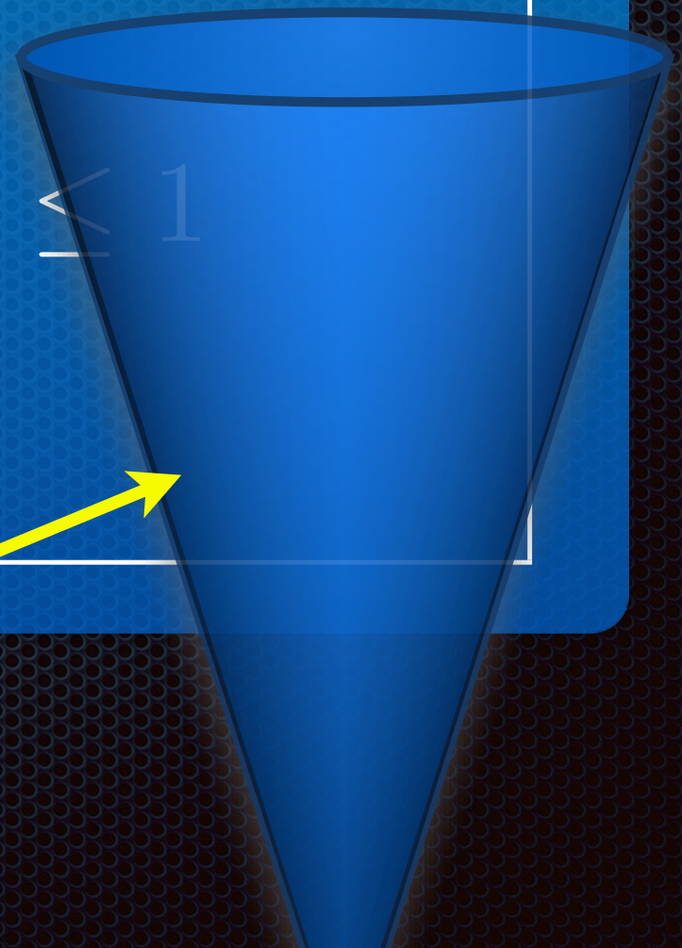
$$(2)\ \mathbf{M} \succeq 0.$$

ensuring positive semi-definiteness

Convex optimization problem



$$i,j)\in\mathcal{D}\ \sqrt{\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}}$$

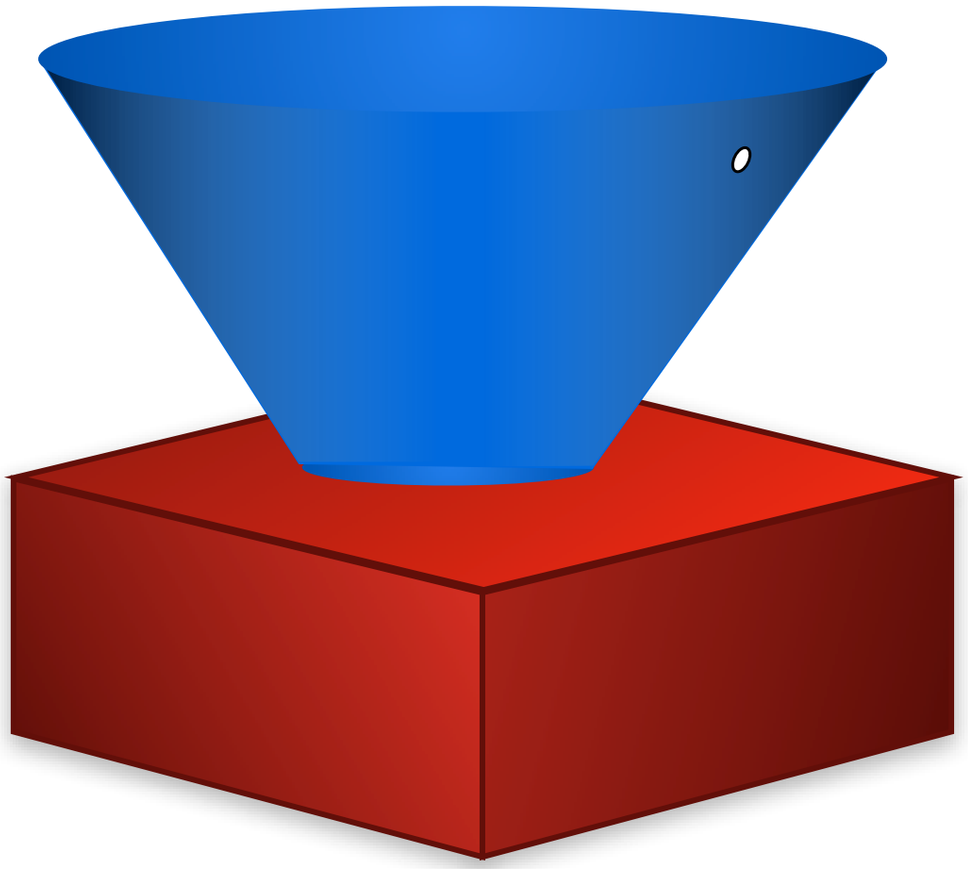


$$\textbf{(1)}\ \sum_{(i,j)\in\mathcal{S}}\vec{x}_{ij}^{\top}\mathbf{M}\vec{x}_{ij}\leq 1$$

$$\textbf{(2)}\ \mathbf{M}\succeq 0.$$

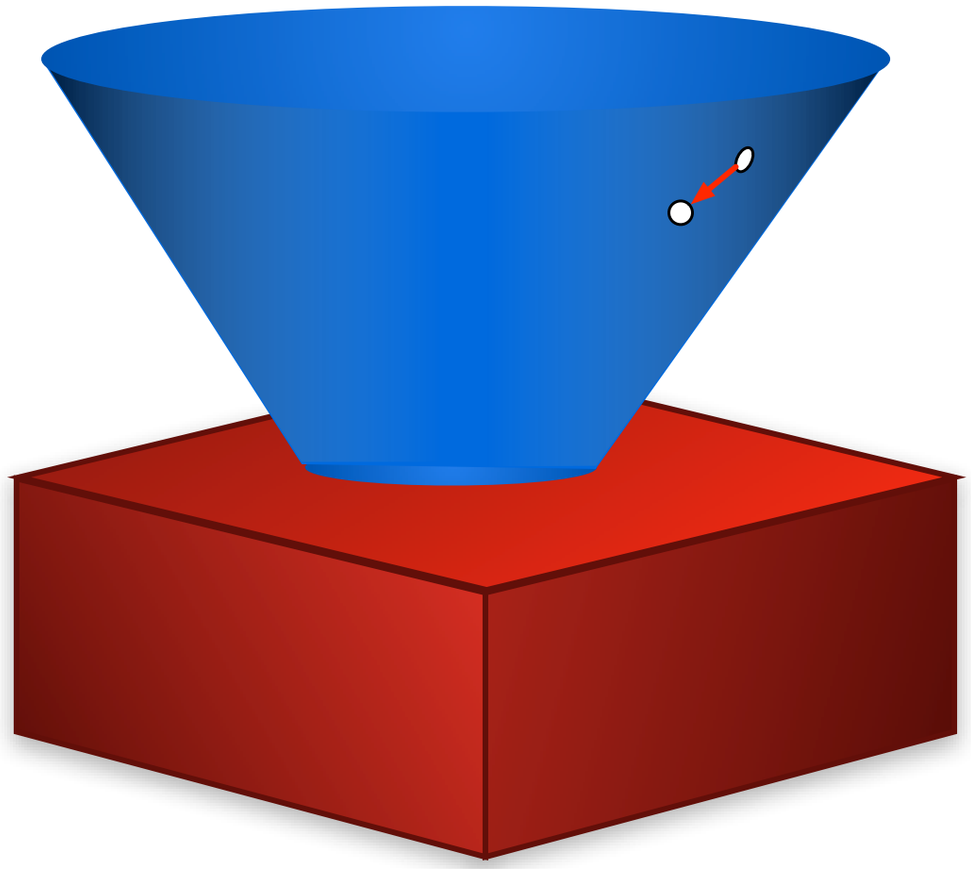
CONVEX

Gradient Alternating Projection



Gradient Alternating Projection

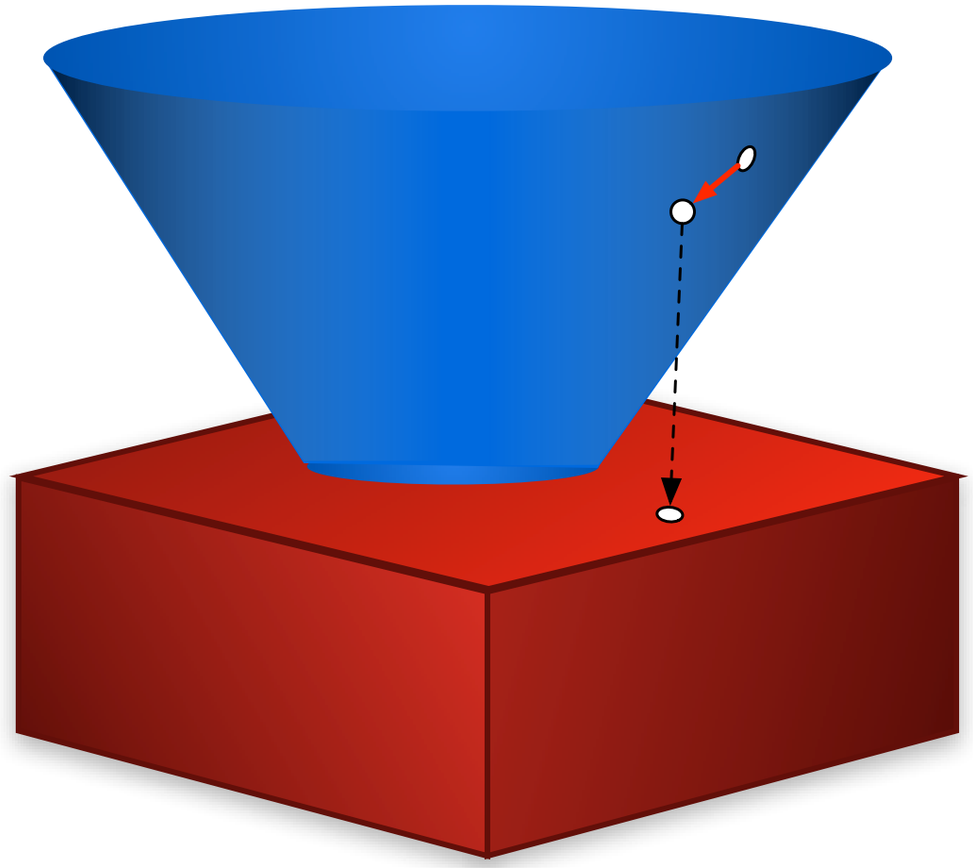
Take step along gradient.



Gradient Alternating Projection

Take step along gradient.

Project onto constraint satisfying sub-space.

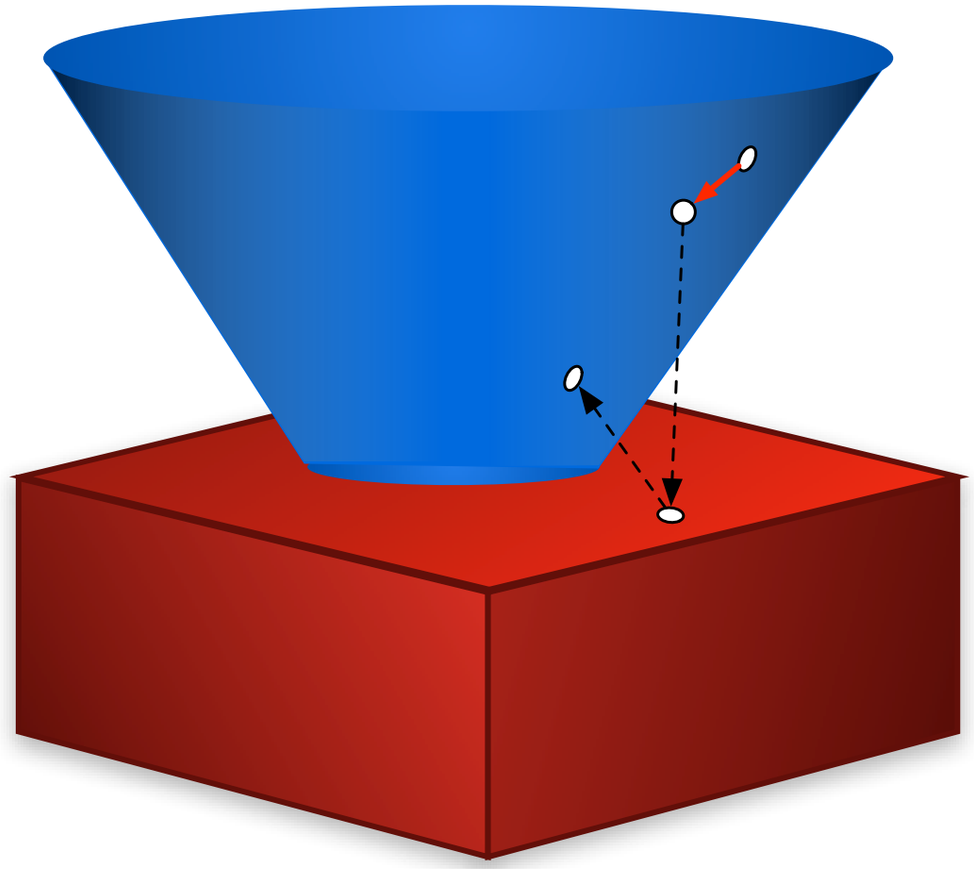


Gradient Alternating Projection

Take step along gradient.

Project onto constraint satisfying sub-space.

Project onto PSD cone.



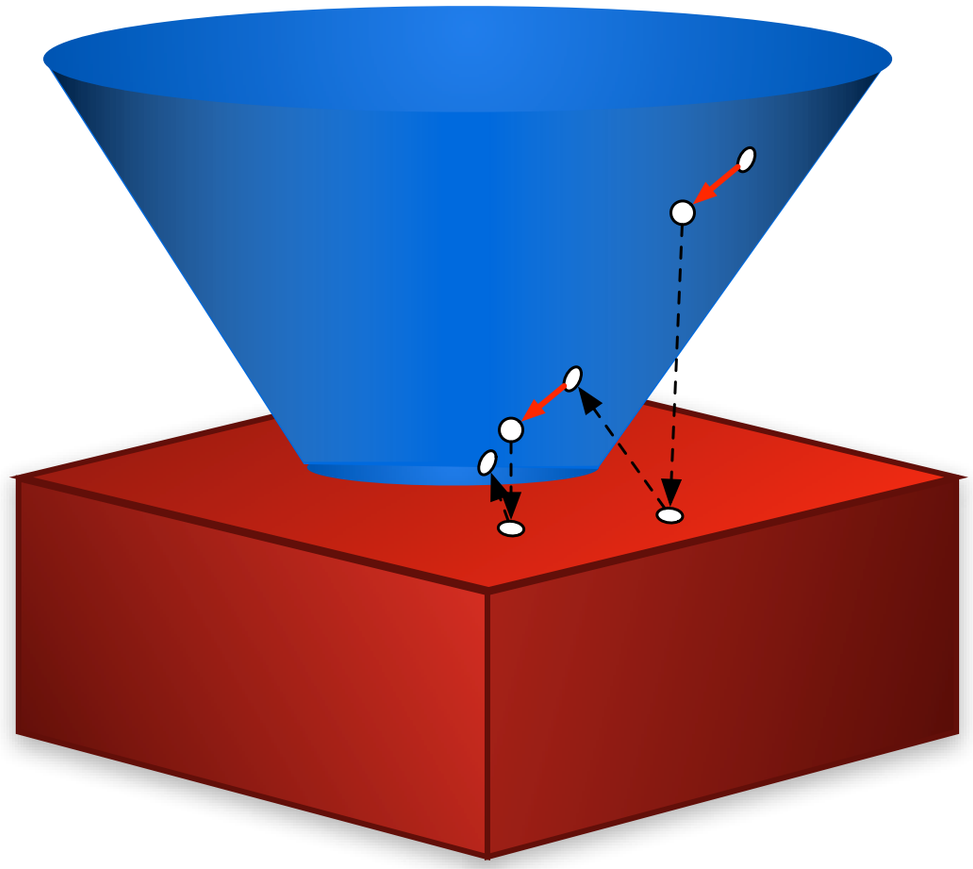
Gradient Alternating Projection

Take step along gradient.

Project onto constraint satisfy sub-space.

Project onto PSD cone.

REPEAT



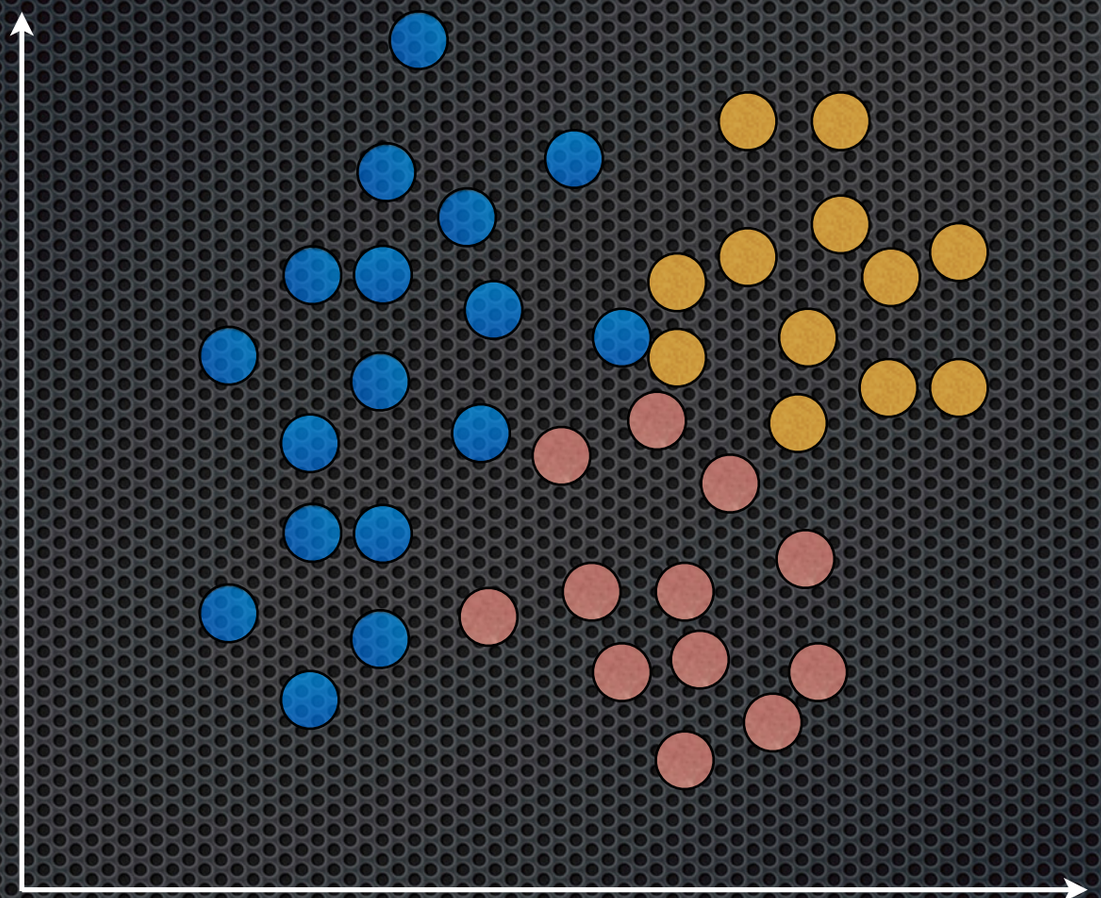
Algorithm is guaranteed to converge to optimal solution

Summary of Xing et al 2002

- **First Metric Learning SDP**
- Learns Mahalanobis metric
- Very suited for clustering
- Can be kernelized
- Optimization problem is convex
- Algorithm is guaranteed to converge
- Assumes data to be uni-modal
- Slow convergence (does not scale to large data)

