

UNIT - 1 (Electronic Materials)

1.1 Introduction

Conducting materials are low resistivity materials, which conduct heat as well as electricity. Electrical conduction is due to free electrons, whereas normal conduction is due to free electrons as well as phonons.

1.1.1 Basic Terminology

Conductors:

Experimental measurements showed that the metals and their alloys exhibit large electrical conductivity in the order of $10^8 \Omega^{-1} m^{-1}$. Hence they are known as conductors. Conducting materials are the materials having high electrical and thermal conductivities. Low resistive materials are also generally known as conducting materials.

Bound electrons:

All the valence electrons in an isolated atom are bound to their parent nuclei which are called as 'bound electrons'.

Free electrons:

In a solid, due to the boundaries of neighbouring atoms overlap each other, the valence electrons find continuity from atom to atom. Therefore, they can move easily throughout the solid. All such valence electrons of its constituent atoms in a solid are called free electrons.

Difference between ordinary gas and free electron gas:

The molecules of ordinary gas are neutral. But, the free electron gas is charged. The density of molecules is smaller than the density of free electrons.

Electric field (E):

The electric field (E) of a conductor having uniform cross section is defined as the potential drop (V) per unit length (l).

$$E = V/l \text{ Vm}^{-1}$$

Current density (j):

Current density (j) is defined as the current per unit area of cross section of an imaginary plane hold normal to the direction of flow of current in a current carrying conductor. If 'I' is the current, and 'A' is the area of cross section, then current density is given by,

$$J = I / A \text{ Am}^{-2}$$

1.2 Conducting Materials

Conducting materials are classified in to three major categories based on the conductivity.

- (i) Zero resistive materials
- (ii) Low resistive materials
- (iii) High resistive materials

(i) Zero resistive materials

The super conductors like alloys of aluminium, zinc, gallium, niobium, etc., are a special class of materials. These materials conduct electricity almost with zero resistance blow transition temperature. Thus, they are called zero resistive materials. These materials are used for saving energy in the power systems, super conducting magnets, memory storage elements etc.,

(ii) Low resistive materials

The metals like silver, aluminium and alloys have high electrical conductivity. These materials are called low resistive materials. They are used as conductors, electrical conduct etc., in electrical devices and electrical power transmission and distribution, winding wires in motors and transformers.

(iii) High resistive materials

The materials like tungsten, platinum, nichrome etc., have high resistive and low temperature co-efficient of resistance. These materials are called high resistive materials. Such a metals and alloys are used in the manufacturing of resistors, heating elements, resistance thermometers.

The conducting properties of solid do not depend on the total number of the electrons available because only the valance electrons of the atoms take part in the conduction. When these valance electrons detached from the orbit they are called free electrons or conduction electrons.

In a metal, the number of free electrons available is proportional to its electrical conductivity. Hence, electronic structure of a metal determines its electrical conductivity.

1.3 Electron theory of solids

We know that the electrons in the outermost orbit of the atom determine the electrical properties in the solid. The free electron theory of solids explains the structure and properties of solids through their electronic structure.

This theory is applicable to all solids, both metals and non-metals. It explains

- (a) The behavior of conductors, semiconductors, and insulators.
- (b) The electrical, thermal and magnetic properties of solids.

So far three electron theories have been proposed.

(i) Classical free electron theory

This theory was developed by Drude and Lorentz. According to this theory, a metal consists of electrons which are free to move about in the crystal molecules of a gas it contains mutual repulsion between electrons is ignored and hence potential energy is taken as zero. Therefore the total energy of the electron is equal to its kinetic energy.

(ii) Quantum free electron theory

Classical free electron theory could not explain many physical properties. In classical free electron theory, we use Maxwell-Boltzmann statistics which permits all free electrons to gain energy. In Sommerfeld developed a new theory, in which he retained some of the features of classical free electron theory included quantum mechanical concepts and Fermi-Dirac statistics to the free electrons in the metals. This theory is called quantum free electron theory. Quantum free electron theory permits only a few electrons to gain energy.

(iii) Zone theory or band theory of solids

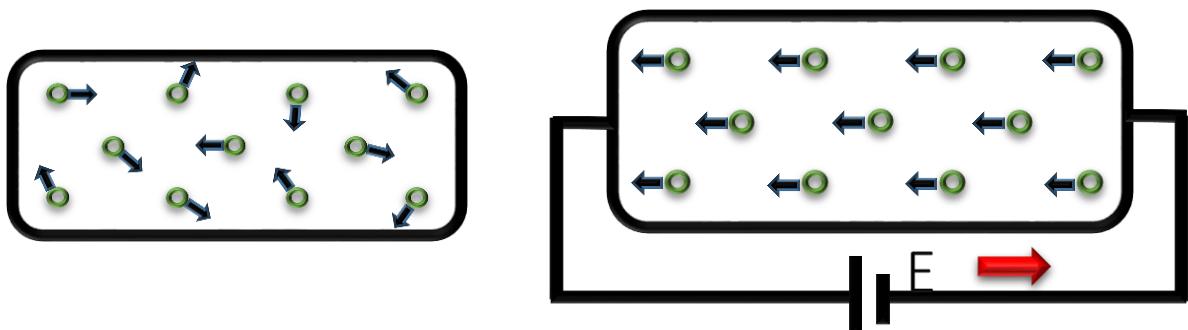
Bloch developed the theory in which the electrons move in a periodic field provided by the Lattice concept of holes, origin of Band gap and effective mass of electrons are the special features of this theory of solids. This theory also explains the mechanism of superconductivity based on band theory.

1.4 Classical free electron theory

Free electron theory of metals was proposed by P. Drude in the year 1900 to explain electrical conduction in metal. This theory was further extended by H.A. Lorentz in the year 1909.

1.4.1 Postulated of free electron theory

- ✓ Metal structure consists positive ion core, with valence electrons moves freely among these ion cores.
- ✓ These valence electrons moves inside metal just like molecules in gas.
- ✓ In the absence of electric field ($E = 0$), free electrons moves in random directions and collide with each other.
- ✓ In the collisions, no loss of energy will observed because collisions are elastic.
- ✓ When electric filed is applied ($E \neq 0$), the free electrons are accelerated in direction opposite to field.
- ✓ Free electrons obey Maxwell-Boltzmann statistics.



1.4.2 Postulates of free electron theory

Drift Velocity (V_d):

The drift velocity is defined as the average velocity acquired by the free electron in particular direction, due to the applied electric field.

$$\text{Drift Velocity} = \frac{\text{Average distance travelled by the electron}}{\text{Time taken}}$$

$$V_d = \lambda/t \text{ ms}^{-1}$$

Mobility (μ):

The mobility is defined as the drift velocity (V_d) acquired by the electron per unit electric field (E).

$$\mu = V_d/E \text{ m}^2\text{V}^{-1}\text{s}^{-1}$$

Mean free path (λ):

The average distance travelled by an electron between two successive collisions is called mean free path.

Mean collision time (τ_c) (or) Collision time

It is the time taken by the free electron between two successive collisions.

$$\tau_c = \lambda/V_d \text{ sec}$$

Relaxation time (τ):

It is the time taken by the electron to reach equilibrium position from disturbed position in the presence of electric field.

$$\tau = l/V_d \text{ sec}$$

Where l is the distance travelled by the electron. The value of relaxation time is of the order of 10^{-14} sec.

Band gap (E_g):

Band gap is the energy difference between the minimum energy of conduction band and the maximum energy of valence band.

Current density (J)

It is defined as the current per unit area of cross section of an imaginary plane holded normal to the direction of the flow of current in a current carrying conductor.

$$\mathbf{J} = I/A \text{ Am}^{-2}$$

1.4.3 Expression for electrical conductivity

- ✓ Electrical conductivity of a semiconductor is different from that of a conductor. The charge carriers in a conductor are electrons. In the case of a semiconductor, both electron and holes are charge carriers.
- ✓ In the absence of an electric field, the motion of these free electrons is completely random like those of molecules in a gas.
- ✓ But when an electric field is applied, the electrons drift in opposite direction to that of applied field with an average velocity called the "drift velocity" (V_d).
- ✓ When electric field is applied, electrons experience a force eE , due to which they are accelerated.

$$\mathbf{F} = e\mathbf{E} \quad \text{But } \mathbf{F} = m\mathbf{a}$$

$$\therefore m\mathbf{a} = e\mathbf{E}$$

$$\therefore \mathbf{a} = (e\mathbf{E})/m \dots \dots \dots (1)$$

As, electrons collision occurs during motion, the electrons will get accelerated momentarily so, the drift velocity will be

$$\therefore V_d = a\tau$$

$$\therefore V_d = \frac{eE}{m}\tau \dots \dots \dots (2)$$

$$\text{Current density } \mathbf{J} = \frac{\mathbf{I}}{A} = neVd \dots \dots \dots (3)$$

$$\therefore J = ne \frac{eE}{m} \tau = \frac{ne^2 E \tau}{m} \dots \dots \dots (4)$$

$$\text{From, electrical conductivity } J = \sigma E \dots \dots \dots (5)$$

$$\therefore \sigma E = \frac{ne^2 E \tau}{m}$$

$$\therefore \sigma = \frac{ne^2 \tau}{m} \dots \dots \dots (6) \ggg \text{Expression for electrical conductivity}$$

Conductivity in terms of $k_B T$.

The relaxation time (τ) in terms of mean free path (λ) and average thermal velocity is given by:

$$\therefore \tau = \frac{\lambda}{v} \quad \dots \dots \dots (7)$$

Now, based on kinetic theory of gases,

$$\therefore \frac{1}{2} m v^2 = \frac{3}{2} k_B T$$

$$\therefore m = \frac{3k_B T}{V^2} \quad \dots \dots \dots (8)$$

Using equation (8) & (7) in (6)

$$\therefore \sigma = \frac{n e^2 \left(\frac{\lambda}{V} \right)}{\frac{3K_B T}{V^2}} = \frac{n e^2 \lambda V^2}{3K_B T V}$$

$$\therefore \sigma = \frac{n e^2 \lambda V}{3K_B T} \quad \dots \dots \dots (9) \text{ (This equation says that electrical conductivity decreases with temperature)}$$

Q.1	A current of 5 A is passing through a metallic wire of cross sectional area $4 \times 10^{-6} \text{ m}^2$. If the density of the charge carriers in the wire is $5 \times 10^{26} / \text{m}^3$, find the drift speed of the electrons. ($e = 1.6 \times 10^{-19}$)
Ans.	I = 5 A, A = $4 \times 10^{-6} \text{ m}^2$, n = $5 \times 10^{26} / \text{m}^3$, $V_d = ?$ The relation between current (I) and drift velocity (V_d) of electron in wire of area (A) is $\therefore I = n e A V_d$

	$\therefore V_d = \frac{I}{neA}$ $\therefore V_d = \frac{5}{(5 \times 10^{26})(1.6 \times 10^{-19})(4 \times 10^{-6})}$ $\therefore V_d = 1.5 \times 10^{-2} \text{ m/s}$
Q.2	A conductor has an electron concentration of $5.9 \times 10^{28} / \text{m}^3$. What density in the conductor corresponds to a drift velocity of 0.625 m/sec. Calculate the mobility of charge carriers. Given $\sigma = 6.22 \times 10^7 \text{ } \Omega \text{-m}^{-1}$.
Ans.	$n = 5.9 \times 10^{28} / \text{m}^3$, $V_d = 0.625 \text{ m/sec}$, $e = 1.6 \times 10^{-19}$, $J = ?$ & $\mu = ?$ $\therefore J = neV_d$ $\therefore J = (5.9 \times 10^{28}) (1.6 \times 10^{-19}) (0.625)$ $\therefore J = 5.9 \times 10^9 \text{ A/m}^2$ $\therefore \sigma = ne\mu$ $\therefore \mu = \frac{\sigma}{ne}$ $\therefore \mu = \frac{(6.22 \times 10^7)}{(5.9 \times 10^{28})(1.6 \times 10^{-19})}$ $\therefore \mu = 6.588 \times 10^{-3} \text{ m}^2/\text{V}\cdot\text{sec}$
Q. 3	Calculate drift velocity of free electrons with a mobility of $3.5 \times 10^{-3} \text{ m}^2 / \text{V}\cdot\text{sec}$ in copper for an electric field strength of 0.5 V/m.
Ans.	$\mu = 3.5 \times 10^{-3} \text{ m}^2/\text{V}\cdot\text{sec}$, $E = 0.5 \text{ V/m}$, $V_d = ?$

	$\therefore \mu = \frac{V_d}{E}$ $\therefore V_d = \mu E$ $\therefore V_d = (3.5 \times 10^{-3}) (0.5)$ $\therefore V_d = 1.75 \times 10^{-3} \text{ m/sec}$
Q. 4	The density of silver $10.5 \times 10^3 \text{ kg m}^{-3}$ assuming that each silver atom provides one conduction electron. The conductivity of silver at 20°C is $6.8 \times 10^7 \Omega^{-1} \text{ m}^{-1}$. Calculate the density and mobility of electron in silver with atomic weight 107.9.
Ans.	<p>Density of silver $d = 10.5 \times 10^3 \text{ kg m}^{-3}$ Conductivity of silver at $20^\circ\text{C} = 6.8 \times 10^7 \Omega^{-1} \text{ m}^{-1}$ Atomic weight $A = 107.9$</p> <p>We know the carrier concentration (n) = $\frac{\text{Avagadro Number} \times \text{Density}}{\text{Atomic weight}}$</p> $\therefore n = \frac{6.023 \times 10^{23} \times 10.5 \times 10^3}{107.9}$ $\therefore n = 5.86 \times 10^{25} \text{ m}^3$ <p>We know, the conductivity is given by $\sigma = ne\mu$</p> $\therefore \mu = \frac{\sigma}{ne}$ $\therefore \mu = \frac{6.8 \times 10^7}{5.86 \times 10^{25} \times 1.6 \times 10^{-19}}$ $\therefore \mu = 7.25 \times 10^{-3} \text{ m}^2 \text{V}^{-1} \text{s}^{-1}$

1.4.4 Expression for Thermal conductivity

The thermal conductivity is defined as the amount of heat flowing through a unit area per unit temperature gradient.

