☆ Mean

Probability is not covered Only Statics and Hypotheses Covered

- Ungroup = $\frac{\Sigma x_i}{n}$
- Group : Simple = $\frac{\Sigma f_i x_i}{\Sigma f_i}$
- Group : Frequency = $a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

→ Note : Less than → Assume upper
$f_i = x_2 - x_1$

More than → Assume Lower
$f_i = x_1 - x_2$

☆ Median

- Ungroup : odd observation = $\left(\frac{n+1}{2}\right)^{th}$ observation
- Ungroup : Even = $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$
- Group : Simple = $M = N/2$
  - → ~~c.f~~
  - → Value of x whose value is greater than $N/2$
- Group : Frequency = $M = l + \left(\frac{N/2 - m}{f}\right) \times c$

→ Median class = Nearest c.f of $N/2$

☆ Mode

- Ungroup = larger frequency
- Group: Simple = Class (x_i) of larger frequency(f
- Group: Frequecney = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$

Modal class = larger frequency

☆ Mean deviation

Ungroup : $M.D = \frac{\Sigma|x-\bar{x})}{n}$

Grouped : $M.D = \frac{\Sigma f_i|x-\bar{x})}{\Sigma f_i}$

☆ Mean deviation about Median

Ungroup : M.D about median $= \frac{\Sigma|x-m|}{n}$

Group : M.D about Median $= \frac{\Sigma f_i|x-m|}{\Sigma f_i}$

☆ MODE = 3(Median) − 2(Mean)

☆ Standard deviation

Ungroup

$$\sigma = \sqrt{\frac{\Sigma (x-\bar{x})^2}{n}}$$

$$\text{Var} = \sigma^2$$
$$= \frac{\Sigma (x-\bar{x})^2}{n}$$

Group

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i}}$$

$$\text{Var} = \sigma^2$$
$$= \frac{\Sigma f_i d_i^2}{\Sigma f_i}$$

☆ Coefficient of Variance

-> $C.V = \frac{\sigma}{\bar{x}} \times 100$

-> In this sum two tables are given we have to find C.v of both table.

-> If $A > B$ mean it is more variable less consistant.

-> If $A < B$ means it is less variable more consistant.

Kalpearson's Coeffcient

→ Covariance : $\mathrm{Cov}(x,y) = \dfrac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$

→ $$r = \frac{\mathrm{Cov}(x,y)}{(\sigma \cdot x)\cdot\cancel{\sigma y}(\sigma y)}$$

$$= \frac{\Sigma(x-\bar{x})(y-\bar{y})/n}{\sqrt{\dfrac{\Sigma(x-\bar{x})^2}{n}}\cdot\sqrt{\dfrac{\Sigma(y-\bar{y})^2}{n}}}$$

$$= \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

LINE

Spearman's Rank corelation method

→ $r = 1 - \dfrac{6\,\Sigma d^2}{n(n^2-1)}$. , Where $d = (R_1 - R_2)^2$
n = number of pair

- If R is positive both have same line of thinking.
- If R is negative both have opposite line of thinking.

☆ Tied Ranks

→ Single tie :

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3 - m)\right]}{n(n^2-1)}$$

Dual tie :

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m_1) + \frac{1}{12}(m_2^3 - m_2)\right]}{n(n^2-1)}$$

where, m = number of tie.

☆ Regression of line

→ y on x	→ x on y
• $y - \bar{y} = r \frac{\sigma y}{\sigma x} (x - \bar{x})$	• $x - \bar{x} = r \frac{\sigma x}{\sigma y} (y - \bar{y})$
Let , $b_{yx} = r \frac{\sigma y}{\sigma x}$	Let, $r \frac{\sigma x}{\sigma y} = b_{xy}$
$y - \bar{y} = b_{yx} (x - \bar{x})$	• $x - \bar{x} = b_{xy} (y - \bar{y})$
• $(y - \bar{y}) = b_{yx} (x - \bar{x})$	• $(x - \bar{x}) = b_{xy} (y - \bar{y})$
where, $b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}$	where, $b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n \Sigma y^2 - (\Sigma y)^2}$

☆ Moment about Mean

- Ungroup : $\mu_r = \Sigma(x_i - \bar{x})^r$

$[r = 1, 2, 3, 4]$

- Group : $\mu_r = \frac{\Sigma f_i (d_i)^r}{\Sigma f_i}$

☆ Moment about origin

- Ungroup : $\mu'_r = \frac{(x_i - a)^r}{n}$
- Group : $\mu'_r = \frac{\Sigma f_i (d_i)^r}{\Sigma f_i}$

☆ Relation between Moment about mean and origin

(i) $\mu_1 = \mu'_1 - \mu'_1$

(ii) $\mu_2 = \mu'_2 - (\mu'_1)^2$

(iii) $\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2(\mu'_1)^3$

(iv) $\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2(\mu'_1)^2 - 3(\mu'_1)^4$

☆ Kalpearson's Coeffcient of Skewness

→ $S_k = \dfrac{\text{Mean} - \text{Mode}}{\sigma}$

$S_k = 0$ → Symetrical
$S_k > 0$ → Positive skewness
$S_k < 0$ → Negative Skewness

Compound Event

→ $P(A/B) = \dfrac{P(A\cap B)}{P(B)}$

→ $P(B/A) = \dfrac{P(A\cap B)}{P(A)}$

→ $P(A\cap B\cap C) = P(A)\cdot P(B/A)\cdot P(C/A\cap B)$

→ $P(\bar{A}\cap B) = P(B) - P(A\cap B)$
→ $P(A\cap \bar{B}) = P(A) - P(A\cap B)$
→ $P(\bar{A}\cap \bar{B}) = 1 - P(A\cap B)$

→ $P(A) = P(B)$ → Equally likely
$P(A\cup B) = 1$ → Exhaustive event
$P(A\cap B) = 0$ → Mutually exclusive
$P(A\cap B) = P(A)\cdot P(B)$ → Independent

Baye's Theorem

$$P(A_1/B) = \frac{P(A_1)\cdot P(B/A_1)}{P(A_1)\cdot P(B/A_1) + P(A_2)\cdot P(B/A_2)\,..}$$

☆ Random Variable

→ $E(x) = \sum x_i P(x_i)$
→ $E(x^2) = \sum (x_i)^2 P(x_i)$

$V(x) = E(x^2) - [E(x)]^2$

* Probability density

$E(x) = \int_{-\infty}^{\infty} x_i \, P(x_i) \cdot dx$

$E(x^2) = \int_{-\infty}^{\infty} x_i^2 \cdot P(x_i) \cdot dx$

$\left[\begin{array}{l} \int_{-a}^{a} \text{odd} = 0 \\ \int_{-a}^{a} \text{even} = 2 \int_{-a}^{0} \text{eve} \end{array} \right.$

$V_x = E(x^2) - [E(x)]^2$

☆ 2-D Random Variable

y \ x	0	1
0		
1		

→ Marginal Probability of x	→ Marginal of y
$P(x=0) = P(0,0) + P(0,1)$	$P(y=0) = P(0,0) + P(1,0)$
$P(x=1) = P(1,0) + P(1,1)$	$P(y=1) = P(0,1) + P(1,1)$

* Condition Probability

$$P\left(x/y\right) = \frac{P(x,y)}{P(y=y)}$$

$$P\left(y/x\right) = \frac{P(x,y)}{P(x=x)}$$

* Joint Probability

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dy\, dx$$

$$\rightarrow \int_{y=0}^{1}\int_{x=2}^{3} xy \; dy\, dx = \int_{y=0}^{1} y \left[\int_{x=2}^{3} x \, . dx\right] . dy$$

* Marginal Probability

$$\rightarrow \text{For } x, \quad f_{x(x)} = \int_{y=-\infty}^{\infty} f(x,y) \cdot dy$$

$$\rightarrow \text{For } y, \quad f_{y(y)} = \int_{x=-\infty}^{\infty} f(x,y)\, dx$$

* Conditional Probability

$$f(x/y) = \frac{f(x,y)}{f_{y}(y)}$$

$$f(y/x) = \frac{f(x,y)}{f_{x}(x)}$$

* Mean, Variance

$\rightarrow$ $E(x) = \int_{-\infty}^{\infty} x \cdot f_{x}(x) \cdot dx$

$E(y) = \int_{-\infty}^{\infty} y \cdot f_{y}(y) \cdot dx$

$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f_{x}(x) \cdot dx$

$E(y^2) = \int_{-\infty}^{\infty} y^2 \cdot f_{y}(y) \cdot dx$

$Var_{(x)} = E(x^2) - [E(x)]^2$

$Var_{(y)} = E(y^2) - [E(y)]^2$

✫ Bionomal Distribution

-> $n \rightarrow$ Very small $\rightarrow n < 30$
$p \rightarrow$ large $\rightarrow$ like (0.1), (0.2)

$$P(x=x) = {}^nC_x \, p^x q^{n-x}$$

- If, $P(x \geq 2) = 1 - P(x<2)$

✫ Bernoulie's Trial

-> Mean $= \mu = np$
Variance $= npq$
S.D $= \sqrt{npq}$

✫ Poison Distribution

-> $n \rightarrow$ very large
$p \rightarrow$ very small [like (0.001)]
-> λ = mean $= np$

$$P(x=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

☆ Normal Distribution

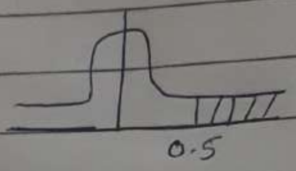
-> μ = average

σ = S.D

$$z = \frac{x - \mu}{\sigma}$$

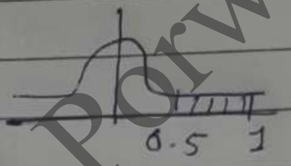
-> Graph : 1

$P(z > 0.5)$



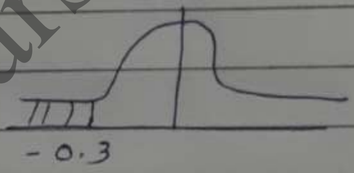
$= 0.5 - P(0 < z < 0.5)$

-> $P(0.5 < z < 1)$



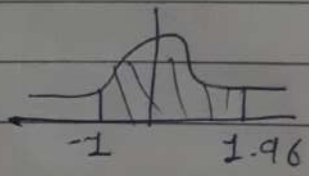
$= P(0 < z < 0.1) - P(0 < z < 0.5)$

-> $P(z < -0.3)$



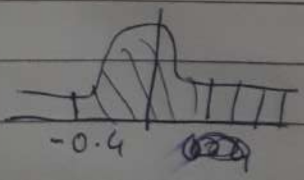
$= 0.5 - P(0 < z < 0.3)$

-> $P(-1 < z < 1.96)$



$= P(0 < z < 1.96) + P(0 < z < 1)$

-> $P(z > -0.4)$



$= 0.5 + P(0 < z < 0.4)$

☆ Least Square Method

* $\underline{y \text{ on } x}$ $\left[\begin{array}{l} y \to \text{Dependent} \\ x \to \text{Independent} \end{array}\right]$

