MATA36 TUTORIAL 3

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Problem 1. Decompose the following fractions.

(1) $\dfrac{9x+4}{3x^2+2x}$

(2) $\dfrac{3z^2+1}{(z+1)(z-5)^2}$

(3) $\dfrac{x^3+x}{x-1}$ (*Hint: Use long division.*)

Problem 2. Solve the following integrals.

(1) $\displaystyle\int \frac{dx}{x^2-a^2}$, $a \in \mathbb{R} \setminus \{0\}$

(2) $\displaystyle\int \frac{\sqrt{x+4}}{x}\,dx$

Problem adapted from the textbook: The German mathematician Karl Weierstrass noticed that if the substitution $t = \tan\left(\frac{x}{2}\right)$ is used, then any rational function of $\sin(x)$ and $\cos(x)$ can be turned into an ordinary rational function of t.

Problem 3. Show the following results.

(1) $\sin(x) = \dfrac{2t}{1+t^2}$

(2) $\cos(x) = \dfrac{1-t^2}{1+t^2}$

(3) $dx = \dfrac{2}{1+t^2}\,dt$

(4) $\displaystyle\int \frac{dx}{3-5\sin x}$

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