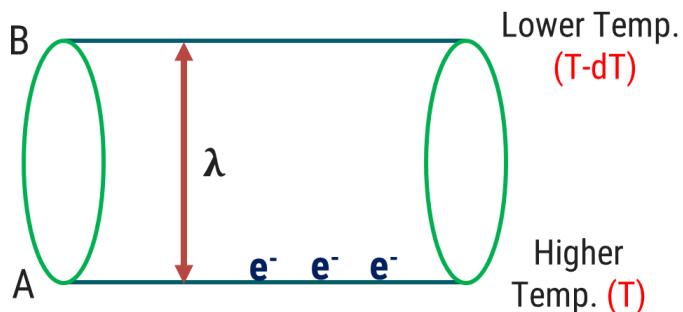
$$\therefore K = -\frac{\Delta Q}{A \left(\frac{dT}{dx}\right)} \frac{\text{Watt}}{\text{meter} \cdot \text{Kelvin}}$$

The negative sign indicates that heat flows hot end to cold end.

ΔQ = rate of heat flow (thermal energy)

A = Cross-sectional area of conductor

(dT/dx) =temperature gradient



$$\text{Average kinetic energy at } A = \frac{3}{2} K_B T$$

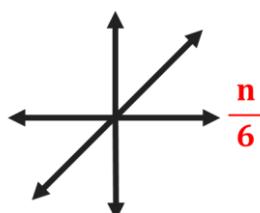
$$\text{Average kinetic energy at } B = \frac{3}{2} K_B (T - dT)$$

The excess energy carried by electrons in traveling from point A to B

$$\frac{3}{2} K_B T = \frac{3}{2} K_B (T - dT)$$

$$= \frac{3}{2} K_B dT$$

The electrons travel in all directions crossing unit area from A to B



$$\text{i. e. } \frac{1}{6} nv$$

So, excess energy transferred from A to B per unit area in unit time is:

$$\begin{aligned} & \frac{1}{6} nv \times \frac{3}{2} K_B dT \\ &= \frac{1}{4} nv K_B dT \end{aligned}$$

Similarly, deficiency of energy from B to A per unit time is:

$$= -\frac{1}{4} nv K_B dT$$

The rate of heat flow can be defined as the net energy transferred from A to B per unit area per unit time:

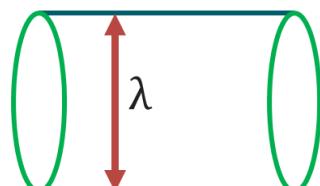
$$\begin{aligned} \therefore \Delta Q &= \frac{1}{4} nv K_B dT - \left(-\frac{1}{4} nv K_B dT \right) \\ \therefore \Delta Q &= nv K_B dT \left(\frac{1}{4} + \frac{1}{4} \right) \\ \therefore \Delta Q &= \frac{1}{2} nv K_B dT \end{aligned}$$

We know thermal conductivity k is defined as:

$$\therefore k = \frac{\Delta Q}{\left(\frac{dT}{dx} \right)}$$

Here, dx can be written as mean free path (λ)

$$\therefore k = \frac{\frac{1}{2} nv K_B dT}{\left(\frac{dT}{\lambda} \right)}$$



$$\text{Thermal conductivity, } k = \frac{1}{2} nv K_B \lambda$$

1.4.5 Wiedemann – Franz law

This law states that the ratio of thermal conductivity to electrical conductivity is directly proportional to the absolute temperature.

$$\text{i. e. } \frac{K}{\sigma} \propto T$$

$$\therefore \frac{K}{\sigma} = L T$$

L = Lorentz number

$$\therefore K = \frac{1}{2} n V K_B \lambda$$

$$\therefore \sigma = \frac{n e 2 \lambda V}{3 K_B T}$$

By, taking ratio

$$\therefore \frac{K}{\sigma} = \frac{\frac{1}{2} n V K_B \lambda}{\frac{n e 2 \lambda V}{3 K_B T}}$$

$$\therefore \frac{K}{\sigma} = \frac{3}{2} \left(\frac{K_B}{e} \right)^2 T$$

$$\therefore \frac{K}{\sigma} = L T$$

Theoretical value of L: $1.12 \times 10^{-8} \text{ W}\Omega/\text{K}^2$

Experimental value of L: $2.44 \times 10^{-8} \text{ W}\Omega/\text{K}^2$

Theoretical and experimental values of L doesn't match with each other.

So, Assumption is wrong that all electrons contributes in thermal conductivity.

1.4.6 Success of free electron theory

- ✓ It verifies ohm's law.
- ✓ It explains the thermal and electrical conductivities of a metal.
- ✓ It helps to deduce Wiedemann – Franz law.
- ✓ It explains optical properties of metals.

1.4.7 Drawbacks of classical free electron theory

- ✓ Theoretical value of specific heat and electronic specific heat of metal based is $\left(\frac{3R}{2}\right)$ is not in agreement with the experimental value ($10^{-4} RT$).
- ✓ The classical free electron theory can't explain electrical conductivity of insulators and semiconductors.
- ✓ According to classical theory, $\frac{K}{\sigma} = T$ is constant at all temperature. But, this is not constant at low temperature.
- ✓ It failed to explain the superconductivity and magnetic susceptibility (χ).
- ✓ The phenomena such as photoelectric effect, Compton Effect and black body radiation can't be explained.
- ✓ According to classical free electron theory, the electrical conductivity (σ) proportional to the free electron density (n). But it failed to explain for different metals.

Q.1	<p>Find the thermal conductivity of copper 20°C with a free electron density of $8.48 \times 10^{28}/\text{m}^3$. The thermal velocity of copper is $1.1536 \times 10^5 \text{ m/sec}$ at 20°C, with a mean free path of 2.813 nm.</p>
Ans.	$n = 8.48 \times 10^{28}/\text{m}^3$, $\lambda = 2.813 \text{ nm} = 2.183 \times 10^{-9} \text{ m}$ $v = 1.1536 \times 10^5 \text{ m/sec}$ $K = ?$ We know, $\therefore K = \frac{1}{2} n V K_B \lambda$ $\therefore K = \frac{1}{2} (8.48 \times 10^{28}) \times (1.1536 \times 10^5) \times (1.38 \times 10^{-23}) \times (2.183 \times 10^{-9})$ $\therefore K = 189.92 \text{ W/m} \cdot \text{K}$

1.5 Quantum free electron theory

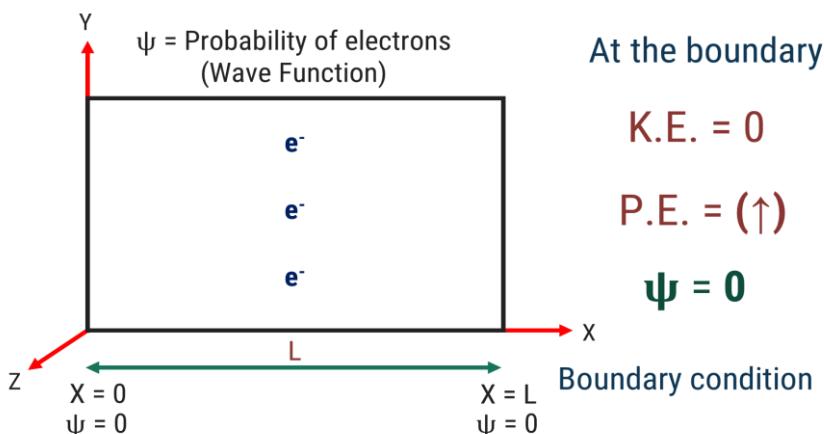
The failure of classical free electron theory paved this way for Quantum free electron theory. It was introduced by Sommerfeld in 1928. This theory is based on making small concepts. This theory was proposed by making small changes in the classical free electron theory and by retaining most of the postulates of the classical free electron theory.

1.5.1 Assumptions (Postulates) of Quantum free electron

1. In a metal the available free electrons are fully responsible for electrical conduction.
2. The electrons move in a constant potential inside the metal. They cannot come out from the metal surface have very high potential barrier.
3. Electrons have wave nature, the velocity and energy distribution of the electron is given by Fermi – Dirac distribution function.
4. The loss of energy due to interaction of the free electron with the other free electron.
5. Electron's distributed in various energy levels according to Pauli Exclusion Principle.

Quantum free electron theory

- ✓ Each metal contains a very large number of electrons and these electrons can move freely inside the metal.
- ✓ Inside of the metal material uniform potential is applied:
 - Constantly moving electrons has only kinetic energy (K.E.) inside the metal while the potential energy (P.E.) is zero.
- ✓ P.E. = 0 can be only possible when,
 - Interaction of free electron with other electron and metallic core of the metal is negligible.



K.E. inside the material is not sufficient to overcome metal boundary.

- ✓ In quantum approach electrons moves in wave form and wave nature of particle is given by Schrodinger.
- ✓ Schrodinger wave equation for 3-dimension:

$$\therefore \frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{2m}{\hbar^2} [E - V]\psi = 0$$

- ✓ Schrodinger wave equation for 1-dimension:

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V]\psi = 0 \dots \dots \dots (1)$$

E = total energy, V = potential energy, m = mass of electron

$$\hbar = \text{modified Planck's constant } \left(\hbar = \frac{h}{2\pi} \right)$$

In our case P.E. = 0, so, total energy (E) of the system is because of only K.E.

$$\therefore \frac{d^2\psi}{dx^2} + \left(\frac{2m}{\hbar^2} E \right) \psi = 0$$

$$\text{Consider, } \left(\frac{2m}{\hbar^2} E \right) = k^2$$

$$\therefore \frac{d^2\psi}{dx^2} + k^2\psi = 0 \dots \dots \dots (2)$$

- ✓ To identify complementary function of ψ :

$$\therefore \psi = [A \cos kx + B \sin kx] \dots \dots \dots (3) \text{ (remember)}$$

Where, k = wave vector, A & B are constants

In order to find the values of A & B we can use two boundary condition at X = 0 and

X = L.

For, (1) X = 0 $\rightarrow \psi = 0$ condition

$$\therefore 0 = [A \cos k0 + B \sin k0]$$

$$\therefore 0 = [A (1) + 0]$$

$$\therefore A = 0$$

Putting this value in equation (3)

$$\therefore \psi = B \sin kx \dots \dots \dots (4)$$

For, (2) $X = L \rightarrow \psi = 0$ condition

$$\therefore \psi = B \sin kL$$

Here, B can't be zero because than whole equation will be zero.

$$\therefore \psi = [A \cos kx + B \sin kx] \dots \dots \dots (3)$$

If ψ becomes zero than probability of having electron inside the metal becomes zero, which is not possible.

This problem can be satisfied by taking $\sin kL$ zero instead of B.

$\sin kL = 0$ only possible at $(\sin n\pi)$ Where, $n = 0, 1, 2, 3, \dots$

So, if we compare $kL = n\pi$ then:

$$\therefore k = \frac{n\pi}{L}$$

Therefore, the wave function will be valid only if:

$$\therefore \psi(x) = B \sin \left(\frac{n\pi}{L} x \right) \dots \dots \dots (5)$$

$$\therefore k^2 = \frac{n^2\pi^2}{L^2}$$

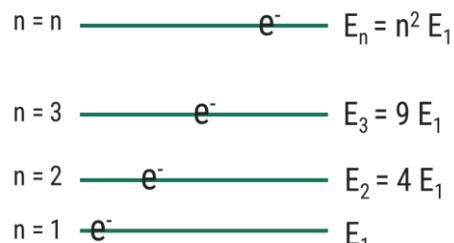
By, comparing the values of k^2

$$\frac{2m}{\hbar^2} E = \frac{n^2\pi^2}{L^2}$$

$$\therefore E_n = \frac{n^2\pi^2\hbar^2}{L^2 2m}$$

$$\therefore E_n = \frac{n^2\pi^2}{L^2 2m} \frac{\hbar^2}{4\pi^2}$$

$$\therefore E_n = \frac{n^2\hbar^2}{8mL^2} \dots \dots \dots (6)$$



According to this equation, electrons will have nth level of energy (E_n) inside the metal.

Value of constant B in equ. 5 obtained by applying the normalization condition to get the value of wave function ψ .

- ✓ Normalizing the wave function means you find the exact form of probability (ψ) of the particle which found somewhere in space is 1.

$$\therefore \int_{-\infty}^{\infty} \Psi^2 dx = 1$$

$$\therefore \int_0^L B^2 \sin^2 \left(\frac{n\pi x}{L} \right) dx = 1$$

$$\therefore \frac{B^2}{2} \int_0^L 1 - \cos \left(\frac{2\pi nx}{L} \right) dx = 1$$

$$\therefore \frac{B^2}{2} \int_0^L 1 dx - \int_0^L \cos \left(\frac{2\pi nx}{L} \right) dx = 1$$

$$\therefore \psi(x) = B \sin \left(\frac{n\pi}{L} x \right) \dots \dots \dots (5)$$

$$\therefore \frac{B^2}{2} [x]_0^L - \left[\sin \frac{2\pi nx}{L} \right]_0^L = 1$$

$$\therefore \frac{B^2}{2} [x]_0^L - \frac{L}{2\pi n} \left[\sin \frac{2\pi nx}{L} \right]_0^L = 1$$

$$\therefore \frac{B^2}{2} [L] - \frac{L}{2\pi n} \left[\sin \frac{2\pi nL}{L} - \sin \frac{2\pi n0}{L} \right]_0^L = 1$$

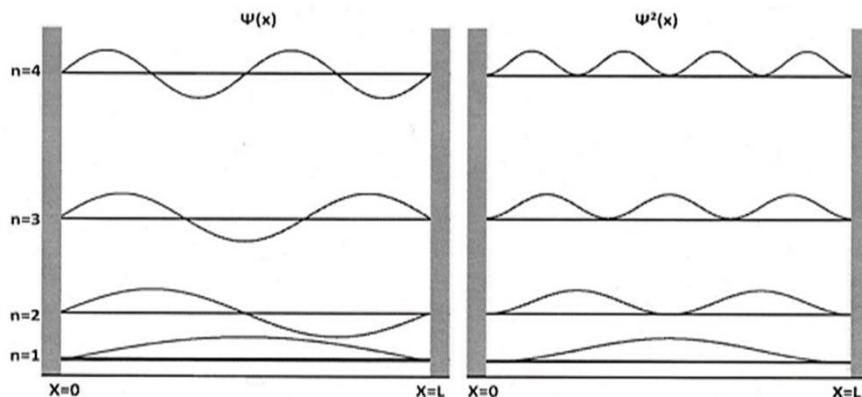
$$\therefore \frac{B^2}{2} [L] = 1$$

Putting this value in equation (5)

$$\therefore \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L}x\right) \dots \dots \dots (7)$$

This equation gives the value of probability of electron in a confined system.

