

2, 3, 4, 5

**2** By differentiating each function verify that  $y_1(t) = e^{-t}$  and  $y_2(t) = \sinh t$  both satisfy the differential equation  $y'' - y = 0$ . Is  $y(t) = Ay_1(t) + By_2(t)$ , where  $A$  and  $B$  are arbitrary constants, also a solution? Why or why not?

**Note:** If you need a refresher on *sinh* (hyperbolic sine) look it up on Wikipedia.

■

**3** Suppose that an object is moving in a straight line with constant acceleration  $a \in \mathbb{R}$ . Use properties of integration to show that the position of the object as a function of time  $t$  is given by

$$p(t) = \frac{1}{2}at^2 + v_0t + p_0,$$

where  $v_0$  and  $p_0$  denote the velocity and position at time  $t = 0$ . Start by observing that acceleration is the second derivative of position, thus,

$$p''(t) = a.$$

Be careful in your solution to rigorously justify each step.

■

4 Verify that

$$y(t) = e^{t^2} \int_0^t e^{-s^2} ds + e^{t^2}$$

is a solution to the differential equation  $y' - 2ty = 1$  with initial condition  $y(0) = 1$ .

**Note:** If you're having trouble differentiating that integral, visit

<http://mathworld.wolfram.com/SecondFundamentalTheoremofCalculus.html>

■

5 Examine Student W's work on the following problem. What did the student do correctly? What mistake(s) did the student make? What is a more correct response to the problem, and what would you say to help the student understand how to correctly complete the problem?

Determine if  $y(x) = x^2 + 1$  is a solution to the initial value problem consisting of  $4yy' = (y')^3 - 3y''x$  and the initial condition  $y(0) = 1$ .

If  $y(x) = x^2 + 1$  then  $y'(x) = 2x$  and  $y''(x) = 2$ .

Plug these into the DE:

$$4yy' = (y')^3 - 3y''x$$

$$4(x^2 + 1) \cdot 2x = (2x)^3 - 3 \cdot 2 \cdot x$$

$$\cancel{8x^3} + 8x = \cancel{8x^3} - 6x$$

$14x = 0$  means  $x = 0$  and that matches  $x_0 = 0$  in the initial condition. ✓

Also  $y(x_0) = y(0) = 1$  so that checks out too. ✓

So  $y(x) = x^2 + 1$  is a solution to the IVP.