Assignment 2

1. $f(n) = n - 10 \quad g(n) = n + 10$

$f(n) = \theta(g(n))$

For large value of n

Big O

$f(n) \leq c \cdot g(n)$

$n - 10 \leq c \cdot (n + 10)$

$c = 1$

$f(n) = O(g(n))$

Omega

$f(n) \geq c \cdot g(n)$

$n - 10 \geq c \cdot (n + 10)$

$c = 1/2$.

$f(n) = \Omega(g(n))$

$\therefore f(n) = \theta(g(n))$

2. $f(n) = n \quad g(n) = n$

$f(n) = \theta(g(n))$

Big O

$f(n) \leq c \cdot g(n)$

$n \leq c \cdot n$

$c = 1$

$f(n) = O(g(n))$

Omega

$f(n) \geq c \cdot g(n)$

$n \geq c \cdot n$

$c = 1$

$f(n) = \Omega(g(n))$

$\therefore f(n) = \theta(g(n))$

3. $64^{\log_2 n} \cdot 32^{\log_2 n} = O(n^5)$

$64^{\log_2 n} \cdot 32^{\log_2 n}$

$= n^{\log_2 64} \cdot n^{\log_2 32}$

$= n^{\log_2 2^6} \; n^{\log_2 2^5}$

$= n^6 \cdot n^5$

for Big O

$f(n) \leq c \cdot g(n)$

$n^{11} \leq c \cdot n^5$ → Not a valid Big O Relation.

$\therefore 64^{\log_2 n} \cdot 32^{\log_2 n} \neq O(n^5)$

4. $\frac{4^n}{2^n} = O(2^n)$

$\frac{4^n}{2^n} = \frac{2^n \times 2^n}{2^n} = 2^n$.

for Big O

$f(n) \leq c \cdot g(n)$

$2^n \leq c \cdot 2^n$ → is a valid Big O

$\therefore f(n) = O(g(n))$

5. $128^{\log_2 n} \cdot n^2 = \theta(n^9)$

$128^{\log_2 n} \cdot n^2$

$= n^{\log_2 128} \cdot n^2$

$= n^{\log_2 2^7} \cdot n^2$

$= n^7 \cdot n^2 = n^9$

Big O

$n^9 \leq c \cdot n^9$

$c = 1$

$f(n) = O(g(n))$

Omega

$n^9 \geq c \cdot n^9$

$c = 1$

$f(n) = \Omega(g(n))$

$\therefore f(n) = \theta(g(n))$