The wave functions corresponding to four lowest states for different values of n and probability of finding the particle are shown in fig.



1.5.2 Advantages of quantum free electron theory

- ✓ It explains the specific heat of materials.
- ✓ It explains Compton Effect, photoelectric effect, blackbody radiation, Zeeman Effect.
- ✓ It gives the correct mathematical expression for the thermal conductivity of metals.
- ✓ It explains the superconductivity.

1.5.3 Drawbacks of quantum free electron theory

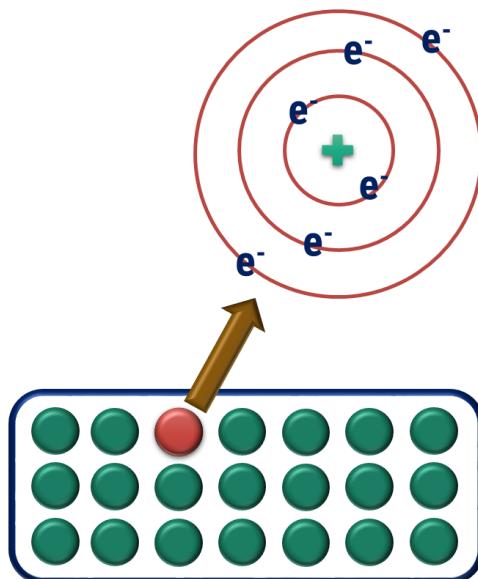
- ✓ This theory fails to explain the positive value of Hall coefficients.
- ✓ It failed to distinguish metal, semiconductor and insulator.
- ✓ It failed to explain lower conductivities of divalent metals than monovalent metals.

1.6 Band Theory or (Energy Band Diagrams)

(a) Formation of energy bands in solids:

- ✓ In a single isolated atom, the electrons in each orbit have definite energy associated with it.

- ✓ In case of solids, the atoms are close to each other, so the energy levels of outermost orbit electrons are affected by the neighbouring atom.
- ✓ When two single or isolated atoms are brought close to each other, the outermost orbit electrons of the two atoms interact with each other.
- ✓ I.e. The electrons in the outermost orbit of one atom experiences an attractive force from the neighbouring atom nucleus.



- ✓ As a result of this, the energy of electrons is no more the same, the energy levels are changed to a value which is higher or lower than the original energy level.
- ✓ The Pauli's exclusive principle allows each energy level to contain only two electrons.
"This grouping of different energy levels is called energy bands".
- ✓ The energy level of the inner orbit electrons are not much affected by presence of neighbouring atoms.

(b) Energy bands in solids:

Valence band:

- ✓ "The energy band formed by grouping the energy levels of valence electrons is called valence band".
- ✓ Electrons in valence band have lower energy than electrons in conduction band.

- ✓ The electrons in valence band are loosely bound to nucleus of atom, they can be moved to conduction band by applying energy.

Forbidden band:

- ✓ “The energy gap that is present between the valence band and conduction band by separating the two energy band is called the forbidden band or forbidden gap.”
- ✓ Electrons can't stay in this band as there is no allowed energy state in this gap.
- ✓ The energy associated with the forbidden gap is called the bandgap energy.

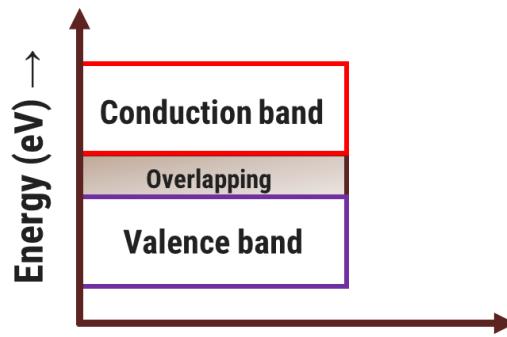
Conduction band:

- ✓ “The energy gap or band formed by grouping the energy levels of free electrons is called conduction band.”
- ✓ It is the uppermost band with free electrons, electrons have higher energy than electron in valence band.
- ✓ The band is completely empty for insulators and partially filled for conductors.
- ✓ The conduction band electrons are not bound to the nucleus of atom.

1.6.1 Types of electronic materials: conductor, Insulators, and semiconductors

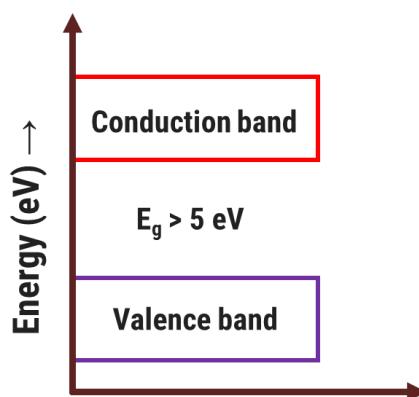
Conductors:

- ✓ “The materials which easily allow the flow of electric current are called conductors.” e.g. copper, iron, silver, etc.
- ✓ The resistivity is of an order of $10^{-9} \Omega\text{m}$ at room temperature.
- ✓ In a conductor, the conduction and valence band overlap each other as shown in figure.
- ✓ A small amount of energy provides enough energy for valence band electrons to move in conduction band.



Insulators:

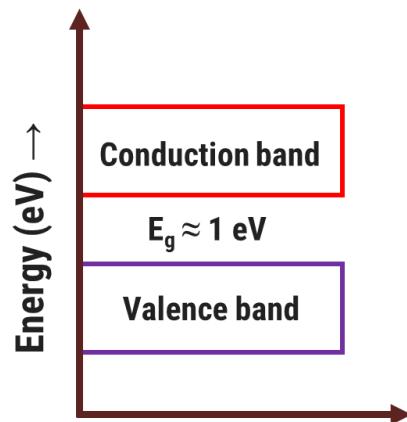
- ✓ “The materials which do not allow the flow of electric current through them are called insulators.”
- ✓ The valence band is completely filled with electrons. The forbidden energy gap is greater than 5 eV.
- ✓ The electrons in valence band can't move to conduction as they are locked up between the atoms.
- ✓ It requires large amount of external energy to move the electrons to conduction band.
- ✓ The resistivity is of approx. 10^3 to 10^{17} Ωm at room temp.



Semi-conductors:

- ✓ “The materials which have electrical conductivity between that of conductors and insulators are called semiconductors.” e.g. Silicon, germanium.
- ✓ The forbidden gap in semiconductors is very small (approx. 1.1 eV).

- ✓ At low temp., valence band is completely filled with electrons and conduction band is completely empty as electrons don't have sufficient energy to move in conduction band.
- ✓ As temp. Increases, electrons gain energy and move to conduction band. I.e. electrical conductivity increases with temperature.
- ✓ So, they have negative temperature coefficient of resistance.



1.6.2 Kroning Penney model

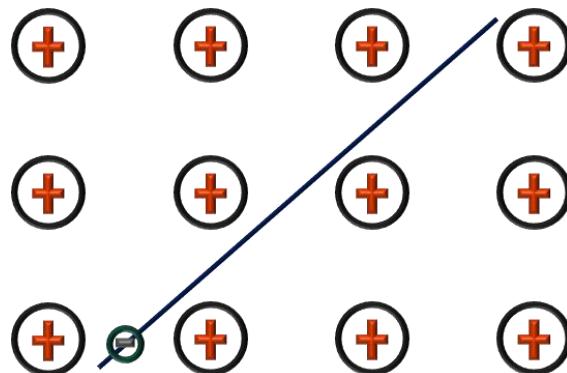
- ✓ It is a simplified model for an electron in one dimensional periodic potential.
- ✓ It demonstrates that a simple one dimensional periodic potential yields energy bands as well as energy bandgaps.
- ✓ It is an instructive tool to demonstrate how band structure can be calculated for periodic potential and how allowed and forbidden energies are obtained using Schrodinger's equation.
- ✓ It also helps in understanding the concept of Effective mass and E-K diagrams.

According to free electron theory:

- ✓ Homogeneous distribution of potential (voltage) electrons in metal. But it doesn't explain it for semiconductor and insulators.

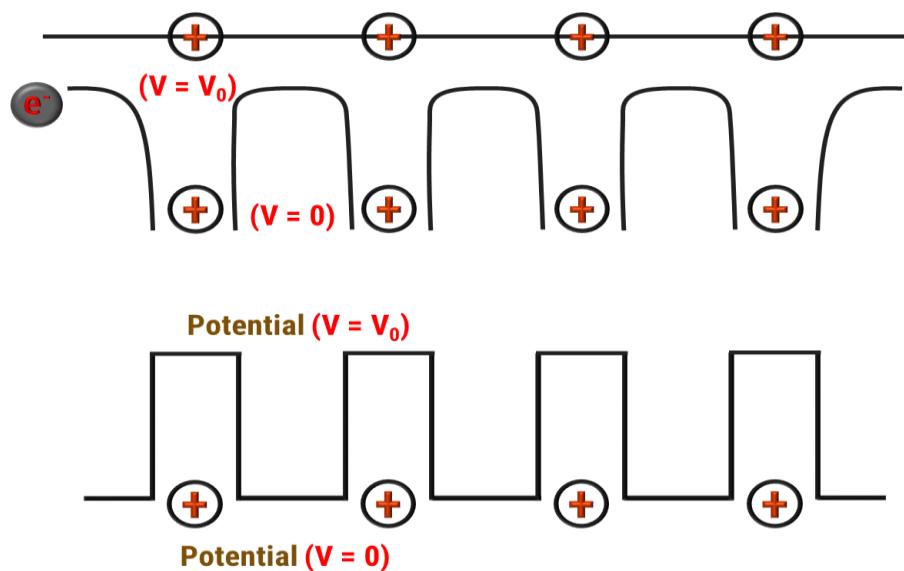
From Band theory:

- ✓ Bloch suggested that: Potential of electrons varies periodically with lattice.
- ✓ Because inside the crystal, there is periodic arrangement positively charged ions, through which the electrons move.

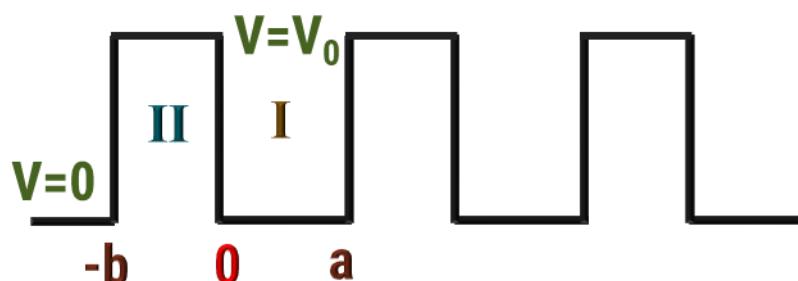


Kroning and Penney suggested:

- ✓ The potential of electron at the positive ion site is zero and is maximum in between the two (i.e. middle of two ions).
- ✓ It was difficult to solve Schrodinger equation with varying potential.
- ✓ Kroning-Penney suggested that take potential as shape of rectangular steps.
- ✓ This model is called Kroning-Penney model.



- ✓ The wave functions associated with this model can be calculated by solving Schrodinger equations for the two regions I and II. (I = Potential well & II = Potential barrier).
- ✓ Two consecutive potential wells are separated by a distance "b" (regions II) and the value of the potential in each of these regions is "V₀".
- ✓ Therefore, potential of electron for the two regions can be expressed as v(x) = 0 for region-I and v(x) = V₀ for region-II.



The Schrodinger's equations are:

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E \psi = 0 \dots \text{(for region - I)} \quad 0 < x < a \dots \dots \dots \text{(1)}$$

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V_0] \psi = 0 \dots \text{(for region - II)} \quad -b < x < 0 \dots \dots \dots \text{(2)}$$

- ✓ Now let us assume that the system is at room temperature and electrons do not acquire sufficient thermal energy to escape from the crystal. Let us define two real quantities α and β such that:

$$\alpha^2 = \frac{2mE}{\hbar^2} \quad \text{and} \quad \beta^2 = -\frac{2m(E - V_0)}{\hbar^2} \dots \dots \dots \text{(3)}$$

$$\therefore \frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0 \dots \text{(for region - I)} \quad 0 < x < a \dots \dots \dots \text{(4)}$$

$$\therefore \frac{d^2\psi}{dx^2} - \beta^2 \psi = 0 \dots \text{(for region - II)} \quad -b < x < 0 \dots \dots \dots \text{(5)}$$

- ✓ The solution (according to Bloch theorem) that will be appropriate for both the regions (I & II) will be of the form of a plane wave e^{ikx} modulated with a periodic function $u_k(x)$:

$$\therefore \psi(x) = e^{ikx} u_k(x)$$

Where $k = \frac{2\pi}{\lambda}$ is propagation vector or wave vector

- ✓ Using Bloch theorem and applying all boundary conditions for region (I & II) for continuity of the wave function, solution of Schrodinger equation is obtained as:

$$\therefore P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \dots \dots \dots (6) \text{ where, } P = \frac{mV_0 ab}{\hbar^2} \text{ (Scattering power)}$$

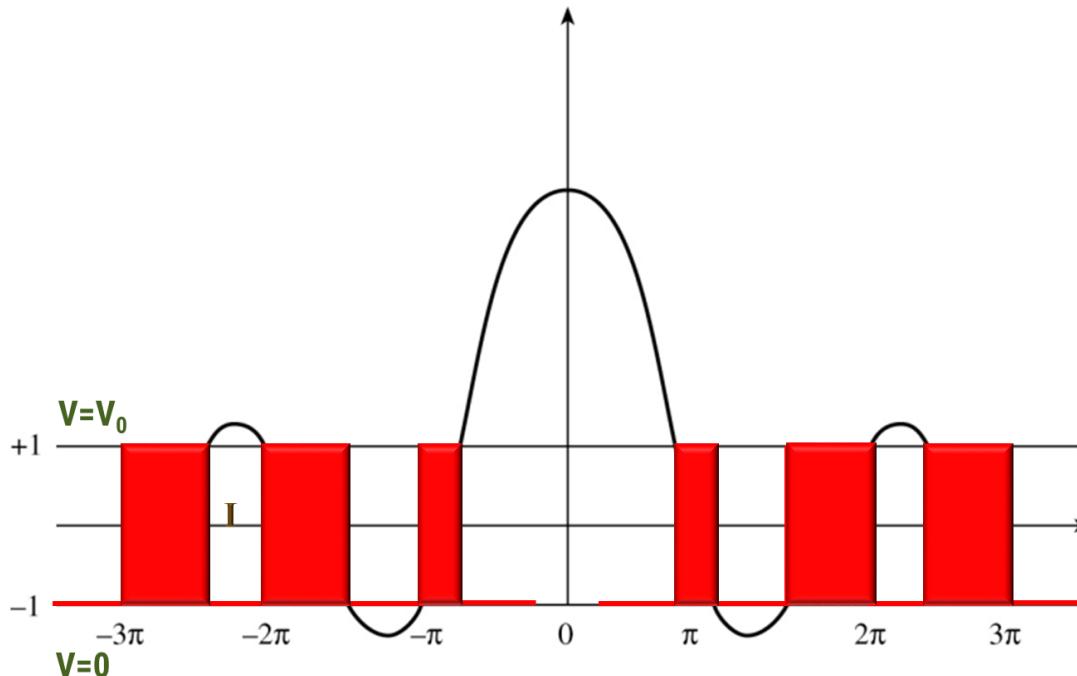
The term $V_0 b$ is called barrier strength.

$$\therefore \cos ka = \pm 1$$

$$\therefore ka = \pm n\pi \quad (n = 1, 2, 3, \dots)$$

$$\therefore k = \frac{\pm n\pi}{a}$$

Graph of Kronig Penny Model:



Findings or conclusions from Kronig – Penny Model:

- ✓ The permissible limit of the term $P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a$ lies between +1 and -1.
- ✓ The shaded portion of the wave shows the bands of allowed energy.
- ✓ By increasing αa , allowed energy state for electron increases, thereby increase in band-width.
- ✓ Increase in band-width leads to distance between electrons and total energy.