→ $y = a + bx$ — ①

$\boxed{\Sigma 1 = n}$

- Multiply by $\Sigma 1$

$$\Sigma y = a\Sigma 1 + b\Sigma x$$
$$\boxed{= na + b\Sigma x}$$

- Multiply eq. ① by x

$$\boxed{\Sigma xy = a\Sigma x + b\Sigma x^2}$$

* $\underline{x \text{ on } y}$ $\left[\begin{array}{l} x \to \text{Dependent} \\ y \to \text{Independent} \end{array}\right]$

→ $x = a + by$ — ①

- Multiply eq. ① by $\Sigma 1$,

$$\Sigma x = a\Sigma 1 + b\Sigma y$$
$$\boxed{= na + b\Sigma y}$$

- Multiply eq. ① by y,

$$\boxed{\Sigma xy = a\Sigma y + b\Sigma y^2}$$

Ex.1 $x = b + ay$

→ $\Sigma x = b\Sigma 1 + a\Sigma y$

$\Sigma xy = b\Sigma y + a\Sigma y^2$

Ex.2 $-bx + y = c$

→ $y = \cancel{b}\, c + bx$

→ $\Sigma y = c\Sigma 1 + b\Sigma x$

$\Sigma xy = c\Sigma x + b\Sigma x^2$

* Question

→ Using least square method find curve

→ By method of least square.

→ Find a straight line

Trick

→ MODE → REG → LINE

For x on y = [y , x] mt

For y on x = [x , y] mt

MODE -> REG -> LINE [x, y]
[p, y]

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Ex.1 A simply supported beam carry load P at it's mid point corresponding value y. P the maximum deflection y is measured find $y = a + bp$

p	y	p^2	Py
100	0.45	10000	45
120	0.55	14400	66
140	0.60	19600	84
160	0.70	25600	112
180	0.80	32400	144
200	0.85	40000	170
$\Sigma P = 900$	$\Sigma y = 3.95$	$\Sigma P^2 = 142{,}000$	$\Sigma Py = 621$

-> $y = a + bp$ — ①

· Multiply eq. ① by $\Sigma 1$,

$$\Sigma y = a \Sigma 1 + b \Sigma P$$

· Multiply eq. ① by ΣP,

$$\Sigma Py = a \Sigma P + b \Sigma P^2$$

-> $6a + 900b = 3.95$ — ②

$900a + 142000b = 621$ — ③

-> Mutiply eq. ② by 150 and
solve eq. ② and ③.

$$900a + 135000b = 592.5$$
$$\underline{900a + 142000b = 621}$$
$$- \quad - \qquad -$$
$$-7000b = 28.5$$
$$b = \frac{28.5}{7000}$$
$$b = 0.0040$$

-> Put value of b in eq. ②,

$$6a + 900(0.0040) = 3.95$$
$$6a = 0.2857$$
$$a = \frac{0.2857}{6}$$

$$a = 0.0476$$

-> Value of (a) anb (b).

$$a = 0.0476$$
$$b = 0.0040$$

-> $y = a + bx$
$= 0.0476 + (0.0040)x$

MODE -> REG -> LINE [y, x]

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Ex. 2 By method of least square find the equation of line $x = a + by$

x	y	y^2	xy
-1	0	0	0
0	0	0	0
1	2	4	2
2	4	16	8
$\Sigma x = 2$	$\Sigma y = 6$	$\Sigma y^2 = 26$	$\Sigma xy = 10$

-> $x = a + by$ — ①

- Multiply eq. ① by $\Sigma 1$ and after Σy

$\Sigma x = a\Sigma 1 + b\Sigma y = na + b\Sigma y$ — ②

$\Sigma xy = a\Sigma y + b\Sigma y^2$

-> $4a + 6b = 2$ — ②

$6a + 20b = 10$ — ③

-> Solving eq. ② and ③,

$a = -0.4545$

$b = 0.6363$

-> $x = a + by$

$= -0.4545 + (0.6363)y$

MODE -> REG -> Line [ω, P]

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Ex. 3 If P is the pull required to lift 100 w by measure of pulling block find a linear law of the for $P = mW + c$ Also find P when $w = 150$.

$x = a + by$

P	w	w^2	Pw
12	50	2500	600
15	70	4900	1050
21	100	10000	2100
25	120	14400	3000
$\Sigma P = 73$	$\Sigma w = 340$	$\Sigma w^2 = 31800$	$\Sigma Pw = 6750$

-> $P = mw + c = c + mw$ — ①

- Multiply eq. ① by Σ1 after Σw

$\Sigma P = c\Sigma 1 + m\Sigma w = nc + m\Sigma w$

$\Sigma Pw = c\Sigma w + m\Sigma w^2$

-> $4c + 340m = 73$ — ②

$340c + 31800m = 6750$ — ③

-> Solving eq. ② and ③,

$c = 2.2758$

$m = 0.1879$

-> $P = mw + c = c + mw = 2.2758 + (0.1879)w$

-> when, $w = 150$

$P = 2.2758 + (0.1879)150 = 30.4608$

☆ 2nd Degree polynomial, [Quadratic, Parabola]

→ $y = a + bx + cx^2$ — ①

$\Sigma y = a\Sigma 1 + b\Sigma x + c\Sigma x^2$ [Multiply by $\Sigma 1$]
$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ [Multiply by Σx]
$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ [Multiply by Σx^2]

→ $y = a + bx^2$

$\Sigma y = a\Sigma 1 + b\Sigma x^2$ [Multiply by $\Sigma 1$]
$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4$ [Multiply by Σx^2]

→ Trick

- MODE → REG → QUAD
  [x, y] m+

- In $y = a + bx^2$,
  We only get total not (a) and (b).

MODE -> REG -> QUAD [x, y]

□ + □ + □ + □ + □ = □

Ex. 4 Fit a 2nd degree polynomial using least square method $y = a + bx + cx^2$

x	y	x^2	x^3	x^4	xy	x^2y
0	1	0	0	0	0	0
1	1.8	1	1	1	1.8	1.8
2	1.3	4	8	16	2.6	5.2
3	2.5	9	27	81	7.5	22.5
4	6.3	16	64	256	25.2	100.8
10	12.9	30	100	354	37.1	130.3

-> $y = a + bx + cx^2$

$\Sigma y = a\Sigma 1 + b\Sigma x + c\Sigma x^2$ [Mul. by $\Sigma 1$]

$\Sigma xy = a\Sigma x + b\Sigma x^2 + c\Sigma x^3$ [Mul. by Σx]

$\Sigma x^2y = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ [Mul by Σx^2]

-> $5a + 10b + 30c = 12.9$ — ①

$10a + 30b + 100c = 37.1$ — ②

$30a + 100b + 354c = 130.3$ — ③

-> Multiply eq. ① with 2 and solving eq. ① and ②.

$$\begin{array}{rrrrr} & 10a + & 20b + & 60c = & 25.8 \\ - & 10a \underset{-}{+} & 30b \underset{-}{+} & 100c \underset{-}{=} & 37.1 \\ \hline & & -10b - & 40c = & -11.3 \end{array}$$ — ④

→ Multiply eq. ② by ③ and solving eq. ② and ③.

$$30a + 90b + 300c = 111.3$$
$$30a + 100b + 354c = 130.3$$
$$- \quad - \quad - \quad -$$

$$-10b - 54c = \cancel{-19.7} \quad -19 \quad \text{— Ⓑ}$$

→ Solving eq. Ⓐ and Ⓑ.

$$-10b - 40c = -11.3$$
$$-10b - 54c = -19$$
$$+ \quad + \quad +$$

$$14c = 7.7$$
$$c = \frac{7.7}{14}$$
$$c = 0.55$$

→ Put value of c in eq. Ⓐ,

$$-10b - 40(0.55) = -11.3$$
$$-10b = 10.7$$
$$b = -1.07$$

→ Put value of (b) and (c) in eq. ①,

$$5a + 10(-1.07) + 30(0.55) = 12.9$$
$$5a = 12.9 - 5.8$$
$$5a = \frac{7.1}{5} \qquad \therefore a = 1.42$$

$$\rightarrow y = 1.42 + (-1.07)x + (0.55)x^2$$

Ex.5 Fit $y = a + bx^2$ using least square method.

→

x	y	x^2	x^4	x^2y
1	1.8	1	1	1
2	5.1	4	16	8
3	8.9	9	81	27
4	14.1	16	256	64
5	19.8	25	625	125
15	49.7	55	979	822.9

→ $y = a + bx^2$

$$\Sigma y = a\Sigma 1 + b\Sigma x^2$$
$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4$$

→ $5a + 55b = 49.7$ — ①
$55a + 979b = 822.9$ — ②

→ Multiply eq. ① by $\overset{11}{\cancel{55}}$ and ~~eq ② by 6 and~~
solving eq. ① and ②.