POLA

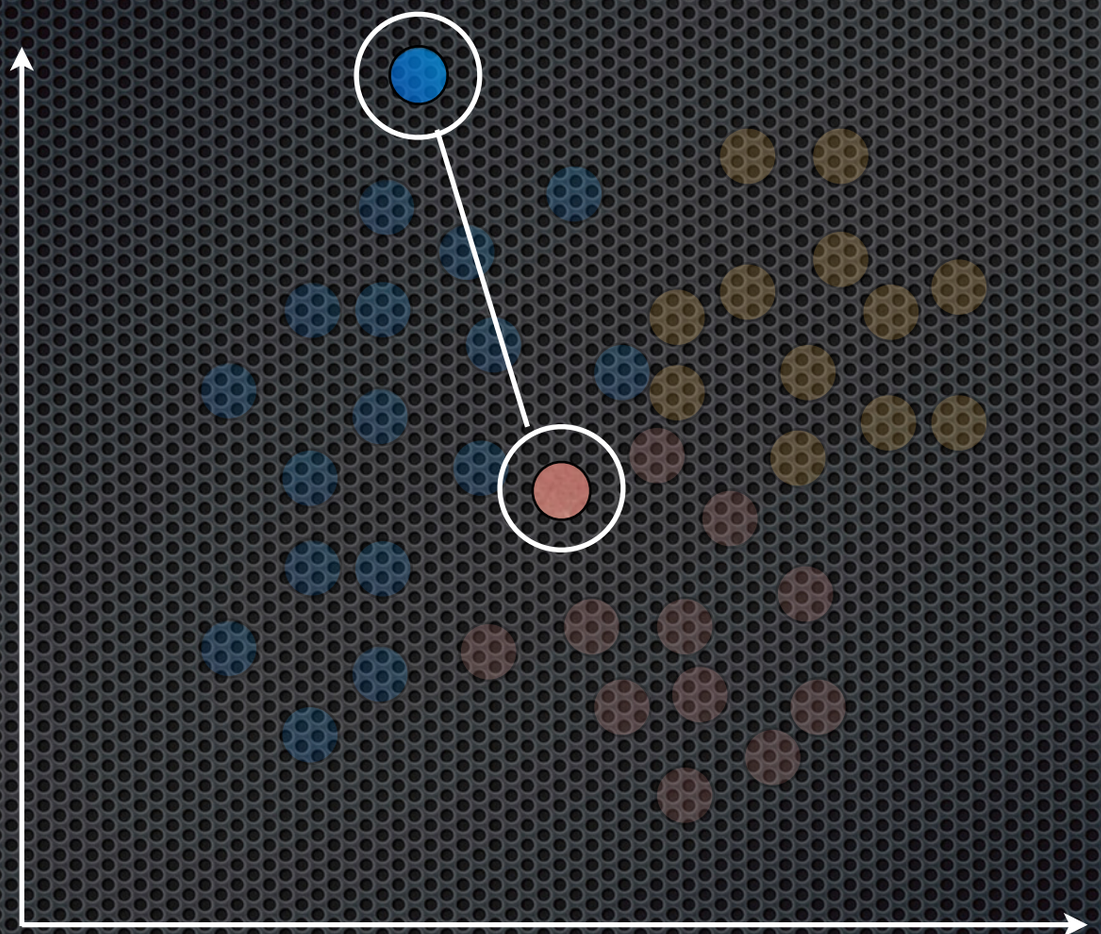
(Pseudo-metric online learning algorithm)



[Shalev-Shwartz et al.; ICML 2004]

POLA

(Pseudo-metric online learning algorithm)

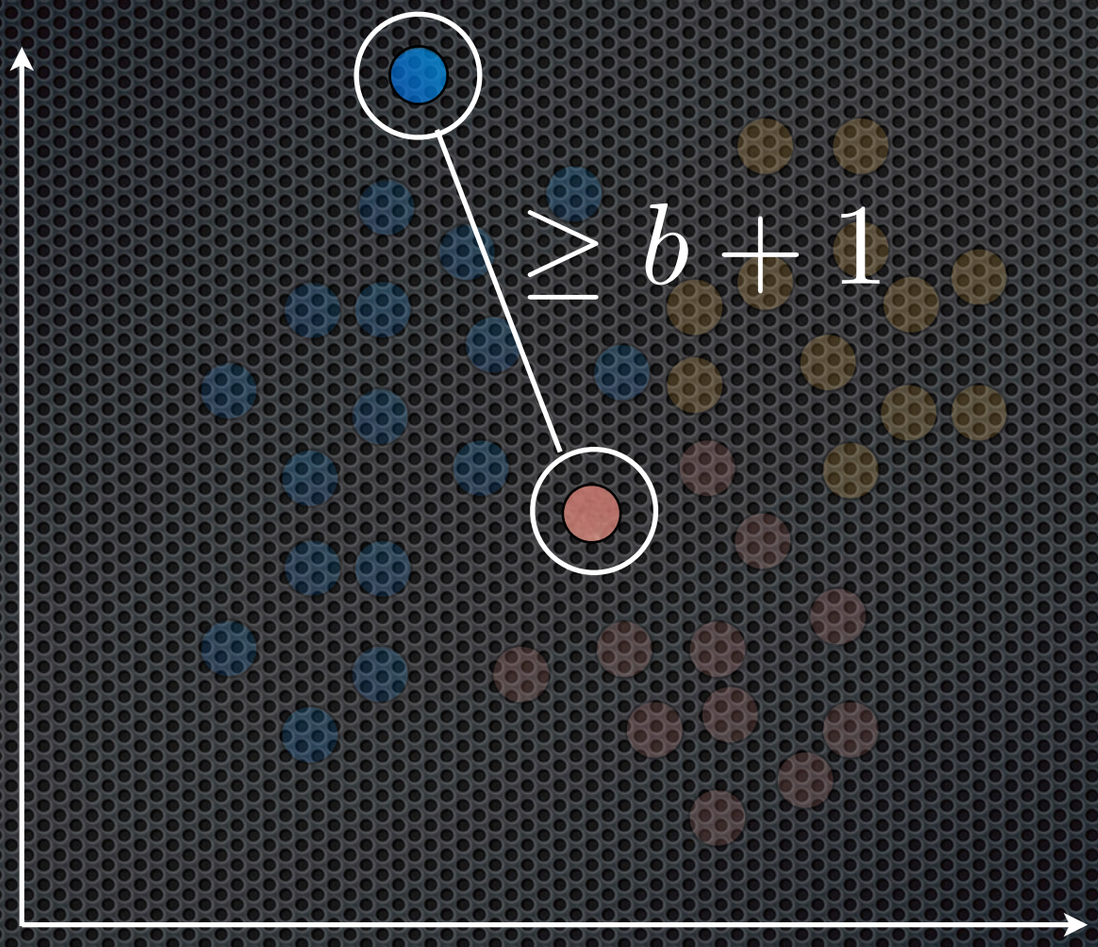


This time the inputs are accessed two at a time.

[Shalev-Shwartz et al.; ICML 2004]

POLA

(Pseudo-metric online learning algorithm)

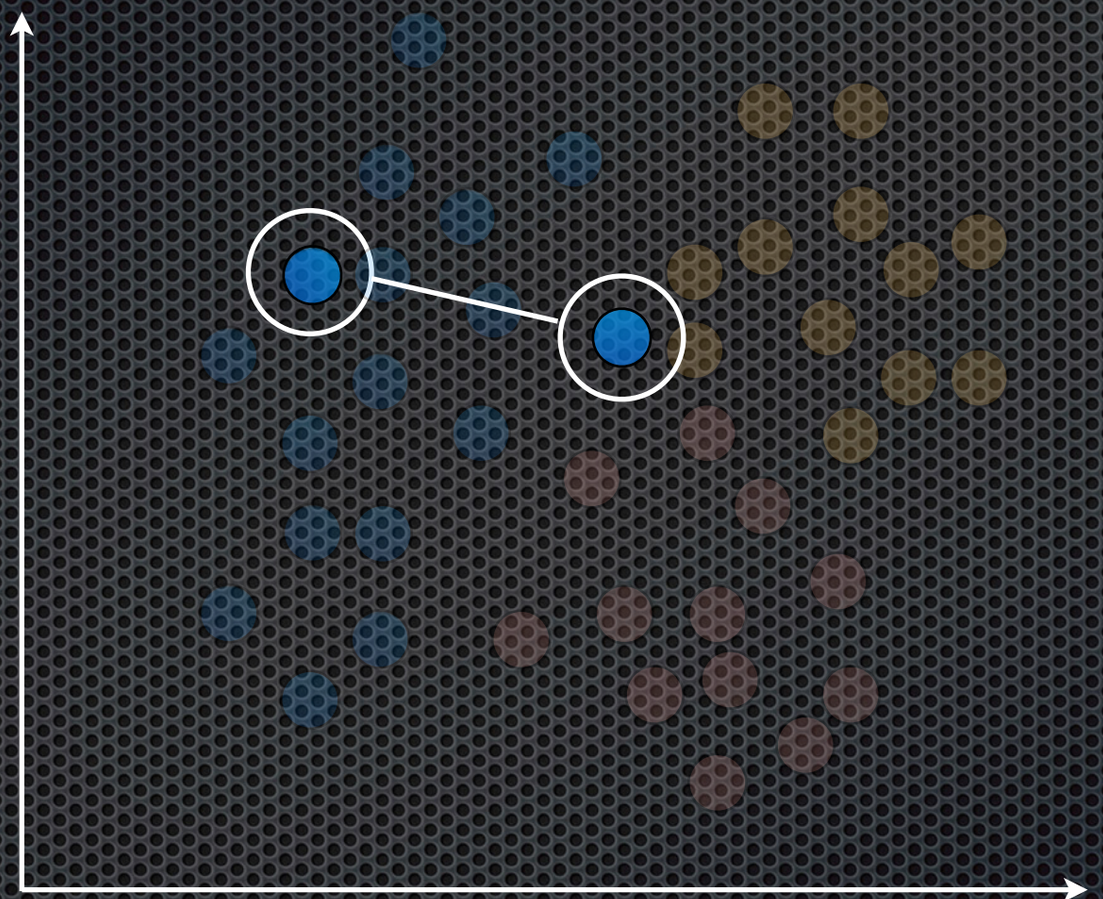


This algorithm is **Differently** labeled inputs are **separated**.

[Shalev-Shwartz et al.; ICML 2004]

POLA

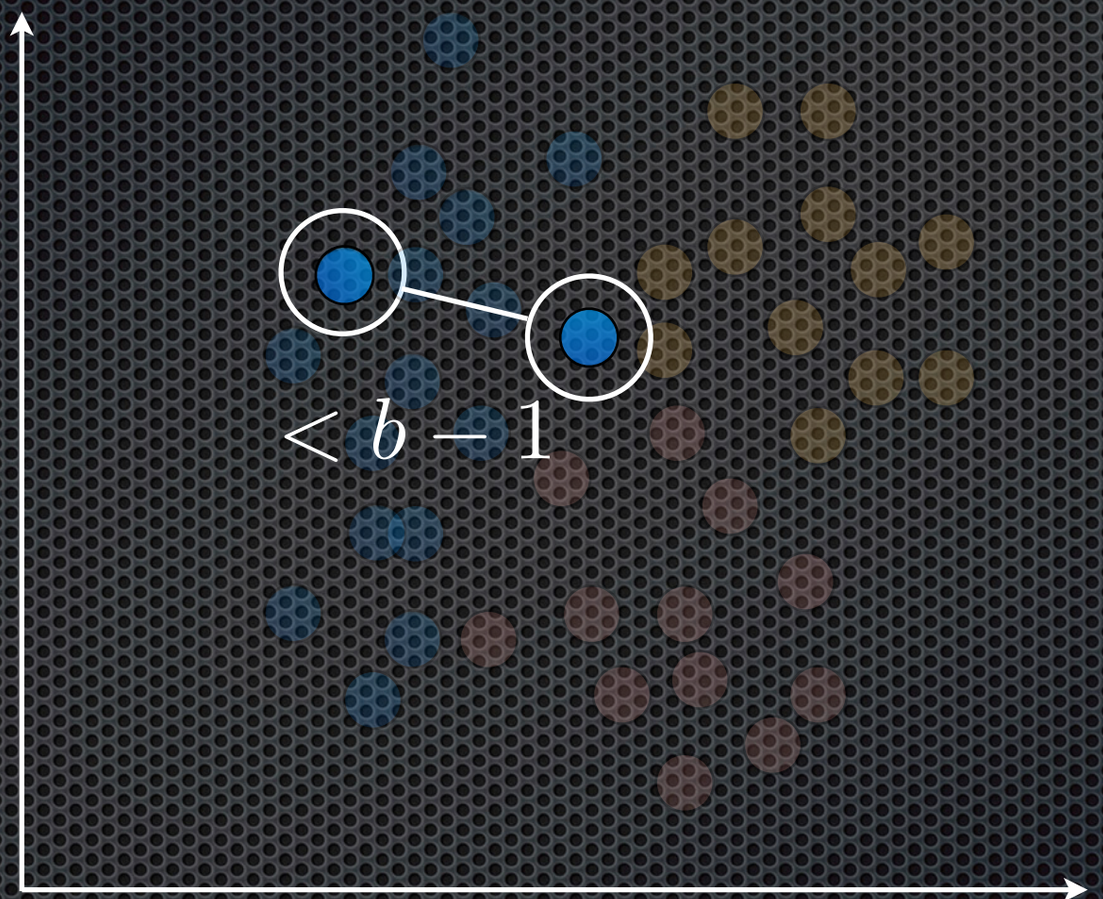
(Pseudo-metric online learning algorithm)



[Shalev-Shwartz et al.; ICML 2004]

POLA

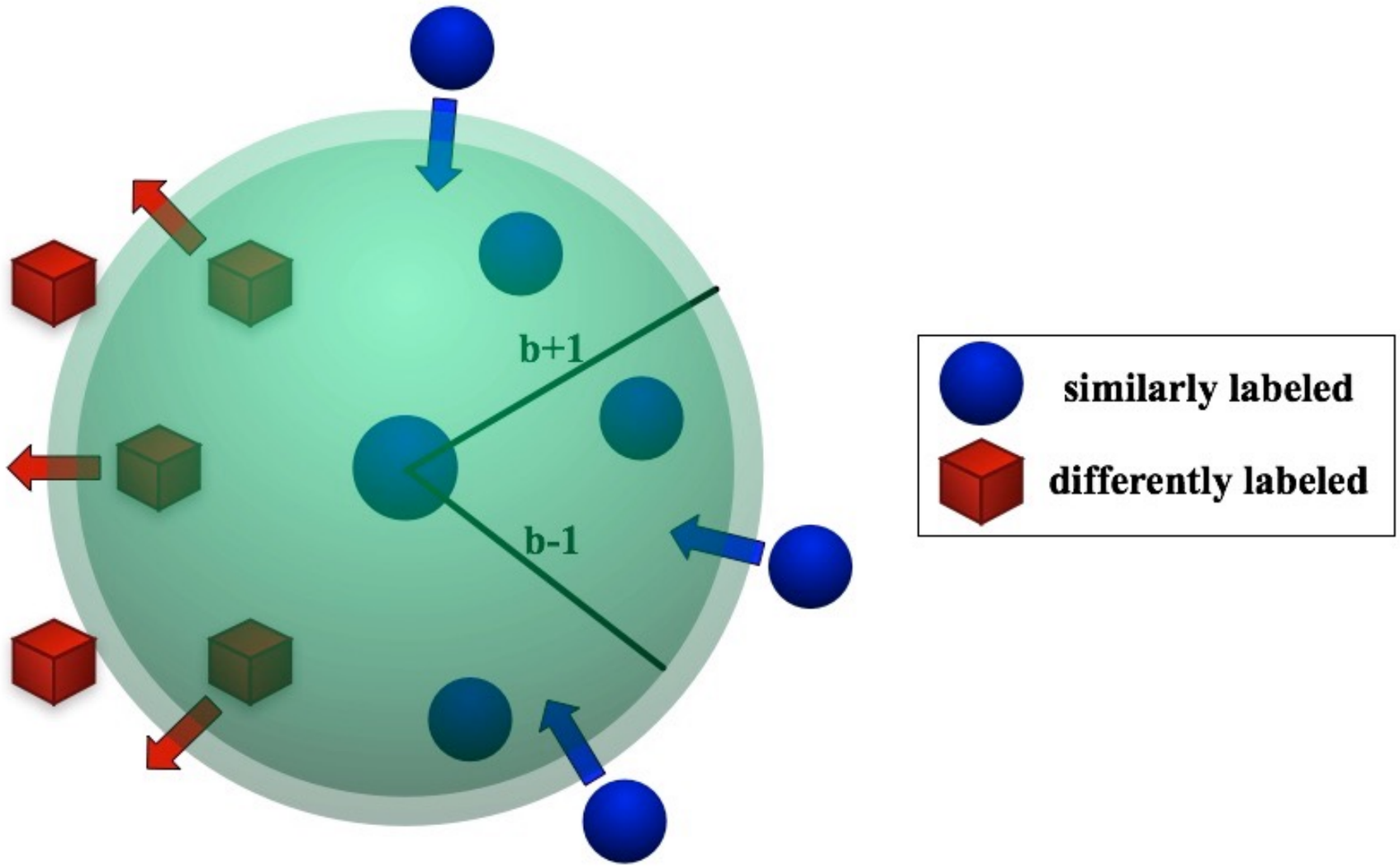
(Pseudo-metric online learning algorithm)



Similarly labeled inputs are **moved closer**.

[Shalev-Shwartz et al.; ICML 2004]

Margin



Convex optimization

At each time t, we get two [illegible]s: $\vec{u}_t$, $\vec{v}_t$

[illegible]nt 1:

$$y_t(b_t - \vec{x}^{\top} \vec{a}_t) \geq$$

Constraint 2:

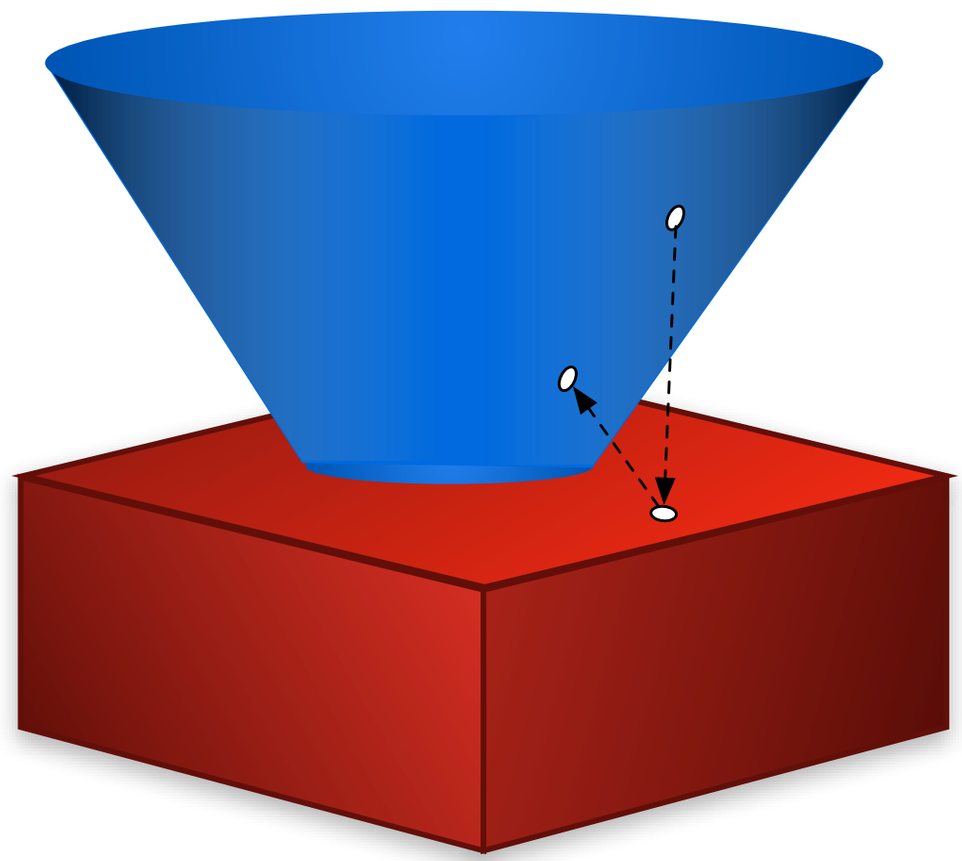
$$\mathbf{M} \succeq 0$$

Both are convex!!