Case (I): $P \rightarrow 0$

$$\therefore \cos \alpha a = \cos ka$$

$$\therefore \alpha a = ka$$

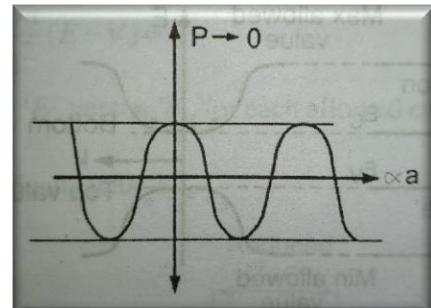
$$\therefore \alpha = k$$

$$\therefore \alpha^2 = k^2 \quad \text{By using equation (3)}$$

$$\therefore k^2 = \frac{2mE}{\hbar^2}$$

$$\therefore E = \frac{k^2 \hbar^2}{2m} \dots \dots (7)$$

$$\therefore E = \frac{P^2}{2m} \dots \dots (8)$$



- ✓ Above equation shows the energy of free electron in conduction band.
- ✓ Energy band is broadened and it is continuous.

Case (II): $P \rightarrow \infty$

$$\therefore P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \dots \dots (6)$$

$$\therefore \sin \alpha a = \frac{\alpha a (\cos ka - \cos \alpha a)}{P}$$

$\therefore \sin\alpha = 0$ sin becomes zero at $n\pi$

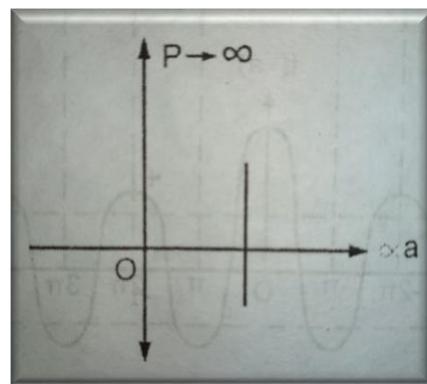
$$\therefore \alpha a = n\pi$$

$$\therefore \alpha^2 = \frac{n^2\pi^2}{a^2}$$

$$\therefore \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2}$$

$$\therefore E = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

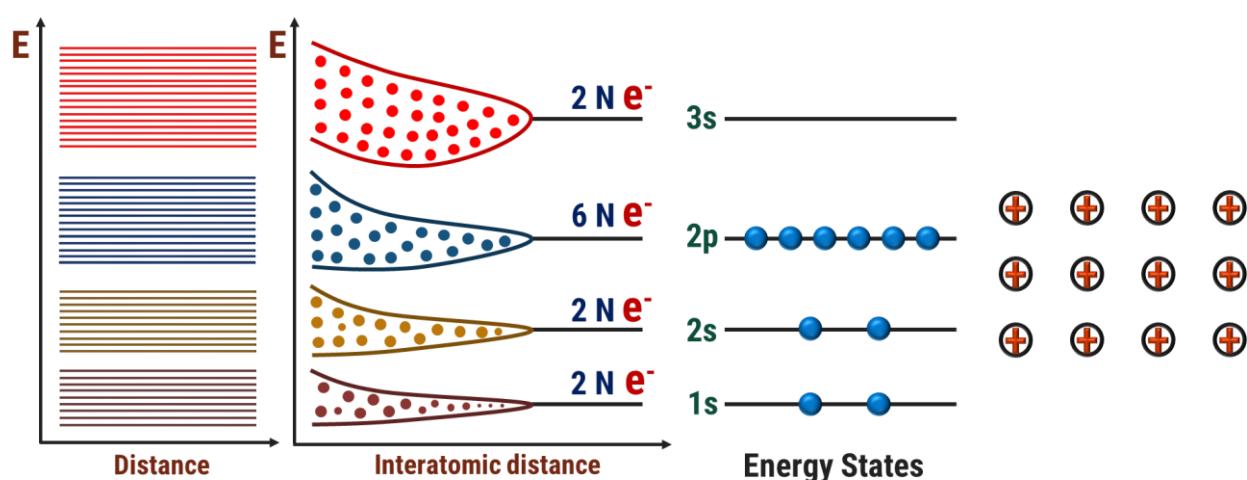
$$\therefore E = \frac{n^2\hbar^2}{8ma^2}$$



- ✓ If P is large, it results in decrease of allowed energy and increase if forbidden gap.
- ✓ The allowed states are compressed to a line spectrum.

1.6.3 Energy band structure of a solid

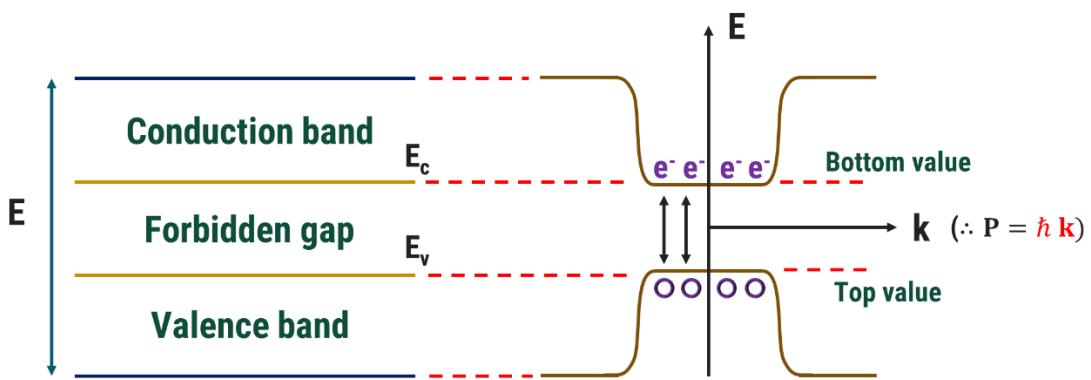
- ✓ A crystal consists of large number of atoms arranged in a regular periodic structure. The extent of energy level splits in the solids depends on the nearness of atoms.
- ✓ Let us assume N identical atoms forms a crystal.



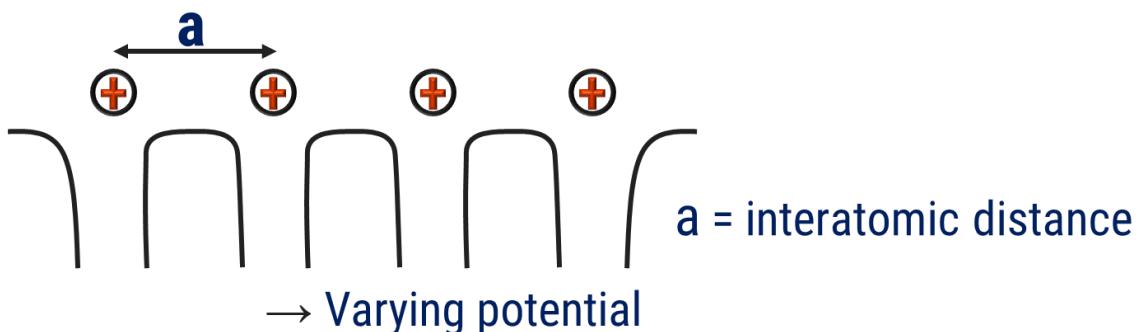
- ✓ The energy levels of the isolated atoms are shown in fig, All the N atoms have identical sets of energy levels.
- ✓ Electron fill the energy levels in each atom independently.
- ✓ As the atoms approaches, a continuous increasing interaction occurs between the atoms.
- ✓ Each of the energy level splits into many distinct level and forms energy bands, as shown in fig.
- ✓ While occupying energy band, electrons starts from the lowest energy level in the band and fill the levels one after another.
- ✓ When $2N$ electron occupy the N levels available in the band, band is said to be completely filled.
- ✓ In case on non-availability of $2N$ electrons, energy band gets **partially filled**.
- ✓ When there are no electrons to occupy the levels, the band remains **vacant**.
- ✓ At room temperature, the kinetic energy of the electron is of the order of $k_B T$ ($\approx 0.026\text{eV}$) which very large is compared to the energy level separation in allowed band.
- ✓ Electrons can easily move into higher vacant levels within the allowed energy band either due to thermal energy or due to small externally applied electric field.
- ✓ The following two conditions are to be fulfilled for electrical conduction to take place in solid.
 1. There should be free electrons available in the solid.
 2. Vacant energy levels should be available above the levels occupied by free electrons.

1.6.4 E – K diagram

“An E – K diagram is a dispersion diagram that shows the relationship between energy and momentum of available states of electrons in a material.”



- ✓ From, the above diagram, we see the bottom value of conduction band corresponding to 'E_c' and top value of valence band corresponds to 'E_v'.
- ✓ From, the E – K diagram, it can be seen that the holes which are closed to the top valence band and electron which are close to the bottom of conduction band, participate in the transition processes.



- ✓ As all atoms are arranged periodically in lattice, we can say that semiconductors are crystalline in nature.
- ✓ As per Bloch's theorem, as the potential varies periodically, probability of finding an electron also varies periodically.

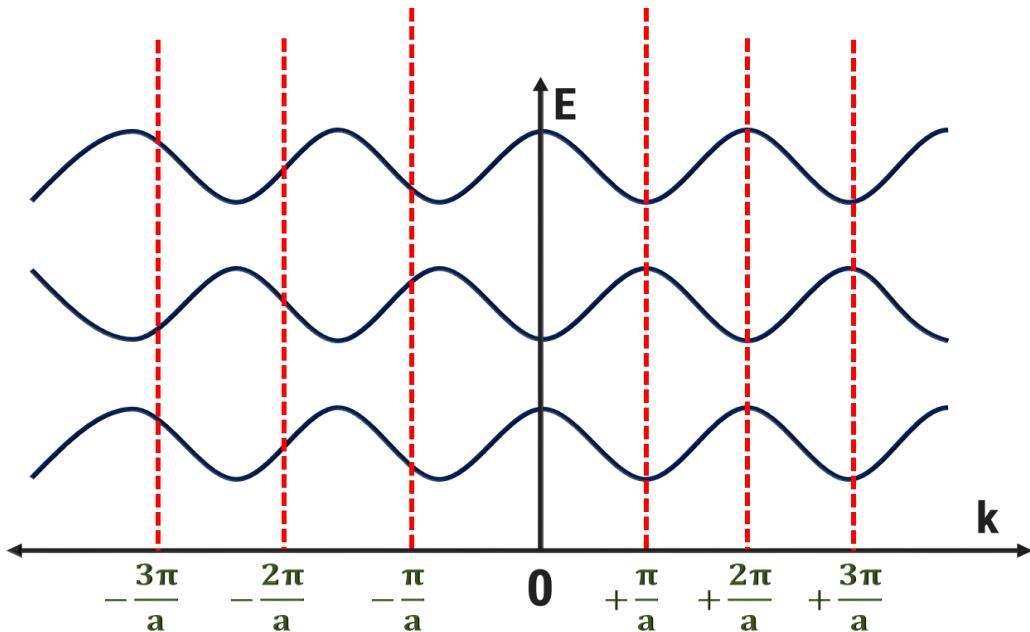
i. e. $\psi_k(r) = u_k(r) e^{ikx}$

e^{ikx} = plane function & $u_k(r)$ periodic function

Using this value in Schrodinger's equation:

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V]\psi = 0$$

- Here, Schrodinger's equation gives periodic graph of 'E' versus 'k' for each allowed energy value.

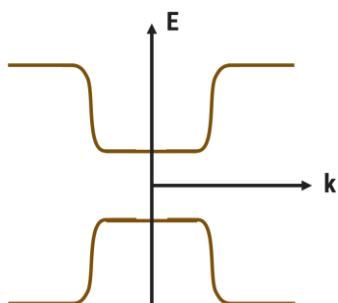


- The discrete energy value of k is in the form of $\pm \frac{\pi}{a}, \pm \frac{2\pi}{a}, \pm \frac{3\pi}{a}, \dots$
- The area between the $+\frac{\pi}{a}$ to $-\frac{\pi}{a}$ is known as **1st Brillouin zone**, and between $+\frac{2\pi}{a}$ to $-\frac{2\pi}{a}$ is known as **2nd Brillouin zone**.

Here this periodic arrangement can be understood by three zones.

