MATA36 TUTORIAL 13 - SESSION 2

PETER CSERNYIK

Problem 1. Prove the following results!

$$\int_{-\pi}^{\pi} \sin(mx)\cos(nx)\,dx = 0$$

$$\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\,dx = \begin{cases} 0 & \text{if } m \neq n \\ \pi & \text{if } m = n \end{cases}$$

Problem 2. Solve the following integrals.

$$\int \frac{1}{x^2\sqrt{x^2-9}}\,dx$$

$$\int \frac{1}{u\sqrt{5-u^2}}\,du$$

$$\int_1^4 2z^5\sqrt{2+9z^2}\,dz$$

You are probably familiar with the definition of the average for a discrete set of data A with N elements.

$$\langle A \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i$$

The continuous case, characterized by a distribution function $f(x)$, is analogous to the discrete case in the following way.

$$\langle f \rangle = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Calculate the average of $f(t) = \frac{\sqrt{t^2-1}}{t}$, $1 \leq t \leq 7$.

Suppose there is a thin wire along the x axis from $-\frac{L}{2}$ to $\frac{L}{2}$. The wire is charged with uniform linear charge density λ. The magnitude of the electric field at point $P = (0, b)$ is the following:

$$E(P) = \int_{-L/2}^{L/2} \frac{\lambda b}{4\pi\epsilon_0 r^3}\,dx$$

, where r is the distance between P and the rod element dx.

Problem 3. Express r in terms of x and constants. Then, solve for the electric field. *Follow-up question: What direction does the electric field point at P?*

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