TCAT Bus Schedule Optimization under Budget Constraints

Yingying Liu (yl3984), Kangmin Cho (kc2344), Wanqi Tang (wt322)

1 Introduction

1.1 Problem Context and System Overview

1.1.1 TCAT introduction

Tompkins Consolidated Area Transit (TCAT) is the public bus system serving Ithaca and the wider Tompkins County area. It is operated by a non-profit agency formed by Cornell University, Tompkins County, and the City of Ithaca, and it runs more than 30 fixed routes connecting campuses, residential neighborhoods, shopping areas, and rural communities (1). Public transit is especially critical in Ithaca because many Cornell students, staff, and local residents do not rely on private cars and instead depend on TCAT for daily commuting (2).

1.1.2 Scope: Route 10 and 81

Within this system, Routes 10 and 81 are two of the most campus-intensive lines. Route 10 is a high-frequency service linking Cornell's central campus and the Ithaca Commons, and it serves as a primary connection for trips between campus, Collegetown, and downtown (3). Route 81 is a daytime campus circulator connecting major North Campus parking and residential areas (including A Lot) with Central Campus and Tower Road, and it is designed to move high volumes of students during class hours (4).

We selected these two routes as the focus of our project for three reasons: (i) they serve heavy student demand and are frequently crowded during peak periods, (ii) they share overlapping campus corridors where allocating buses to one route can affect congestion and capacity on the other, and (iii) they are operationally important yet sufficiently narrow in scope to model in detail within a semester-long course project.

Our study considers a typical weekday during the academic semester, when Cornell is in session and ridership is high. We focus on daytime service (approximately 7:00 AM to 7:00 PM), when most class-related travel occurs on Routes 10 and 81. We divide this horizon into short time blocks (20–30 minutes each). For each block, we track passenger demand, operating cost, and service frequency.

1.1.3 Research question

This project focuses on the scheduling of TCAT Routes 10 and 81 during a typical weekday and addresses the following overarching research question:

> ***How should TCAT allocate buses and driver-hours across Routes 10 and 81 over the day to minimize total system cost—operating cost, driver cost, passenger waiting cost, and unmet-demand penalties—while satisfying fleet, labor, and coverage constraints and remaining robust to uncertainty in passenger demand?***

More concretely, the analysis explores sub-questions such as:

- If TCAT could assign additional buses to Routes 10 and 81, how many buses are needed on each route to substantially reduce unmet demand, and where do diminishing returns set in?
- Is the system primarily constrained by budget, fleet size, or available driver-hours?
- What is the "price of robustness" when we design schedules to be feasible under demand uncertainty, in terms of extra cost versus reduced risk of severe unmet demand?

1.2 Systems Thinking Perspective

Our project involved multiple stakeholder groups:

- TCAT Operations & Planning Team: Schedulers, dispatchers, and operations managers who design daily timetables and decide how many buses to assign to Routes 10 and 81 under limited resources.
- TCAT Bus Drivers and Unions: Drivers whose working hours, shifts, and workload are captured through driver-hour constraints and labor cost terms in the model.
- Passengers on Routes 10 and 81: Cornell undergraduates, graduate students, staff, faculty, and campus visitors who rely on these buses to move between North Campus, Central Campus, Collegetown, and downtown. Local residents and workers commute between the Ithaca Commons and Cornell's campus.
- Institutional Sponsors and Local Governments: Cornell University, the City of Ithaca, and Tompkins County, which jointly oversee and fund TCAT and care about both financial sustainability (operating and labor costs) and equitable service coverage across key zones.
- Broader Community & Environment: Residents and community organizations that benefit from reliable transit (reduced congestion, improved accessibility) and from shifting trips away from private cars toward public transportation.

1.2.1 Stakeholder analysis

Routes 10 and 81 share the same buses, drivers, and budget. They cannot be improved separately. When we assign an extra bus to Route 10, it means one less bus is available for Route 81.

This creates a problem: making one route better usually makes the other route worse. If we add more buses to Route 10, passengers traveling between North Campus, Central Campus, and Collegetown will wait less and have more space. But this means Route 81 passengers going between campus and downtown will wait longer and face more crowding. The opposite happens when we add more buses to Route 81 instead.

This means TCAT must make a difficult choice: they need to provide good service to both routes fairly, not just focus on one. Our analysis shows this trade-off clearly. It demonstrates how different ways of dividing buses between Routes 10 and 81 affect both the operating costs and how many passengers cannot be served.

1.2.2 Objective Definition

Since Routes 10 and 81 share the same buses and drivers, our goal is to maximize daily net benefit on a typical school day. "Net benefit" means the total value the bus system provides to the community minus what it costs to run. We look at the whole network instead of just one route at a time, because decisions about one route affect the other—what helps some riders may hurt others.

Net benefit goes up when TCAT moves more riders on time and efficiently. It goes down when operating costs increase or when poor service creates problems for passengers. We measure net benefit by combining two factors: agency costs and passenger impacts. Agency costs include bus expenses and driver wages. Passenger impacts include the costs of poor service (longer wait times) and unmet demand (crowded buses, missed trips, and unfair access during busy times). Net benefit goes up when TCAT moves more riders on time and efficiently. It goes down when operating costs increase or when poor service creates problems for passengers. We measure net benefit by combining two factors: agency costs and passenger impacts. Agency costs include bus expenses and driver wages. Passenger impacts include longer wait times and unmet demand. Unmet demand means crowded buses, missed trips, and unfair access during busy times.

This approach highlights a key trade-off: with limited buses and drivers, adding more service to one route means less service on the other. By maximizing total net benefit, we find the best allocation for both routes combined, rather than just moving problems from one route to the other. Section 2 explains the math behind this objective, but the main idea is to use our limited buses and drivers to provide the best overall weekday service, while reducing the number of passengers who can't get on a bus.

2 Model Formulation

2.1 Assumptions and Parameters

Please refer to Table 1 for a summary of the assumptions and parameters.