- Periodic Zone
- Reduced Zone
- Extended Zone

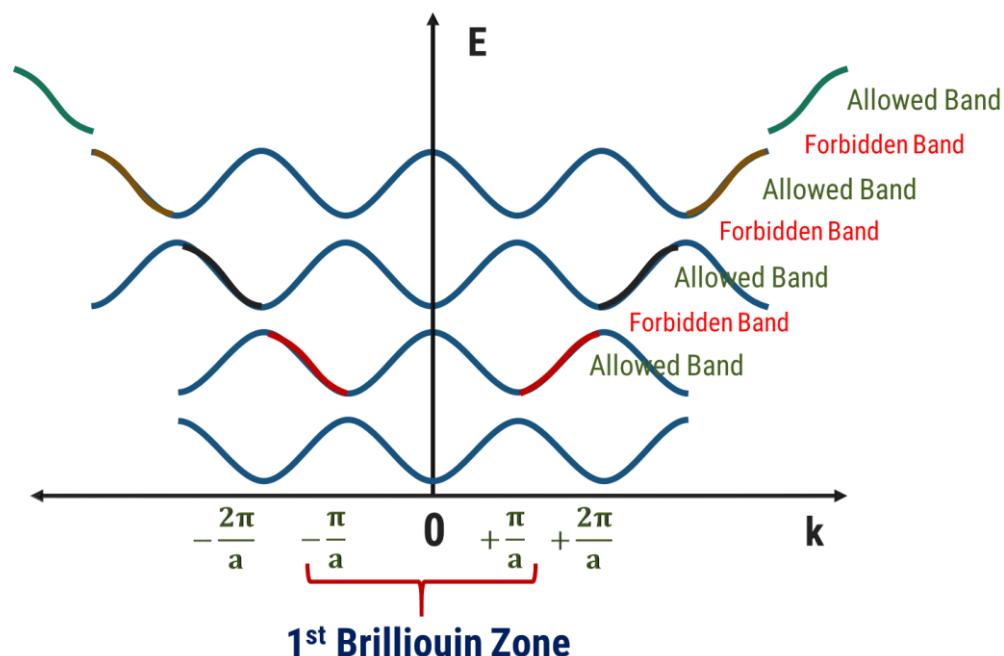
As the variation of E Vs k is periodic, we can restrict ourself to only one period. I.e $+\frac{\pi}{a}$ to $-\frac{\pi}{a}$. The final reduced zone diagram is as below.



(According to De – Broglie's theorem, $P = \hbar k$)

Energy of the particle is given by: $E = \frac{p^2}{2m}$, $E = \frac{\hbar^2 k^2}{2m}$

Extended Zone:



- ✓ From the graph, it is clear that electrons have allowed energy values in the region or zone extending from $k = -\pi/a$ to π/a . This is called first Brillouin zone. After a break in energy values, called forbidden band, the electrons have another allowed zone of energy values in the region extended from $k = -\pi/a$ to $-2\pi/a$ and from $k = \pi/a$ to $2\pi/a$. This zone is called the second Brillouin zone.
- ✓ Similarly, the higher order Brillouin zones can be defined.

Q.1	Consider two-dimensional square lattice of side 3.0 Å. At what electron momentum values do the sides of first Brillouin zone appear? What is the energy of free electron with this momentum?
Ans.	We know, $p = \hbar k$ Here, \hbar = Reduced Plank constant k = wave number

p = momentum
 a = Side of the lattice

For 1st Brillouin zone

$$k = \pm \frac{\pi}{a}$$

$$\therefore p = \hbar \frac{\pi}{a} = \frac{h}{2a}$$

$$\therefore p = \frac{6.6 \times 10^{-34}}{2 \times 3 \times 10^{-10}} = 1.1 \times 10^{-24} \text{ kg} \cdot \text{m/sec}$$

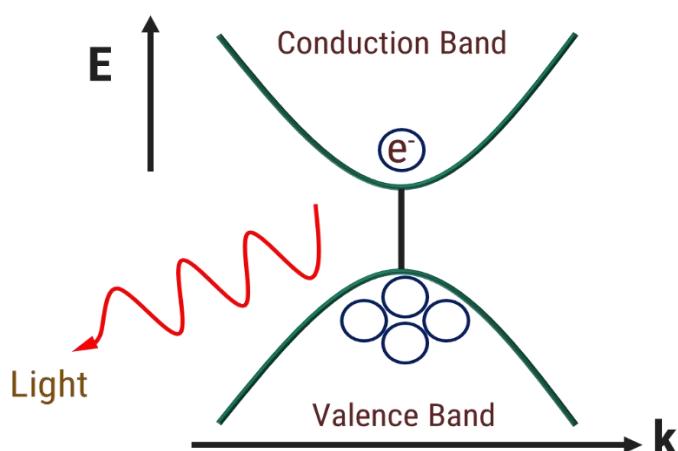
$$\text{Therefore, Energy (E)} = \frac{p^2}{2m}$$

$$\therefore E = \frac{(1.1 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}} = 6.6 \times 10^{-19} \text{ Joule}$$

$$\therefore E = \frac{6.6 \times 10^{-19}}{1.6 \times 10^{-19}} = 4.2 \text{ eV}$$

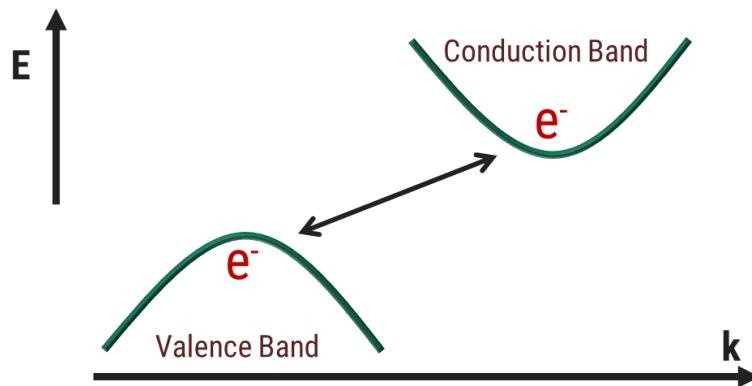
1.6.5 Direct and Indirect bandgap semiconductors as per E - K Diagram

Direct bandgap semiconductor:



- ✓ A direct band gap semi-conductor is the one in which maximum energy level of valence band aligns with the minimum energy level of conduction band with respect to momentum. e.g. GaAs
- ✓ In a semi-conductor, the minimum energy state in conduction band and maximum energy state in valance band is characterized by crystal momentum and wave vector 'k' (propagation constant).
- ✓ In a direct bandgap semiconductor, the k-vectors are same for conduction band minima and valence band maxima.
- ✓ Whenever an electron from conduction band recombines with a hole from valence band, the energy and momentum, both should be conserved.
- ✓ The energy difference between conduction band and valence band is released in the form of a photon.
- ✓ The k-vectors of electron and hole are same. So, we say that momentum is also conserved.
- ✓ We say the energy is conserved by emitting a photon.

Indirect bandgap semiconductor:



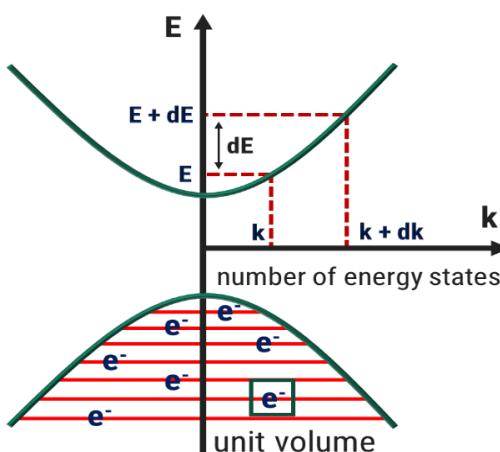
- ✓ An indirect bandgap semiconductor is the one in which maximum energy level of valence band and minimum energy level of valence band and minimum energy level of conduction band are not aligned with respect to momentum. e.g., Si, Ge.
- ✓ In an indirect bandgap semiconductor, the k-vectors are different for conduction band minima and valence band maxima.
- ✓ In an indirect bandgap semiconductor, there is a difference in momentum. The recombination process can occur only after the momentum align.

- ✓ Due to crystal imperfections in certain semiconductors, the electron in the conduction band loses energy and momentum at the same time.
- ✓ The electron actually passes through an intermediate state and transfers its momentum to the crystal lattice.
- ✓ In this case, the energy is emitted in the form of heat. The transition is non-radiative in nature.

1.6.6 Difference between direct and Indirect bandgap semiconductors

DBG semiconductor	IBG semiconductor
It is one in which maximum energy level of valence band aligns with the minimum energy level of conduction band with respect to momentum.	It is the one in which maximum energy level of valence band and minimum energy level of valence band and minimum energy level of conduction band are not aligned with respect to momentum.
In this direct recombination takes place with energy equal to the difference between energy of recombining particles.	In this due to a difference in momentum, first momentum is conserved by release of energy and only when the two momenta are aligned, recombination occurs.
The probability of radiative recombination is very high.	The probability of radiative recombination is almost negligible.
Efficiency factor is high.	Efficiency factor is low.
They are preferred for making optical devices like LED's	Cannot be used to make optical device.

1.6.7 Density of states



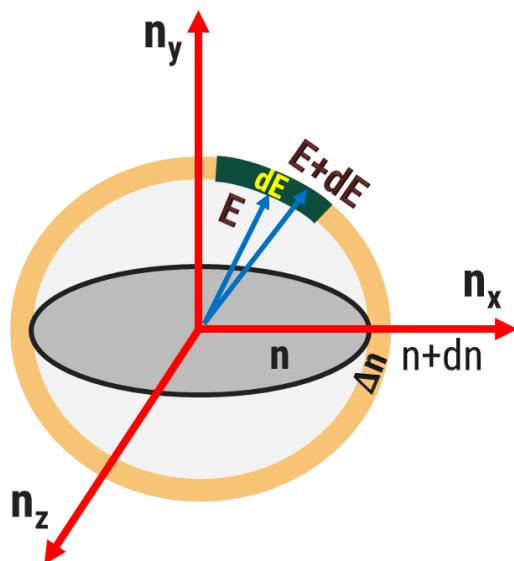
- ✓ Density of states: The number of energy states per unit volume of a metal.
- ✓ It is used to calculate the number of charge carriers per unit volume of any solid.

$$N(E)dE = \frac{\text{Number of energy states between } E \text{ and } E + dE}{\text{Volume of the metal}}$$

$$N(E)dE = \frac{D(E)dE}{V} \dots \dots \dots (1)$$

Density of states $N(E)dE$ is defined as the number of available electron states per unit volume in an energy interval (dE).

Derivation:



- ✓ Let us consider a sphere of radius "n" in space with quantum numbers n_x, n_y, n_z .
- $$n^2 = n_x^2 + n_y^2 + n_z^2$$
- ✓ The sphere is further divided into many shells represents a particular combination of quantum numbers and represents particular energy value.
- ✓ Let us consider two energy values E and $E+dE$ can be found by finding the number of energy states between the shells of radius n and $n+dn$ from the origin.
- ✓ Since the quantum numbers are positive integers, n values can be defined only in the positive octant of the n -space.

The number of energy states within a sphere: $n = \frac{4}{3}\pi n^3$

- ✓ The number of available energy states within the sphere of radius "n" due to one octant.

$$\therefore n = \frac{1}{8} \left[\frac{4}{3} \pi n^3 \right]$$

- ✓ Similarly the number of available energy states within the sphere of radius $n+dn$ corresponding energy.

$$\therefore n + dn = \frac{1}{8} \left[\frac{4}{3} \pi (n + dn)^3 \right]$$

The number of available energy states between the shells of radius n and $n + dn$ (or) between the energy levels E and $E + dE$.

$$\therefore D(E)dE = \frac{1}{8} \left[\frac{4}{3} \pi (n + dn)^3 - \frac{4}{3} \pi n^3 \right]$$

$$\therefore D(E)dE = \frac{1}{8} \left[\frac{4}{3} \pi (n^3 + dn^3 + 3n^2dn + 3dn^2n - n^3) \right]$$

- ✓ Since the higher powers of dn is very small, dn^2 and dn^3 terms can be neglected.

$$\therefore D(E)dE = \frac{1}{8} \left[\frac{4}{3} \pi (3n^2dn) \right] \dots \dots \dots (2)$$

$$\therefore D(E)dE = \frac{\pi n^2 dn}{2} \dots \dots \dots (3)$$

- ✓ We know that allowed energy values is

$$\therefore E = \frac{n^2 h^2}{8 m a^2}$$

$$\therefore n^2 = \frac{8 m a^2 E}{h^2} \dots \dots \dots (4)$$

$$\therefore n = \sqrt{\frac{8 m a^2 E}{h^2}} \dots \dots \dots (5)$$

Differentiating equation (4) w.r.t 'n'

$$\therefore n \, dn = \frac{8 m a^2 \, dE}{2h^2} \dots \dots \dots (6)$$

Substituting (5) & (6) in equation (3)

$$\therefore D(E)dE = \frac{\pi}{2} n \cdot n \, dn$$

$$\therefore D(E)dE = \frac{\pi}{2} \left(\frac{8 m a^2 E}{h^2} \right)^{\frac{1}{2}} \left(\frac{8 m a^2}{2h^2} \right) dE$$

$$\therefore D(E)dE = \frac{\pi}{4} \left(\frac{8 m a^2}{h^2} \right)^{\frac{1}{2}} \left(\frac{8 m a^2}{h^2} \right)^1 E^{\frac{1}{2}} dE$$

$$\therefore D(E)dE = \frac{\pi}{4} \frac{(8 m a^2)^{\frac{3}{2}}}{(h^2)^{\frac{3}{2}}} E^{\frac{1}{2}} dE$$

$$\therefore D(E)dE = \frac{\pi}{4 h^3} (8 m)^{\frac{3}{2}} a^3 E^{\frac{1}{2}} dE$$

✓ Volume of the metal, $V = a^3$

$$\therefore D(E)dE = \frac{\pi}{4 h^3} (8 m)^{\frac{3}{2}} V E^{\frac{1}{2}} dE$$

✓ For unit volume of metal $V = 1m^3$

$$\text{From equation (1)} \therefore N(E)dE = \frac{D(E)dE}{V}$$

$$\therefore N(E)dE = \frac{\frac{\pi}{4 h^3} (8 m)^{\frac{3}{2}} V E^{\frac{1}{2}} dE}{V}$$

$$\text{Density of states} \therefore N(E)dE = \frac{\pi}{4 h^3} (8 m)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