Alternating Projection

Initialize inside PSD cone

Project onto constraint - satisfying hyperplane and back

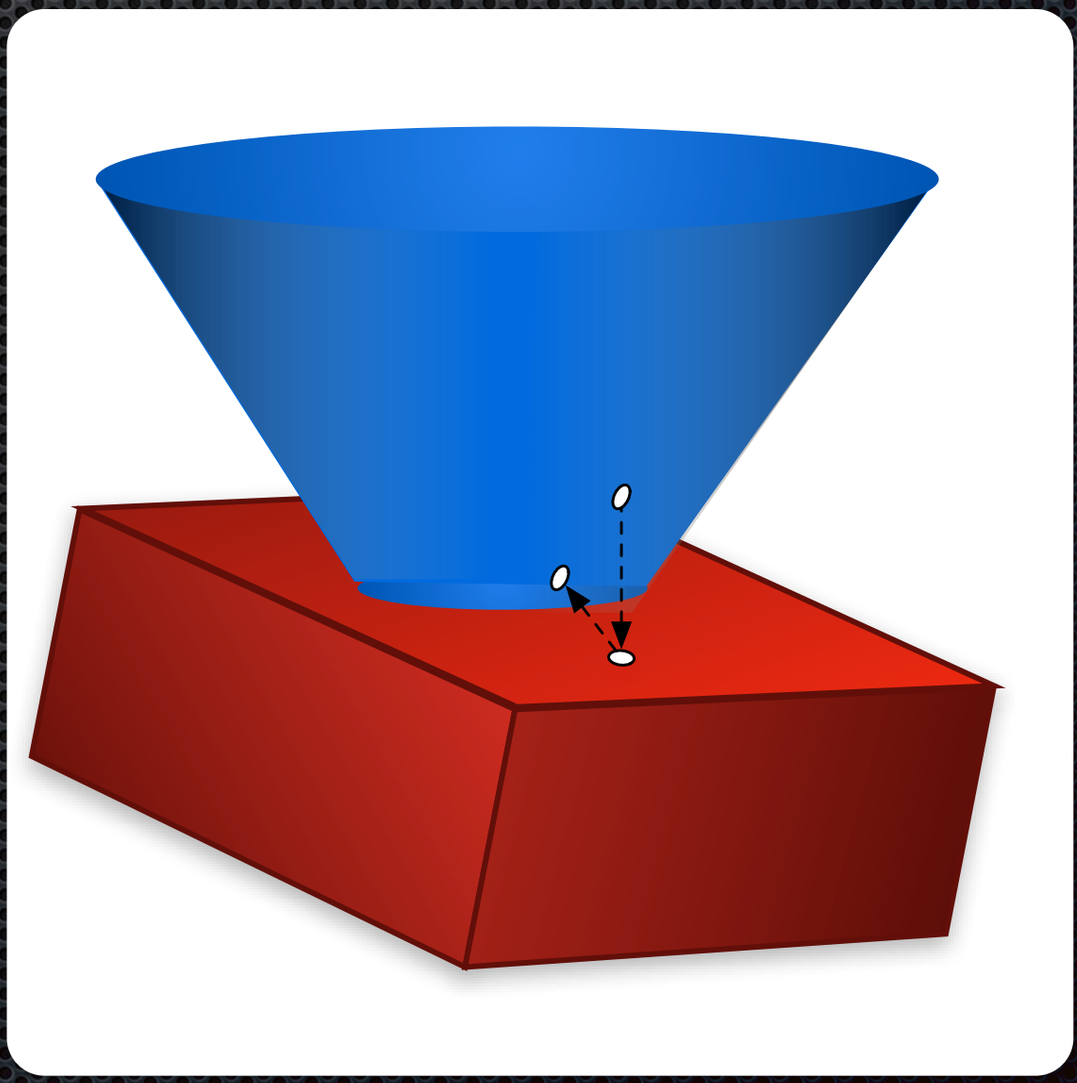


Alternating Projection

Initialize inside PSD cone

Project onto constraint - satisfying hyperplane and back

Repeat with new constraints

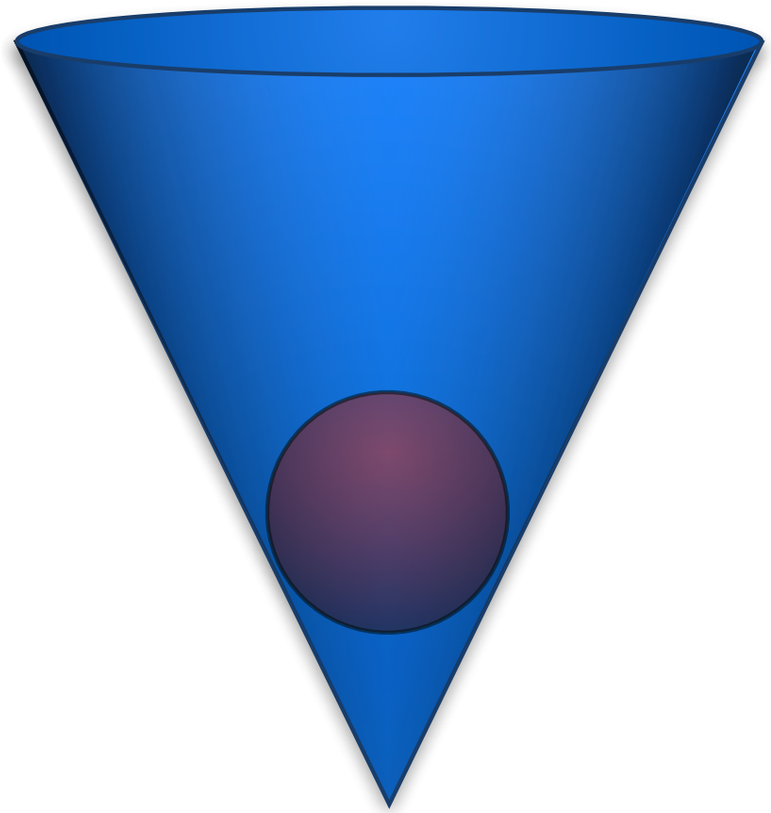


Alternating Projection

Initialize inside PSD cone

Project onto constraint - satisfying hyperplane and back

Repeat with new constraints

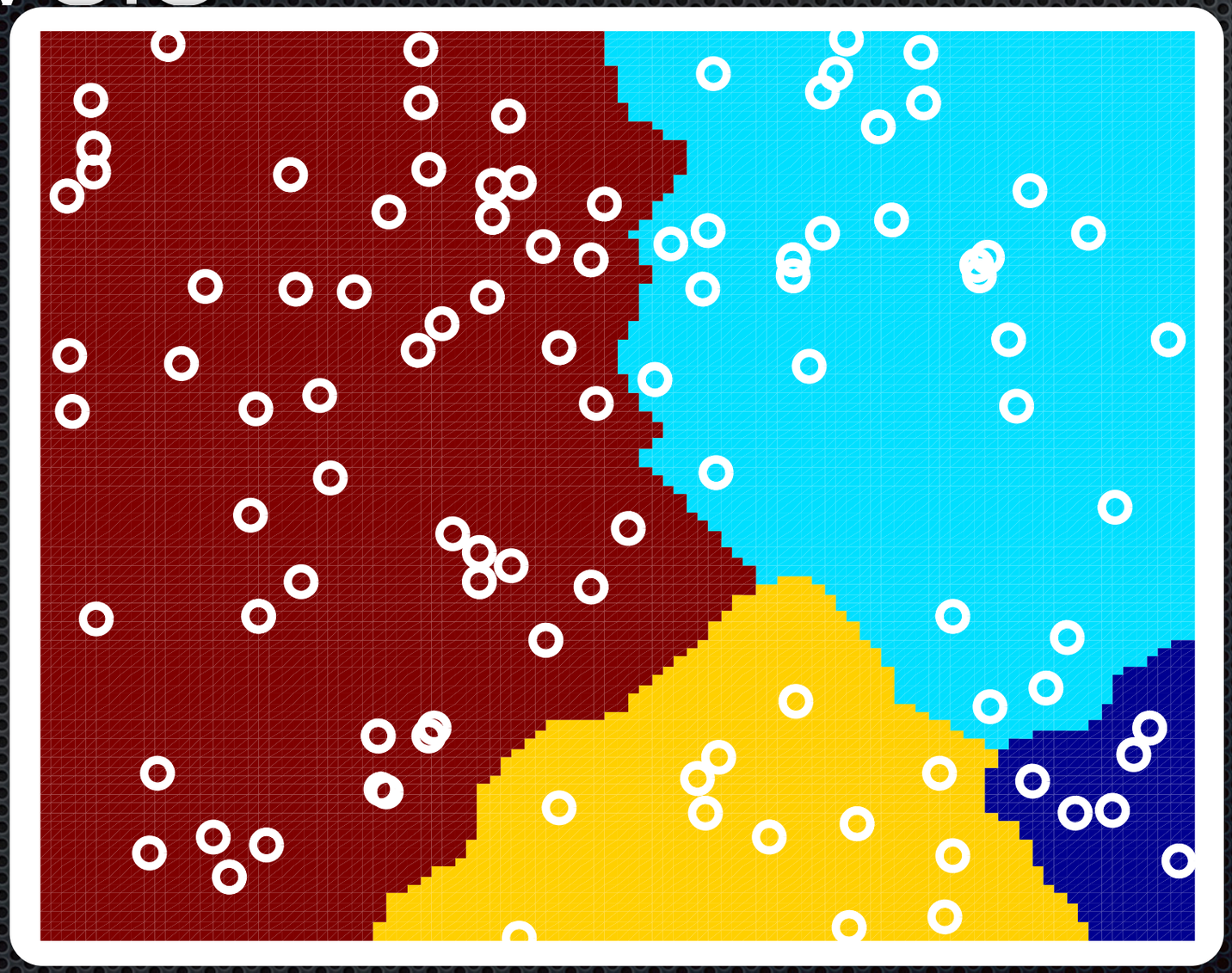


If solution exists, algorithm converges inside intersection.

Summary of POLA

- Learns Mahalanobis metric
- **Online algorithm**
- Can also be kernelized
- **Introduces a margin**
- Algorithm converges if solution exists
- Assumes data to be unimodal

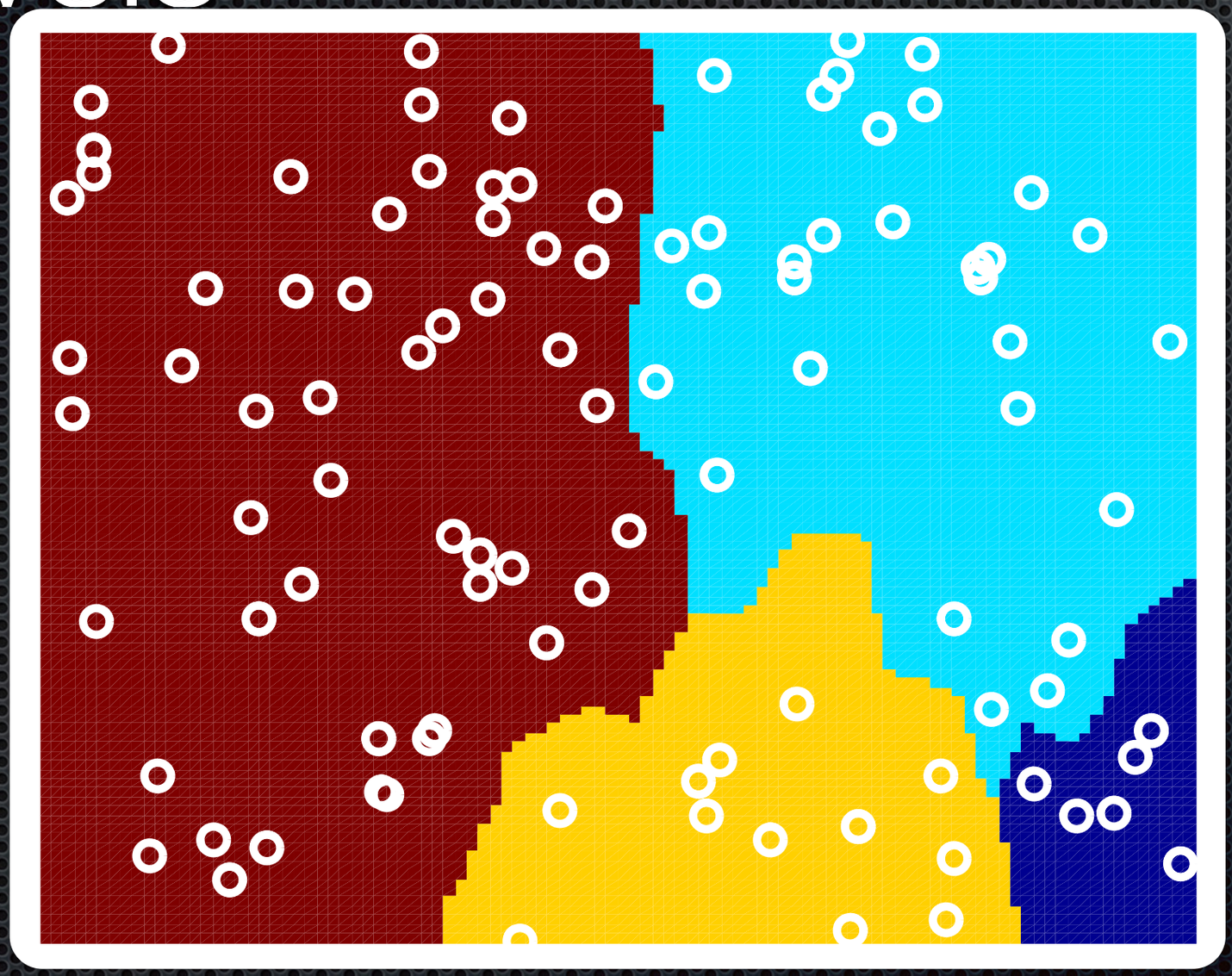
Neighborhood Component Analysis



Distance metric for visualization and kNN

[Goldberger et. al. 2004]

Neighborhood Component Analysis



Distance metric for visualization and kNN

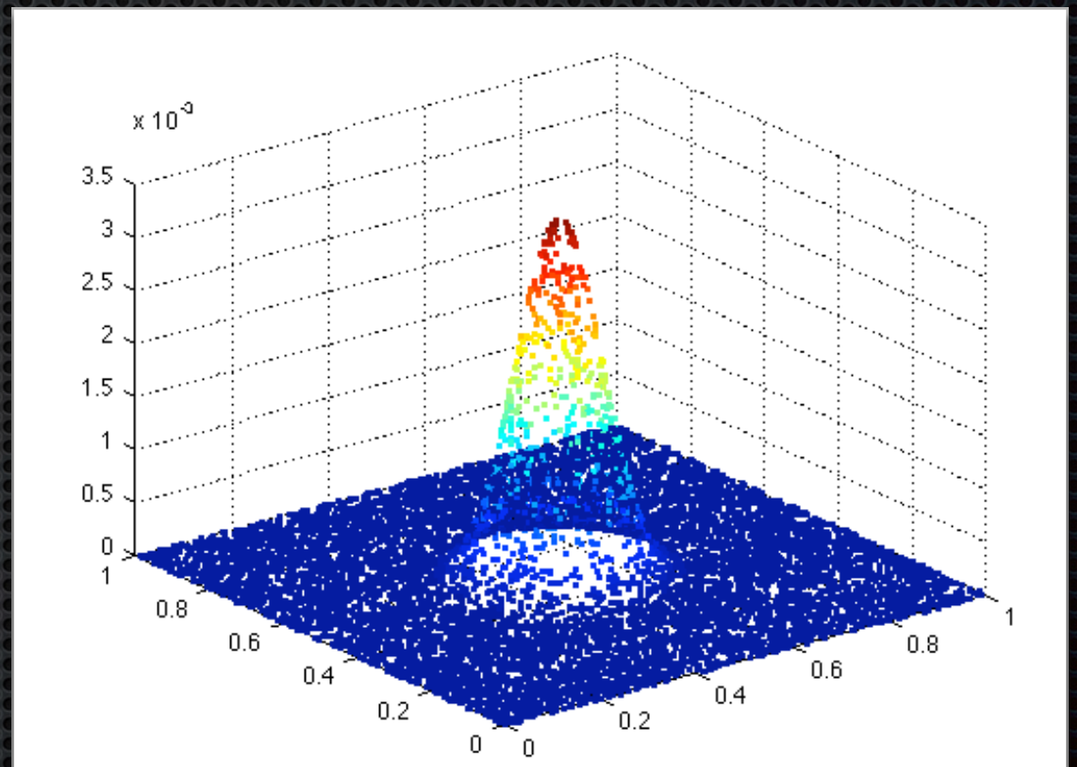
[Goldberger et. al. 2004]

Stochastic Neighborhood

- Pick neighbor randomly

$$p_{ij} = \frac{1}{Z} \exp(-\|\mathbf{A}(\vec{x}_i - \vec{x}_j)\|^2$$

Neighbor distribution



Stochastic Neighborhood

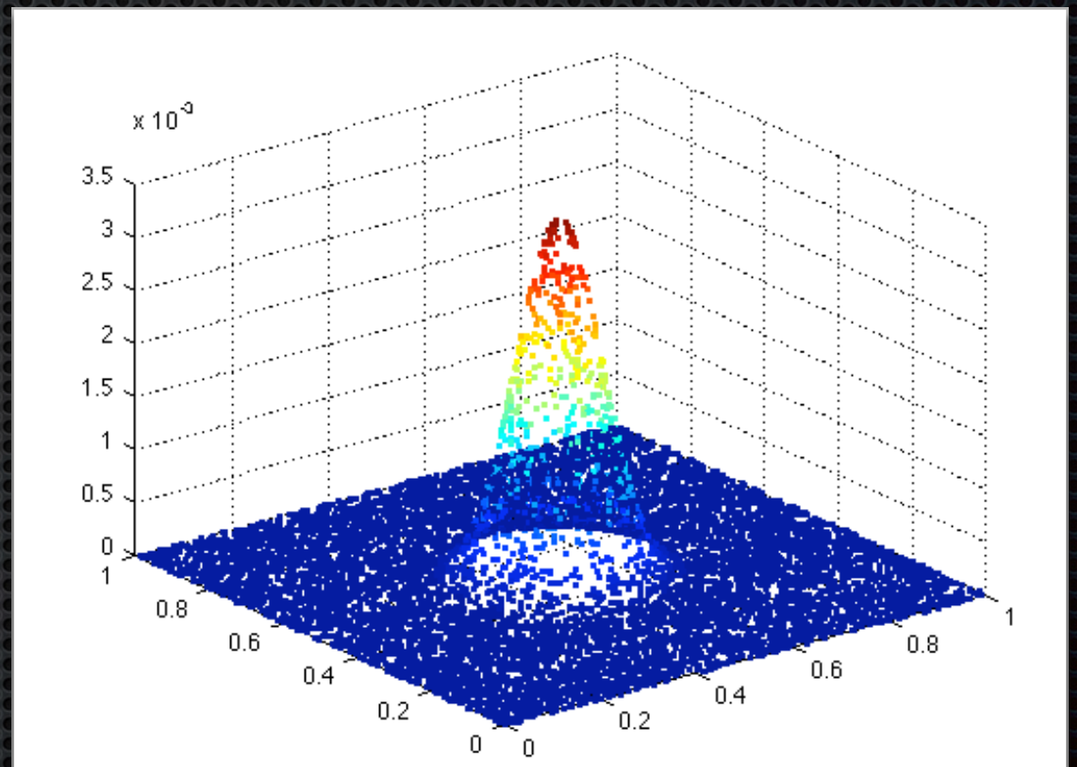
- Pick neighbor randomly

$$p_{ij} = \frac{1}{Z} \exp(-\|\mathbf{A}(\vec{x}_i - \vec{x}_j)\|^2$$

- Maximize expected number of *correct* neighbors

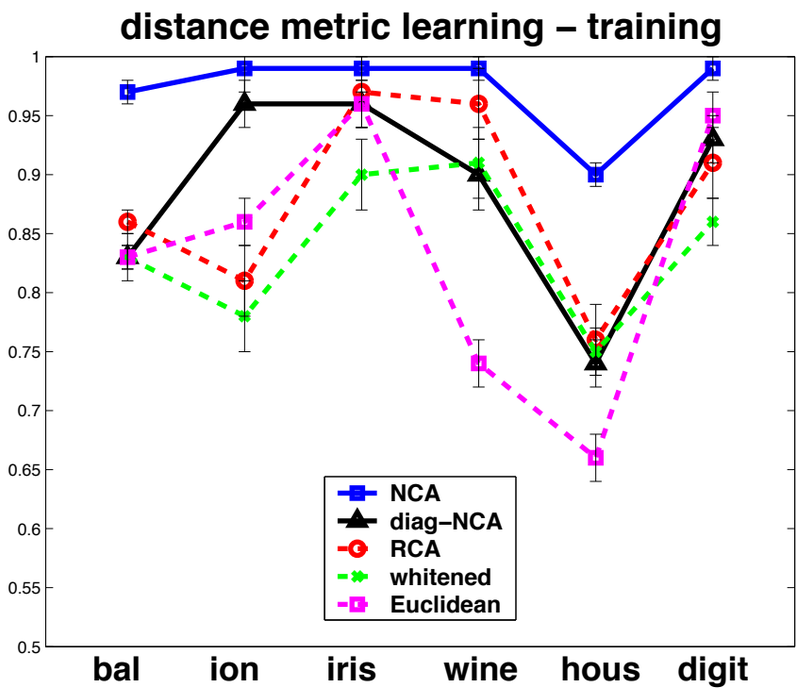
$$\max_A \sum_{(i,j) \in S} p_{ij}$$

Neighbor distribution



(Unconstraint optimization problem - non-convex)