Table 1: Assumptions and parameters.

Symbol	Description
	Basic parameters
λ_{rt}	Expected passenger demand on route r during period t.
F	Total number of buses in the fleet.
L_{rt}	Duration (hours) of one trip on route r in period t.
c	Operating cost per bus-hour (fuel and maintenance).
p	Average fare per passenger ($1.5).
Δt	Period duration (hours).
	Drivers
H^{driver}	Total number of driver-hours TCAT can staff.
w	Driver wage cost per driver-hour.
	Buses
F	Total number of available buses (currently: 55).
H	Maximum operating hours per bus per day.
C	Passenger capacity per bus.
	Passengers
θ	Penalty value per unserved passenger.
γ	Weight (cost) per crowded passenger.
β	Waiting cost per passenger.

2.2 Mathematical Model (MILP)

Decision variables.

- $x_{r,t}$: number of running buses on route r during period t.
- $u_{r,t}$: unmet demand on route r in period t.

Objective function. Minimize the combination of operating cost, driver cost, waiting cost, and unmet-demand penalty:

$$\min \quad \sum_{r,t} c\, x_{r,t} + \sum_{t} w\, z_t\, \Delta_t - \sum_{r,t} \beta_{r,t}\, x_{r,t} + \theta \sum_{r,t} u_{r,t} \tag{1}$$

$$= \min \quad \sum_{r,t} (c - \beta_{r,t})\, x_{r,t} + \sum_{t} w\, z_t\, \Delta_t + \theta \sum_{r,t} u_{r,t}. \tag{2}$$

Waiting time cost.

$$WT_{r,t} = \beta_{r,t}\, x_{r,t}. \tag{3}$$

Here, $\beta_{r,t}$ is a coefficient that represents the marginal waiting-time benefit of adding one more bus. $x_{r,t}$ is the number of passing buses in period t. For example, if the bus frequency (headway) is every 15 minutes, then the expected waiting-time per person is 7.5 minutes (can apply Monte Carlo here, random distribution). Suppose the passenger demand is 120 passengers/hour. Then the baseline waiting time per bus is

$$WT_{\text{base}} = 120 \times 7.5 = 900 \text{ passenger/minutes.} \tag{4}$$

If we add one extra trip, the new headway is 12 minutes, and the expected waiting-time per person is 6 minutes (randomly distributed). The new waiting cost is $120 \times 6 = 720$ passengers/hour. Thus,

$$\beta_{r,t} = \Delta WT = 900 - 720 = 180. \tag{5}$$

Constraints.

- To satisfy the demand constraint:

$$u_{r,t} \geq \lambda_{r,t} - C\, x_{r,t} \qquad \forall r, t. \tag{6}$$

- To consider the budget constraint:

$$\sum_{r,t} c\, x_{r,t} + \sum_{t} w\, z_t\, \Delta_t - p \sum_{r,t} \big(\lambda_{r,t} - u_{r,t}\big) \leq B^{\text{total}}. \tag{7}$$

- To consider the labor capacity constraint: the total driver-hour cannot exceed the available labor hour and one bus at least has one driver.

$$\sum_{t} z_t\, \Delta_t \leq H^{\text{driver}}. \tag{8}$$

$$z_t \geq \sum_{r} x_{r,t} \qquad \forall t. \tag{9}$$

- To consider the capacity of available bus-hour:

$$\sum_{r} \sum_{t} L_{r,t}\, x_{r,t} \leq FH. \tag{10}$$

- Positive requirements for all variables:

$$x_{r,t} \geq 0, \quad u_{r,t} \geq 0, \quad z_t \geq 0 \qquad \forall r, t. \tag{11}$$

2.3 Solution Approach

Following the mathematical model in Section 2.2, we introduce the Uncertain Linear Optimization framework. The above model assumed that the given nominal data were exact, the resulting nominal optimal solution is what is recommended for use, hoping that small data uncertainties will not affect the feasibility and optimality significantly. However, there is uncertainty in real-world bus scheduling problems that can severely impact the optimality of nominal solution. Especially, uncertainty can stem from common prediction errors like from fluctuating passenger demand.

Unlike deterministic approaches that rely on fixed point forecasts, Robust Optimization constructs a solution that remains feasible for any realization of the uncertain parameters within a specified set. Specifically, we utilize the Budget of Uncertainty approach introduced by Bertsimas and Sim (2004) for a flexible trade-off between the robustness of the schedule and the conservativeness of the solution.

2.3.1 Uncertain Linear Optimization Framework

We consider the bus scheduling problem as an instance of a general Uncertain Linear Optimization problem. This is defined as a collection of linear programming problems where the data is not fixed but varies within a specified uncertainty set U. The general formulation is given by:

$$\left\{\, c^T x + d : Ax \leq b,\ \forall (A, b) \in U \,\right\}, \tag{12}$$

where x is the vector of decision variables, c and d are the objective function coefficients and are assumed to be deterministic here, A is the constraint matrix, b is the right-hand side vector, and U is the uncertainty set.

2.3.2 Model of Data Uncertainty

We model the uncertainty in the constraint coefficients using a symmetric interval around a nominal forecast (box uncertainty set). For a given constraint i, let J_i be the set of coefficients subject to uncertainty. Each uncertain parameter $\tilde{a}_{ij}$ is modeled as:

$$\tilde{a}_{ij} = \bar{a}_{ij} + \hat{a}_{ij}\, z_{ij}, \quad \forall j \in J_i. \tag{13}$$

where $\tilde{a}_{ij}$ is the actual uncertain value of the parameter, $\bar{a}_{ij}$ is the nominal (point forecast) value of the parameter, $\hat{a}_{ij}$ is the maximum deviation (half of the width) from the nominal value, and z_{ij} is a scaled random variable representing the deviation which takes values in the range $[-1, 1]$.