~~$330a + 3025b = 2733.5$~~
~~$330a + 5874b = 4437.4$~~
~~$- \quad - \quad -$~~
~~$-2849b = -2203.9$~~

→ $55a + 605b = 546.7$

$$\begin{array}{r} 55a + 979b = 822.9 \\ - \quad - \quad\quad - \\ \hline -374b = -276.2 \end{array}$$

$$b = \frac{276.2}{374}$$

$$b = 0.73$$

→ Put value of b in eq. ①,

$$5a + 55(0.73) = 49.7$$
$$5a = 9.55$$
$$a = 1.97$$

→ $y = a + bx^2$

$$= 1.97 + (0.73)x^2.$$

✱ Fitting Of Exponantial And Power

1) $y = ae^{bx}$

→ take log both side

$$\log y = \log (a \cdot e^{bx})$$
$$\ln y = \ln a + \ln e^{bx}$$
$$\ln y = \ln a + bx \ln e$$
$$\ln y = \ln a + bx$$

· Let, $\boxed{\begin{aligned} \ln y &= Y \\ \ln a &= A \end{aligned}}$

$$Y = A + bx$$

→ $\boxed{\begin{aligned} \Sigma Y &= A\Sigma 1 + b\Sigma x \\ \Sigma xY &= A\Sigma x + b\Sigma x^2 \end{aligned}}$

→ get, $\boxed{\begin{aligned} A &\rightarrow \ln a = A \\ &\therefore a = e^{A} \end{aligned}}$

$\boxed{b \rightarrow}$

→ $\boxed{A, b}$

→ MODE → REG → EXP

(2) $y = ab^x$

→ take log both side

$$\ln y = \ln(a \cdot b^x)$$
$$\ln y = \ln a + \ln b^x$$
$$\ln y = \ln a + x \ln b$$

→ Let, $\boxed{\begin{aligned} \ln y &= Y \\ \ln a &= A \\ \ln b &= B \end{aligned}}$

$$Y = A + Bx$$

→ $\boxed{\begin{aligned} \Sigma Y &= A \Sigma 1 + B \Sigma x \\ \Sigma xY &= A \Sigma x + B \Sigma x^2 \end{aligned}}$

→ get,

~~A → ln a = A~~

$\therefore a = e^A$

$B \Rightarrow \ln b = B$

$\therefore b = e^B$

→ $\boxed{A, B}$

→ MODE → REG → EXP

i) $y = ax^b$

→ take log both side

$$\ln y = \ln(a \cdot x^b)$$
$$\ln y = \ln a + \ln x^b$$
$$\ln y = \ln a + b \ln x$$

→ Let , $\boxed{\begin{aligned} \ln y &= Y \\ \ln a &= A \\ \ln x &= X \end{aligned}}$

$$Y = A + bX$$

$$\Sigma Y = A\Sigma 1 + b\Sigma X$$
$$\Sigma XY = A\Sigma X + b\Sigma X^2$$

get , $\boxed{\begin{aligned} &A \rightarrow \ln a = A \\ &\quad \therefore a = e^A \\ &b \rightarrow \end{aligned}}$

$\boxed{A , b}$

Trick

MODE → REG → PWR

MODE -> REG -> EXP $[\overset{}{\underset{a}{A}}, b]$

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Ex. 6 Fit Exponential curve $y = ae^{bx}$.

x	y	$Y(\ln y)$	x^2	xy
0	150	5.01	0	0
2	63	4.14	4	126
4	28	3.33	16	112
6	12	2.49	36	72
8	5.6	1.72	64	44.8
20		16.69	120	50.30

-> $y = ae^{bx}$

take log both side

$\ln y = \ln a + \ln e^{bx}$

$\ln y = \ln a + bx \ln e$

-> let $\ln y = Y$

$\ln a = A$

$Y = A + bx$

-> $\Sigma Y = A \Sigma 1 + b \Sigma x$

$\Sigma xY = A \Sigma x + b \Sigma x^2$

-> $5A + 20b = 16.69$ — ①

$20A + 120b = 50.30$ — ②

-> Multiply eq. ① by 4 and solving eq. ① and ②,

$$20A + 80b = 66.76$$
$$\underline{\overset{-}{20A} \overset{-}{+} 120b \overset{-}{=} 50.30}$$
$$-40b = -16.46$$
$$b = -0.4115$$

→ Put value of b in eq. ①

$$5A + 20(-0.4115) = 16.69$$
$$5A = 24.92$$
$$A = 4.984$$

→ $A = \ln a$

$a = e^{A}$

$a = 146.28.$

→ $y = a e^{bx}$

$$y = (146.28)\, e^{(-0.4115)x}$$

MODE → REG → EXP [a, B]

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Ex. 7 Fit curve $y = ab^x$ for following data.

x	y	ln y (Y)	x^2	xy
1	87	4.46	1	87
2	97	4.57	4	194
3	113	4.72	9	339
4	129	4.85	16	516
5	202	5.30	25	1010
6	195	5.27	36	1170
7	193	5.26	49	1351
28		34.47	140	142.275

→ $y = ab^x$

take log both side

$\ln y = \ln a + \ln b^x$

$\ln y = \ln a + x \ln b$

→ Let, $\ln y = Y$

$\ln a = A$

$\ln b = B$

$Y = A + Bx$

→ $\Sigma Y = A\Sigma 1 + B\Sigma x$

$\Sigma xY = A\Sigma x + B\Sigma x^2$

→ $7A + 28B = 34.47$ — ①

$28A + 140B = 742.25$ — ②

→ Multiply eq. ① by 4 and solve eq. ① and ②.

$$\begin{aligned} 28A &+ 112B = 137.88 \\ 28A &+ 140B = 142.25 \\ - \quad &- \quad\quad - \end{aligned}$$

$$28B = 4.37$$

$$B = 0.15$$

→ Put value of B in eq. ①,

$7A + 28(0.15) = 34.47$

$7A = 34.47 - 4.2$

$7A = 30.27$

$A = 4.3$

→ $A = 4.3$ $\qquad$ $B = 0.15$

$\ln a = A$ $\qquad$ $\ln b = B$

$a = e^{A}$ $\qquad$ $b = e^{0.15}$

$a = e^{4.300}$

$a = 73.74$ $\qquad$ $b = 1.16$

→ $y = ab^{x}$

$y = (73.74)(1.16)^{x}$

MODE -> REG -> PWR [a, b]

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Ex. 8 Fit a curve $y = ax^b$ from following data.

x	y	X	Y	x^2	xY
20	22	2.99	3.09	8.94	9.23
16	11	2.77	2.34	7.67	6.62
10	120	2.30	4.78	5.29	10.99
11	89	2.39	4.48	5.71	10.70
14	56	2.63	4.02	6.91	10.57
		13.107	18.790	34.678	48.318

-> $y = ax^b$

take log both side,

$\ln y = \ln a + \ln x^b$

$\ln y = \ln a + b \ln x$

Let, $\ln y = Y$

$\ln a = A$

$\ln x = X$

$Y = A + bX$

$\Sigma Y = A \Sigma 1 + b \Sigma X$

$\Sigma xY = A \Sigma x + b \Sigma x^2$

-> $5A + 13.107 b = 18.790$ — ①

$13.107A + 34.678 b = 48.318$ — ②

-> Solving eq. ① and ②.

$A = 11.60$

$\ln a = 11.60$

$a = e^{11.60}$

$a = 109240.66$

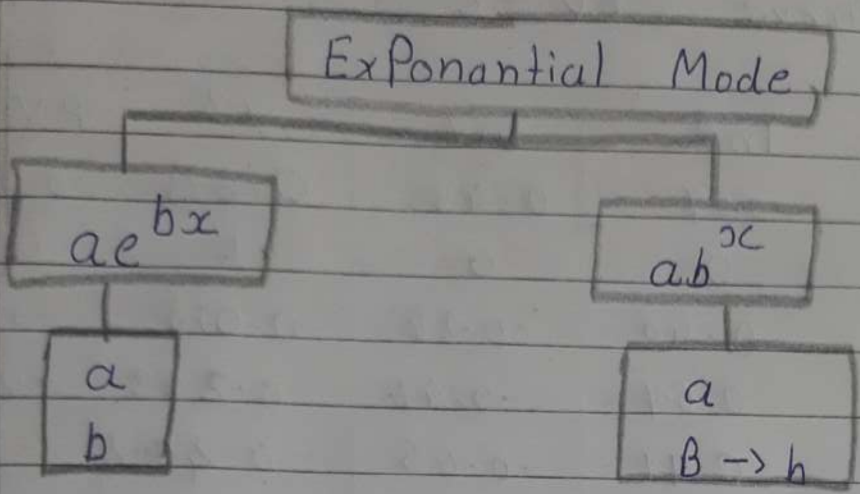
$B = 1.095$

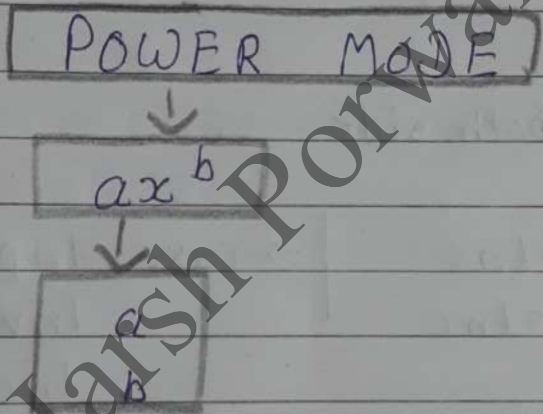
$\ln b = -1.095$

$b = e^{-1.095}$

$b = -2.99$

-> $y = (109240.66) x^{(-2.99)}$


ExPonantial Mode
ae^{bx}
a
b
ab^{x}
a
B -> b


POWER MODE
ax^{b}
a
b

Central Tendency

Arithmetic Mean

> For Ungrouped data :

$$\bar{x} = \frac{\Sigma x_i}{n}$$

$$= \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

> For Gouped data :

(i) Simple data : $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

(ii) Frequency data : $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

where, a = assumed mean
$d_i = x_i - a$
$x_i = \frac{\text{Total of class}}{2}$

Que :

- Average word.
- Arithmetic Mean.
- Number find per day

Less than = Assume upper
$f_i = x_2 - x_1$

More than = Assume lower
$f_i = x_1 - x_2$

Ex.10 Find out average wages for the consumption of the building for the wages paid for different worker.