Results


distance metric learning – training
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
NCA
diag–NCA
RCA
whitened
Euclidean
bal
ion
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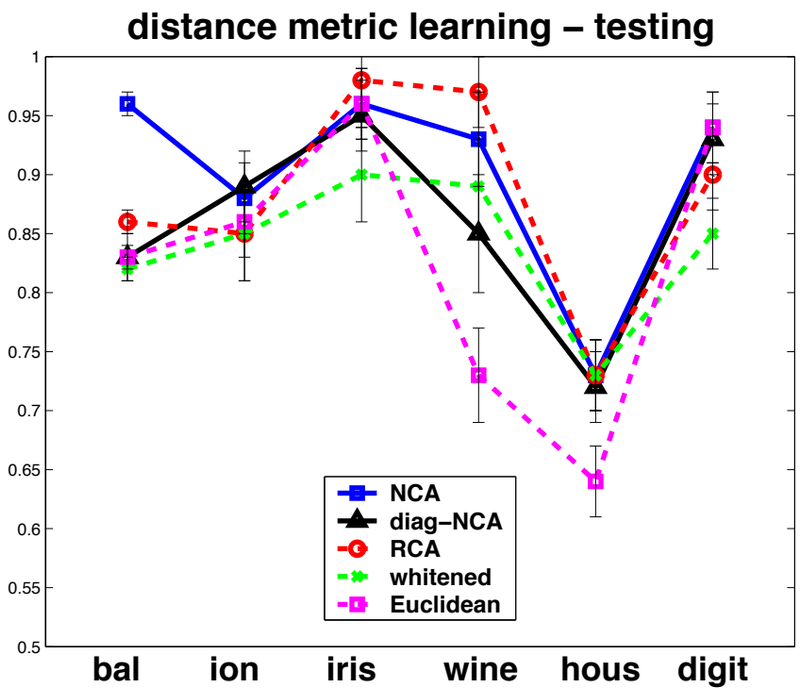
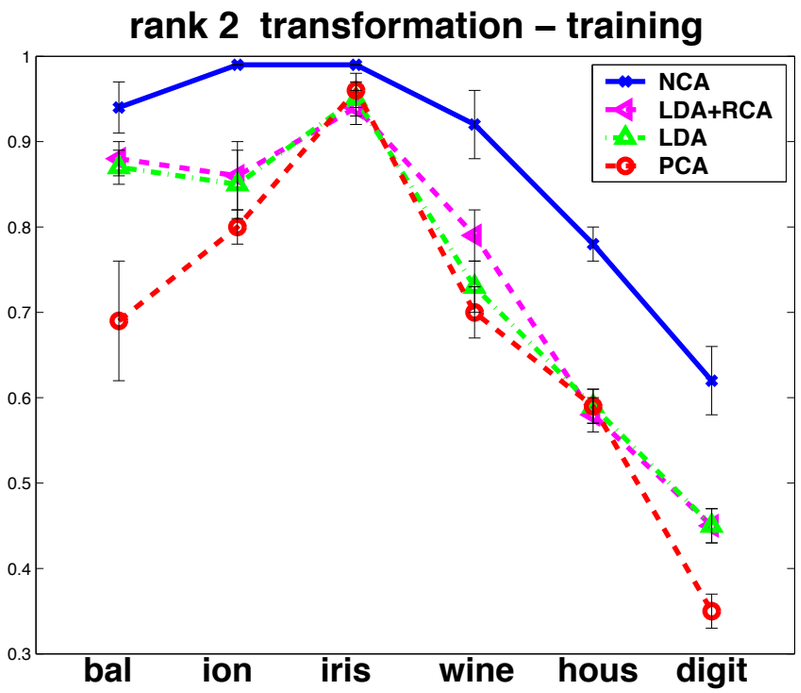
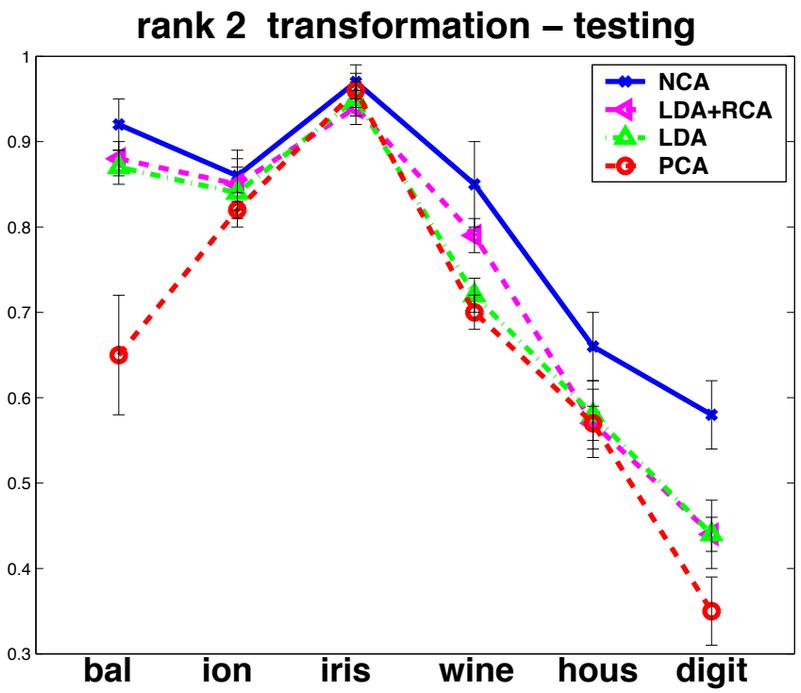
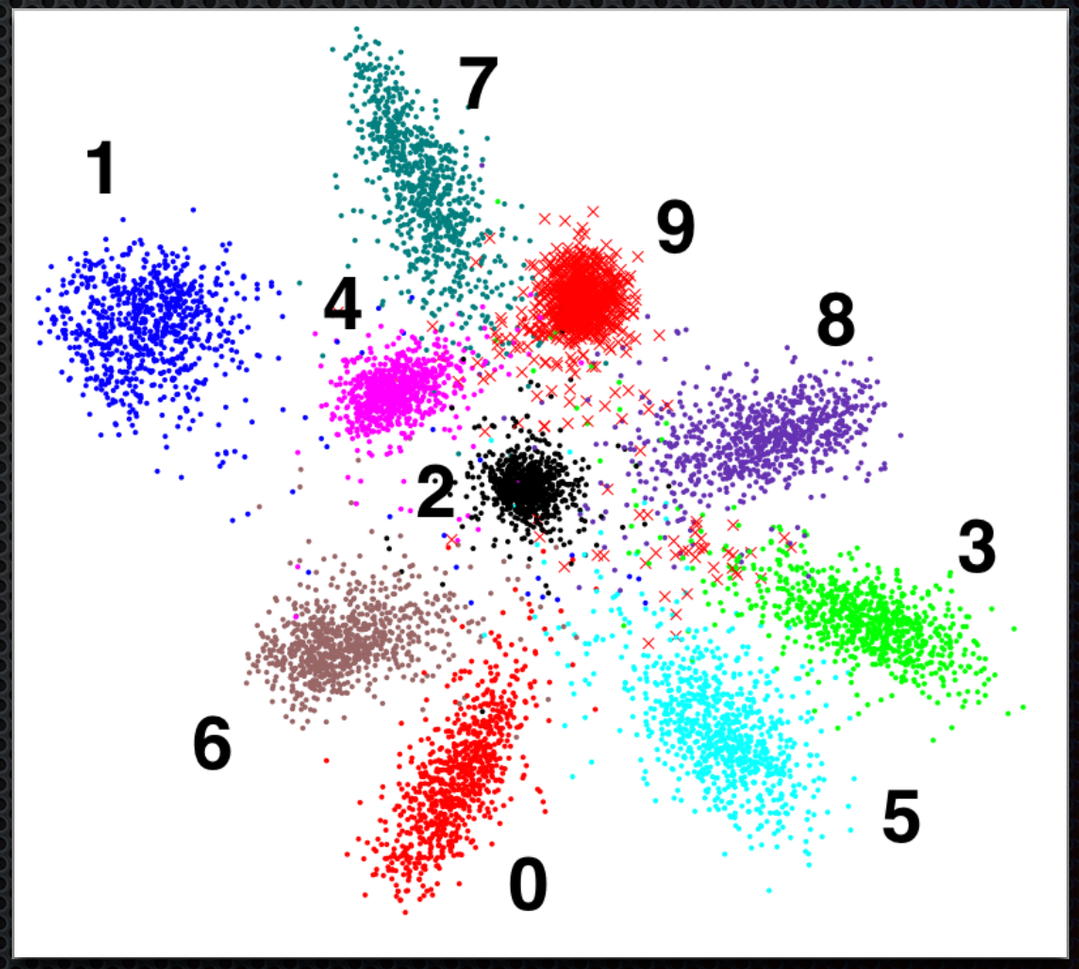

distance metric learning – testing
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
NCA
diag–NCA
RCA
whitened
Euclidean
bal
ion
iris
wine
hous
digit


rank 2 transformation – training
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
NCA
LDA+RCA
LDA
PCA
bal
ion
iris
wine
hous
digit


rank 2 transformation – testing
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
NCA
LDA+RCA
LDA
PCA
bal
ion
iris
wine
hous
digit

NCA-Extensions

- [Globerson and Roweis; NIPS 2005]
  - **Convex** loss function
- [Weinberger and Tesauro; AISTATS 2007]
  - Kernel **regression**
- [Salakhutdinov and Hinton; AISTATS 2007]
  - **Non-linear** mapping (DBN)



MNIST handwritten digits embedding - (Courtesy of Salakhutdinov and Hinton)

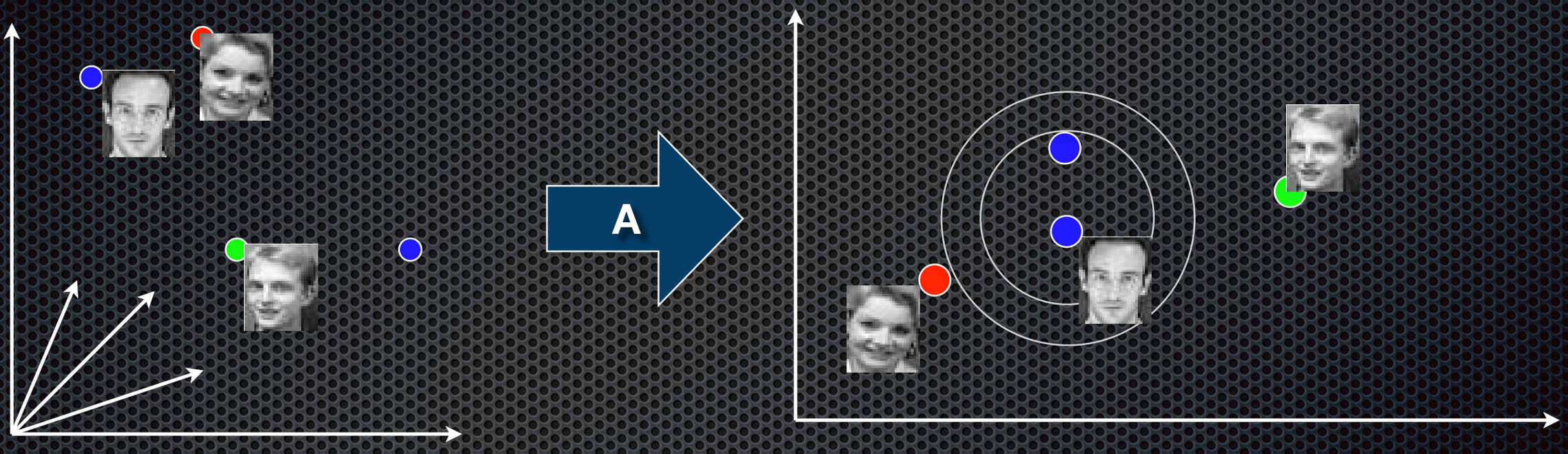
Summary of NCA

- Mahalanobis metric for k-NN
- Also, indirectly, learns parameter k
- Can be used to reduce dimensionality
- NCA optimization is unconstrained
- **NCA can handle multi-modal data** (not MCML)
- NCA objective is not convex
- Normalization becomes expensive for large data

Large Margin Nearest Neighbor

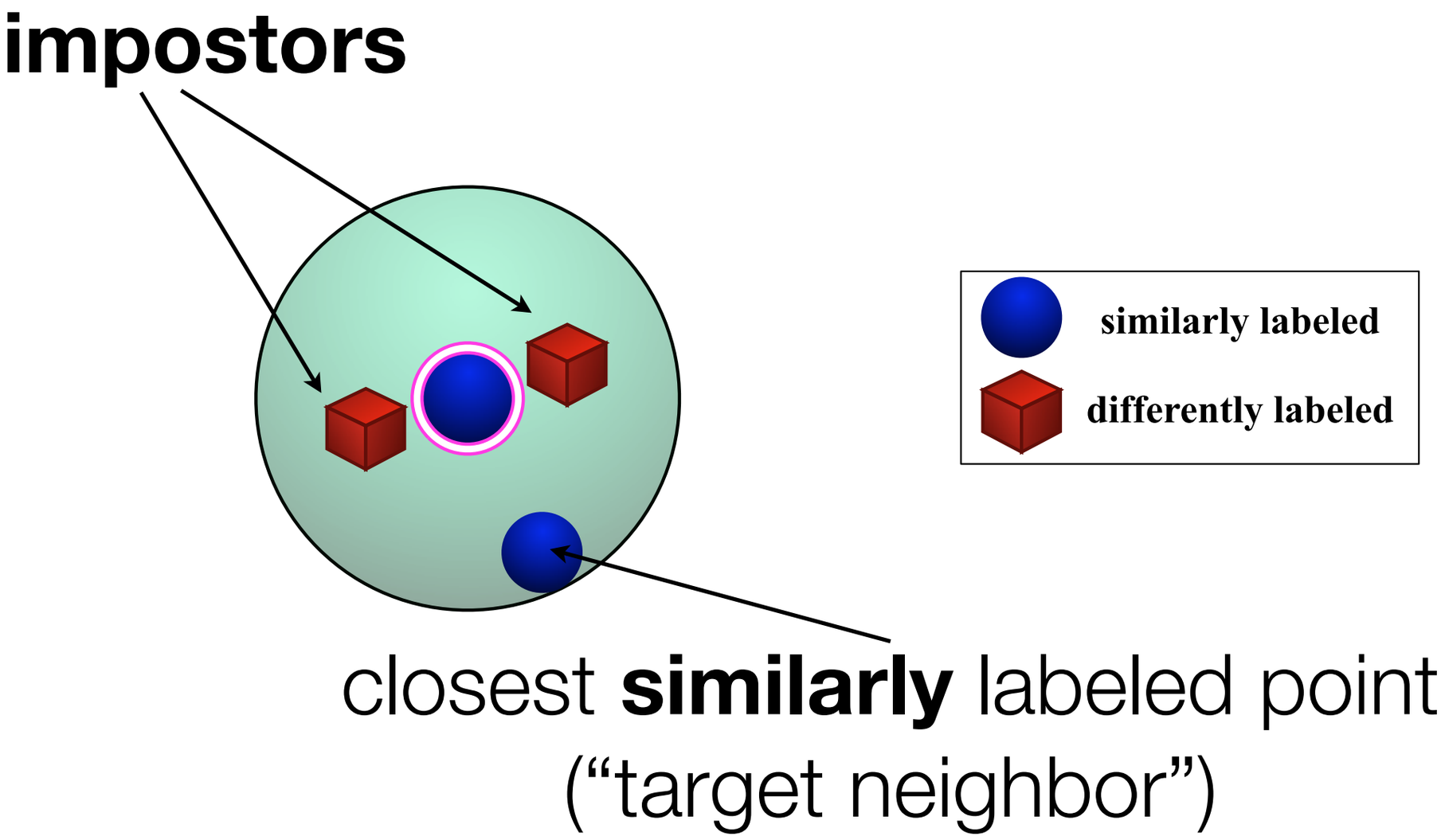
[Weinberger et al.; NIPS 2006]

Large Margin Nearest Neighbor

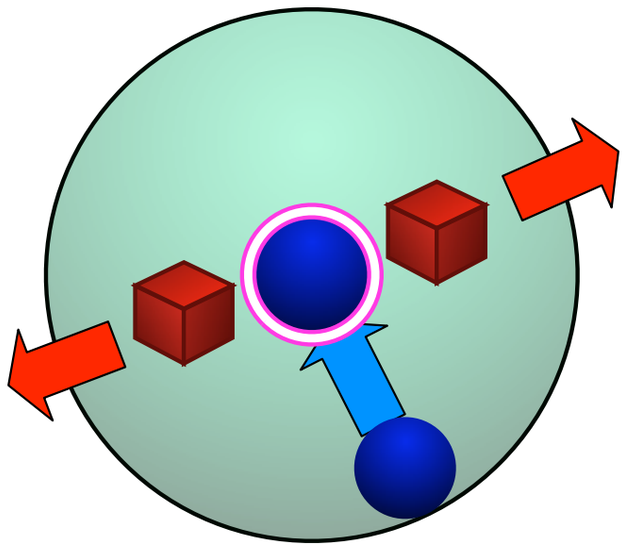


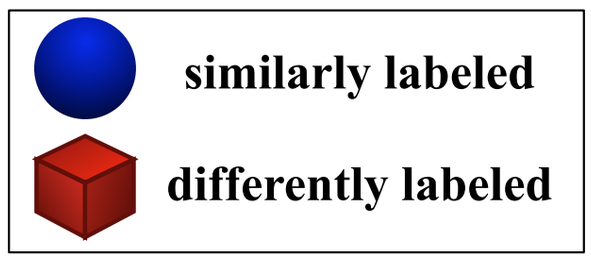
Intuition: Each data point should be closer to the **closest similar** point than to **any different one**.

Intuition


impostors
similarly labeled
differently labeled
closest **similarly** labeled point
("target neighbor")

Intuition

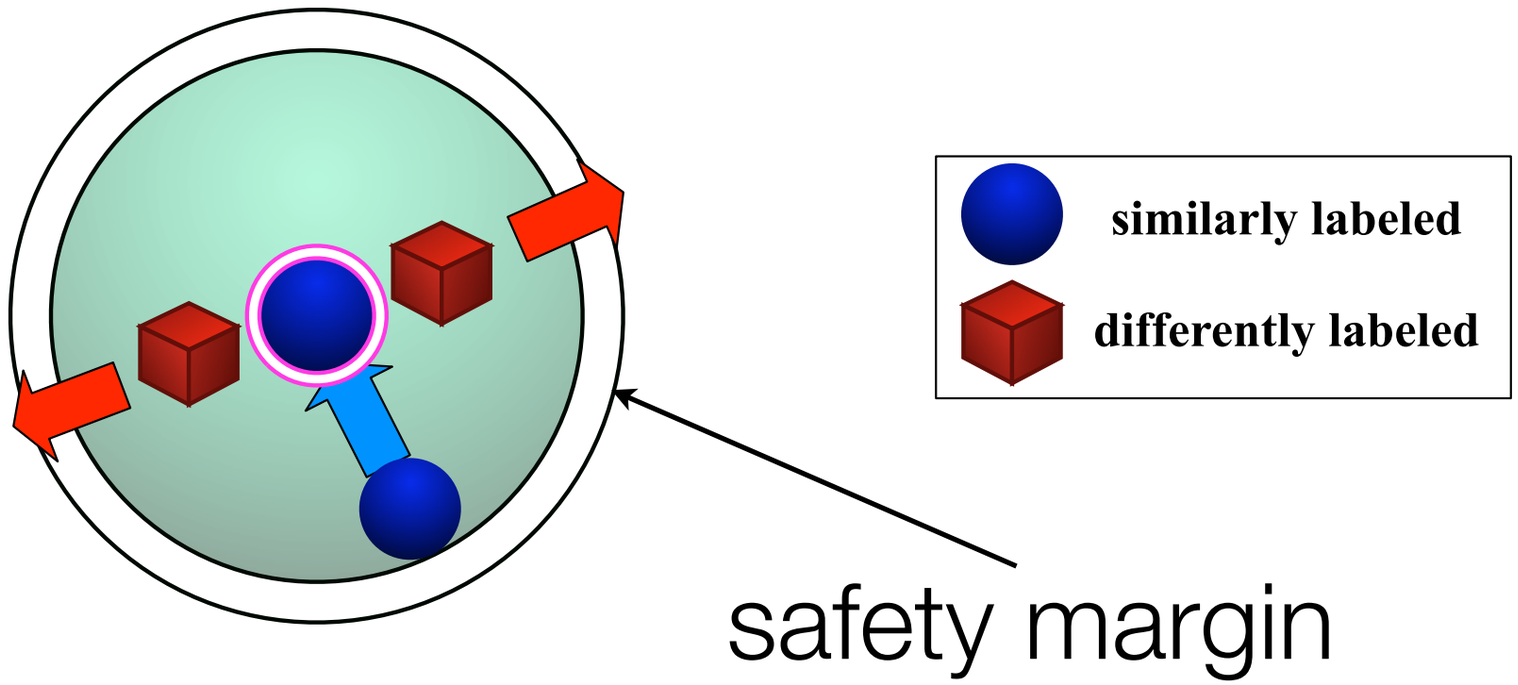




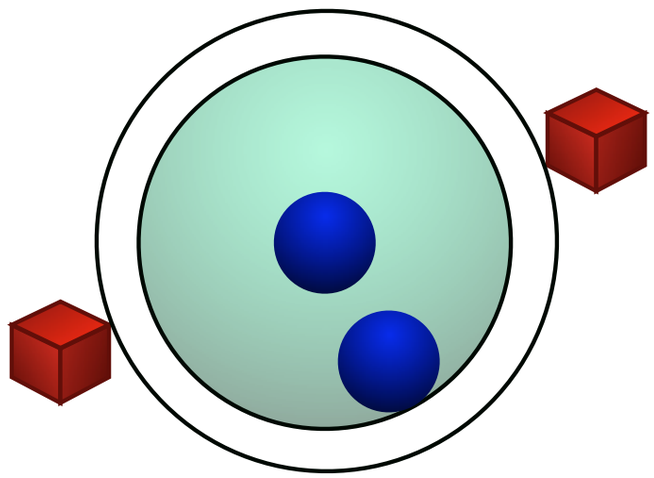
1. pull target neighbors closer

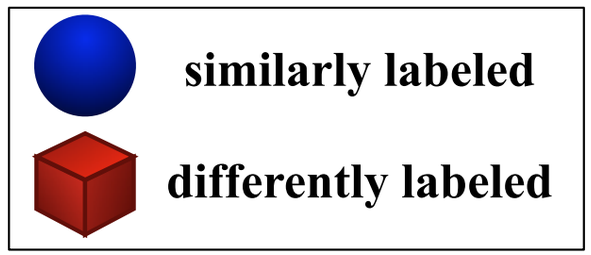
2. push impostors away

Intuition



Clean neighborhood!