2.3.3 Budget of Uncertainty

To avoid being overly conservative by assuming the worst-case scenario for every parameter simultaneously (i.e., $z_{ij} = 1$ for all cases), we impose a *Budget of Uncertainty* for each constraint i, denoted as Γ_i. The uncertainty set Z is restricted such that the total scaled deviation of parameters in constraint i does not exceed Γ_i:

$$Z_i = \left\{ z \;\middle|\; |z_{ij}| \le 1,\ \forall j \in J_i,\ \sum_{j \in J_i} |z_{ij}| \le \Gamma_i \right\}. \tag{14}$$

where Γ_i is the budget of uncertainty for constraint i, taking values in $[0, |J_i|]$. If $\Gamma_i = 0$, the model ignores uncertainty (becomes a nominal case). If $\Gamma_i = 1$, the model considers the worst-case scenario as all parameters deviate to their maximum. If $0 < \Gamma_i < |J_i|$, the model protects against a subset of worst-case deviations and balances between reliability and cost.

2.3.4 Linear Robust Counterpart

With strong duality, the uncertain problem with the budget constraint can be reformulated into a linear optimization model, known as the *Robust Counterpart*. We treat the objective function as certain and make the constraints robust. The complete Robust Counterpart for a general linear optimization problem with a Box Uncertainty Set and a Budget of Uncertainty is given by:

$$\begin{aligned}
\max \quad & c^T x && (15)\\
\text{s.t.} \quad & \sum_j a_{ij} x_j + \sum_{j \in J_i} q_{ij} + \Gamma_i p_i \le b_i, \quad \forall i, && (16)\\
& p_i + q_{ij} \ge \hat{a}_{ij}\, y_j, \quad \forall i,\ \forall j \in J_i, && (17)\\
& -y_j \le x_j \le y_j, \quad \forall j, && (18)\\
& q_{ij} \ge 0, \quad \forall i,\ \forall j \in J_i, && (19)\\
& p_i \ge 0, \quad \forall i, && (20)\\
& y_j \ge 0, \quad \forall j. && (21)
\end{aligned}$$

Here, $c^T x$ represents the original objective function, which is to minimize the total cost. $\sum_j a_{ij} x_j$ is the nominal value of the constraint (using point forecasts), and $\sum_{j \in J_i} q_{ij} + \Gamma_i p_i$ is the function added to the constraint to account for the worst-case deviation allowed by the budget Γ_i. p_i and q_{ij} are dual variables derived from the inner maximization problem of the uncertainty set. Finally, y_j is the auxiliary variable used to bound the absolute value of the decision variables ($|x_j|$), ensuring the model remains linear.

2.3.5 Computational Implementation

The above mathematical model was implemented using Python 3.12.7 within a Jupyter Notebook environment. The computational framework consists of the following key components:

- The linear optimization framework was constructed using Google OR-Tools (`pywraplp`). This library served as the primary interface for defining decision variables, objective functions, and robust counterpart constraints. The underlying solver was utilized to handle the Mixed-Integer Linear Programming (MILP) characteristics of the fleet allocation problem.

- The SciPy library was used to calculate and compare the boundaries of different uncertainty sets.

- Post-optimization analysis including demand-supply comparisons and Pareto frontiers were performed using Matplotlib.

All computational experiments were performed on a standard local machine, ensuring that the proposed robust solution is computationally tractable.

3 Computational Experiments

3.1 Base Case Analysis (Current System)

Based on the number of buses currently available at TCAT, the total fleet size is set to $F = 3$. We run the proposed optimization model under this base-case setting, and the results are shown in Figure 1. In Figure 1, solid lines represent the actual number of buses deployed in operation, while dashed lines denote the corresponding passenger demand time series. The red curves correspond to Route 81, which primarily serves campus commuters, and the blue curves correspond to Route 10, which connects residential areas with the campus. Focusing on these two routes allows us to better examine the interaction between different types of commuter bus services.

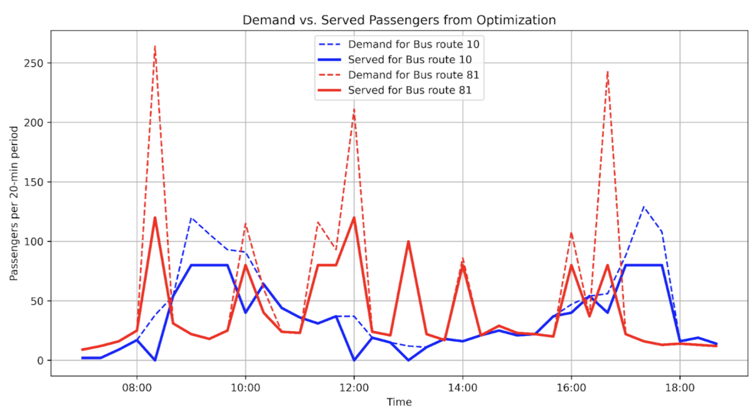

Figure 1: Demand vs. served passengers from optimization.

As shown in figure above, the current fleet size is sufficient to satisfy passenger demand when the peak periods of the two routes do not overlap. However, when peak demands occur simultaneously, the system is unable to meet the demand on both routes. For instance, around 12:00 PM, both routes experience high demand, and neither can be fully served under the current fleet size. Addressing this issue requires an expansion of the available fleet, implying that TCAT would need to acquire additional buses. The optimization results under alternative fleet-size scenarios are discussed in Section 3.2.

Figure 2 illustrates the cost components aggregated over 20-minute intervals, including waiting-time benefit, unmet-demand cost, driver cost, and operating cost. The waiting-time benefit appears as a negative value, reflecting the fact that increasing the number of deployed buses reduces passenger waiting time and, consequently, the associated waiting cost. Among all components, unmet-demand cost constitutes the largest share, particularly during peak travel periods. This result highlights the importance of increasing the available number of buses in order to effectively reduce the overall system cost over time.