Wages (x_i)	No.of worker (f_i)	$f_i x_i$
100	3	300
240	5	1200
300	6	1800
400	9	3600
500	2	1000
	25	7900

→ $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$= \frac{7900}{25}$

$\bar{x} = 316$

→ Trick : MODE → SD
x_i ; f_i

Ex. 11 Find out A.M for given data.

Class	fi	xi	di	fidi
0-30	8	15	-60	-480
30-60	13	45	-30	-390
60-90	22	a=75	0	0
90-120	27	105	30	810
120-150	18	135	60	1080
150-180	7	165	90	630
	95			1650

-> Assume, $a = 75$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 75 + \frac{1650}{95}$$

$$\bar{x} = 92.3684$$

-> Trick : MODE -> SD

x_i ; f_i

-> we get $\bar{x}$

-> for, $f_i d_i = \bar{x} - a \times \Sigma f_i$

Ex-12 Find A.M for given data.

Capital less than	No. of company
5	20
10	27
15	29
20	38
25	48
30	53

Less than

take → 0

$f_i = x_2 - x_1$

→

x_i	f_i	x_i	d_i	$f_i d_i$
0 - 5	20	2.5	-10	-200
5 - 10	7	7.5	-5	-35
10 - 15	2	$\boxed{a = 12.5}$	0	6
15 - 20	9	17.5	5	45
20 - 25	10	22.5	10	100
25 - 30	5	27.5	15	75
	53			-15

→ Assume, $a = 12.5$

$$\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 12.5 + \frac{-15}{53}$$

$$= 12.2168$$

→ Trick : Same as previous Example

Ex. 13 Find A.M for given data.

Marks more than	No. of student
0	60
10	56
20	40
30	20
40	10
50	3

more than

take -> 60

$f_i = x_1 - x_2$

->

x_i	f_i	x_i	d_i	d_i	f_id_i
0 - 10	4	5	-15	-20	-80
10 - 20	16	15	-5	-10	-160
20 - 30	20	$\boxed{a=25}$	0	0	0
30 - 40	10	35	5	10	100
40 - 50	7	45	1(	20	140
50 - 60	3	55		30	90
	60				90

-> Assume, $a = 25$

$$\bar{x} = a + \frac{\Sigma f_id_i}{\Sigma f_i}$$

$$= 25 + \frac{90}{60}$$

$$\bar{x} = 26.5$$

-> Trick : Same as Ex. 11

☆ Median

* Ungrouped data :

→ when no. of observation are odd

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

→ when no. of observation are even

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right) \text{observation}}{2}$$

* Grouped data :

(i) Simple data : $M = \frac{N}{2}$

Mediam = cumulative frequency is just greater than $N/2$.

(ii) Grouped data : $M = l + \left(\frac{\frac{N}{2} - m}{f}\right) \times c$
(Frequency data)

where, l = Lower class of Median class
m = upper c.f. of median class
f = frequency of median class
c = class difference.

→ Median class = Nearest value of $\frac{N}{2}$.

Ex.14 Find out Median of following data

x	f	c.f
5	3	3
10	4	7
15	5	12
20	6	18
25	2	20
	20	

→ $M = \frac{N}{2}$

$= \frac{20}{2\cancel{10}}$

$= 10$

M = Nearest value of $N/_2$

M = 15

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Ex. 15 Find out Median for following data

class	Frequency	C.f
0 - 30	8	8
30 - 60	13	21
60 - 90	22	43
90 - 120	27	70 → Median class
120 - 150	18	88
150 - 180	7	95
	95	

→ $M = N/2 = 95/2 = 47.5$

→ $l = 90$, $m = 43$, $f = 27$, $c = 30$

→ Median $= l + \left(\frac{\frac{N}{2} - m}{f}\right) \times c$

$= 90 + \left(\frac{47.5 - 43}{27}\right) \times 30$

$= 90 + 5$

Median $= 95$

Median class = nearest value of $\frac{N}{2}$.

Ex.16 Find out Median for following data.

Marks	Student	C.f
10 - 14	4	4
15 - 19	6	10
20 - 24	10	20
25 - 29	5	25 → Median class
30 - 34	7	32
35 - 39	3	35
40 - 44	9	44
45 - 49	6	50
	50	

→ $M = N/2 = 50/2 = 25$

→ $l = \frac{24+25}{2} = 24.5$

$f = 5$, $m = 20$, $c = 5$

→ $\text{Median} = l + \left(\frac{\frac{N}{2} - m}{f}\right) \times c$

$= 24.5 + \left(\frac{25 - 20}{5\cancel{0}}\right) \times 5$

Median = 29.5

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MODE

* Ungrouped data :

Mode = larger frequency
= larger repeated frequency

* Grouped data :

(i) Simple data :

Mode = class of larger frequency
= (x_i) of larger (f_i)

(ii) Frequency data :

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times c$$

where, l = lower bond of modal class
f_1 = frequency of modal class
f_0 = frequency of Just above modal class
f_2 = frequency of Just below modal class
c = class difference

→ Modal class = larger frequency.

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Ex.17 Find Mode for following data.

class	frequency	
0 - 10	5	
10 - 20	5	
20 - 30	20	
30 - 40	20	→ f_0
40 - 50	32	→ f_1 → Modal
50 - 60	14	→ f_2
60 - 70	14	

→ $l = 40, \; f_0 = 20, \; f_1 = 32, \; f_2 = 14, \; c = 10$

→ $$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$$

$$= 40 + \left(\frac{32 - 20}{2(32) - 20 - 14}\right) \times \cancel{4}\, 10$$

$$= 40 + 4$$

Mode = 44

Relation between Mean, Median, Mode 18

→ Mode = 3(Median) - 2(Mean)

→ Mean = $\frac{1}{2}$ [3(median) - mode]

→ Median = $\frac{1}{3}$ [Mode + 2(Mean)]

→ If, Mean ≈ Median ≈ Mode
then distribution is symmetrical.

* Frequency data :

→ Always find Median and Mode.
We get mean from above relation.

* Simple data :

→ Always find Mean and Mode.
We get Median from above relation.

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Ex.18 Find Mean, Median and Mode.

x	f	fx
1	4	4
2	7	14
3	8	24
4	10	40
5	6	30
6	6	36
7	4	28
8	2	16
9	2	18
10	1	10
	50	220

→ Mean $= \frac{\Sigma fx}{\Sigma f} = \frac{220}{50}$

$= 4.4$

→ Mode = Value of larger frequency = 4

→ Median $= \frac{1}{3}$ [Mode + 2 (Mean)]

$= \frac{1}{3}\left[4 + 2(4.84)\right]$

$= 12.12/3$

Median $= 4.26$

Ex.14 Find out Mean, Median, Mode.

Mark	Student	C-f
0 - 5	5	5
5 - 10	2	7
10 - 15	3	10
15 - 20	4 → f_0	14
20 - 25	7 → f_1	21 → Median
25 - 30	3 → f_2	24
30 - 35	4	28
35 - 40	2	30
40 - 45	2	32
45 - 50	1	33

→ $N/2 = 33/2 = 16.5$, $l = 20$, $m = 14$, $f = 7$,
$c = 5$, $f_1 = 7$, $f_0 = 4$, $f_2 = 3$

→ $\text{Median} = l + \left(\frac{N/2 - m}{f}\right) \times c$

$= 20 + \left(\frac{16.5 - 14}{7}\right) \times \overset{5}{\cancel{10}}$

$= 21.78$

→ For Mode: $l = 20$, $f_0 = 4$, $f_1 = 7$,
$f_2 = 3$, $c = 5$

→ Median clas

→ $\text{Mode} = l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$

$= 20 + \left(\dfrac{7 - 4}{2(7) - 4 - 3}\right) \times 5$

$= 20 + 2.14$

$\text{Mode} = 22.14$

→ $\text{Mean} = \dfrac{1}{2}\left[3(\text{median}) - 2(\overset{\text{mode}}{\cancel{\text{mod}}})\right]$

$= \dfrac{1}{2}\left[3(21.78) - 22.14\right]$

$= 21.7$

→ Here, Mean $\approx$ Median $\approx$ Mode
then distribution is symmetrical.

☆ Despersion

* Types of desperion

(i) Range
(ii) Mean deviation
(iii) Standard deviation

* Range

Ex. Measure range of 5 students.
20, 25, 30, 32, 40

→ Range = 40 - 20
= 20

Ex. Price of 4 book
200, 300, 450, 600

→ Range = 600 - 200
= 400

* Mean deviation

→ For ungrouped data :

$$M.D = \frac{\Sigma|x - \bar{x}|}{n}$$

→ For Grouped data :

$$M.D = \frac{\Sigma f_i |x - \bar{x}|}{n}$$

* Mean deviation about Median

→ For ungrouped data :

$$M.D \text{ about Median} = \frac{\Sigma|x - M|}{n}$$

→ For Grouped data :

$$M.D \text{ about Median} = \frac{\Sigma f_i |x - M|}{n}$$

Ex.20 Find out Mean deviation and M.D about Median. data
35, 40, 25, 20, 30

→

x	$\lvert x - \bar{x} \rvert$	$\lvert x - M \rvert$
35	5	10
40	10	15
25	5	0
20	10	5
30	0	5
	30	25

→ $\bar{x} = 35+40+25+20+30 / \cancel{2}5 = 30$

→ Mean deviation $= \frac{\Sigma \lvert x - \bar{x} \rvert}{n}$

$= \frac{30}{5}$

$= 6$

→ $M = \left(\frac{\cancel{6}\,n+1}{2}\right)^{th}$ observation

$= 3^{th}$ observation

$= 25$

→ M.D about median $= \frac{\Sigma \lvert x - M \rvert}{n}$

$= \frac{25}{5}$

$= 5$

Ex.21 Find M.D and M.D about Median.