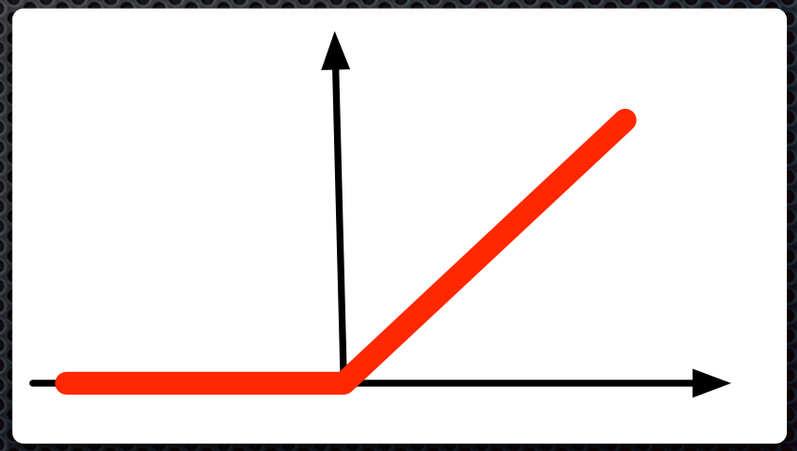
Notation

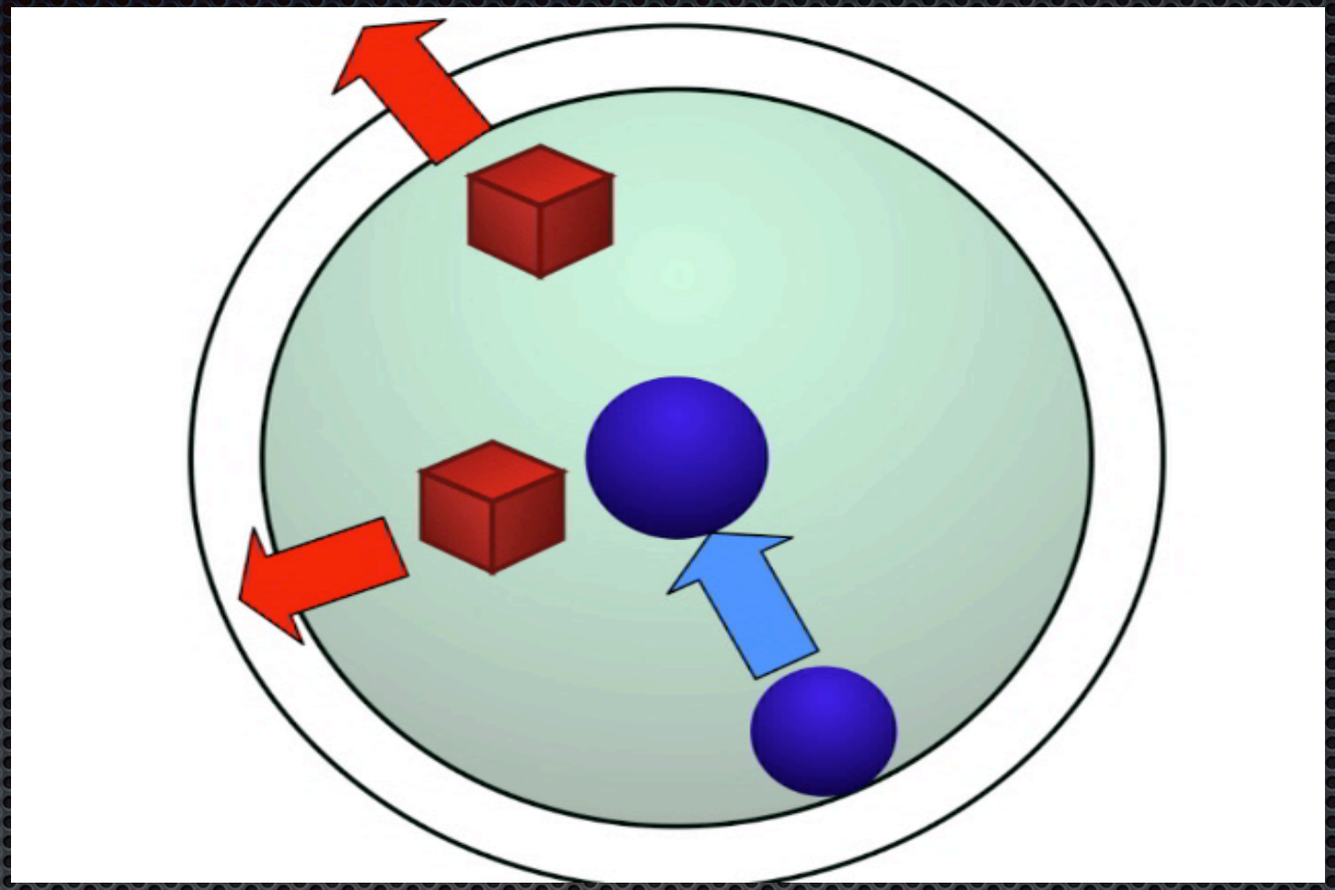
$$D = \{(i,l) \mid \vec{x}_i, \vec{x}_l \text{ are } \textit{dissimilar}\}$$

$$j \rightsquigarrow i : \quad \vec{x}_j \text{ is a } \textit{target neighbor} \text{ of } \vec{x}_i$$

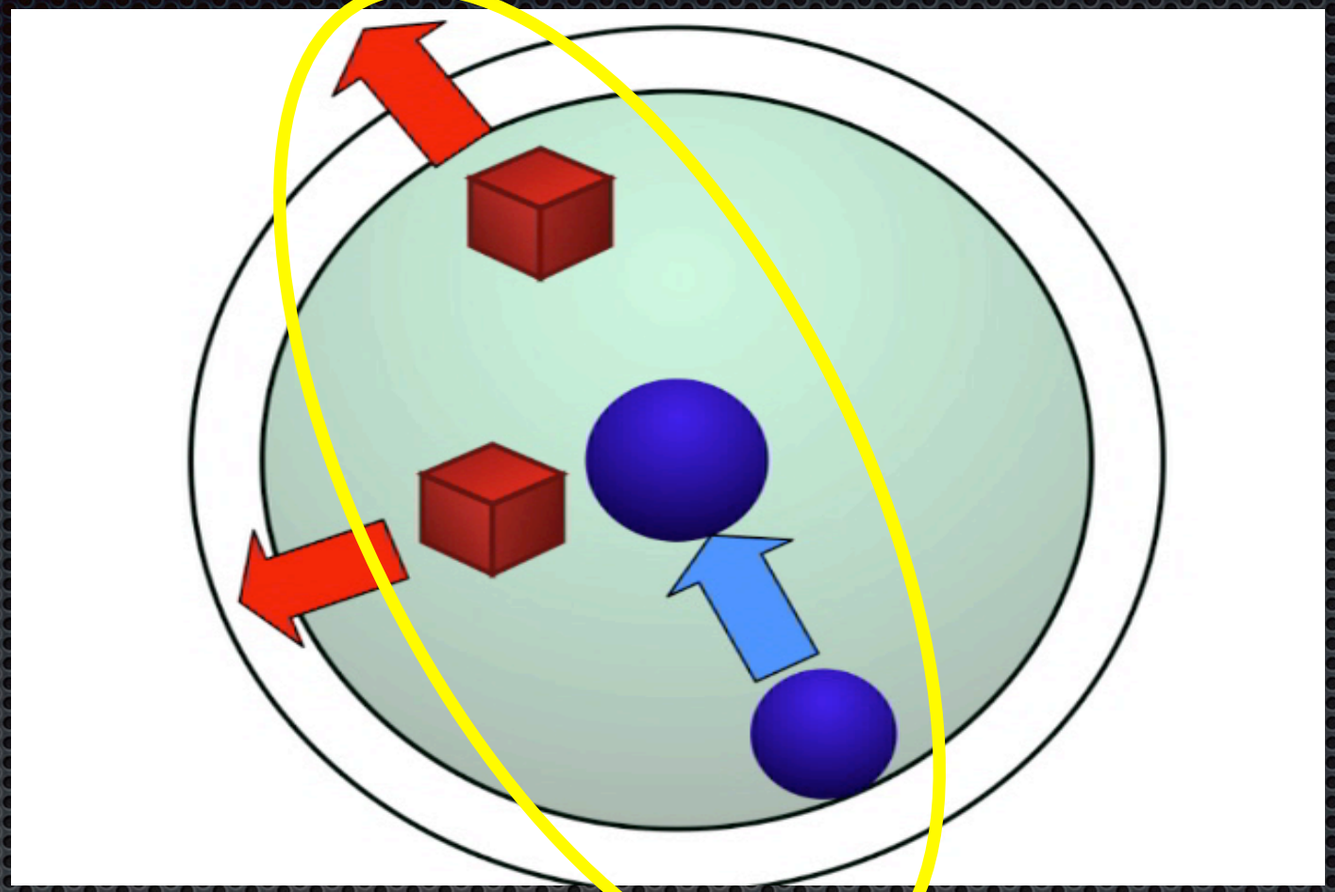
$$[a]_+ = max(a, 0)$$

(hinge-loss)



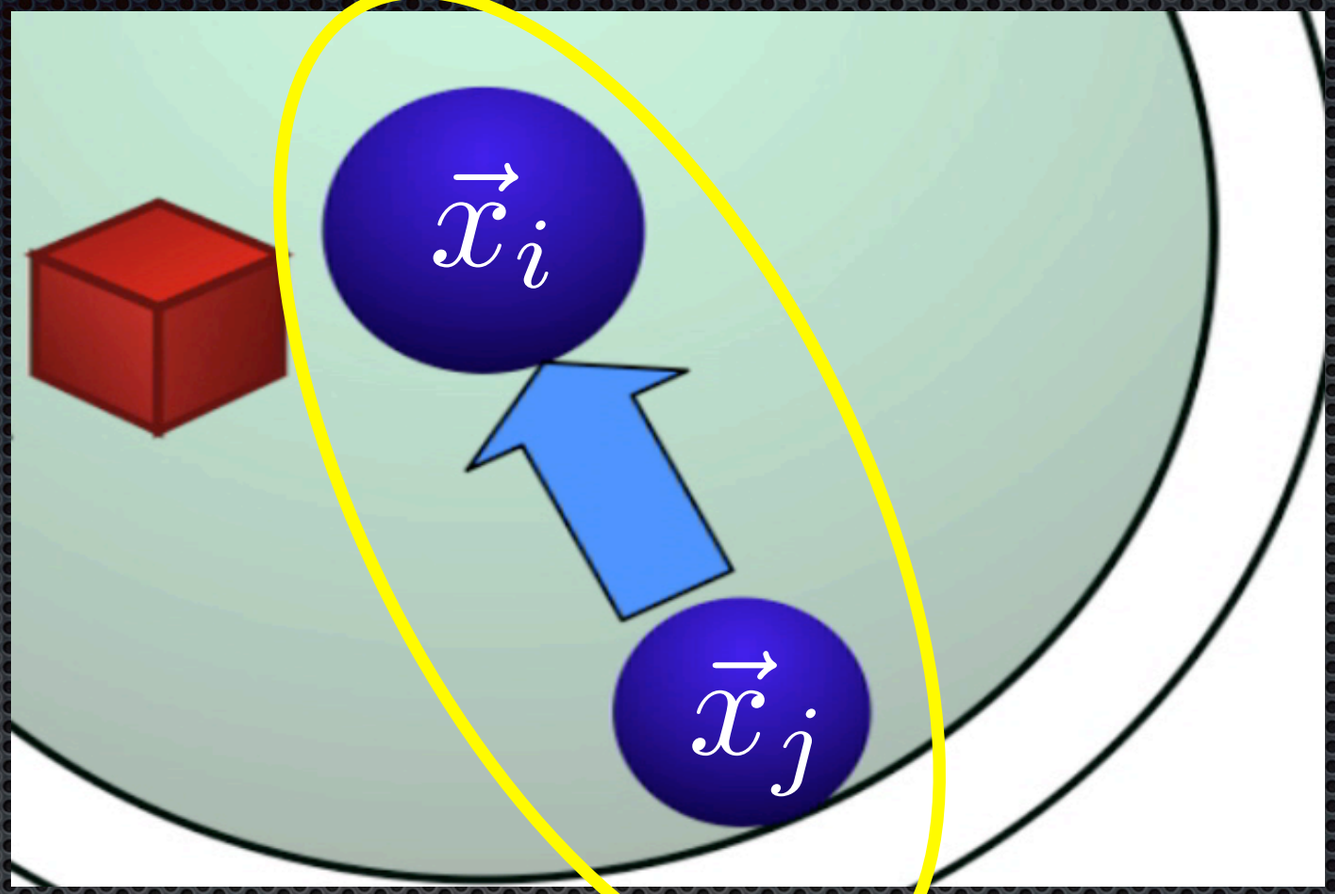


$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{pull}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2$$



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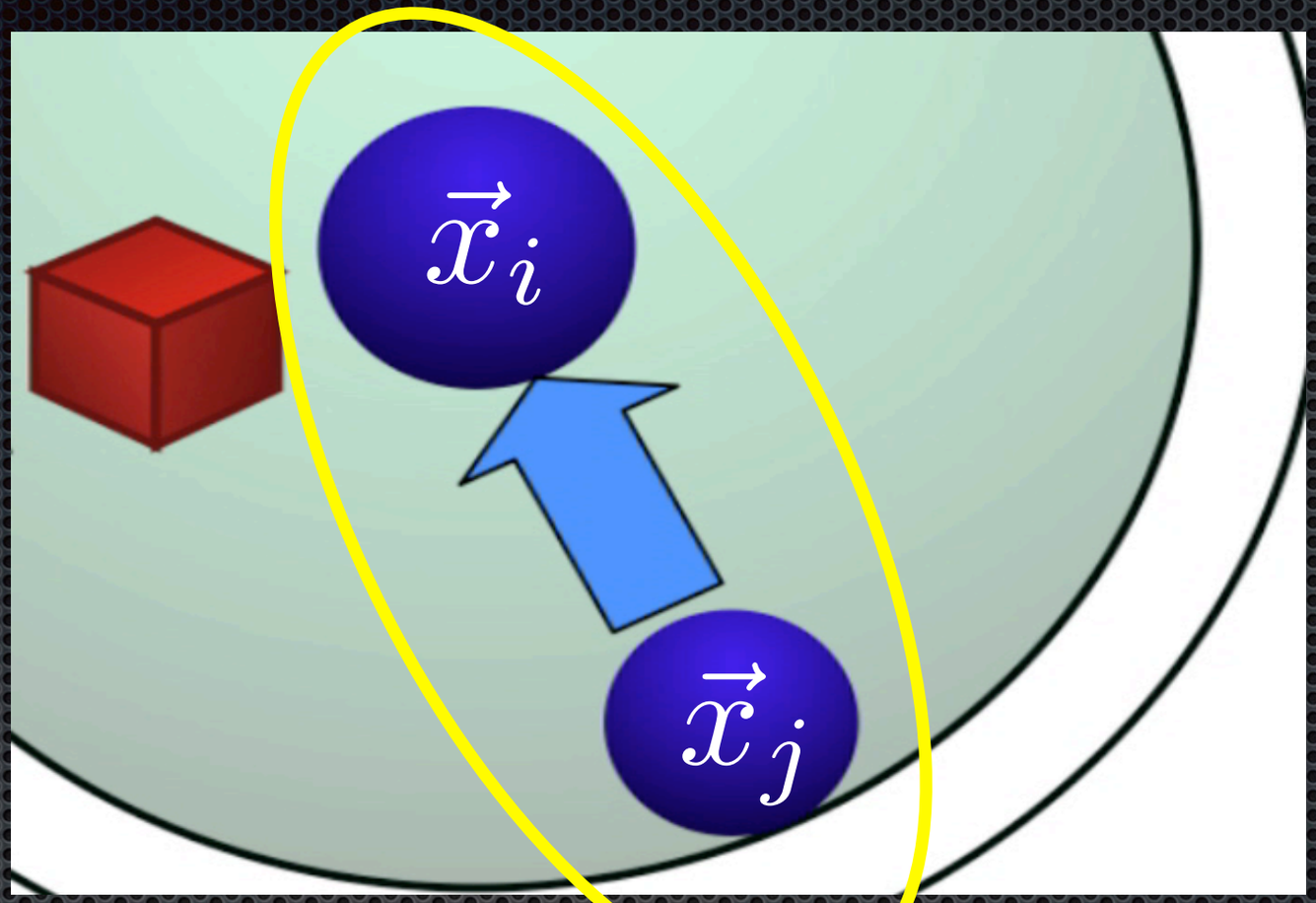
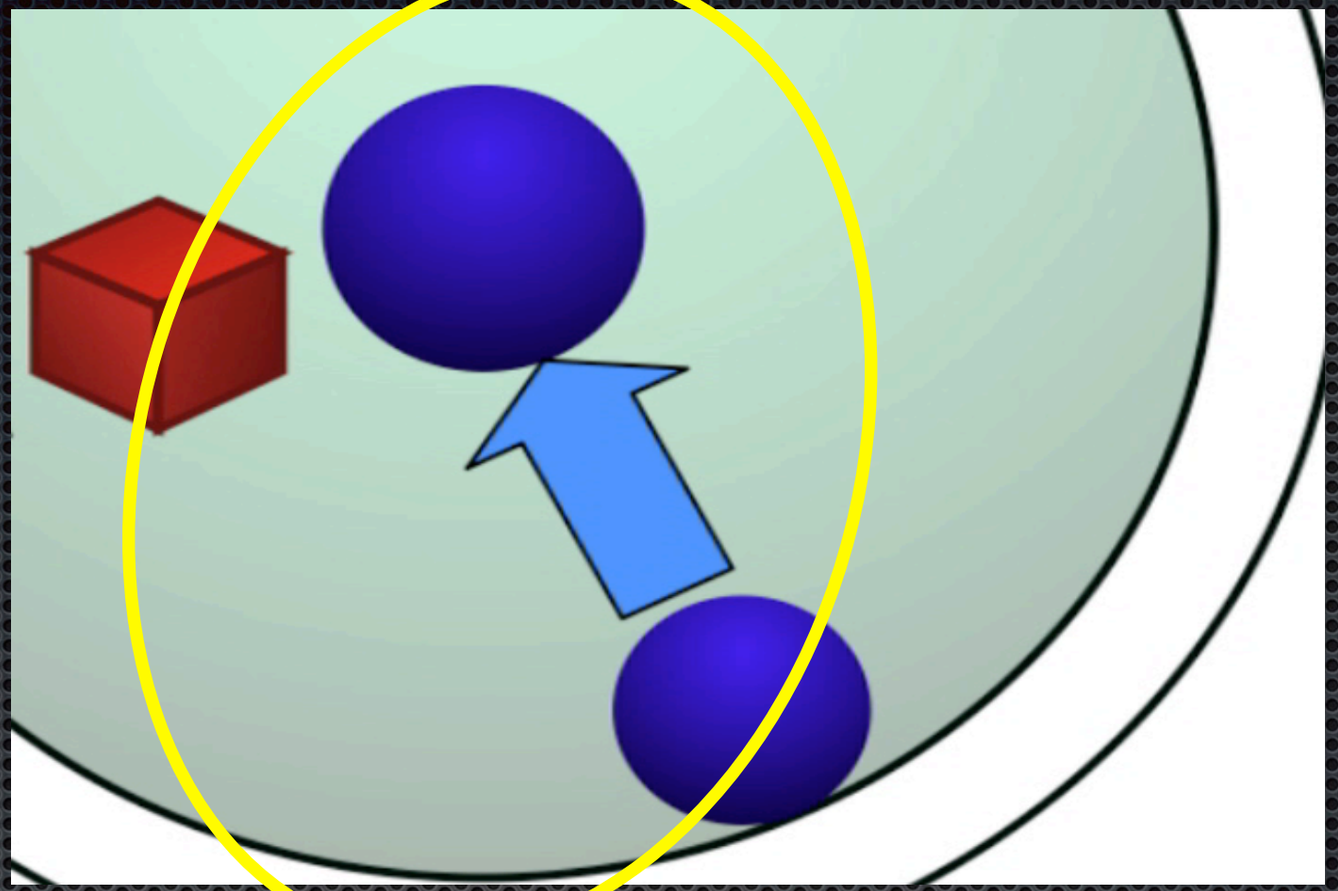
Target neighbors