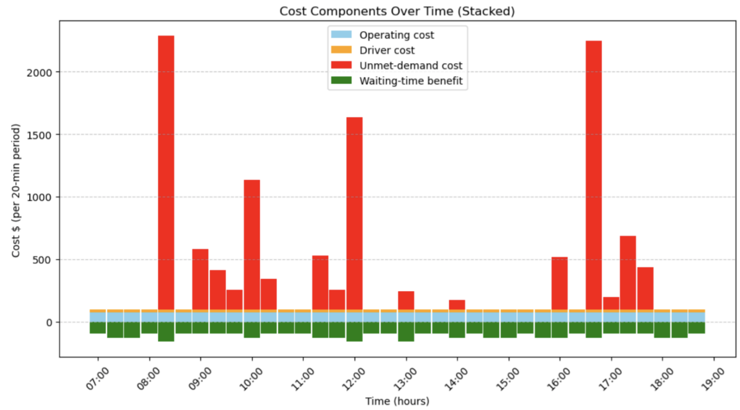

Figure 2: Cost Components Over Time (Stacked)

3.2 Pareto Optimality and Fleet Optimization

Following the robust formulation in Section 2.3, we conducted a Pareto analysis to determine the optimal fleet size. This analysis shows the trade-off between operational cost (number of buses and budget) and service quality (unmet passenger demand) which helps stakeholders to visualize the cost of reliability.

3.2.1 Budget and Physical Constraints

Initial experiment of Pareto analysis in Figure 3 focused on relaxing the daily budget constraint while keeping the fleet size fixed at the current level of $F = 3$ buses. The results indicate a completely flat Pareto frontier where increasing the daily budget beyond the current level yields zero reduction in unmet demand.

This demonstrates that the current system is constrained in capacity, not in budget. The solver utilizes all three available buses to their maximum capacity during peak hours but still leaves approximately 861 passengers unserved daily. Therefore, simply allocating more funds without acquiring new vehicles is ineffective. The physical fleet is the true bottleneck.

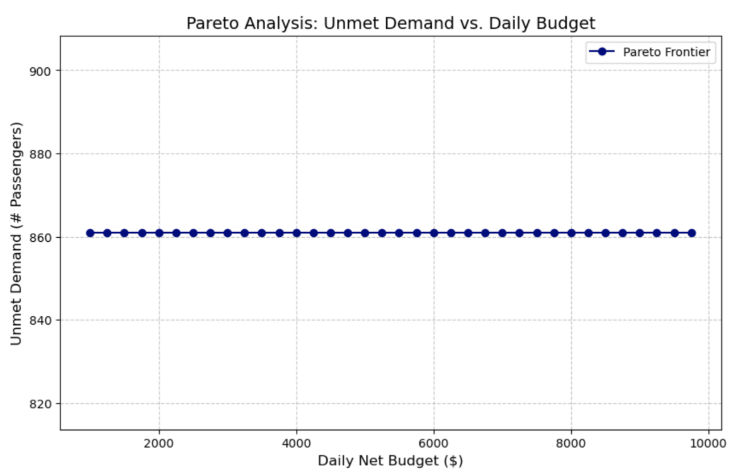

Figure 3: Pareto analysis of unmet demand versus daily budget. The blue line represents its Pareto frontier.

3.2.2 Fleet Size Sensitivity Analysis

To identify the optimal fleet number, we iteratively increased its number (F) from 3 to 15 buses and analyzed the impact on service gaps and financial performance. As shown in Figure 4 and Table 2, the number of unmet passengers drops as the fleet size increases from 3 to 8 buses. The curve begins to flatten around 8-9 buses, suggesting that further additions yield diminishing returns in terms of service quality. Also, while a larger fleet maximizes service, it results in higher operating and labor costs. Figure 5 and Table 3 reveal that the net financial result (i.e., Revenue – Cost) peaks at 4 buses. Beyond this point, the incremental operating costs exceed the additional fare revenue.

Table 2: Result of Pareto Optimality analysis on the number of buses versus cost and service quality.

Number of Buses	Operational Cost ($)	Unmet Demand
3	1200.00	861.00
4	1600.00	431.00
5	2000.00	260.00
6	2400.00	132.00
7	2800.00	43.00
8	3200.00	3.00
9	3600.00	0.00
10	4000.00	0.00

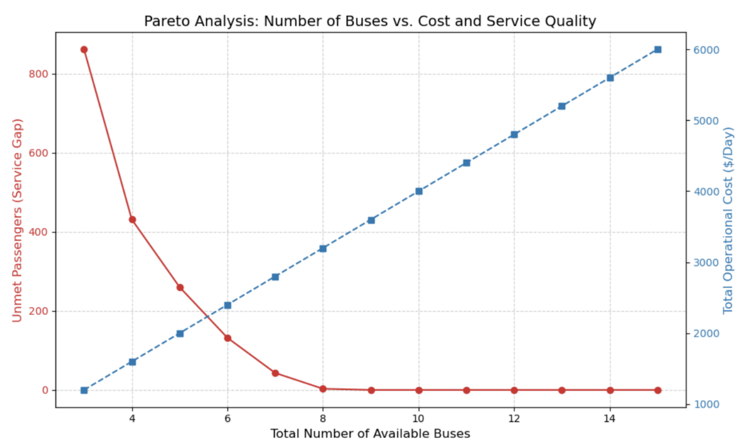

Figure 4: Pareto analysis of number of buses versus cost and service quality. Blue line shows the increments in total operational cost ($/Day), and red line shows the decreases in unmet passengers (service gap) per the number of total available buses. Pareto frontier is represented by the red line.

Table 3: Result of Pareto Optimality analysis on the number of available buses versus unmet passengers and total operational cost.