→

x	f	$f_i x_i$	cf	$\lvert x-\bar{x}\rvert$	$f_i\lvert x-\bar{x}\rvert$	$\lvert x-m\rvert$	$f_i\lvert x-m\rvert$
2	2	4	2	5.5	11	4	8
5	8	40	10	2.5	20	1	8
6	10	60	20	1.5	15	6	0
8	7	56	27	0.5	3.5	2	14
10	8	80	35	23.5	20	4	32
12	5	60	40	4.5	22.5	6	30
	40	300			92		92

→ $\bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{300}{40} = 7.5$

→ $M = N/2 = 40/2 = 20$

$\therefore M = 6$

→ $M.D = \dfrac{\Sigma f_i \lvert x - \bar{x}\rvert}{n}$

$= \dfrac{92}{40}$

$= 2.3$

→ M.D about Median $= \dfrac{\Sigma f_i \lvert x - M\rvert}{n}$

$= \dfrac{92}{40}$

$= 2.3$

Ex.22 Find out standard deviation of following data.
12, 6, 7, 3, 15, 10, 18, 5

x	$(x-\bar{x})$	$(x-\bar{x})^2$
12	2.5	6.25
6	-3.5	12.25
7	-2.5	6.25
3	-6.5	42.25
15	5.5	30.25
10	0.5	0.25
18	8.5	72.5
5	-4.5	20.25
76		190

→ $\bar{x} = \frac{\Sigma x_i}{n} = \frac{76}{8} = 9.5$

→ $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$= \sqrt{\frac{190}{8}}$

$= 4.8733$

→ Trick: MODE → SD → x

$(x-\bar{x})^2 = \sigma x^2 \times n$

Ex 23 Find out S.D for following data.

x_i	f_i	$f_i x_i$	$d_i (x_i - \bar{x})$	d_i^2	$f_i d_i^2$
5	7	35	-9	81	567
10	4	40	-4	16	64
15	6	90	1	1	6
20	3	60	6	36	108
25	5	125	11	121	605
	25	350			1350

$\rightarrow$ $\bar{x} = \dfrac{\Sigma x_i f_i}{\Sigma f_i}$

$= \dfrac{350}{25} = 14$

$\rightarrow$ $\sigma = \sqrt{\dfrac{\Sigma f_i d_i^2}{\Sigma f_i}}$

$= \sqrt{\dfrac{1350}{25}}$

$= 7.3484$

$\rightarrow$ Trick : MOD $\rightarrow$ SD $\rightarrow$ x ; y

for : $f_i d_i = \sigma^2 \times \Sigma f_i$

Ex.24 Find out standard deviation for following data

class	f_i	x_i	d_i	f_id_i	d_i^2	$f_id_i^2$
0 - 30	9	15	-120	-1080	14400	129600
30 - 60	17	45	-90	-1530	8100	137706
60 - 90	43	75	-60	-2580	3600	154806
90 - 120	82	105	-30	-2460	900	73800
120 - 150	81	$\boxed{135}$	0	0	0	0
150 - 180	44	165	30	1320	900	39600
180 - 210	24	195	60	1440	3600	86400
	300			-4890		542193

$$\rightarrow \bar{x} = a + \frac{\Sigma f_id_i}{\Sigma f_i}$$

$$= 118.7$$

$$\rightarrow \sigma = \sqrt{\frac{\Sigma f_id_i^2}{\Sigma f_i}}$$

$$= \sqrt{\frac{542143}{300}}$$

$$= 42.5124$$

☆ Coefficient of Variation

→ $$C.V = \frac{\sigma}{\bar{x}} \times 100$$

- If C.V is greater that means it is more variable and less consistant.

- If C.V is ~~greater~~ lower that means it is less variable and more consistant.

Ex.25 The A.M of Run scored by 3 batsman A, B and C in score are 50, 48 and 12. The S.d of their runs are 15, 12 and 2 who is more consistant.

→ (A)

$\bar{x} = 50$

$\sigma = 15$

→ $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{15}{50} \times 100$

$= 30\%$

(B)

$\bar{x} = 48$

$\sigma = 12$

→ $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{12}{48} \times 100$

$= 25\%$

(C)

$\bar{x} = 12$

$\sigma = 2$

→ $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{2}{12} \times 100$

$= 16.68\%$

→ That's why C is more consistant.

Ex. 26 The run scored by two batsman in 9 matches are given which is more consistant.

A	85	20	62	28	74	5	69	4	13
B	72	4	15	30	59	15	44	27	26

-> [A]

x_A	$(x_A - \bar{x})^2$
85	2025
20	400
62	484
28	144
74	1156
5	1225
69	841
4	1296
13	729
	8300

[B]

x_B	$(x_B - \bar{x})^2$
72	1521
4	841
15	324
30	9
59	676
15	324
44	256
27	36
26	49
	4036

-> $\bar{x} = \frac{360}{9} = 40$

-> $\sigma = \sqrt{\frac{\Sigma(x_A - \bar{x})^2}{n}}$

$= 30.3681$

-> $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{30.3681}{40} \times 100$

$= 75.92\ \%$

-> $\bar{x} = \frac{297}{9} = 33$

-> $\sigma = \sqrt{\frac{\Sigma(x_B - \bar{x})}{n}}$

$= 21.1765$

-> $C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{21.1765}{33} \times 100$

$= 64.16$

-> B is more consistant.

x.27 Goal scored by two teams A and B in football season were us follow which team is more consistant.

No. of goal scored in match	No. of match played by A	No. of match played by B.
0	27	12
1	9	9
2	8	6
3	5	5
4	4	3

-> [A]

x	f	fx	d_i	$f_id_i^2$
0	27	6	-1.056	30.10
1	9	9	-0.656	0.02
2	8	16	0.944	7.12
3	5	15	1.944	18.89
4	4	16	2.994	34.66
	53			90.83

[B]

x	f	fx	d_i	$f_id_i^2$
0	17	0	-1.2	24.48
1	9	9	-0.2	0.36
2	6	12	0.8	3.84
3	5	15	1.8	16.2
4	3	12	2.8	23.52
	40			68.4

-> $\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{56}{53} = 1.056$

-> $\bar{x} = \frac{48}{46} = 1.2$

-> $\sigma = \sqrt{\frac{\Sigma f_id_i}{\Sigma f_i}} = \sqrt{\frac{90.83}{53}}$

$= 1.3091$

-> $\sigma = \sqrt{\frac{\Sigma f_id_i}{\Sigma f_i}} = \sqrt{\frac{68.4}{40}}$

$= 1.3076$

-> $C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.30}{1.05} \times 100$

$= 123.96\%$

-> $C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.30}{1.2} \times 100$

$= 108.96\%$

-> B is more consistant.

☆ Covariance

→ $$\mathrm{Cov}(x,y) = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{n}$$

☆ Kal pearson's Coefficient Method

→ $$r = \frac{\mathrm{Cov}(x,y)}{\sigma_x \cdot \sigma_y}$$

where , $\sigma_x = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

$\sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})/n}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}}$$

$$r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2} \cdot \sqrt{\Sigma(y-\bar{y})^2}}$$

→ Trick : MODE → REG → LINE

$\sqrt{\Sigma(x-\bar{x})^2} = \sigma_x^2 \cdot (n)$

$\sqrt{\Sigma(y-\bar{y})^2} = \sigma_y^2 \cdot (n)$

Ex. 28 Find out the correlation coeffcient for given data -

x	y	$(x-\bar{x})$	$(y-\bar{y})$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
1	6	-3	-4	9	16	12
2	8	-2	-2	4	4	4
3	11	-1	1	1	1	1
4	9	0	-1	0	1	0
5	12	1	2	1	4	2
6	10	2	0	4	0	0
7	14	3	4	9	16	12
28	70			28	42	29

→ $\bar{x} = \frac{\Sigma x_i}{n}$

$= 28/7$

$= 4$

→ $\bar{y} = \frac{\Sigma y_i}{n}$

$= 70/7$

$= 10$

→ $r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$

$= \frac{29}{\sqrt{28}\cdot\sqrt{42}}$

$= 0.8456$

→ It is positive correlation.

→ Trick : MODE → ~~3~~ REG → LINE

$\sqrt{\Sigma(x-\bar{x})^2} = \sigma_x^2 \cdot (n)$

$\sqrt{\Sigma(y-\bar{y})^2} = \sigma_y^2 \cdot (n)$

Ex.29 The coefficient of correlation between two variable x and y is 0.48. covariance is 36. The variance of x is 16. Find s.d. of y.