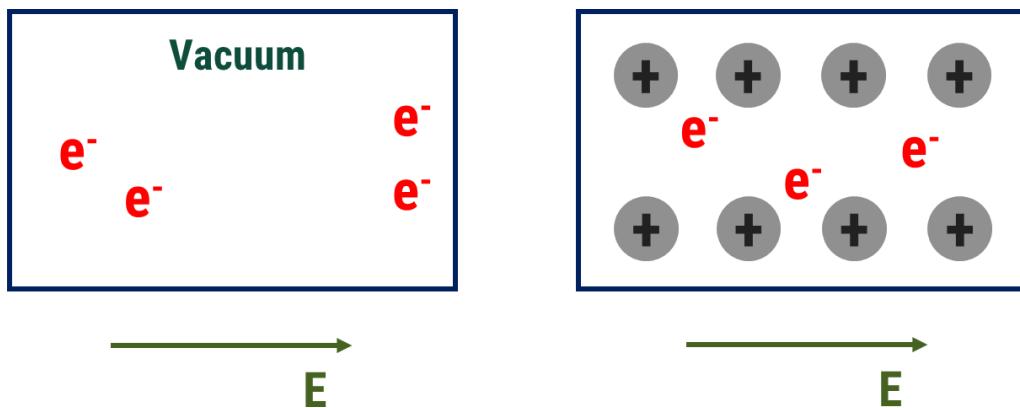
✓ Each electron energy level can accommodate two electrons as per Pauli's exclusion principle. (Spin up and Spin down = 2 (e) × density of states).

$$\therefore N(E)dE = 2 \times N(E)dE$$

$$\therefore N(E)dE = \frac{\pi}{2 h^3} (8 m)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

1.6.8 Effective mass of electron

- ✓ Experimentally measured values shows that for some solids the mass is larger, while incase of other solids it is slightly smaller then free electron mass.
- ✓ This experimentally determined electron mass is known as effective mass (m^*).



$$\therefore F_{\text{ext}} = m a$$

$$\therefore a = \frac{F_{\text{ext}}}{m}$$

$$\therefore F = F_{\text{ext}} + F_{\text{internal}}$$

$$\therefore F_{\text{ext}} = m^* a$$

$$\therefore a = \frac{F_{\text{ext}}}{m^*}$$

- ✓ Consider mass of electron as effective mass (m^*).
- ✓ Consider mass as per quantum mechanical approach.
- ✓ Then we can neglect internal force in the crystal system, where atoms having periodic arrangement.
- ✓ So, Periodic potential will be applied on each electrons.
- ✓ We see that same magnitude of electric field (E) is applied to both electrons (In vacuum and inside the crystal), but the electron accelerates at a different rate inside the crystal, due to varying (periodic) potential inside the crystal.
- ✓ The electron mass is altered in this case due to the varying potential.
- ✓ "This altered mass is called effective mass of electron."

Derivation:

- Let E be the electric field applied to the electron inside the crystal.

The force experienced by it is

$$\therefore F_{\text{ext}} = m^* a$$

- According to De-Broglie hypothesis electron behaves like wave. Here, velocity acquired by electron is known as group velocity (V_g):

$$\therefore V_g = \frac{d\omega}{dk}, \text{ Where, } \omega = \text{angular frequency}$$

$$\therefore V_g = \frac{d}{dk} 2\pi \frac{E}{h} \dots \dots \dots (1)$$

- differentiating equ. (1) w.r.t (t)

$$\therefore \frac{dV_g}{dt} = \frac{2\pi}{h} \frac{d}{dt} \left(\frac{dE}{dk} \right)$$

$$\therefore a = \frac{1}{\hbar} \frac{d}{dt} \left(\frac{dE}{dk} \right)$$

$$\therefore a = \frac{1}{\hbar} \frac{d}{dk} \left(\frac{dE}{dt} \right) \quad (\text{Energy and Time are dependent term, that's why we exchange it.})$$

$$\therefore a = \frac{1}{\hbar} \frac{d}{dk} \left(\frac{dE}{dk} \frac{dk}{dt} \right) \dots \dots \dots (2)$$

$$\therefore P = \hbar k$$

- differentiate above equ. w.r.t (t)

$$\therefore \frac{dP}{dt} = \frac{d}{dt} \hbar k$$

$$\therefore F = \hbar \frac{dk}{dt}$$

$$\therefore \frac{dk}{dt} = \frac{F}{\hbar} \dots \dots \dots (3)$$

- ✓ Substitute this value in equ. (2)

$$\therefore a = \frac{1}{\hbar} \frac{d}{dk} \left(\frac{dE}{dk} \frac{F}{\hbar} \right)$$

$$\therefore a = \frac{F}{\hbar^2} \frac{d^2 E}{dk^2}$$

$$\therefore F = \left[\frac{\hbar^2}{\frac{d^2 E}{dk^2}} \right] a \dots \dots \dots (4)$$

- ✓ Comparing equ. (4) with $F = m a$

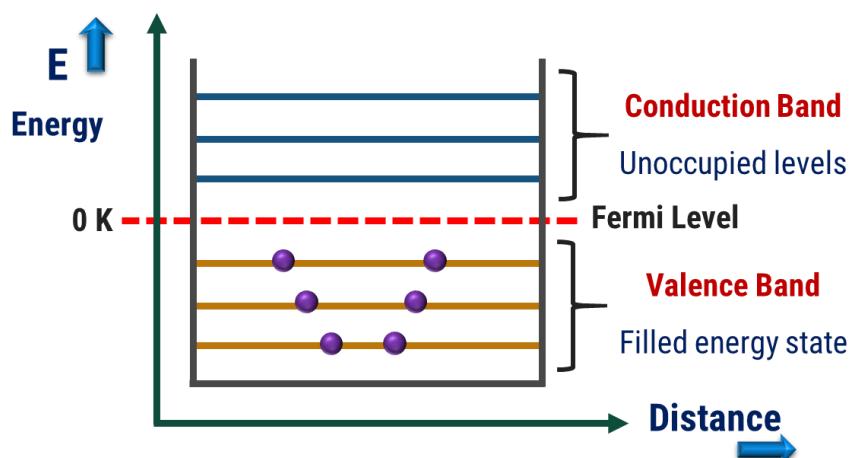
$$\therefore m^* = \frac{\hbar^2}{\frac{d^2 E}{dk^2}} \quad \text{Expression of effective mass of an electron.}$$

- ✓ It is clear from the above equation that effective mass of an electron depends on E – K diagram.

1.6.9 Fermi level and fermi function

Fermi level: “ The highest energy level which an electron can occupy at absolute zero (0 K) or (- 273°C) temperature is called fermi level.”

Fermi energy: “ The energy possessed by electrons in the fermi level at absolute zero temperature is called fermi energy.”



Fermi function:

Also known as **Fermi – Dirac distribution function**.

Fermi distribution function is used for half integer spin particles (fermions) like electrons, which obey Pauli's exclusive principle.

Statement: It is an expression for the distribution of electrons among the energy levels as a function of temperature, the probability of finding an electron in a particular energy state of energy E is given by:

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

Where, $f(E)$ = Fermi function

E_f = Fermi energy

T = Absolute temperature (0 K)

k_B = Boltzmann constant

- ✓ From above equation, we can say that a system is characterized by its temperature and fermi energy (E_f).
- ✓ For a filled energy level $f(E) = 1$ and for unfilled level $f(E) = 0$.

Case I: At T = 0 K

- ✓ At $T = 0$ K, Electron occupy lowest energy first, followed by next higher levels as per Pauli's exclusive principle.

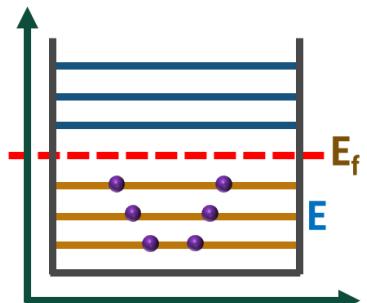
(a) At T = 0 K, $E < E_f$

For energy level E, lying below E_f , $(E - E_f)$ makes negative value.

$$\therefore f(E) = \frac{1}{1 + e^{-\infty}} = \frac{1}{1 + 0} = 1$$

$$\therefore f(E) = 1$$

- ✓ This implies that all energy levels below E_f are completely filled.



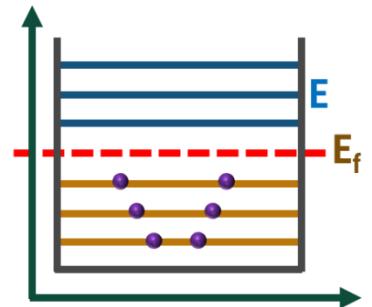
(b) At T = 0 K, E > E_f

For energy level E, lying above E_f, (E - E_f) makes positive value.

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

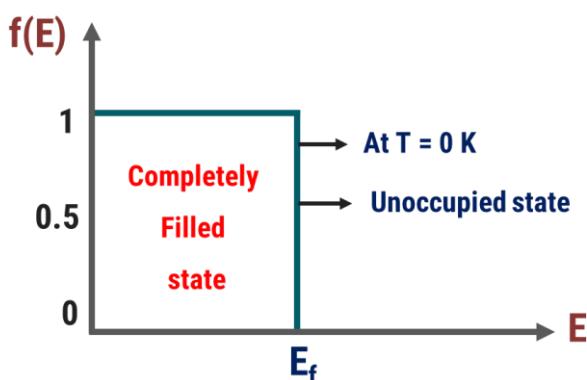
$$\therefore f(E) = \frac{1}{1 + e^\infty} = \frac{1}{1 + \infty} = 0$$

$$\therefore f(E) = 0$$



- ✓ This implies that all energy levels above E_f are completely Vacant.
- ✓ There is no possibility of electron occupying an energy level above E_f.

Variation of f (E) at T = 0 K

**Case II: At T > 0 K**

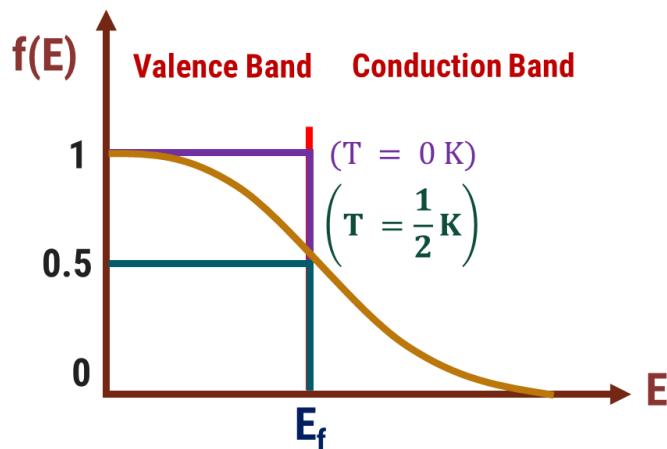
- ✓ At room temperature, the probability starts reducing from 1 for values of E close to E_f, but larger than E_f.
- ✓ At T > 0 K, for E = E_f, exponential function becomes zero.

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

$$\therefore f(E) = \frac{1}{1 + e^0} = \frac{1}{1 + 1} = \frac{1}{2}$$

$$\therefore f(E) = 0.5$$

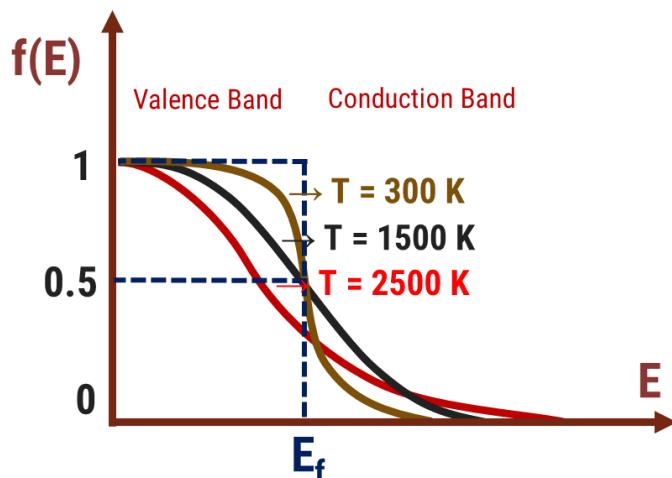
- ✓ f(E) = 0.5 implies that probability of occupancy of an electron is 50% at any temperature above 0 K.



- ✓ For $E = E_f$, the value of $f(E)$ falls off to zero rapidly as shown in the figure above.

Case III: At $T = \text{very high temperature}$

- ✓ At room temperature, it can be seen from the figure that the transition between completely filled states and completely empty states is rather gradual than abrupt.
- ✓ i.e. $f(E)$ changes from 1 to 0 more gradually.