Buses	Served	Revenue($)	Operational Cost($)	Net Profit($)
3	2595	3892	1200	2692
4	3025	4538	1600	2938
5	3196	4794	2000	2794
6	3324	4986	2400	2586
7	3413	5120	2800	2320
8	3453	5180	3200	1980
9	3456	5184	3600	1584
10	3456	5184	4000	1184

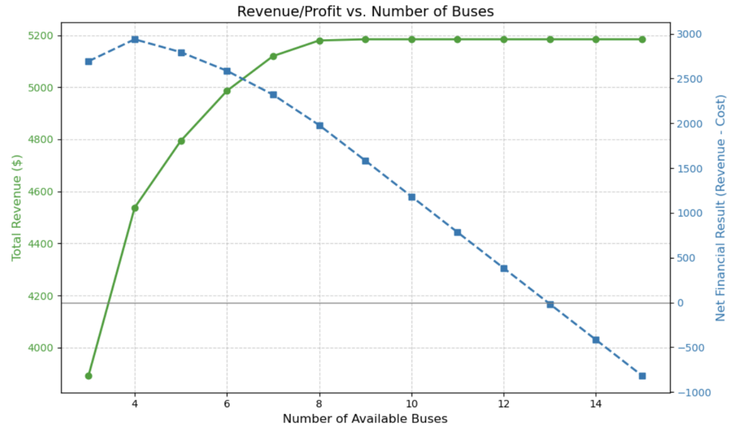

Figure 5: Pareto analysis of revenue (or profit) versus the number of buses. The blue line shows net financial result (Revenue – Cost) and the red line shows the total revenue earned per number of available buses. Both lines are useful for optimal decision-making with regard to the trade-off between service reliability and operational cost.

3.2.3 Optimal Trade-off Selection

Balancing the conflicting objectives of maximizing service reliability (goal for a public agency) and minimizing cost (goal for a private operator), we identified a fleet size of $F = 6$ buses as the optimal strategic choice. This decision is driven by the following three factors:

1. **It provides a safety margin for profitability.** Financial sustainability is a critical constraint for TCAT. As illustrated in Figure 5, maximizing pure profit would limit the fleet to just 4 buses. However, expanding to 6 buses keeps a substantial financial buffer. Our analysis verifies that at $F = 6$, the agency maintains a daily net profit exceeding $2,500. This creates a profit safety margin that protects the agency against minor operational variances while still funding the expanded service, whereas increasing the fleet to 8 or more buses causes profitability to drop below viable levels.

2. **It gives high marginal utility of the sixth bus.** Compared to the solution at $F = 5$, a fleet size of 6 reveals a high return on investment for the sixth vehicle. At $F = 5$, the system leaves approximately 260 passengers unserved. Meanwhile, at $F = 6$, the unserved passenger count drops to 132 (around a 49.23% reduction). This indicates that the sixth bus is not only providing spare capacity but is actively capturing a significant volume of demand that would otherwise be abandoned.

3. **It handles the peak demand on Route 81.** Figure 6 illustrates that the solver allocates all 6 buses exclusively to Route 81 during extreme peak periods (e.g., 8:20 and 16:40). This suggests that by securing a fleet of 6, TCAT ensures it can fully saturate the campus route during these brief but critical windows that a 5-bus fleet cannot fully absorb.

Under this solution, the system achieves a net socioeconomic objective value of $-5{,}479.60$, which significantly outperforms the current 3-bus system (+7,535.00 cost penalty). This negative value reflects the substantial waiting time savings benefit relative to the cost terms in the objective function. In addition, the total number of unmet passengers decreases from 861 to 132 per day, which is equivalent to an 84.67% reduction in service denials. In terms of cost, the total operating cost increases to $1,800.00 from $900.00 in the base case, and the labor cost also increases to $600.00 from $300.00. However, despite the doubled operational costs, the massive reduction in unmet demand penalties and passenger waiting costs justifies the investment from a systems perspective.

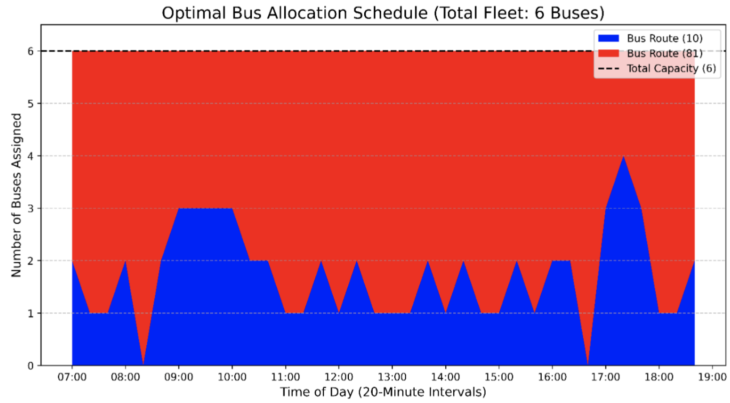

Figure 6: Optimal bus allocation schedule of total fleet of 6 buses during a time of day with 20-minute intervals for two Bus Routes 10 and 81, represented by blue and red color, respectively.

3.2.4 Dynamic Fleet Allocation Schedule

The optimization model leverages the flexibility of the 6-bus fleet to dynamically reallocate resources based on time-varying demand patterns. The optimal schedule for the simulation largely shows three patterns:

- **Morning Rush:** The system allocates maximum resources to Route 81 with 6 buses to handle the intense influx of students moving between North and Central Campus.
- **Mid-Day Balance:** The allocation shifts to a 1:5 split favoring Route 81, maintaining high frequency on campus while keeping Route 10 operational.
- **Evening Commute:** The pattern reverses, with 4 buses shifted to Route 10 to facilitate the departure of staff and students returning to downtown Ithaca.

This dynamic shifting ensures that the fixed fleet of 6 buses is always deployed with the highest marginal utility, which is a strategy impossible under the current 3-bus constraint.