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

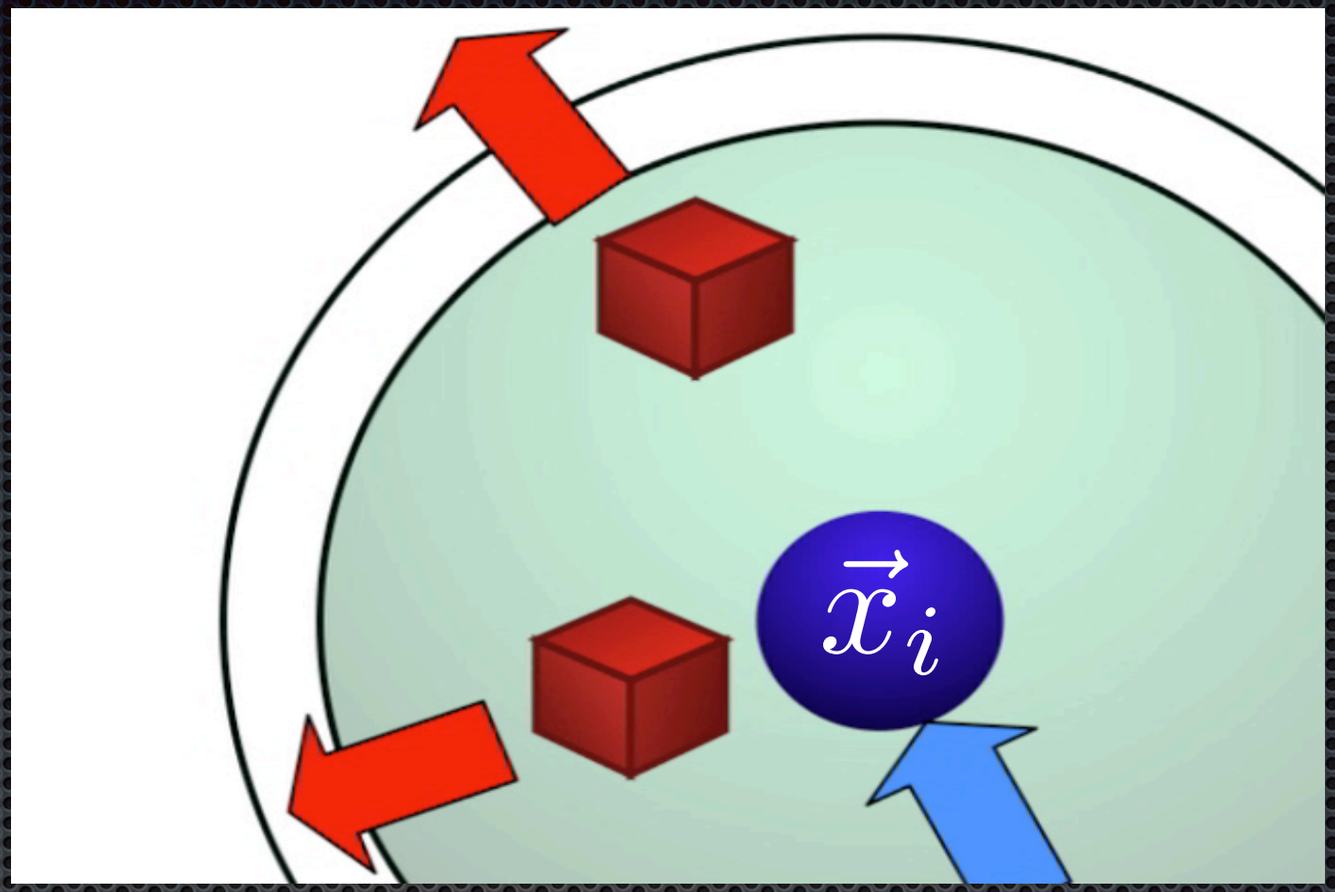
$$\mathcal{L}_{pull}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2$$

Distance to target neighbor



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$



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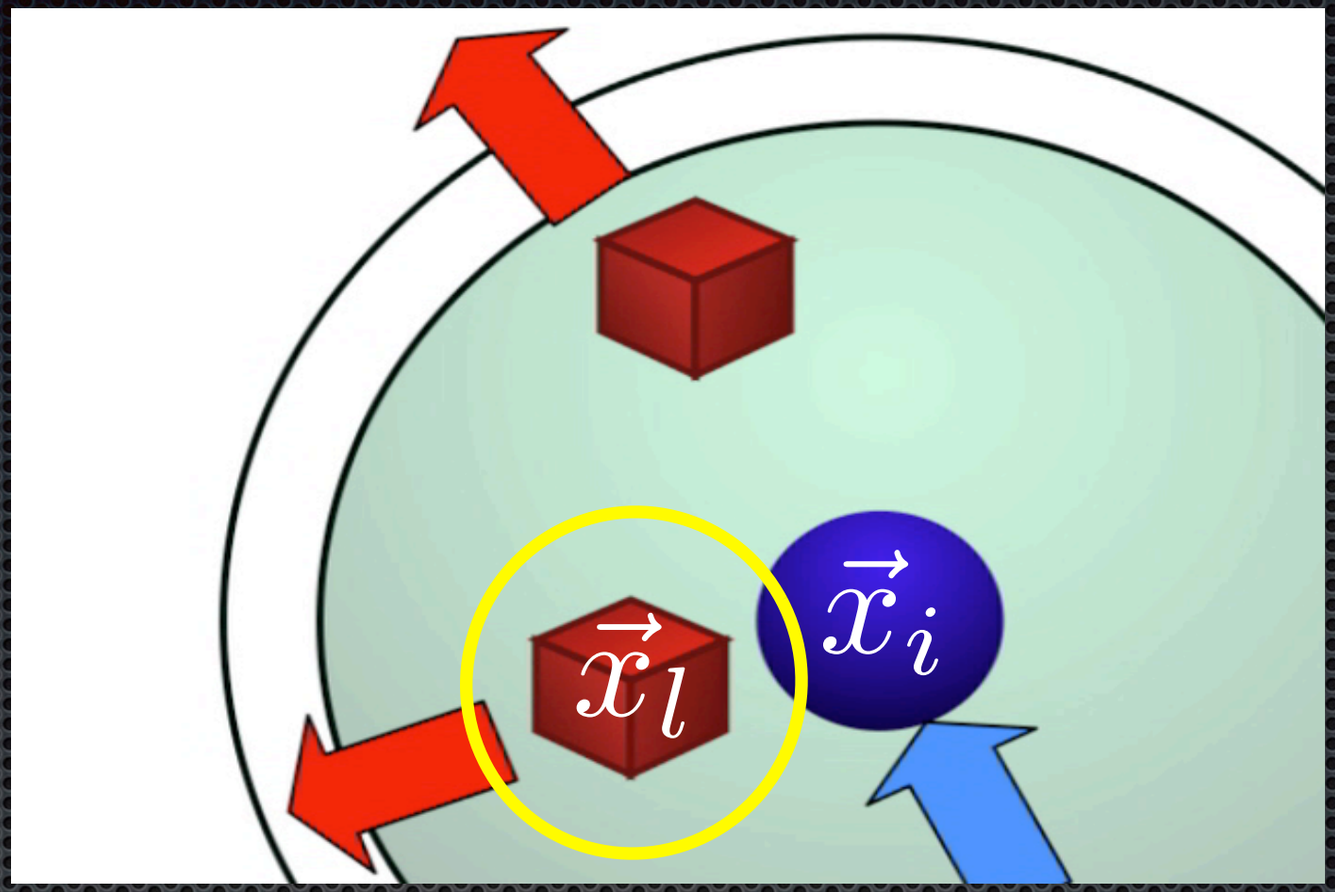
target neighbor



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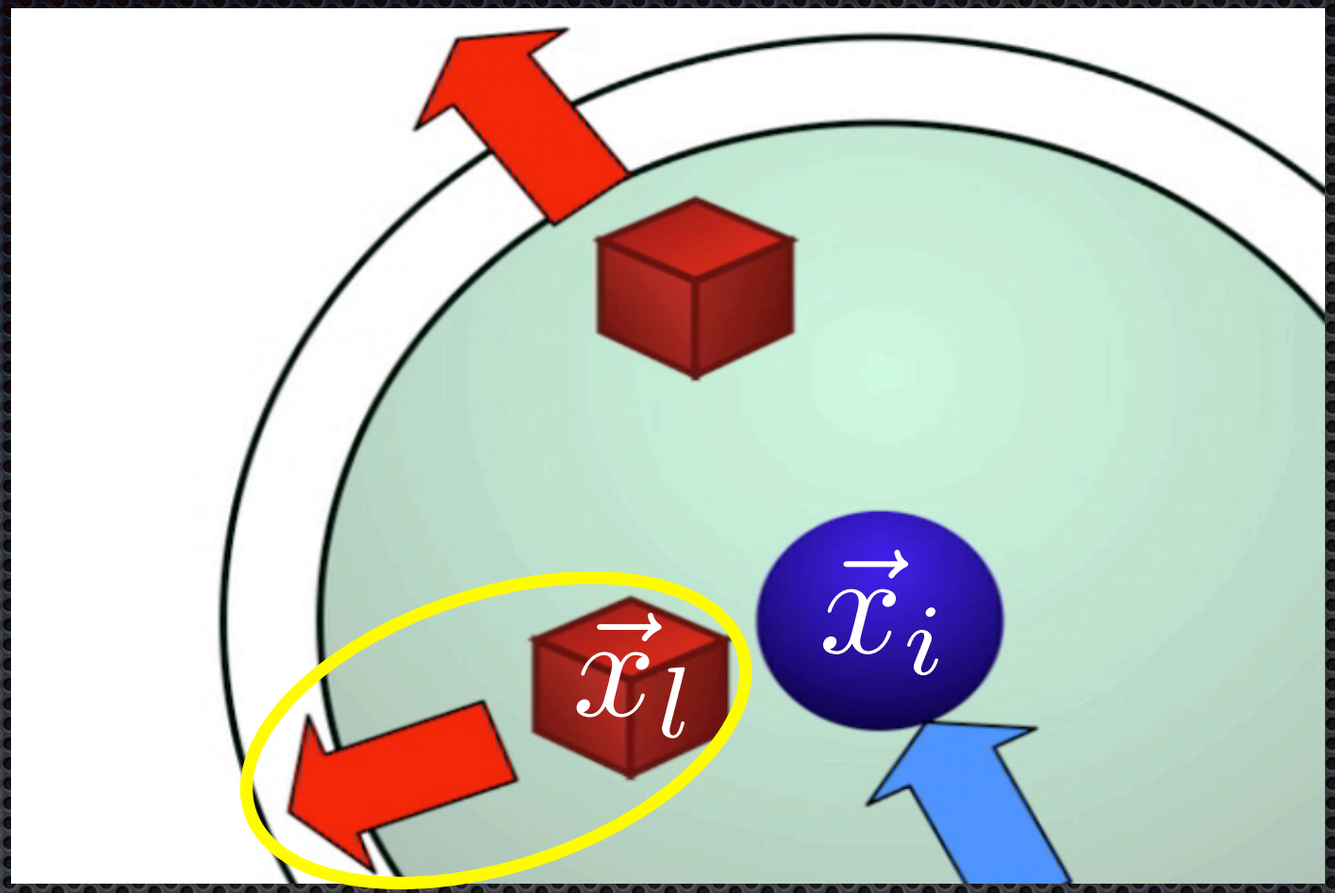
$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$