→ $r = 0.48$

$cov(x,y) = 36$

$Var(x) = 16 \qquad \therefore \sigma_x = \sqrt{16} = 4$

$\sigma_y = (?)$

→ $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y}$

$0.48 = \frac{36}{4 \cdot \sigma_y}$

$\sigma_y = 18.75$

Ex.30 Given $n=10$, $\sigma_x = 5.4$, $\sigma_y = 6.2$ and sum of product of deviation from the mean y-x and y is 66. Find coefficient correlation

→ $n = 10, \quad \sigma_x = 5.4, \quad \sigma_y = 6.2,$

$\Sigma(x-\bar{x})(y-\bar{y}) = 66, \quad r = (?)$

→ $Cov(x,y) = \frac{\Sigma(x-\bar{x})\cancel{\Sigma}\cdot(y-\bar{y})}{n} = \frac{66}{10} = 6.6$

→ $r = \frac{cov(x,y)}{\sigma_x \cdot \sigma_y} = \frac{6.6}{(5.4)(6.6)}$

$r = 0.1971$

Ex.31 Find n if $r = 0.5$, $\Sigma(x-\bar{x})(y-\bar{y}) = 120$, $\sigma_y = 8$, $\Sigma(x-\bar{x})^2 = 90$

$$\rightarrow \quad r = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

$$\sqrt{\Sigma(y-\bar{y})^2} = \frac{\Sigma(x-\bar{x})\cdot(y-\bar{y})}{r\sqrt{\Sigma(x-\bar{x})^2}}$$

$$= \frac{120}{0.5(\sqrt{90})} = 25.2982$$

$$\rightarrow \text{Now,} \quad \sigma_y = \sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}$$

$$\sigma_y^2 = \frac{\Sigma(y-\bar{y})^2}{n}$$

$$n = \frac{\Sigma(y-\bar{y})^2}{\sigma_y^2}$$

$$= \frac{(25.2982)^2}{(8)^2}$$

$$n = 10$$

Ex.32 For calculation of coefficient $\Sigma x = 25$, $\Sigma y = 5$, $\Sigma x^2 = 650$, $\Sigma y^2 = 290$, $\Sigma xy = 335$, $n = 10$. On verification it was found the pair $(x = 10, y = 4)$ was coppied wrongly the correct value is $(x = 12, y = 9)$. find coefficient of correlation.

→ $\Sigma x = 25 - 10 + 12 = 27$

$\Sigma y = 5 - 4 + 9 = 10$

$\Sigma x^2 = 650 - (10)^2 + (12)^2 = 694$

$\Sigma y^2 = 290 - (4)^2 + (9)^2 = 355$

$\Sigma xy = 355 - (10 \times 4) + (12 \times 9) = 403$

→ $r = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2}\sqrt{\Sigma y^2}}$

$= \dfrac{403}{\sqrt{694} \cdot \sqrt{355}}$

$= 0.8119$

☆ Spearman's Rank Correlation Method

→ $$r = 1 - \frac{6 \cdot \Sigma d^2}{n(n^2-1)}$$

Where, d = difference between R_1, R_2
n = number of pairs

→ If r is positive both have same line of thinking.

→ If r is negative both have opposite line of thinking.

Ex. 33 Two Judges in beauty contest rank the 12 contestent as follow what degree of agreement is there between two judges.

→

R_1	R_2	$d^2 \boxed{(R_1 - R_2)^2}$
1	12	121
2	9	49
3	6	9
4	10	36
5	3	4
6	5	1
7	4	9
8	7	1
9	8	1
10	2	64
11	11	0
12	1	121
		416

→ $r = 1 - \frac{6 \cdot \Sigma d^2}{n(n^2-1)} = 1 - \frac{6(416)}{12((12)^2-1)}$

$= -0.4545$

→ Both Judges have opposite line of thinking.

Ex. 34 Competition in music test where rank by 3 Judges in following order. Determine which pair of Judges ~~same~~ has nearest approach.

→

R_1	R_2	R_3	$d_1^2 = (R_1-R_2)^2$	$d_2^2 = (R_2-R_3)^2$	d^3 $(R_1-R_3)^2$
1	3	6	4	9	25
6	5	4	1	1	4
5	8	9	9	1	16
10	4	8	36	16	4
3	7	1	16	36	4
2	10	2	64	64	0
4	2	3	4	1	1
9	1	10	64	~~64~~ 81	1
7	6	5	1	1	4
8	9	7	1	4	1
			200	214	60

→ For R_1 and R_2,

$$r_1 = 1 - \frac{6\,\Sigma d_1^2}{n(n^2-1)}$$

$$= 1 - \frac{6(200)}{10(10^2-1)} = 1 - \frac{1200}{990}$$

$$= -0.2121$$

→ For R_2 and R_3,

$$r_2 = 1 - \frac{6\Sigma d_2^2}{n(n^2-1)}$$

$$= 1 - \frac{6(214)}{10((10)^2-1)}$$

$$= -0.2969$$

→ For R_1 and R_3,

$$r_3 = 1 - \frac{6.\Sigma d_3^2}{n(n^2-1)}$$

$$= 1 - \frac{6(60)}{10((10)^2-1)}$$

$$= 1 - \frac{3600}{990}$$

$$= 0.6363$$

→ R_1 and R_3 has nearest approach.

→ Rank is always less than or equal to n

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Ex.35 Student got following percentag of marks in math and physics. Find Rank correlation of coefficient.

→

x	y		x	y	d^2
8	84		10	3	49
36	5		7	10	9
98	91		1	1	0
25	60		9	6	9
75	68		4	4	0
82	62		3	5	4
92	86		2	2	0
62	58		6	7	1
65	35		5	9	16
35	49		8	8	0
					88

→ $r = 1 - \frac{6 \cdot \Sigma d^2}{n(n^2-1)}$

$= 1 - \frac{6 \cdot (88)}{10((10)^2-1)}$

$= 1 - \frac{528}{990}$

$= 0.4646$

→ Both Subject have same percentage of mark

☆ Tied Rank

-> If there is a tie between two or more individual ranks the rank is divided among individual.

-> If there is tie between one rank then $\frac{1}{12}(m^3-m)$ is added to Σd^2.

furture n tie then add $\frac{1}{12}(m^3-m)$ for each tie.

-> Single tie :

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m^3-m)\right]}{n(n^2-1)}$$

-> Dual tie :

$$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3-m) + \frac{1}{12}(m_2^3-m)\right]}{n(n^2-1)}$$

Ex. 36 Obtain Rank correlation of coefficient.

→

x	y
1 10	12 1
2 12	18 2
4.5 18	25 4
4.5 18	25 4
3 15	50 6
6 40	25 4

x	y	d^2
1	1	0
2	2	0
4.5	4	0.25
4.5	4	0.25
3	6	9
6	4	4
		13.5

→ $m_1 = 2$, $m_2 = 3$

→ $$r = 1 - \frac{6\left[\Sigma d^2 + \frac{1}{12}(m_1^3 - m) + \frac{1}{12}(m_2^3 - m_2)\right]}{n(n^2-1)}$$

$$= 1 - \frac{6\left[13.5 + \frac{1}{12}(8-2) + \frac{1}{12}(27-3)\right]}{6((6)^2-1)}$$

$$= 1 - \frac{6[13.5 + 0.5 + 2]}{6(35)}$$

$$= 0.4571$$

☆ Regression Of line

→ Regression y on x,

$$y = a + bx$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

where, $\bar{x}$ = A.M of x , σ_x = S.d of x
$\bar{y}$ = A.M of y , σ_y = S.d of y
r = correlation coefficient

→ Let, $r \frac{\sigma_y}{\sigma_x} = b_{yx}$

$$y - \bar{y} = b_{yx} (x - \bar{x})$$

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→ Regression x on y,

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$\bar{x}$ and $\bar{y}$

$\rightarrow x = a + by$

where, $\bar{x}$ = A.M of x
$\bar{y}$ = A.M of y
σ_x = S.d of x
σ_y = S.d of y
r = correlation coeffcient

→ Let, $r \frac{\sigma_x}{\sigma_y} = b_{xy}$

$$x - \bar{x} = b_{xy} (y - \bar{y})$$

method.

☆ Relation between Correlation, coeffcient and Regression coefficient:

→ $b_{yx} = r\frac{\sigma_y}{\sigma_x}$, $b_{xy} = r\frac{\sigma_x}{\sigma_y}$

Now, $b_{yx} \cdot b_{xy} = r \cdot \frac{\sigma_y}{\sigma_x} \times r \cdot \frac{\sigma_x}{\sigma_y}$

$$b_{yx} \cdot b_{xy} = r^2$$

$$r = \sqrt{b_{yx} \cdot b_{xy}}$$

* Regression :

y on x	x on y
$y - \bar{y} = b_{yx}(x - \bar{x})$	$x - \bar{x} = b_{xy}(y - \bar{y})$
where, $b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$	$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2}$

Ex.37 Find out regression y on x.

x	y	x^2	xy
2	9	4	18
5	6	25	30
6	8	36	48
3	5	9	15
4	4	16	16
20	32	90	127

→ $\bar{x} = {}^{20}/_{6} = 4$

$\bar{y} = {}^{32}/_{5} = 6.4$

→ Regression coeffcient y on x

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{5(127) - (20)(32)}{5(90) - (20)^2}$$

$$= -0.1$$

→ Regression y on x

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 6.4 = -0.1(x - 4)$

$y - 6.4 = -0.1x + 0.4$

$y = 6.8 - 0.1x$

Ex.38 following data give us the experience of machine operator and their perfomance calculate regression line of perfomance on experience and perfomance of operator if he had 11 year of experience.

x	y	xy	y^2
23	5	115	25
43	6	258	36
53	7	371	49
63	8	504	64
73	9	657	81
83	10	830	100
338	45	2735	355

→ $\bar{x} = 338/6 = 56.33$

$\bar{y} = 45/6 = 7.5$

→ Regression coeffcient x on y

$$b_{xy} = \frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \frac{6(2735) - (338)(45)}{6(335) - (45)^2}$$

$= 11.42$

→ Regression x on y

$(x - 56.33) = 11.42(y - 7.5)$

$x = 11.42y - 29.38$

$x = -29.38 + 11.42y$

→ If he had 11 year experience

$x = -29.38 + 11.42(11)$

$= 96.24$

Ex.39 Two regression lines are given,

$8x - 10y + 66 = 0$

$40x - 18y = 214$ find (1) b_{yx} (2) b_{xy} (3) r (4) $\bar{x}$ and $\bar{y}$

→ $8x - 10y = 66$

$8x - 66 = 10y$

$y = (0.8)x - 6.6$

→ $40x - 18y = 214$

$40x = 18y + 214$

$x = 0.45y + 5.35$

→ $b_{yx} = 0.8$

→ $b_{xy} = 0.45$

→ (iii) $r = \sqrt{b_{xy} \cdot b_{yx}}$

$= \sqrt{0.45 \times 0.8} = 0.6$

→ (iv) $8x - 10y = -66$ — ①

$40x - 18y = 214$ — ②

→ Multiply eq. ① by 5 and do elimination method.

$$\begin{aligned} 40x - 50y &= -330 \\ 40x - 18y &= 214 \\ \hline -32y &= -544 \\ \bar{y} &= 17 \end{aligned}$$

→ $8x - 10(17) = -66$

$8x - 170 = -66$

$\bar{x} = 13$

☆ Measure of Skewness

→ A measure of skewness give the extent and direction of skewness.

