Math 45 - Section —
HW 2
Tuesday, March 8, 2016

1, 2, 3

1 Complex numbers are helpful when expressing oscillatory behavior using trigonometric functions. Euler's formula helps us to convert from complex exponentials like $e^{i\theta}$ to trigonometric functions like cosine and sine:

$$e^{i\theta} = \cos\theta + i\sin\theta.$$

How do we convert from cosine and sine into complex exponentials? Consider that

$$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta.$$

Add the two equations above and solve for $\cos\theta$. Subtract to solve for $\sin\theta$.

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2 Simplify each of the following expressions, assuming that a, t, ω are real numbers.

Example: $\Re\left(e^{i\omega t}\right) = \Re\left[\cos(\omega t) + i\sin(\omega t)\right] = \boxed{\cos(\omega t)}$

(a) $\Im\left(e^{i\omega t}\right) =$

(b) $\Im\left(e^{-i\omega t}\right) =$

(c) $\Re\left(e^{(a+i\omega)t}\right) = \Re\left(e^{at}e^{i\omega t}\right) = \Re\left[e^{at}\cos(\omega t) + ie^{at}\sin(\omega t)\right] =$

(d) $\Im\left(e^{(a+i\omega)t}\right) =$

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3 For each of the following ordinary differential equations, indicate its order, whether it is linear or nonlinear, whether it is autonomous or non-autonomous, and whether it is driven or undriven.

(a) $\dfrac{dg}{dx} + g^3 = 0$

(b) $\ddot{y}(t) + e^{t\,y(t)} = \cosh t$ **Note:** $\ddot{y}(t)$ is the same as $y''(t) = \dfrac{d^2y}{dt^2}$.

(c) $r^2R''(r) + rR'(r) - 5R(r) = 0$

(d) $\ddot{\theta} + \sin\theta = 0$

(e) $f''' = f' + x\,f + 4\sin(x)$

(f) $\dfrac{y'}{y} = 7$ **Note:** If possible, rewrite this DE so that it is linear. If not, explain why.

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