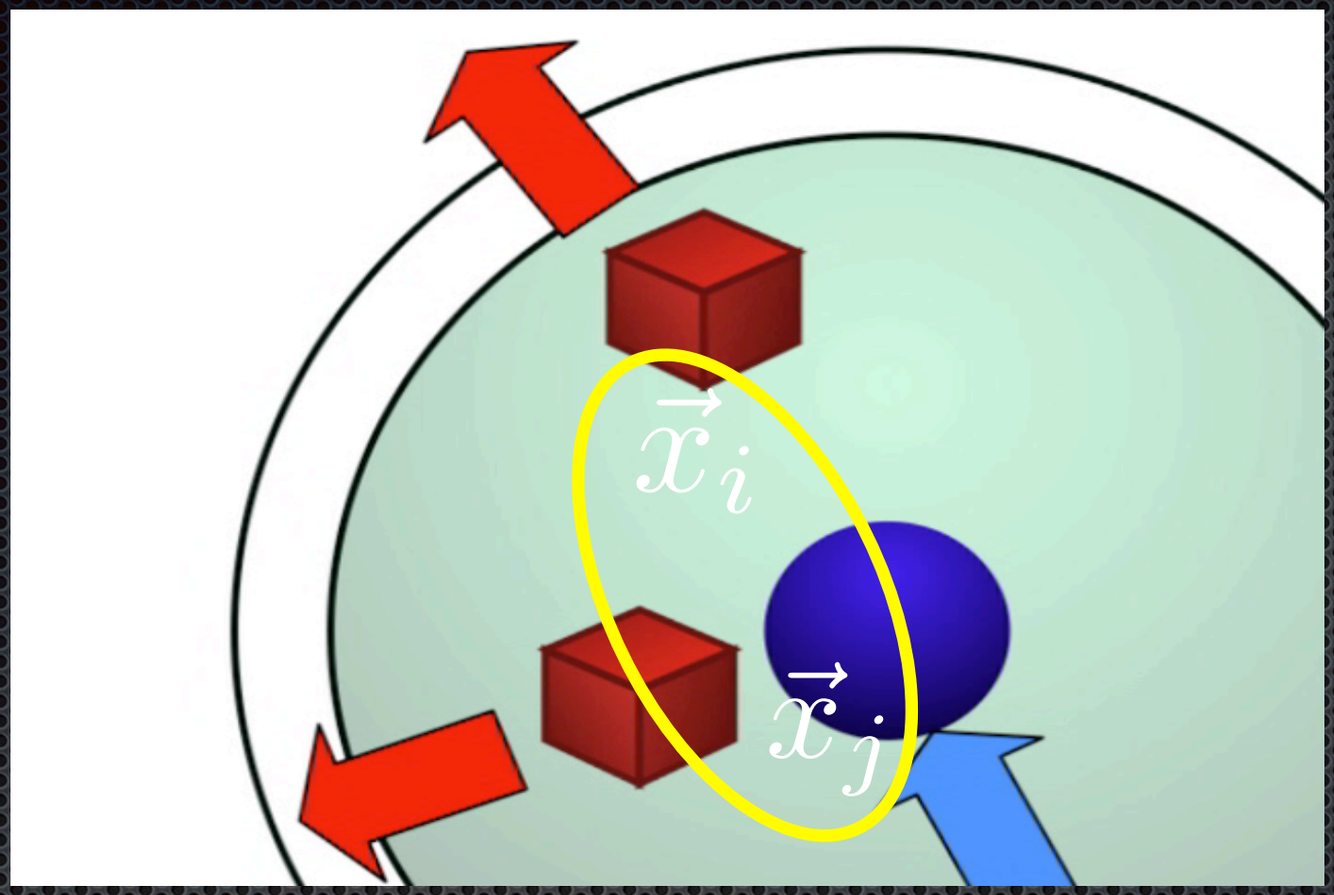
impostor



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$

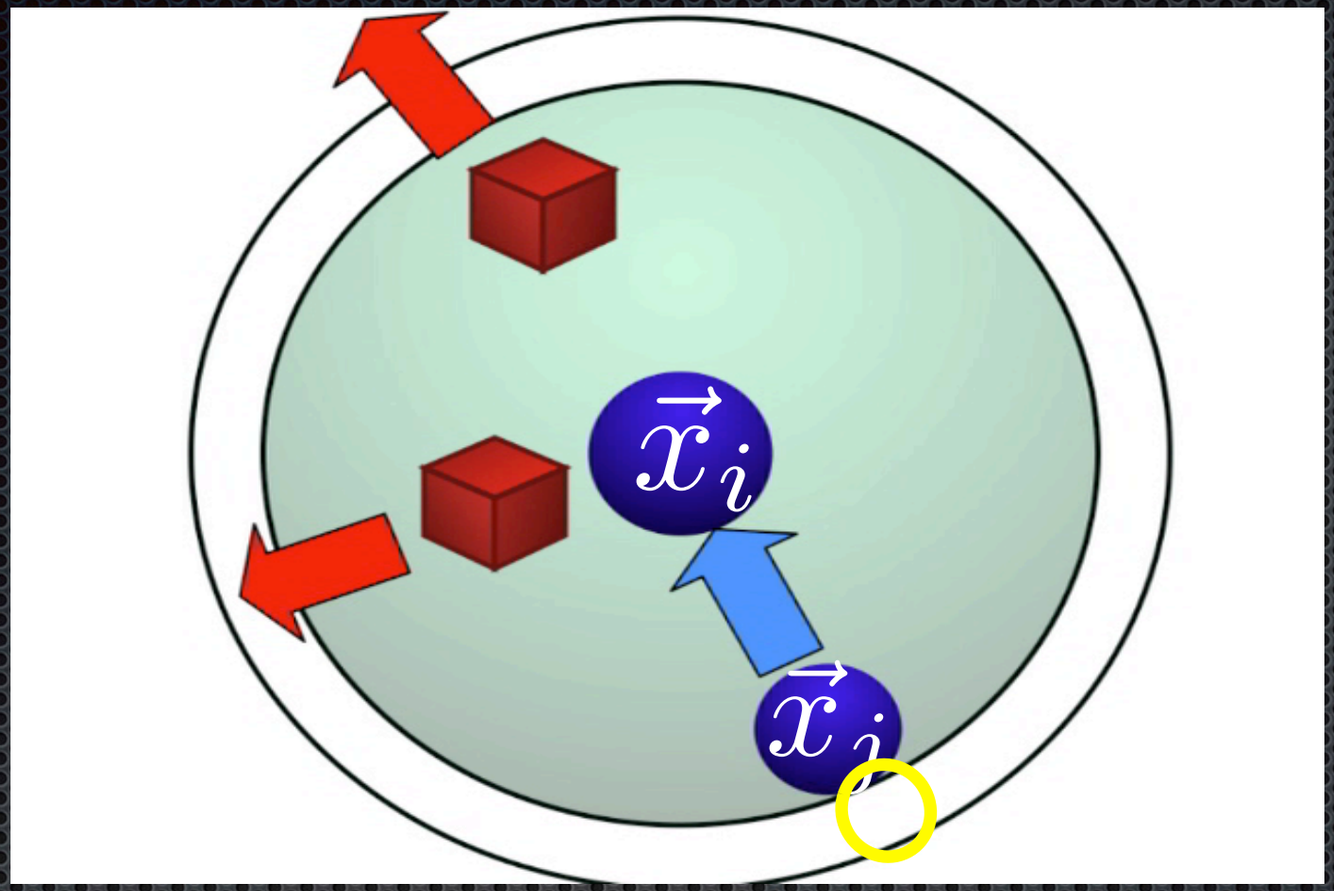
hinge loss



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$

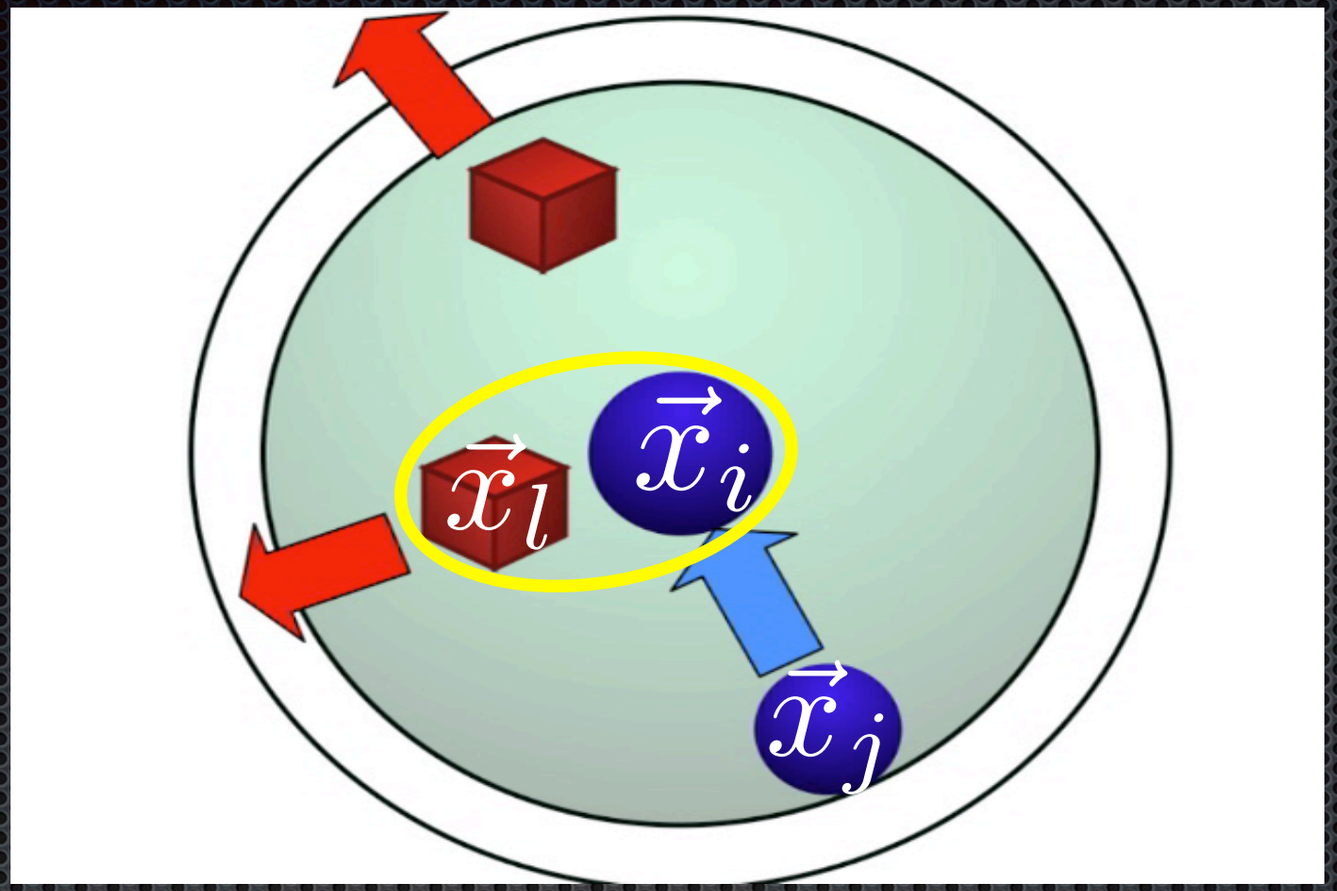
neighborhood radius



$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$

safety margin

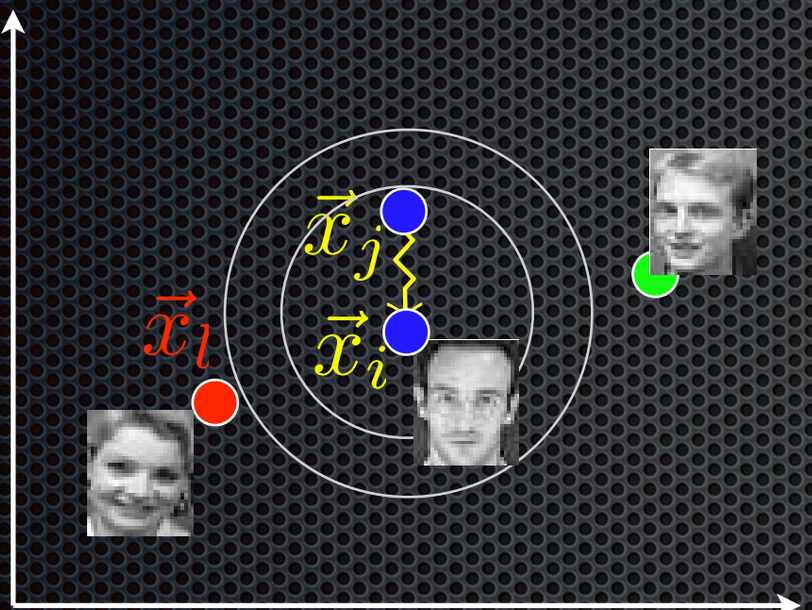


$$\min_{\mathbf{L}} \quad \mathcal{L}(\mathbf{L}) = \mathcal{L}_{pull}(\mathbf{L}) + \mathcal{L}_{push}(\mathbf{L})$$

$$\mathcal{L}_{push}(\mathbf{L}) = \sum_{j \rightsquigarrow i} \sum_{(i,l) \in D} [1 + \|\mathbf{L}(\vec{x}_i - \vec{x}_j)\|_2^2 - \|\mathbf{L}(\vec{x}_i - \vec{x}_l)\|_2^2]_+$$

distance to impostor

Large margin nearest neighbor (LMNN)



Semi-definite program

$$\min_{\mathbf{M}} \sum_{i,j \rightsquigarrow i} \|\vec{x}_i - \vec{x}_j\|^2_{\mathbf{M}} + \mu \sum_{i,j,l} \xi_{ijl}$$

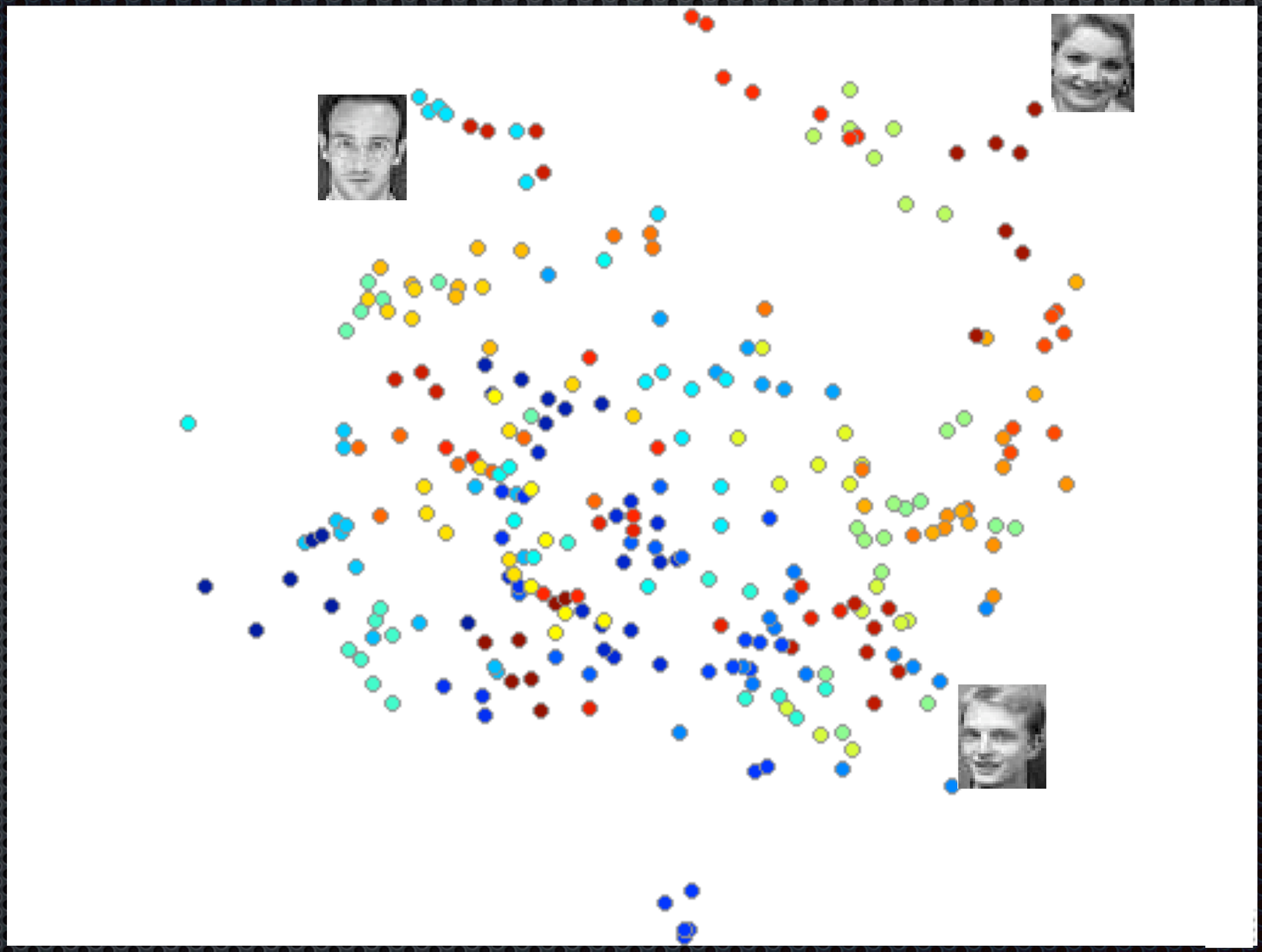
$$\text{subject to:} \quad \forall j \rightsquigarrow i, (l,i) \in D$$

$$\|\vec{x}_i - \vec{x}_j\|^2_{\mathbf{M}} + 1 \leq \|\vec{x}_i - \vec{x}_l\|^2_{\mathbf{M}} + \xi_{ijl}$$

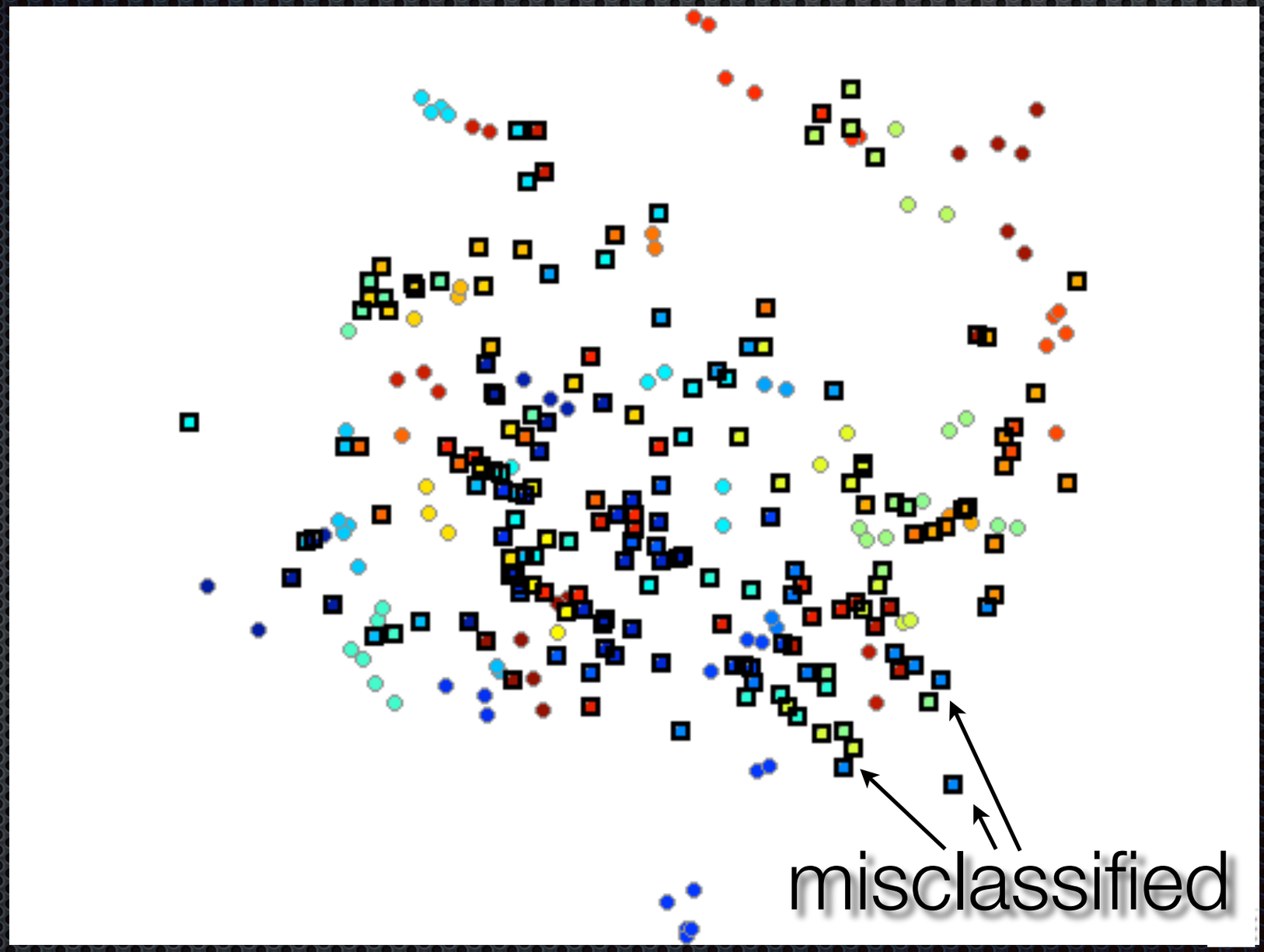
$$\mathbf{M} \succeq 0, \ \xi_{ijl} \geq 0$$

Special-purpose solver [Weinberger 2009]
MNIST: 3.2 Billion constraints in 20 minutes