3.3 Robust Optimization

Following the Pareto Optimality, we conducted Robust Optimization to evaluate the schedule's resilience against demand fluctuations. Passenger demand was chosen for our uncertainty analysis because prediction errors like future demands are one of its major sources. This section details the selection of the uncertainty set, the quantitative trade-off between nominal and robust performance, and the sensitivity of the system to varying levels of forecast error.

3.3.1 Selection of Uncertainty Set

Before applying robust constraints to the TCAT network, we first evaluate which geometric shape of uncertainty would best model the risks inherent in transit operations. We tested a theoretical toy model comparing Nominal, Box, Ellipsoidal, and Worst-case sets to determine their conservativeness.

The toy model results in Figure 7 demonstrated that the Box Uncertainty Set provides the optimal balance for this specific application. While the Ellipsoidal set often yields smoother solutions in financial portfolio problems, it tends to underestimate the risk of simultaneous spikes in a transport network (i.e., when a single event causes demand to surge across multiple routes at once). Therefore, we selected the Box Uncertainty Set for the TCAT scheduling problem. This choice provides a balanced trade-off. It is sufficiently conservative to protect against independent, simultaneous demand fluctuations unlike the risky nominal solution, but avoids the extreme conservativeness of the absolute worst-case scenario often associated with unbounded uncertainty. The actual application of the Box Uncertainty Set is illustrated in Figure 8.

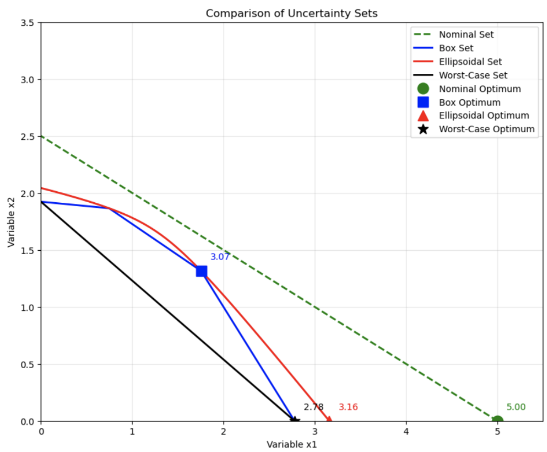

Figure 7: Toy model of four different types of Uncertainty Sets. (1) Red dash line represents Nominal Set, (2) blue solid line shows Box Uncertainty Set, (3) red solid line displays Ellipsoidal Uncertainty Set, and (4) black solid line described the Worst-Case Set.

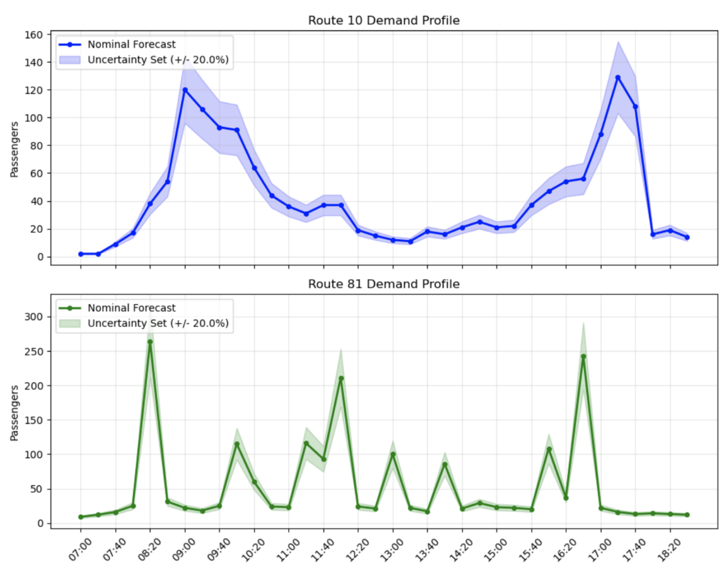

Figure 8: Demand profiles of Routes 10 and 81 applying the Box Uncertainty Set of ±20%.

3.3.2 Nominal and Robust Schedule

We solved the full fleet optimization problem under a demand uncertainty level of ±20%. The comparison between the Nominal Schedule (optimized for point forecasts) and the Robust Schedule reveals the cost of reliability. As shown in Table 4, the system's objective value (net socioeconomic benefit) drops from $-4{,}379.60$ in the nominal case to $-3{,}128.40$ in the robust case. This reduction represents a Price of Robustness of 42.91% which indicates that to guarantee service during a 20% demand surge, the system must sacrifice nearly half of its potential theoretical efficiency to maintain slack in the fleet.

One interesting phenomenon observed in Table 5 is that the average unmet passenger count rises slightly in the Robust Schedule (105.73) compared to the nominal (93.99). This counterintuitive result occurs because the robust solver is optimizing for the worst-case scenario, not the average. It reallocates buses to cover potential bottlenecks that might never happen in an average day, leading to slightly lower efficiency in nominal reality but guaranteeing survival in the worst reality. We discuss the implications of robust solution in detail in Section 3.3.3.

Table 4: Results summary of out-of-sample testing on total cost

	Nominal Schedule	Robust Schedule
Average Total Cost	-5074.49	-4815.85
Maximum Total Cost	-3408.99	-3489.32

Table 5: Results summary of out-of-sample testing on unmet demand

	Nominal Schedule	Robust Schedule
Average Unmet Passengers	93.99	105.73

3.3.3 Out-of-Sample Testing

To validate the solutions, we conducted an out-of-sample test with the Nominal and Robust schedules against 10,000 random demand scenarios. The resulting histograms (frequency versus outcome) reveal critical differences in the risk profiles of the two approaches.

In Fig. 9, the total cost histogram visualizes the financial performance distribution. The nominal schedule is centered around a more favorable average indicating higher potential profit. However, its distribution is wider, implying high volatility. Meanwhile, the Robust Schedule shows a narrower spread which means lower variance. Especially, the robust distribution demonstrates superior worst-case performance, given the Nominal profit drops to -\$3,408.99 whereas the Robust schedule never falls below -\$3,489.32. This confirms that while the Robust solution sacrifices the best-case peaks, it guarantees a higher financial floor ensuring the agency maintains better profitability during worst-case demand shocks.