Absolute skewness = |Mean - Mode|

→ If mean > mode → Skewness is positive.
If mean < mode → Skewness is negative.

☆ Kalpearson's coefficient of skewness

→ $$S_K = \frac{\text{Mean} - \text{Mode}}{\sigma}$$

$$S_K = \frac{\text{Mean} - [3(\text{median}) - 2(\text{mean})]}{\sigma}$$

$$\therefore \; S_K = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

where, $S_K = 0$ → Symmetrical
$S_K > 0$ → Positive Skewness
$S_K < 0$ → Negative Skewness

4.3 Find Kalpearson coefficient of skewness

→

x	$(x-\bar{x})$	$(x-\bar{x})^2$
3	-1	1
4	0	0
8	4	16
2	-2	4
4	0	0
4	0	0
3	-1	1
4	0	0
7	3	9
18	14	196
9	5	25
11	87	49
		301

→ $\bar{x} = 77/12 = 6.14$ → Mode = 4

→ $\sigma = \sqrt{\Sigma(x-\bar{x})^2/n} = \sqrt{301/12} = 4.38$

→ $S_k = \dfrac{\text{Mean} - \text{Mode}}{\sigma}$

$= \dfrac{6.41 - 4}{4.38}$

$= 0.5509$

→ Positive skewness.

Ex. 24 Find Kalpearsons of skewness for given data.

x	f_i	f_ix_i	d_i	$f_id_i^2$
20	7	140	-2.88	58.06
21	12	252	-1.88	42.41
$\boxed{22}$	25	550	-0.88	19.36
23	10	230	0.12	0.144
24	8	192	1.12	10.03
25	6	150	2.22	29.57
26	8	208	3.22	82.94
27	3	81	4.22	53.42
28	1	28	5.22	27.24
	80	1831		311.98

→ $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1831}{80} = 22.88$

→ Mode = 22

→ $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i}} = \sqrt{\frac{311.98}{80}} = 1.9748$

→ $S_k = \frac{\text{Mean} - \text{Mode}}{\sigma}$

$= \frac{22.88 - 22}{1.9748}$

$= 0.4494$

→ Positive skew

Ex.45 Find Kalpearson's coeffcient for skewness for the following data.

→

x	f	x_i	d_i	f_id_i	$f_id_i^2$
0-10	18	5	-20	-360	7200
10-20	20 f_0	15	-10	-200	2000
20-30	30 f_1	$\boxed{25}$	0	0	0
30-40	25 f_2	35	10	250	~~150~~2500
40-50	12	45	20	240	4800
					16500

→ $f_0 = 20, \quad f_1 = 30, \quad f_2 = 25, \quad c = 10, \quad l = 20$

→ $\bar{x} = a + \frac{\Sigma f_id_i}{\Sigma f_i} = 24.33$

→ $\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times c$

$= 20 + \left(\frac{30 - 20}{2(30) - 20 - 25}\right) \times 10$

$= 20 + (6.66)$

$= 26.66$

$\sigma = \sqrt{\frac{\Sigma f_id_i^2}{\Sigma f_i}}$

$= \sqrt{\frac{16500}{105}}$

$= 12.53$

→ $S_k = \frac{\text{Mean} - \text{Mode}}{\sigma}$

$= \frac{24.33 - 26.66}{12.53}$

$= -0.18$

→ Negative skewness

☆ Large Sample when [Mean, S.D given]

* Working rule :

(i) Null hypothesis : $\mu = \mu_0$

(ii) Alternative hypothesis : $\mu \neq \mu_0$

(iii) level of significant : Select or given

(iv) • When S.d (σ) is given : $z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$

• When S-d of sample (s) given : $z = \dfrac{\bar{x} - \mu}{(s/\sqrt{n})}$

(v) Critical value : $|z_\alpha| =$

(vi) Decision : $|z| < |z_\alpha|$ → Accepted
$|z| > |z_\alpha|$ → Rejected

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Ex.1 An amubulance service claims that it takes on the average 10 minutes to reach it's destination in emergency. A sample of 36 calls has mean of 11 minutes and the variance of 16 minutes. Test the claim at 0.05 level of significant.

→ $n = 36$, $\mu = 10$ min, $\bar{x} = 11$ min,
Variance $= 16$ $\therefore S = \sqrt{16}$ $\therefore S = 4$

(i) Null hypothesis : $\mu = 10$ min.
[An ambulance service claim that it taken average 10 min in emergency]

(ii) Alternative hypothesis : $\mu \neq 10$ min [Two tail]
[An ambulance service does not taken average 10 min in emergency]

(iii) Level of significant : 0.05

(iv) Test significant : $z = \bar{x} - \mu / (S/\sqrt{n})$
$= \frac{11-10}{(4/6)} = 1.5$

(v) Critical value $= z_{0.05} = 1.96$

(vi) Decision : $|z| < |z_{0.05}|$
→ This hypothesis accepted at 5% level of significant.

☆ Test for difference of Mean

[When, 2 Mean and 2 S.d. given]

* Working Rule.

(i) Null hypothesis : $\mu = \mu_0$

(ii) Alternative hypothesis : $\mu \neq \mu_0$ [Two tail] or $\mu < \mu_0$ [Left tail] or $\mu > \mu_0$ [Right tail]

(iii) Level of significant : 0.05

(iv) Test statics :

- When population S.d (σ_1, σ_2) given :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{n_2^2}}}$$

- When Sample S.d (s_1, s_2) given :

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1^2} + \frac{s_2^2}{n_2^2}}}$$

(v) Critical value : Given or select 0.05 value.

(vi) Decision :

$|z| < |z_\alpha| \rightarrow$ Accepted

$|z| > |z_\alpha| \rightarrow$ Rejected.

Ex.2 The mean of sample of size 1000 and 2000 are 67.5 and 68 respectively. The sample can be regard as drawn from the same population of s.d 2.5.

→ $n_1 = 1000$ $\bar{x}_1 = 67.5$ $\sigma_1 = 2.5$
$n_2 = 2000$ $\bar{x}_2 = 68$ $\sigma_2 = 2.5$

(i) Null hypothesis : $\mu_1 = \mu_2$
[Sample drawn from same sample of population with 2.5 s.d.]

(ii) Alternative hypothesis : $\mu_1 \neq \mu_2$
[Sample drawn from population does not have s.d of 2.5]

(iii) level of significant : $\alpha = 0.05$

(iv) Test statics : $z = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\sigma_1^2/n_1^2 + \sigma_2^2/n_2^2}}$

$$= \frac{67.5 - 68}{\sqrt{\dfrac{(2.5)^2}{(1000)^2} + \dfrac{(2.5)^2}{(1000)^2}}} = -5.16$$

(v) Critical value : $|Z_{0.05}| = 1.96$
(vi) Desicison : $|z| > |Z_{0.05}|$

- Null hypothesis is rejected at 5% level of significant.

Ex.3 The average marks scored by 32 boys is 72 with s.d. of 8 while 36 girls girls is 70 with s.d. 6. Test at 1% level of significant wheather boys perfome better than girls.

→ $n_1 = 32$, $\bar{x}_1 = 72$, $s_1 = 8$
$n_2 = 36$, $\bar{x}_2 = 70$, $s_2 = 6$

(i) Null hypothesis : $\mu_1 = \mu_2$

(ii) Alternative hypothesis : $\mu_1 > \mu_2$ [Right tail]

Ex.4 A simple sample of 6400 english men has a mean of 170 cm and s.d of 6.4 cm. while a sample of heights of 1600 American has a mean of 172 cm with s.d of 63 cm. Do the data indicates americans are taller than the english men.

→ $n_1 = 6400$, $\bar{x}_1 = 170$ cm , $s_1 = 6.4$ cm
$n_2 = 1600$, $\bar{x}_2 = 172$ cm , $s_2 = 6.3$ cm

(i) Null hypothesis : $\mu_1 = \mu_2$

(ii) Alternative hypothesis : $\mu_1 < \mu_2$ [Left tail]

☆ Difference of standard deviation

[when there is no Mean only two n_1 and two S.d given]

$$\begin{bmatrix} n_1 = & , & s_1 = \\ n_2 = & , & s_2 = \end{bmatrix}$$

(i) Null hypothesis : ~~H~~ $\sigma_1 = \sigma_2$

(ii) Alternative hypothesis : $\sigma_1 \neq \sigma_2$

(iii) Level of significant : Given or 0.05

(iv) Test static :

- σ_1, σ_5 given :

$$z \neq \cancel{\sigma_1 - \sigma_2} \qquad Z = \frac{s_1 - s_2}{\sqrt{\frac{\sigma_1^2}{2n_1} + \frac{\sigma_2^2}{2n_2}}}$$

- s_1, s_2 given : $z = \dfrac{s_1 - s_2}{\sqrt{\frac{s_1^2}{2n_1} + \frac{s_2^2}{2n_2}}}$

(v) Critical value : $|Z_{0.05}| =$

(vi) Decision : $|Z| < |Z_{0.05}|$ → Accepted
$|Z| > |Z_{0.05}|$ → Rejected

Ex. 5 Examine weather the two sample for while the data are given in following table. Could have been drawn from population with same s.d.?