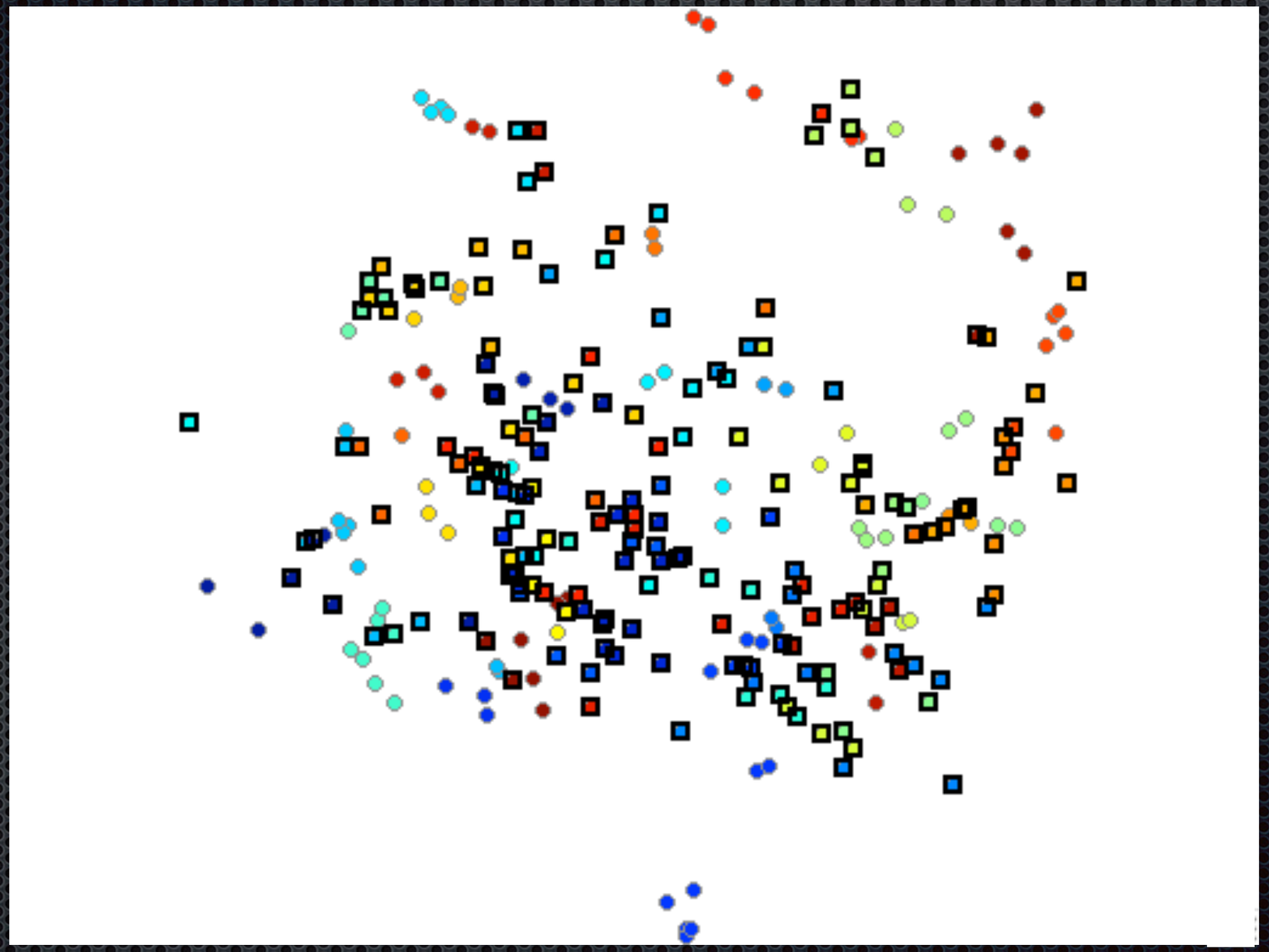
Faces data set



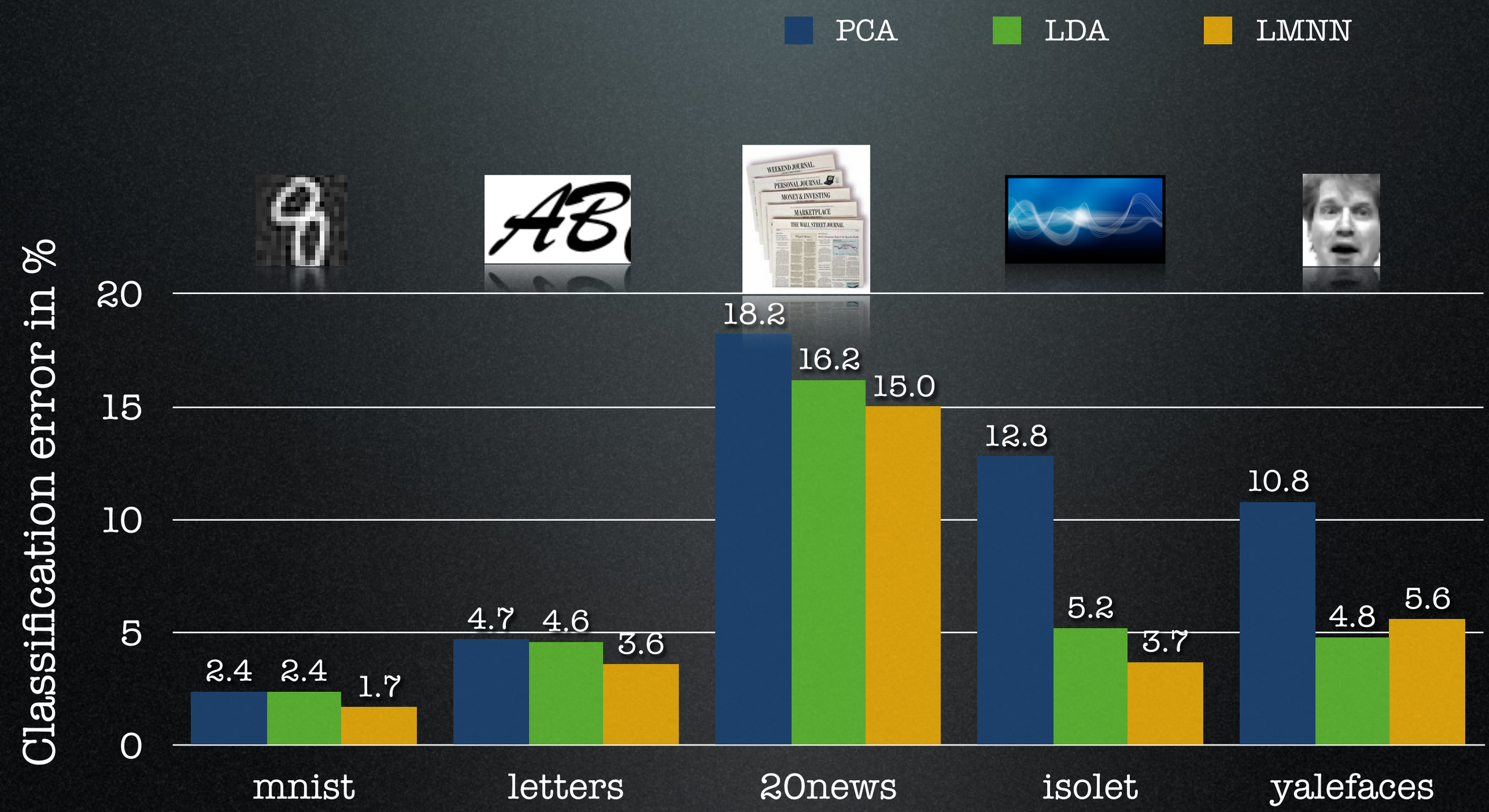
Faces data set



Faces data set

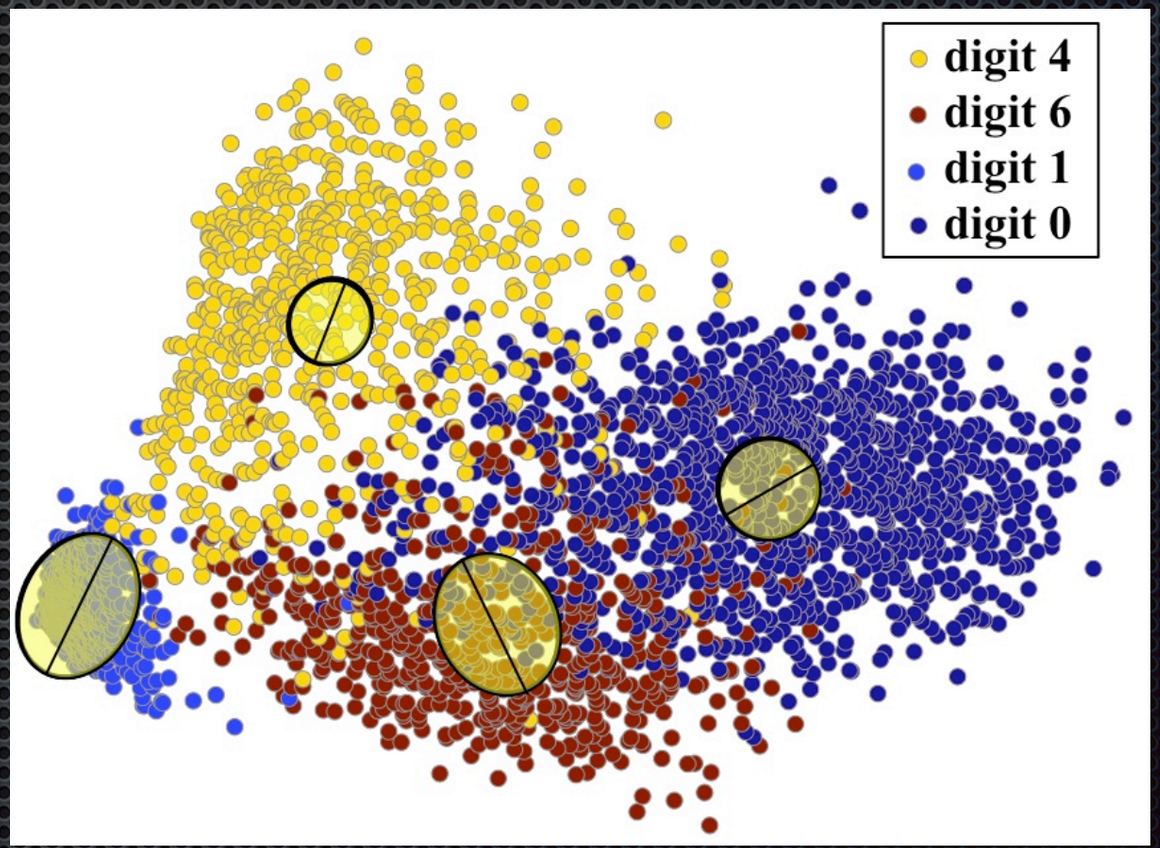


RESULTS


PCA
LDA
LMNN
Classification error in %
20
15
10
5
0
18.2
16.2
15.0
12.8
10.8
5.6
4.8
5.2
4.7
4.6
3.6
3.7
2.4
2.4
1.7
mnist
letters
20news
isolet
yalefaces

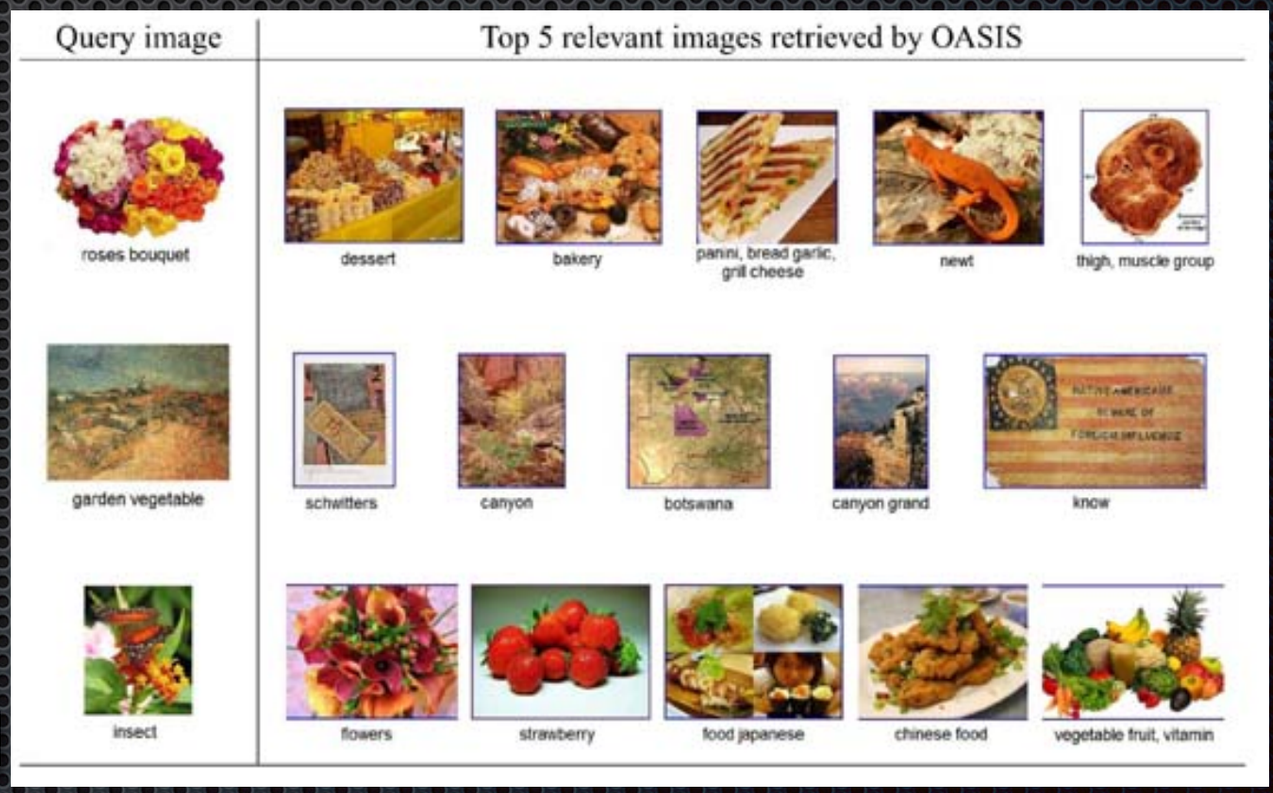
Extensions ...

- Torresani and Lee, [NIPS 2006]
  - **Kernelization** (significantly slower but more powerful)
  - **Linear dimensionality reduction** (potentially better classification results, but breaks convexity)
- Weinberger and Saul, [ICML 2008]
  - Many **locally linear metrics** instead of a single global metric.
  - **Fast nearest neighbor retrieval** with ball-tree data structures.
- Kumar et al., [ICCV 2007]
  - i-LMNN metric **invariant** to **polynomial** input **transformations**



Extensions … (cnt'd)

- Shen et al., [NIPS 2008]
  - Optimize through **boosting**
  - Weak learners: rank 1 PSD matrices
- Chechik et al., [NIPS 2009]
  - **Online version**.
  - Scales to **gigantic** data sets.
  - Relaxation of SDP constraint
- Parameswaran and Weinberger [NIPS 2010]
  - **Multi-class** adaptation
  - **Transfer learning**

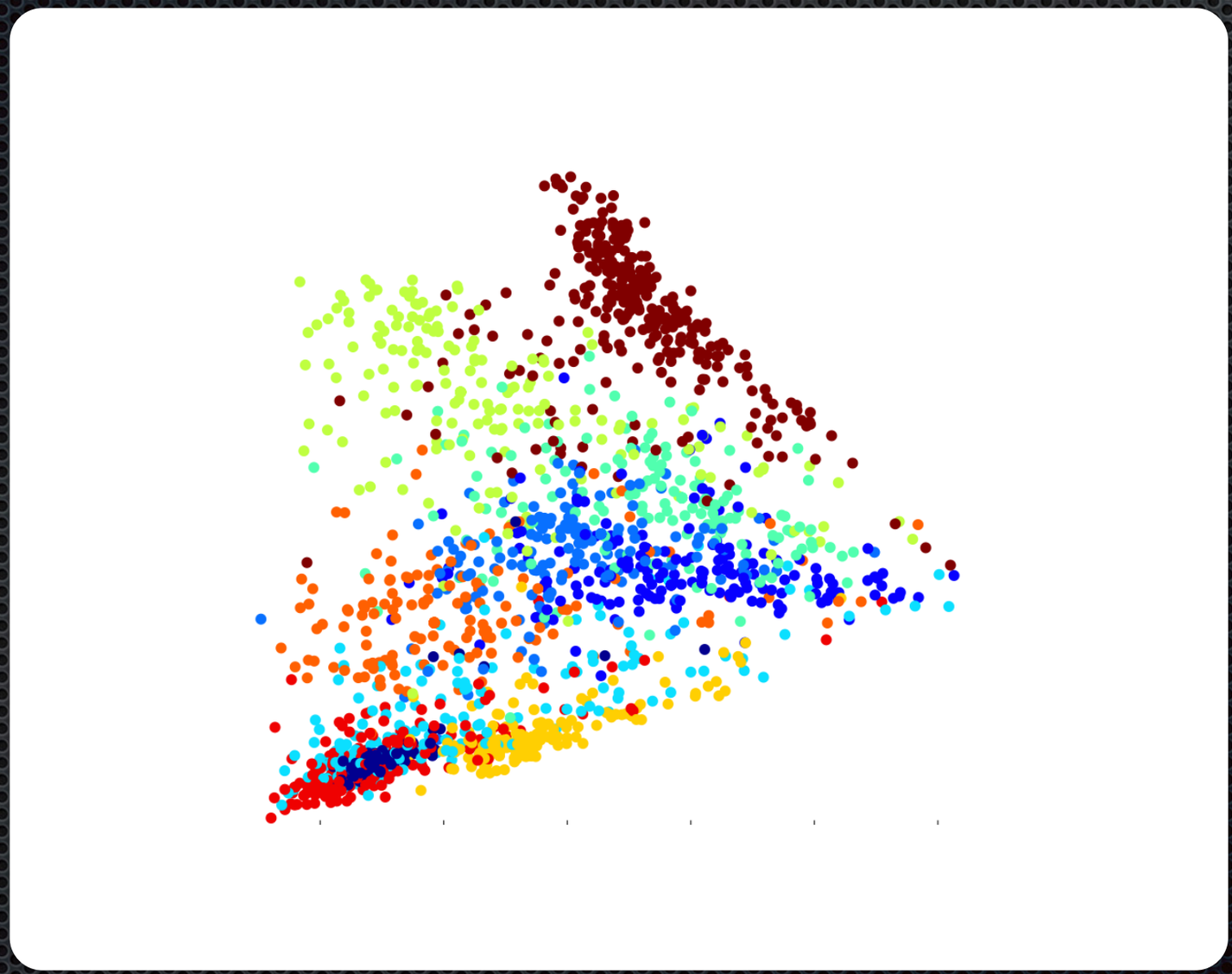


[Courtesy of Chechik et al.]

Summary of LMNN

- Learns Mahalanobis metric
- **Local margins**
- Optimization problem is convex
- Algorithm is guaranteed to converge
- Scales well -- generally best for large data
- Requires “target neighbors”

Information Theoretic Metric Learning (ITML)



[Davis et al. ICML 2007]

ITML Optimization

Define some "initial" Mahalanobis distance $\mathbf{M}_0$
(e.g. Euclidean $\mathbf{M}_0 = \mathbf{I}$)

Convex optimization:

$$\min_{\mathbf{M}} \operatorname{tr}(\mathbf{M}\mathbf{M}_0^{-1}) - \log\det(\mathbf{M}\mathbf{M}_0^{-1})$$

$$(\vec{x}_i - \vec{x}_j)^\top \mathbf{M}(\vec{x}_i - \vec{x}_j) \leq s,\ \forall (i,j) \in S$$

$$(\vec{x}_i - \vec{x}_\ell)^\top \mathbf{M}(\vec{x}_i - \vec{x}_\ell) \geq d,\ \forall (i,\ell) \in D$$

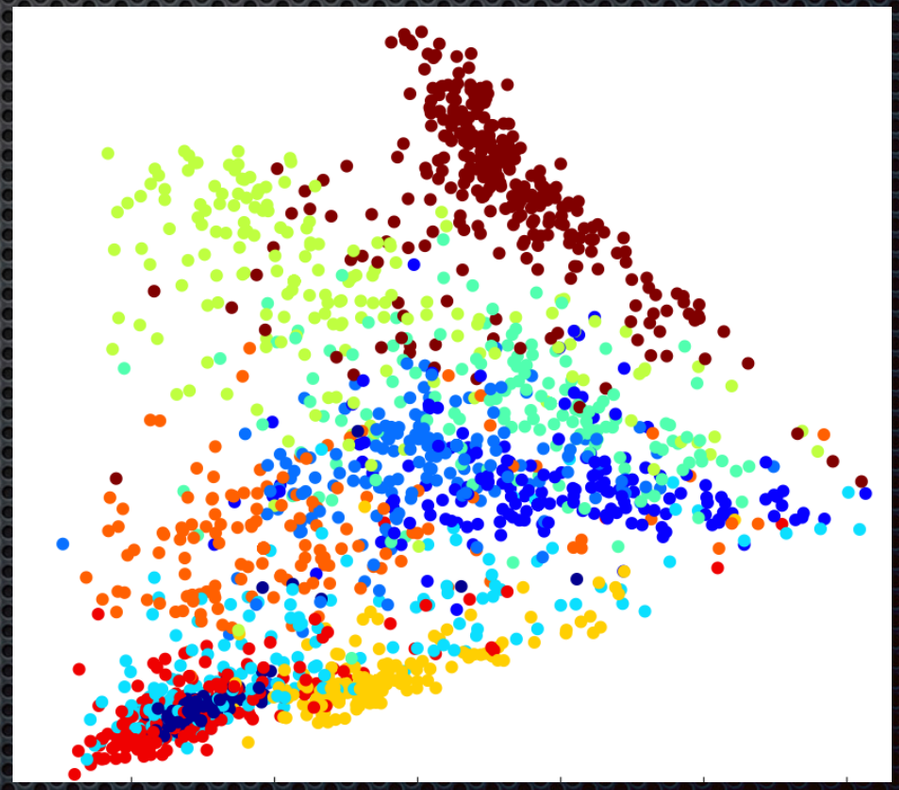
Stein's Loss

$$\min_{\mathbf{M}} \operatorname{tr}(\mathbf{M}\mathbf{M}_0^{-1}) - \log\det(\mathbf{M}\mathbf{M}_0^{-1})$$

- Keep metric **close to initial guess** (e.g. Euclidean)
- **Automatically preserves positive semidefiniteness**
- Allows cheap rank-1 updates

Extensions ...

- P. Jain, B. Kulis, I. S. Dhillon, K. Grauman., [NIPS 2009]
  - **Online** formulation
  - **Local Sensitive Hashing** for fast similarity search
- Z. Lu, P. Jain, I. S. Dhillon. [ICML 2009]
  - **Geometry aware** metric learning.
  - Data set **Visualization**.
- P. Jain, B. Kulis, I. S. Dhillon. [NIPS 2010]
  - Link with **kernel learning**.
  - Generalization of **regularizers**



[Zhengdong et al.]

Summary of ITML

- Mahalanobis metric for k-NN
- Performs well in practice
- Simple updates
- Scales well (online algorithm)
- **Convex without PSD constraint**
- Assumes unimodal data

A few more things ...

Many more Algorithms...

- Semi-supervised M. learning (EM update) **[Bilenko et al.; ICML'04]**
- Metric Learning for SVM: SVML **[Zu et al., TR'11]**
- Siamese neural networks **[Chopra et al., CVPR'05]**
- SVM-style feature reweighing **[Schultz & Joachims, NIPS'02]**
  - Generalization **[Kwok and Tsang; ICML'03]**
- Discr. Adaptive Nearest Neighbors **[Hastie & Tibshirani; PRMI'96]**
- Generative local metric learning for n.n. **[Noh et al.; NIPS'10]**
- ... many more

About High Dimensionality...

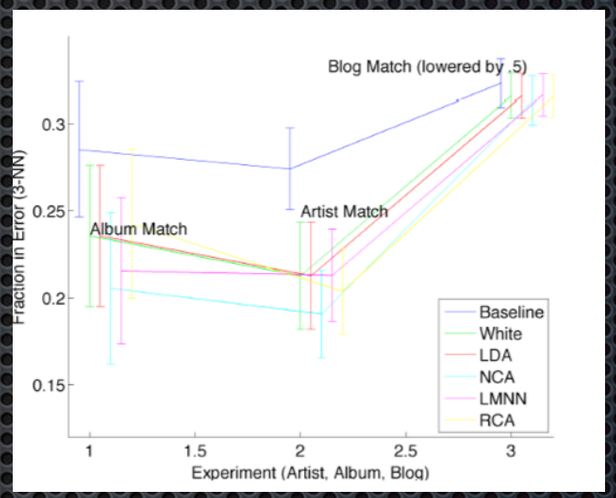
- Mahalanobis matrix is dxd
- BUT:
  - All algorithms from this talk can be kernelized
  - Kernelization simple: Everything in terms of dot-products
  - Close relationship between **kernel**- and **metric** learning

[Chatpatanasiri et al.; ArXiV 2008]

Applications

Face Verification
[Chopra et al. 2005]

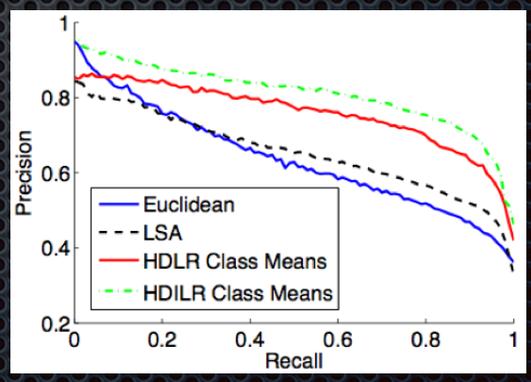
Object Recognition
[Fromme et al. 2007]



Music Similarity
[Slaney et al. 2008]

Pedestrian detection
[Dikmen et al. 2010]

Pose estimation
[Jain et al. 2010]



Text retrieval
[Davis et al. 2008]

When use what?

- **Best accuracy**: LMNN, ITML
- **Online**: LMNN*, ITML*
- **Large-Scale**: LMNN, ITML
- **Small data**: NCA,MCML
- **Clustering**: MMC

Outlook

- Loose ends:
  - Non-linear metrics (so far neural nets and boosting)
  - Algorithms beyond nearest neighbors
  - Unified theory (noteworthy start [Jain et al. 2011])
  - Weaker supervision

Summary

- Metric Learning is **fun**!
- Linear transformation: Mahalanobis metric
  - Many algorithms, different strengths weaknesses
  - Very diverse applications
- Think twice before you use the Euclidean metric.
- Still a lot of open problems ...