In another histogram of Fig. 10 compares the operational consistency of the fleets. The Nominal distribution exhibits a heavy right tail, indicating a significant probability of days with extreme service failures (i.e., high unmet demand). On the other hand, the Robust distribution is more tightly clustered. Although the average unmet demand is slightly higher, the Robust Schedule eliminates the extreme failure days where passengers are abandoned uncontrollably. This reveals that the robust solution offers stable daily operation where service gaps are known and manageable, rather than fluctuating severely between perfect service and serious overcrowding.

3.3.4 Budget of Uncertainty Analysis

Finally, we performed a sensitivity analysis on the Budget of Uncertainty to determine the maximum volatility the 6-bus fleet can sustain before becoming economically viable. Fig. 11 tracks the objective value as the uncertainty level increases from 0% to 50%. At 0% uncertainty, the system operates at maximum profitability (Objective = -5,479.60). As the level increases, the robust constraints tighten and force the solver to spend

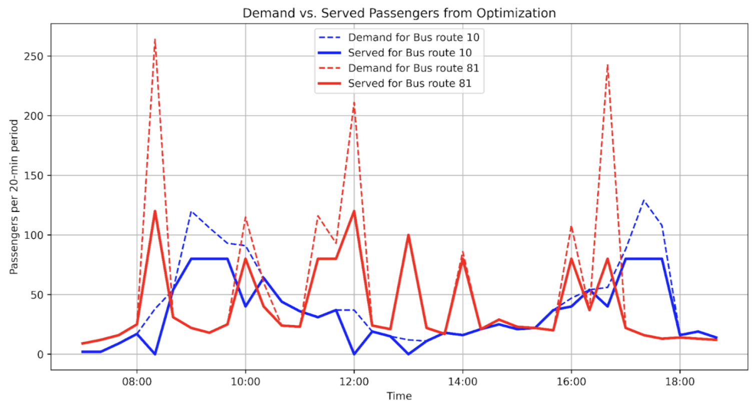

Figure 9: Histograms of out-of-sample testing of the nominal solution (red) and robust solution (blue) with respect to the total cost. The test was conducted for 10,000 trials.

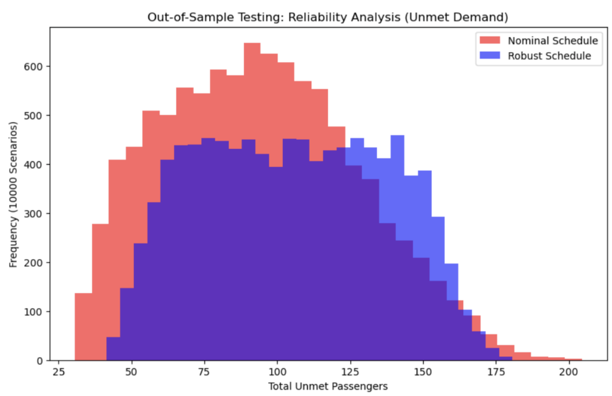

Figure 10: Histograms of Out-of-Sample testing of Nominal solution (red) and Robust solution (blue) with respect to the total unmet demand. The test was conducted for 10,000 trials.

more objective value on protection. The analysis identifies a critical tipping point at 44.2%. The system remains profitable up to 44.2% uncertainty, but the objective flips to positive, which indicates a net loss.

This defines the operational boundary for TCAT. The current fleet configuration is sustainable only if demand prediction errors are kept below 44.2%. If the environment becomes more volatile than this threshold, the operational costs of protection outweigh the socioeconomic value of the transit service.

4 Conclusions and Recommendations

4.1 Summary of Insights

The comprehensive modeling and computational experiments on the TCAT network have provided critical insights regarding the system's operational limits and potential for optimization. First, physical capacity is the primary bottleneck. The analysis of the current system F=3 demonstrated that the service gaps are driven by physical fleet limitations, rather than financial budget constraints. The solver utilizes the existing 3 buses to their maximum capacity. Consequently, simply increasing the budget without acquiring additional vehicles yields zero improvement in service quality.

Also, while the Pareto Optimality suggests service quality continues to improve up to 9 buses, the 6th bus represents a critical tipping point. The addition of this single unit reduces unmet passenger demand by

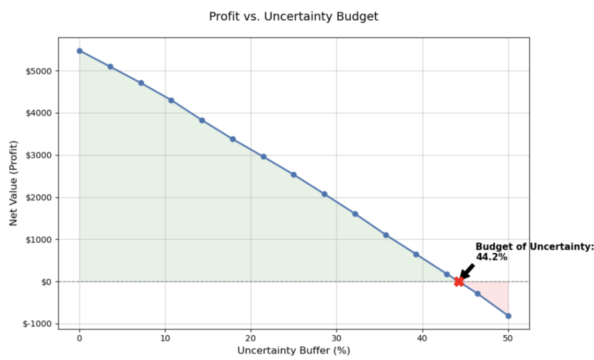

Figure 11: Budget of Uncertainty between profit versus uncertainty budget, found to be at 44.2%.

nearly 50% compared to the 5-bus scenario. This indicates that the 6th bus is essential for capturing the peak-hour demand overflow that the current fleet abandons.

However, when we introduce uncertainty in demand and implement a Robust Schedule, it comes at a significant theoretical cost, a 42.91% reduction in the objective value compared to the Nominal Schedule. Nevertheless, this Price of Robustness pays for operational stability and effectively cuts off the tail of the risk distribution, eliminating the extreme worst-case scenarios.

Finally, The Budget of Uncertainty identifies a clear operational boundary. The proposed 6-bus system remains financially viable as long as demand prediction errors remain below 44.2%. Beyond this volatility threshold, the cost of protecting the schedule outweighs the benefits provided by the service.