	Size	S.d
Sample 1	1000	5
Sample 2	2000	7.

(i) Null hypothesis : $\sigma_1 = \sigma_2$

[Sample drawn from population with same s.d]

(ii) Alternative hypothesis : $\sigma_1 \neq \sigma_2$

[Sample drawn from population does not have same s.d.]

(iii) Level of significant : $\alpha = 0.05$

(iv) Test static : $|z| = \dfrac{S_1 - S_2}{\sqrt{\dfrac{\sigma_1^2}{2n_1} + \dfrac{\sigma_2^2}{2n_2}}} = \left| \dfrac{100 - 200}{\sqrt{\dfrac{(5)^2}{2(1000)} + \dfrac{(7)^2}{2(2000)}}} \right.$

$= 1$

(v) Critical value : $|z_{0.05}| = 1.96$

(vi) Decision : $|z| < |z_{0.05}|$

Null hypothesis is accepted at 5% level of significant.

✩ ① Difference of Performance

$$\left[\begin{array}{ll} P_1 = \quad , & n_1 = \\ P_2 = \quad , & n_2 = \end{array}\right]$$

(i) Null hypothesis : $P_1 = P_2$

(ii) Alternative hypothesis : $P_1 \neq P_2$ [Two tail]
$P_1 > P_2$ [Right tail]
$P_1 < P_2$ [left tail]

(iii) Level of significant : Given or select

(iv) Test static :-

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where, $p = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2}$

$q = 1 - p$

(v) Critical value : $|Z_\alpha| =$

(vi) Decision : $|z| < |Z_\alpha| \rightarrow$ Accepted
$|z| > |Z_\alpha| \rightarrow$ Rejected.

Ex.6 Random sample of 400 men and 600 women were asked weather they would like to have a fly over near their residency. 200 men and 325 women in favour in proposal. Test the hypothesis that proposal of men and women in favour of the proposanl are same as 5% level of significant.

→ $n_1 = 400$, $n_2 = 600$

$P_1 = 200/400 = 1/2 = 0.5$, $P_2 = 325/600 = 0.541$

→ $$P = \frac{n_1P_1 + n_2P_2}{n_1+n_2} = \frac{400(0.5) + 600(0.541)}{400+600} = 0.525$$

$q = 1 - P = 1 - 0.525 = 0.475$

(i) → Null hypothesis : $P_1 = P_2$

[Men and women are favour in proposal of fly over near their residency]

(ii) → Alternative hypothesis : $P_1 \neq P_2$

[Men and women does not in favour of proposal ~~hav~~ of flyover near residency]

(iii) → Level of significant : $\alpha = 0.05$

(iv) Test Statics :

$$z = \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$= \frac{0.5 - 0.541}{\sqrt{(0.525)(0.475)\left(\frac{1}{400} + \frac{1}{600}\right)}}$$

$$= 1.27$$

(v) Critical value : $z_{0.05} = 1.96$

(vi) Decision :

$$|z| < |z_\alpha|$$

∴ Null hypothesis is accepted at 5% level of significant.

Small

☆ When $\bar{x}$, s and n given check μ.

(i) Null hypothesis : $\mu = \mu_0$

(ii) Alternative hypothesis : $\mu \neq \mu_0$ [Two tail]
$\mu > \mu_0$ [Right tail]
$\mu < \mu_0$ [left tail]

(iii) Level of significant : given or select.

(iv) Test static : $t = \dfrac{\bar{x} - \mu}{\left(\dfrac{s}{\sqrt{n-1}}\right)}$

(v) Critical value : $v = n-1$

$t_{0.05}(v) =$

(vi) Decision :

$|t| < |t_\alpha|$ → Accepted
$|t| > |t_\alpha|$ → Rejected

Ex. 7 The mean life of sample of 25 bulb is
8 found to 1550 hrs with s.d of 120 hrs.
The company manufacture claim that
the bulb average life is atleast
1600 hrs. Is the claim accepted
at 5 % level of significant.

→ $n = 25$, $\bar{x} = 1550$ hrs, $s = 120$ hrs

(i) Null hypothesis : $\mu = 1600$ hrs.

[company claim that bulb average life is 1600 hrs]

(ii) Alternative hypothesis : $\mu > 1600$ hrs

[Bulb average life is greater than 1600 hrs]

(iii) Level of significant : $\alpha = 0.05$

(iv) Test Statics : $t = \dfrac{\bar{x} - \mu}{(s/\sqrt{n-1})} = \dfrac{1550 - 1600}{(120/\sqrt{25-1})}$

$= -2.04 \quad \therefore |t| = 2.04$

(v) Critical value : $v = n - 1 = 24$

$t_{0.05}(24) = 1.711$

(vi) Decision :

$|t| > |t_{0.05}|$

- Test is rejected at 5 % level of significant.

☆ F - Test [Variance Given]

(i) Null hypothesis : $\sigma_1^2 = \sigma_2^2$

(ii) Alternative hypothesis : $\sigma_1^2 > \sigma_2^2$

(iii) Level of significant : Given or choose.

(iv) Test Statics :

$$F = \frac{S_1^2}{S_2^2} \quad \text{where} \quad S_1^2 > S_2^2 ,$$

$$F = \frac{S_2^2}{S_1^2} \quad \text{where} \quad S_2^2 > S_1^2$$

where, $S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1}$ or $S_1^2 = \frac{n_1 s_1^2}{n_1 - 1}$

$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 1}$ or $S_2^2 = \frac{n_2 s_2^2}{n_2 - 1}$

(v) Critical value : $v_1 = n_1 - 1$, $v_2 = n_2 - 1$,

$F_{0.05}(v_1, v_2) =$

(vi) Decision : $|F| < |F_\alpha|$ → Accepted

$|F| > |F_\alpha|$ → Rejected

પ્રશ્ન પેટા પ્રશ્ન ક્રમાંક

Ex.7 π

પ્રશ્ન પેટા પ્રશ્ન ક્રમાંક

Ex.8 In 2 independent sample of sizes 8 and 10. The sum of square of deviation of the sample value from respective means were 84.4 and 102.6 Test weather the difference of variance if the populatation is signifancant or not use at 0.05 level of significant.

→ $n_1 = 8$, $\Sigma(x-\bar{x})^2 = 84.4$

$n_2 = 10$, $\Sigma(y-\bar{y})^2 = 102.6$

→ $S_1^2 = \frac{\Sigma(x-\bar{x})^2}{n_1 - 1} = \frac{84.4}{7} = 12.058$

$S_2^2 = \frac{\Sigma(y-\bar{y})^2}{n_2 - 2} = \frac{102.6}{9} = 11.4$

(i) Null hypothesis : $\sigma_1^2 = \sigma_2^2$

[There is no difference in variance]

(ii) Alternative hypothesis : $\sigma_1^2 > \sigma_2^2$

[There is a difference in variance]

(iii) Test statics : $F = \frac{S_1^2}{S_2^2} = \frac{12.058}{11.4}$

$= 1.557$

(iv) Critical value : $v_1 = 8 - 1 = 7$

$v_2 = 10 - 1 = 9$

$$F_{0.05}(7,9) = 3.29$$

(v) ~~Test statistics :~~ ~~F~~

Decision : $|Z| < |Z_\alpha|$

- Null hypothesis is accepted at 5% of level of significant.

Ex.9 The S.d. calculated from 2 random sample of size 9 and 13 are 2.1 and 1.8 respectively. Can the sample be regarded as drawn from normal populations with same S.d. ?

→ $n_1 = 9$, $S_1 = 2.1$

$n_2 = 13$, $S_2 = 1.8$

→ $$S_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{9(2.1)^2}{8} = 4.96$$

$$S_2^2 = \frac{n_2 s_2^2}{n_1 - 1} = \frac{13(1.8)^2}{12} = 3.51$$

(i) Null hypothesis : $\sigma_1^2 = \sigma_2^2$

[Sample drawn from normal population with same s.d.]

(ii) Alternative hypothesis : $\sigma_1^2 > \sigma_2^2$

[Sample drawn from normal population with different s.d.]

(iii) Level of significant : $\alpha = 0.05$

(iv) Test static : $F = \frac{S_1^2}{S_2^2} = \frac{4.96}{3.57}$

$= 1.41$

(v) Critical value : $v_1 = n_1 - 1 = 8$

$v_2 = n_2 - 2 = 12$

$F_{0.05}(8,12) = 2.85$

(vi) Decision : $|F| < |F_\alpha|$

- Null hypothesis accepted at 5% level of significant.

☆ Chi - Square (χ^2) test

Ex.10 A dice can thrown 132 times and following frequency are observed. Test hypothesis that dice is unbiased.

→

Observation frequency	Expected frequency (fe)	$\frac{(f_0-f_e)^2}{f_e}$
15	22	2.23
20	22	0.18
25	22	0.41
15	22	2.23
29	22	2.23
28	22	1.04
		8.92

→ $f_e = \frac{132}{6} = 22$

(i) Null hypothesis : Dice is unbiased.

(ii) Alternative hypothesis : Dice is not unbiased.

(iii) Level of significant = 0.05

(iv) Test static : $\chi^2 = \Sigma (f_0 - f_e)^2 / f_e = 8.92$

(v) Critical value : $v = n - 1 = 5$.

$\chi^2_{0.05}(5) = 11.07$

(vi) Decision $|\chi^2| < |\chi^2_{0.05}|$

- Null hypothesis is accepted at 5% level of significant.