- ✓ As temperature (T) increases, electron may get an energy of an order kBT and go to the higher vacant state.
- ✓ Relation between fermi energy E_f , fermi velocity V_f temperature T_f and mean free path λ is given by:

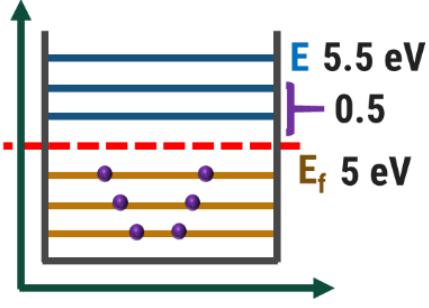
Fermi velocity: $V_f = \sqrt{\frac{2E_f}{m}}$

Fermi temperature: $T_f = \frac{E_f}{k_B}$

The mean free path: $\lambda = \tau V_f$

Q.1	Evaluate the Fermi function for energy $k_B T$ above the Fermi energy.
Ans.	<p>$E - E_F = k_B T, \quad f(E) = ?$</p> <p>We know Fermi Function $\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{k_B T}\right)}}$</p> <p>For an energy $K_B T$ above Fermi energy:</p> $E - E_F = k_B T$ $\therefore f(E) = \frac{1}{1 + e^1} = \frac{1}{1 + 2.78}$ <p>Fermi distribution function $f(E) = 0.269$</p>
Q.2	Calculate the fermi velocity and mean free path for conduction electrons, given that its fermi energy is 11.63 eV and relaxation time for electrons is 7.3×10^{-15} sec.
Ans.	<p>$E_f = 11.63 \text{ eV} = 11.63 \times 1.6 \times 10^{-19} \text{ Joule},$</p> <p>$\tau = 7.3 \times 10^{-15} \text{ sec},$</p> <p>$V_f = ?,$</p> <p>$\lambda = ?$</p> $\therefore V_f = \sqrt{\frac{2 E_f}{m}}$

	$\therefore V_f = \sqrt{\frac{2 \times (11.63 \times 1.6 \times 10^{-19})}{9.11 \times 10^{-31}}}$ $\therefore V_f = \sqrt{4.085 \times 10^{12}}$ $\therefore V_f = 2.02 \times 10^6 \text{ m/sec}$ <p>The mean free path: $\lambda = \tau V_f$</p> $\therefore \lambda = \tau V_f$ $\therefore \lambda = (7.3 \times 10^{-15}) (2.02 \times 10^6)$ $\therefore \lambda = 1.47 \times 10^{-8} \text{ m}$ $\therefore \lambda = 14.75 \text{ nm}$
Q.3	Calculate the fermi energy and fermi temperature in a metal. The fermi velocity of electrons in the metal is $0.86 \times 10^6 \text{ m/sec}$.
Ans.	$V_f = 0.86 \times 10^6 \text{ m/sec}$ $E_f = ?$ $T_f = ?$ $E_f = \frac{1}{2} m V_f^2$ $\therefore E_f = \frac{1}{2} (9.11 \times 10^{-31}) (0.86 \times 10^6)^2$ $\therefore E_f = 3.36 \times 10^{-19} \text{ Joule}$ $\therefore E_f = \frac{3.36 \times 10^{-19}}{1.6 \times 10^{-19}}$ $\therefore E_f = 2.105 \text{ eV}$

	<p>Fermi temperature: $T_f = \frac{E_f}{k_B}$</p> $\therefore T_f = \frac{3.36 \times 10^{-19}}{1.38 \times 10^{-23}}$ $\therefore T_f = 24.41 \times 10^3 \text{ K}$
Q.4	<p>Using fermi function, evaluate the temperature at which there is 1% probability that an electron in a solid will have an energy 0.5 eV above E_f of 5 eV.</p>
Ans.	<p>$E = 5.5 \text{ eV}$ $E_f = 5 \text{ eV}$ $f(E) = 1\% = 0.01$ $T = ?$</p> $\therefore E - E_f = 5.5 - 5 = 0.5 \text{ eV}$ $\therefore E - E_f = 0.5 \times (1.6 \times 10^{-19}) \text{ J}$ $\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$ $\therefore f(E) \left(1 + e^{\left(\frac{E - E_f}{k_B T}\right)} \right) = 1$ $\therefore f(E) + f(E) e^{\left(\frac{E - E_f}{k_B T}\right)} = 1$ $\therefore f(E) e^{\left(\frac{E - E_f}{k_B T}\right)} = 1 - f(E)$ $\therefore e^{\left(\frac{E - E_f}{k_B T}\right)} = \frac{1 - f(E)}{f(E)}$ $\therefore e^{\left(\frac{E - E_f}{k_B T}\right)} = \frac{1 - f(E)}{f(E)}$ <p>Taking logarithm on both side</p> 

$$\therefore \frac{E - E_f}{k_B T} = \ln \left[\frac{1 - f(E)}{f(E)} \right]$$

$$\therefore \frac{E - E_f}{k_B T} = \ln(1 - f(E)) - \ln(f(E))$$

$$\therefore \frac{1}{k_B T} = \frac{\ln(1 - f(E)) - \ln(f(E))}{E - E_f}$$

$$\therefore k_B T = \frac{E - E_f}{\ln(1 - f(E)) - \ln(f(E))}$$

$$\therefore T = \frac{E - E_f}{k_B [\ln(1 - f(E)) - \ln(f(E))]}$$

$$\therefore T = \frac{0.5 \times (1.6 \times 10^{-19})}{1.38 \times 10^{-23} [\ln(1 - 0.01) - \ln(0.01)]}$$

$$\therefore T = \frac{8 \times 10^{-20}}{(1.38 \times 10^{-23}) [-0.01005 - (-4.6051)]}$$

$$\therefore T = \frac{8 \times 10^{-20}}{6.341 \times 10^{-23}}$$

$$\therefore T = 1261.597$$

$$\therefore T = 1.261 \times 10^3 \text{ K}$$

Q.5	Find the probability with which an energy level 0.02 eV above fermi level will be occupied at room temperature of 300 K and at 1000 K.
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Ans.	$E - E_f = 0.02 \text{ eV} = 0.02 \times 1.6 \times 10^{-19} \text{ J}$ Probability of occupancy at 300 K
------	--

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right)}}$$

$$\therefore f(E) = \frac{1}{1 + e^{(0.7729)}}$$

$$\therefore f(E) = \frac{1}{1 + 2.166}$$

$$\therefore f(E) = 0.315$$

Probability of occupancy at 1000 K

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

$$\therefore f(E) = \frac{1}{1 + e^{\left(\frac{0.02 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 1000}\right)}}$$

$$\therefore f(E) = \frac{1}{1 + e^{(0.2318)}}$$

$$\therefore f(E) = \frac{1}{1 + 1.2609}$$

$$\therefore f(E) = 0.442$$

1.6.10 Carrier concentration in metals

- ✓ Number of electrons per unit volume can be obtained by multiplying density of states with probability function.

$$n_c = f(E) \times N(E)dE$$

$N(E)dE$ = density of states

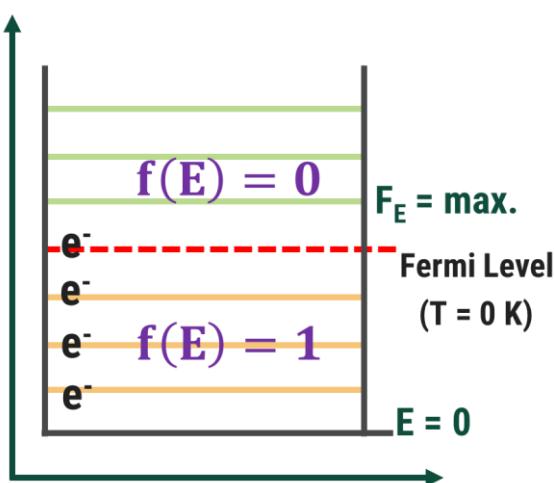
$f(E)$ = fermi function

$$\therefore n_c = f(E) \times \frac{\pi}{2 h^3} (8 m)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

$$\therefore n_c = \frac{\pi}{2 h^{(3 \times \frac{2}{2})}} (8 m)^{\frac{3}{2}} E^{\frac{1}{2}} dE \times f(E)$$

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8 m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE \times f(E)$$

Calculation of fermi energy at T = 0 K



- ✓ We will calculate the energy state in the order of increasing energy starting from $E = 0$ to the highest filled state E_f , at $T = 0 \text{ K}$.

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8 m}{h^2}\right)^{\frac{3}{2}} E^{\frac{1}{2}} dE \times f(E)$$

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} \int_0^{E_F} E^{\frac{1}{2}} dE \times f(E)$$

At T = 0 K, f(E) = 1

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} \int_0^{E_F} E^{\frac{1}{2}} dE \times f(E)$$

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} \int_0^{E_F} E^{\frac{1}{2}} dE \times f(E)$$

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} \left[\frac{E^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^{E_F}$$

$$\therefore n_c = \frac{\pi}{2} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} \frac{2}{3} E_F^{\frac{3}{2}}$$

$$\therefore n_c = \frac{\pi}{3} \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} E_F^{\frac{3}{2}} \dots \dots \dots (1)$$

Re-arranging the equation 1, we can define fermi energy

$$\therefore \left(\frac{3n_c}{\pi} \right) = \left(\frac{8m}{h^2} \right)^{\frac{3}{2}} E_F^{\frac{3}{2}} \dots \dots \dots (2)$$

Multiply equ.2 with power 2/3

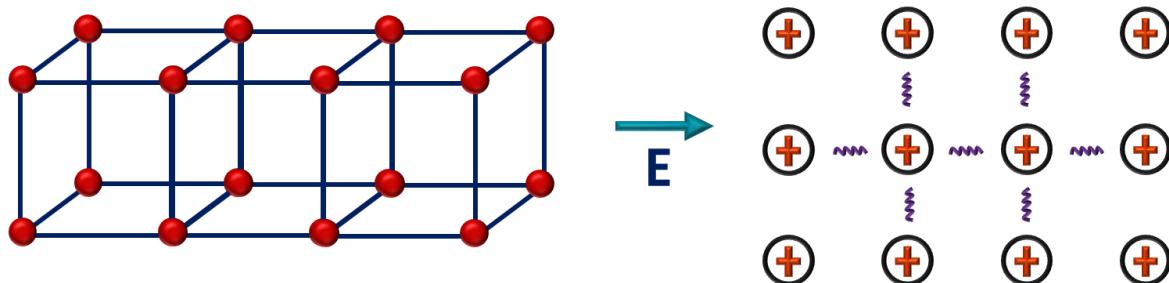
$$\therefore \left(\frac{3n_c}{\pi} \right)^{\frac{2}{3}} = \left(\frac{8m}{h^2} \right)^{\frac{3}{2} \times \frac{2}{3}} E_F^{\frac{3}{2} \times \frac{2}{3}}$$

$$\therefore E_F = \left(\frac{h^2}{8m} \right) \left(\frac{3n_c}{\pi} \right)^{\frac{2}{3}}$$

Q.1	Calculate the fermi energy for sodium. Given atomic weight 23 gm/mole and density of sodium 0.97 gm/cm ³ .
Ans.	<p>Atomic weight = 23 gm/mole = 0.023 Kg/mole = 23×10^{-3} Kg/mole Density of sodium = $0.971 \text{ gm/cm}^3 = 0.971 \times 10^3 \text{ Kg/m}^3$ $E_F = ?$</p> <p>Charge carrier concentration (n_c) = $\frac{\text{No. of atoms}}{\text{Volume}}$</p> <p>$\therefore n_c = \frac{\text{Avegadro no.} \times \text{density}}{\text{Atomic weight}}$</p> $\therefore n_c = \frac{(6.023 \times 10^{23}) \times (0.971 \times 10^3)}{23 \times 10^{-3}}$ $\therefore n_c = 2.541 \times 10^{28} \text{ atoms/m}^3$ $\therefore E_F = \left(\frac{h^2}{8m} \right) \left(\frac{3n_c}{\pi} \right)^{\frac{2}{3}}$ $\therefore E_F = \left(\frac{(6.62 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31}} \right) \left(\frac{3 \times 2.541 \times 10^{28}}{3.14} \right)^{\frac{2}{3}}$ $\therefore E_F = (6.022 \times 10^{-38}) (2.427 \times 10^{28})^{\frac{2}{3}}$ $\therefore E_F = (6.022 \times 10^{-38}) (8.38 \times 10^{28})$ $\therefore E_F = 5.049 \times 10^{-19} \text{ J}$

1.6.11 Phonon

- ✓ A solid crystal consists of atoms bound into a specific three-dimensional pattern called lattice.
- ✓ The atoms behaves as if they are connected by tiny springs and so their own thermal energy (internal energy present in a system by virtue of temperature) or outside forces makes the lattice vibrate.
- ✓ This generates mechanical waves that carry heat and sound through the material.
- ✓ A packet of these waves can travel throughout the crystal with definite energy and momentum.
- ✓ These waves are treated as a particle called “phonons”.



- ✓ Phonons acts like bosons, they can be created or destroyed in collisions.
- ✓ Phonons exist with discrete amount of energy, given by

$$E = \hbar \omega = h \nu$$

Where, $\omega/2\pi$ = frequency of vibrations, h = plank constant, ν = frequency

- ✓ Phonons can carry heat and sound and they play a major role in determine heat capacities of solid and liquids.

UNIT - 2 (Semiconductors)

2.1 Introduction of semiconductors:

- ✓ A semiconductor has electrical conductivity between that of a conductor and an insulator. Semiconductors differ from metals in their characteristic property of decreasing electrical resistivity with increasing temperature. Semiconductors can also display properties of passing current more easily in one direction than the other, and sensitivity to light.
- ✓ Because the conductive properties of a semiconductor can be modified by controlled addition of impurities or by the application of electrical fields or light, semiconductors are very useful devices for amplification of signals, switching, and energy conversion. The comprehensive theory of semiconductors relies on the principles of quantum physics to explain the motions of electrons through a lattice of atoms.
- ✓ Semiconductors are the foundation of modern electronics, including radio, computers, and telephones. Semiconductor-based electronic components include transistors, solar cells, many kinds of diodes including the light-emitting diode (LED), the silicon controlled rectifier, photo-diodes, digital analog integrated circuits. Increasing understanding of semiconductor materials and fabrication processes has made possible continuing increases in the complexity and speed of semiconductor devices, an effect known as Moore's Law.

2.2 Properties of semiconductor

- ✓ The resistivity of semiconductors lies between a conductor and an Insulator. (It varies from 10^{-4} to $0.5 \Omega m$).
- ✓ At 0 K it behave as insulator.
- ✓ They have negative temperature Coefficient of resistance. (when the temperature is increased large number of charge carriers are produced due to breaking of covalent bonds and hence these electrons move freely and gives rise to conductivity)
- ✓ In semiconductors, both electrons and holes are charge carriers.
- ✓ If we increase the temperature of semiconductor, its electrical conductivity also increases.
- ✓ They have an empty conduction band and almost filled valence band 0 K.
- ✓ They are formed by a covalent bonds.

- ✓ They have small energy gap (or) band gap.
- ✓ Semiconductors are material having electrical conductivity considerably greater than that of an insulator but significantly lower than that of a conductor.
- ✓ Germanium (Ge) and Silicon (Si) are Elemental semiconductors and are widely used in semiconductor devices.
- ✓ Gallium Arsenide (GaAs), Indium Phosphide (InP), Cadmium Sulphide (CdS), etc are known as **compound semiconductors**.