4.2 Recommendations

Based on these insights in Section 4.1, we propose the following strategic and operational recommendations to TCAT management:

- Expand the active fleet allocation for Route 10 and 81 from 3 to 6 buses. This configuration is the optimal Pareto compromise. It reduces daily unmet demand by approximately 85% (from 861 to 132) while guaranteeing a daily net profit margin of over $2,500, ensuring financial sustainability alongside public service mandates.
- Adopt the dynamic allocation schedule generated by the optimization model, rather than static route arrangements. Demand on Routes 10 and 81 is asynchronous and the model shows that shifting capacity is crucial. Specifically, allocating all buses to Route 81 during short peak periods and rebalancing to Route 10 during the evening commute. This flexibility allows the agency to serve peak loads that would otherwise be impossible to meet with a static arrangement.
- Implement the Robust Schedule rather than the Nominal Schedule. While the latter appears more profitable on average, it is fragile. The Robust Schedule can be considered as an insurance policy ensuring consistent reliability. For a public transit agency, avoiding severe overcrowding days is often more valuable than maximizing theoretical average efficiency.
- Establish a monitoring protocol to track the variance between forecasted and actual ridership. Our Budget of Uncertainty analysis indicates a safety limit of 44.5% uncertainty. If real-world demand fluctuations consistently exceed this percentage, the current operational model becomes economically unstable, and a fundamental re-evaluation of fare structures or subsidy levels will be required.

4.3 Future Work on Transportation Network

As the transportation system becomes more complex and incorporates a larger number of bus routes, additional decision variables and constraints must be introduced. With respect to decision variables, we propose introducing z_t, representing the number of drivers on duty during time period t. In the current model, we assume $x_t = z_t$ because the number of optimized routes is limited. However, in a realistic TCAT operating environment, there are 34 bus routes and 55 available buses. Under such conditions, the number of buses in operation and the number of drivers on duty should be modeled separately.

In terms of constraints, expanding the set of bus routes necessitates explicit consideration of transportation network coverage. This requires incorporating policy-related parameters associated with spatial service coverage. To this end, Ithaca can be divided into multiple zones, such as the Cornell campus, College town, Downtown, and Lansing. Two additional parameters are then introduced to represent spatial coverage requirements:

- $F_z^{\min}$: minimum trips covering zone z.
- δ_{zr}: whether route r serves zone z (1: serve, 0: non-serve).

Regional policy and spatial coverage constraints:

$$\sum_r \delta_{zr} \sum_t x_{rt} \geq F_z^{\min}, \qquad \forall z, \tag{22}$$

$$\sum_t x_{rt} \geq f_r^{\text{peak}}, \qquad \forall r. \tag{23}$$

These extensions will allow the model to better capture realistic operational requirements and policy objectives in large-scale public transportation networks.

In our study, the Robust Optimization approach effectively handles the TCAT schedule against demand uncertainty. However, it also protects against all realizations within the uncertainty set including extreme worst-case scenarios that may have a negligible probability of occurring. This worst-case orientation can lead to overly conservative solutions (as shown in the extremely high Price of Robustness), potentially allocating more buses or budget than is necessary for typical daily operations.

To address this, future study can transit to Chance-Constrained Optimization. Unlike Robust Optimization that requires the constraints to hold for every realization $\zeta \in U$, Chance-Constrained Optimization requires the constraints to hold with a high probability of $1 - \epsilon$.

In this framework, we replace the hard robust constraints with probabilistic ones. We assume the uncertain demand parameters $\tilde{a}_{ij}$ follow a probability distribution P. The optimization problem is reformulated as:

$$\begin{aligned} \min_x \quad & c^\top x \\ \text{s.t.} \quad & \mathbb{P}\left(\sum_j \tilde{a}_{ij} x_j \leq b_i \right) \geq 1 - \epsilon_i, \ \forall i, \\ & x \in X, \end{aligned} \tag{24}$$

where $P(\cdot)$ is the probability measure defined over the uncertain area, ϵ_i is the risk level, and therefore $1 - \epsilon_i$ becomes the confidence level required for constraint i.

One major challenge with chance constraints can be that feasible regions defined by probabilistic inequalities are non-convex and difficult to compute, especially when the exact distribution of demand is unknown or complex. To solve this problem, we can employ robust counterparts of chance constraints.

This theory poses that a chance constraint can be safely approximated by a deterministic Robust Optimization problem with a specifically tuned uncertainty set, U_ϵ.

With this approach, TCAT can trade off service reliability against operational cost. Instead of asking "What is the cost to survive in the worst possible case" in Robust Optimization, the company can ask "What is the cost to guarantee service 95% or 99% of the time", leading to a more economic allocation of the fleet.

References

[1] Wikipedia contributors, "Tompkins Consolidated Area Transit," *Wikipedia, The Free Encyclopedia*, last edited Nov. 19, 2025. [Online]. Available: `https://en.wikipedia.org/wiki/Tompkins_Consolidated_Area_Transit`. Accessed: Dec. 16, 2025.

[2] Reshma N., "Tips for Getting Around Campus," Cornell Office of Undergraduate Admissions (Community Blog), May 20, 2024. [Online]. Available: `https://admissions.cornell.edu/community/blog/tips-for-getting-around-campus`. Accessed: Dec. 16, 2025.

[3] Transit, "TCAT 10 bus — Cornell–Commons," Transit app. [Online]. Available: `https://transitapp.com/en/region/ithaca/tcat/bus-10`. Accessed: Dec. 16, 2025.

[4] Transit, "TCAT 81 bus — Cornell Daytime Campus Service," Transit app. [Online]. Available: `https://transitapp.com/en/region/ithaca/tcat/bus-81`. Accessed: Dec. 16, 2025.

Data and Code Availability

All data and codes used for the model formulation and computational simulation are available in separate Jupyter Notebook files.