MATA36 Tutorial 0013 - Session 1

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Welcome to the first tutorial! For every tutorial, I will prepare a worksheet like this, which we will solve together. The worksheets will be available on https://github.com/CsPeti05/MATA36-Tutorials. Integration techniques are extremely important, especially for students in the physical sciences. Most upper year physics courses require full mastery of these techniques and fast execution. This is what we will work for today!

Problem 1. - *Warm up!*

(a) Solve the following integrals using "basic" integration.

$$\int_0^1 (x+4)^2 \, dx$$

$$\int_0^\pi \sin(2x) + 5e^x + \sec^2(x) \, dx$$

$$\int \frac{1}{4+4z^2} \, dz$$

(b) Solve the following integrals using u-substitution.

$$\int \frac{4x+3}{2x^2+3x+2} \, dx$$

$$\int \frac{5x}{\sqrt{x^2+3}} \, dx$$

$$\int_{-1}^1 x^2 \cos(x^3+1) + xe^{x^2+\pi} \, dx$$

(c) Solve the following integrals using integration by parts.

$$\int_0^\pi x^2 \cos(4x) \, dx$$

$$\int_1^e \ln x \, dx$$

Problem 2. - *Solve the following integrals!*

$$\int \arctan(x) \, dx$$

$$\int 5 \arctan\left(\frac{6}{w}\right) \, dw$$

$$\int \ln^2(t) \, dt$$

$$\int_0^1 \sin^2 x \, dx$$

$$\int \sec^6(k) \tan^6(k) \, dk$$

Follow-up question: Based on the previous integral, what might be the general strategy for integrals of the following form?

$$\int \sec^m(x) \tan^n(x)\, dx$$

, where $m, n \in \mathbb{Z}^+$.

You might be familiar with the definition of work in one dimension:

$$W = \int_{x_1}^{x_2} F\, dx \tag{1}$$

, where F is a force and we integrate over the interval $[x_1, x_2]$. Using this definition, one can show that in thermodynamics, eq. (1) implies that[a]

$$W = -\int_{V_1}^{V_2} P\, dV$$

, where V is the volume and P is the pressure as a function of V. Another important piece of physics that you will need for the next problem is the ideal gas law, which you might remember from high school. For ideal gases (e.g., in a container),

$$PV = NkT$$

, where P is pressure, V is volume, N is the number of particles in the container, k is the Boltzmann constant and T is the temperature.

[a]There is one more technical assumption to be made in this case, which is that the process needs to be *quasistatic*, i.e., the system is in thermal equillibrium at all times.

Problem 3. Using the ideal gas law, derive an expression for the work done on an ideal gas as it is compressed from V_2 to V_1.