

If $y(x) = x^2 + 1$ then $y'(x) = 2x$ and $y''(x) = 2$.

Plug these into the DE:

$$4yy' = (y')^3 - 3y''x$$
$$4(x^2 + 1) \cdot 2x = (2x)^3 - 3 \cdot 2 \cdot x$$

$$\cancel{8x^3} + 8x = \cancel{8x^3} - 6x$$

$14x = 0$ means $x = 0$ and that matches $x_0 = 0$ in the initial condition. ✓

Also $y(x_0) = y(0) = 1$ so that checks out too. ✓

So $y(x) = x^2 + 1$ is a solution to the IVP.