- ✓ These compound semiconductors which are formed from the combinations of the elements of groups III and V [Gallium phosphide (GaP), Gallium arsenide (GaAs), Indium phosphide (InP) Indium arsenide (InAs)] or group II and VI [Magnesium oxide (MgO), Magnesium silicon (MgSi) Zinc oxide (ZnO), Zinc sulphide (ZnS)] and are widely used in fabrication of optoelectronic devices. Such as LASER, LEDs etc...
- ✓ Semiconductors consist two charge carriers, namely electrons and holes, for conduction.
- ✓ The electrical conductivity of a pure semiconductor is significantly low and not be used in device fabrication.
- ✓ **These pure semiconductors are known as intrinsic semiconductors.**

- ✓ Through the technique of doping, the conductivity of a semiconductor can be increased in magnitude to a desire value for conduction.
- ✓ **Doped semiconductors are known as extrinsic semiconductors.**

- ✓ The remarkable feature of these extrinsic semiconductors is that current is transported in them by two different charge carriers , electrons and holes, and through two different processes , drift and diffusion.
- ✓ Extrinsic semiconductors are widely used in fabrication of solid state devices.
P-N junction diode, Transistors, Capacitors, Integrated circuits, etc.

2.3 Types of semiconductor

They are classified on the basis of type of energy emission:

(1) Direct and indirect bandgap semiconductor:

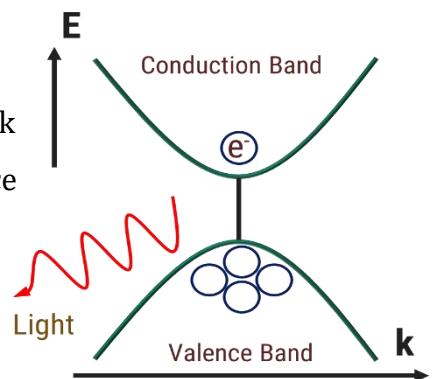
(2) Intrinsic and Extrinsic semiconductor:

(1) Direct and indirect bandgap semiconductor:

(a) Direct bandgap semiconductor:

- ✓ In this type of semiconductor, when an excited electron falls back into valance band, the electron and holes recombine to produce light (release a photon).

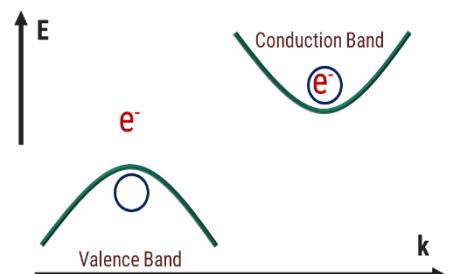
i.e. $e^- + \text{hole} \rightarrow h\nu$ (photon)



- ✓ This process is called radiative recombination. (also known as spontaneous emission).

(b) Indirect bandgap semiconductor:

- ✓ In this type of semiconductor, when an excited electron falls back into valance band, the electron and holes recombine to produce light (release a photon).



- ✓ In a indirect bandgap semiconductor, when an excited electron falls back into the valence band, electrons and holes recombine to generate heat and this heat is dissipated in the material.

i.e. $e^- + \text{hole} \rightarrow \text{heat}$

- ✓ This process is known as non-radiative recombination.

(2) Intrinsic and Extrinsic semiconductor:

(a) Intrinsic semiconductor:

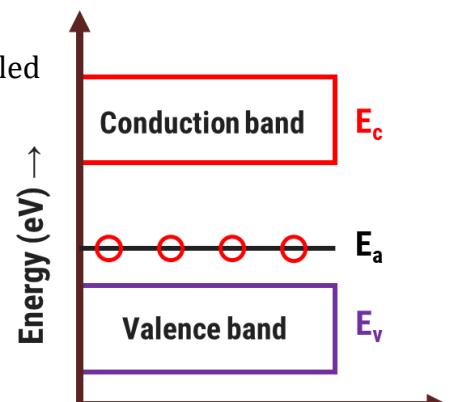
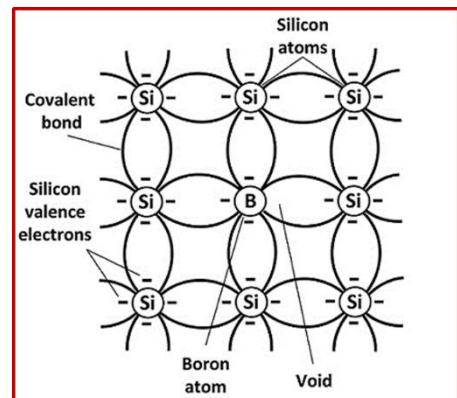
- ✓ A semiconductor in extremely pure form, without the addition of impurities is known as intrinsic semiconductors. Its electrical conductivity can be changed due to thermal excitation.
- ✓ At 0K the valance band is completely filled and the conduction band is empty.
- ✓ The carrier concentration (i.e.) electron density (or) hole density increases exponentially with increase in temperature.

(b) Extrinsic semiconductor:

- ✓ A semiconductor in extremely impure form, with the addition of impurities is known as extrinsic semiconductors. Extrinsic semiconductor can be of two type:
 - (I) P – type semiconductors
 - (II) N – type semiconductors

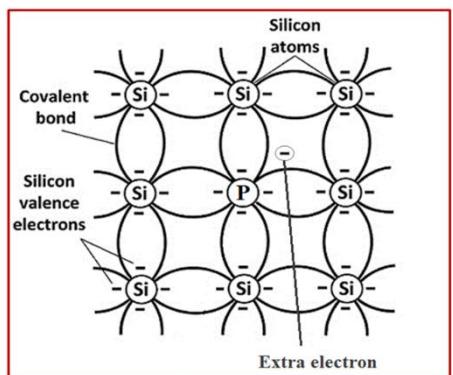
(I) P – type semiconductors:

- ✓ It is formed by doping a trivalent impurity in Si or Ge. e.g. Ga, In, B.
- ✓ Let us assume, a trivalent element B is added to an intrinsic semiconductor Si. All the valence electrons of B will form covalent bonds with neighbouring Si atoms as shown in fig.
- ✓ The dopant is in need of an electron to complete its fourth covalent bond formation with Si. Thus holes acts as acceptors of electrons.
- ✓ These holes have slightly higher energy and creates an energy level called acceptor level just above valence band.
- ✓ As the dopant atoms accept electrons, they are also called Acceptors.
- ✓ An electron must gain energy of an order of E_a in order to create hole in valence band.
- ✓ The acceptor atoms get negatively ionized after accepting electrons from valence band at room temperature. This is how holes are created in valence bands.
- ✓ This is why holes are majority charge carriers in p – type semiconductors. At sufficient high temperatures, additional electron hole pairs are generated due to braking of covalent bond.

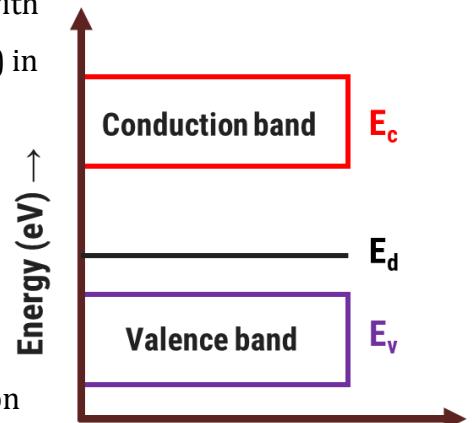


(II) N – type semiconductors:

- ✓ It is formed by doping a Pentavalent impurity in Si or Ge. e.g. P, As, Sb.
- ✓ Let us assume, a pentavalent element P is added to an intrinsic semiconductor Si.



- ✓ All the valence electrons of P will form covalent bonds with neighbouring Si atoms, leaving an extra electron (fifth electron) in the unbounded state as shown in fig.
- ✓ This extra electron is weakly bounded to the atom and enters into an energy level in donor state, just below the conduction band.
- ✓ As these electrons are not tightly bound to the atom, all such electrons at room temperature can get excited into conduction band, even for small amount of external energy.
- ✓ As the pentavalent atom donates electrons to conduction band, they are also called donor atoms.
- ✓ E_d is the minimum energy required for electron to enter in conduction band. So, in this type of semiconductors, free electrons are the majority charge carriers.



2.3.1 Difference between intrinsic and extrinsic semiconductors

Intrinsic semiconductor	Extrinsic semiconductor
It is pure semiconductor without impurity.	Impurities are added in this semiconductors.
The number of free electrons in conduction band and holes in valence band are same/equal.	Number free electrons and holes are not same.
Electrical conductivity is low.	Electrical conductivity is high.
Electrical conductivity is a function of temperature only.	Electrical conductivity is a function of temperature and impurities both.
Examples are crystalline forms of pure silicon (Si) and germanium (Ge).	Examples are silicon and germanium doped with impurities like Boron, Phosphorous, etc...

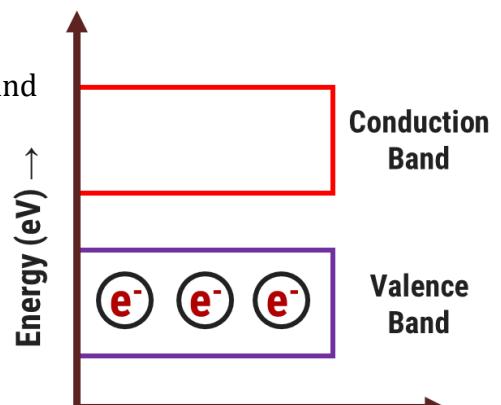
2.3.2 Difference between P-type and N-type semiconductors

P-type semiconductors	N-type semiconductors
In this type of semiconductor impurities like boron, aluminium, and gallium are added.	In this type of semiconductor impurities like Phosphorus, arsenic, antimony are added.

Holes are majority carriers and electrons are minority carriers.	Electrons are majority carriers and holes are minority carriers.
Density of holes is much greater than density of electrons i.e. $n_h > n_e$.	Density of electrons is much greater than density of holes i.e. $n_e > n_h$.
The acceptor energy level is close to valence band and away from conduction band.	The donor energy level is close to conduction band and away from valence band.
Fermi level lies between acceptor level and valence band.	Fermi level lies between donor level and conduction band.
Impurity level creates a vacancy of electrons i.e. holes	Impurity atom provides an extra electron.

2.4 Equilibrium carrier statistics

- ✓ The energy gap between valence and conduction band is relatively very small. Hence, at room temperature, some electrons may possess enough thermal energy to cross over the band gap and enter conduction band.
- ✓ These excited electrons leave behind a vacancy called 'Hole'.
- ✓ In an intrinsic electrons, for every excited electrons, moving to conduction band there is a hole created in valence band.
- ✓ Thus, in an intrinsic semiconductor: $n_e = n_h$ (density of electron = density of holes)
- ✓ Here, when an electron moves to fill a hole, another hole is created at original electron source.
- ✓ When a voltage is applied, electrons in conduction band accelerate towards positive terminal and holes in valence band move towards negative terminal.
- ✓ So, we can say that conduction takes place due to the movement of both charge carriers.
- ✓ At a temperature T, charge carriers possess an average kinetic energy (E) and the mean thermal velocity v_{th} ,
 \therefore Drift velocity: $v_d = \mu e$
- ✓ As we denote the drift velocity of electron with v_{de} and that of hole with v_{dh} and mobility of electron and hole with μ_e and μ_h respectively,
- ✓ Current density due to electrons: $J_e = n_e e v_{de} = n_e e \mu_e E$
- ✓ Current density due to electrons: $J_h = n_h e v_{dh} = n_h e \mu_h E$



- ✓ From ohm's law $J = \sigma E$, electronic and hole conductivities are

$$\sigma_e = n_e e \mu_e$$

$$\sigma_h = n_h e \mu_h$$

- ✓ The intrinsic conductivity $\sigma_i = (n_e e \mu_e + n_h e \mu_h)$

Carrier concentration: Let us now calculate the electron concentration (n_e), in the conduction band and the hole concentration (n_h) in valence band.

- ✓ Definition: The number of electrons in the conduction band per unit volume (n_e) and the number of holes in the valence band per unit volume (n_h) of the material is known as carrier concentration or density of charge carriers.

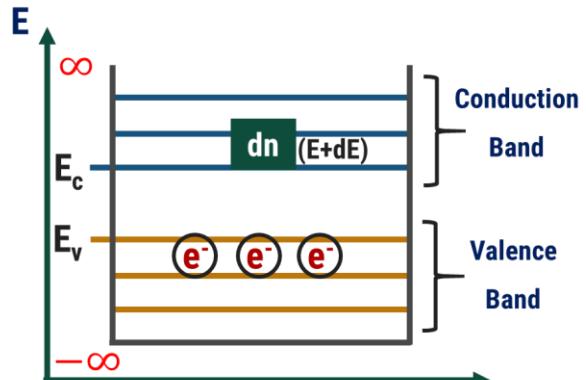
2.4.1 Carrier Concentration: Density of electrons in conduction band

Calculation of electron density:

- ✓ Let dn be the number of electrons whose energy lies in the energy interval E and $(E + dE)$ in the conduction band. Then,

$$\therefore dn = N(E) dE \cdot f(E)$$

- ✓ $N(E) dE$ = density of states
- ✓ $f(E)$ = probability function for electrons



$$\therefore n_e = \int_{E_c}^{\infty} dn = \int_{E_c}^{\infty} N(E) dE \cdot f(E)$$

$$\therefore n_e = \int_{E_c}^{\infty} N(E) dE \cdot f(E) \dots \dots \dots (1)$$

$$\therefore N(E)dE = \frac{\pi}{2} \left(\frac{8m_e^*}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

$$\therefore N(E)dE = \frac{\pi}{2} \left(\frac{8m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_F)^{\frac{1}{2}} dE \dots \dots \dots (2)$$

$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{k_B T}\right)}}$$

- ✓ At any temp, energy required by electron to move in conduction band is higher than $k_B T$.

$$E - E_F \gg k_B T = \frac{E - E_F}{k_B T} \gg 1 = e^{\frac{E - E_F}{k_B T}} \ggg 1$$

$$\therefore f(E) = \frac{1}{e^{\left(\frac{E - E_F}{k_B T}\right)}}$$

$$\therefore f(E) = e^{-\left(\frac{E - E_F}{k_B T}\right)}$$

$$\therefore f(E) = e^{\left(\frac{E_F - E}{k_B T}\right)} \dots \dots \dots (3)$$

- ✓ Put the value of equation (2) & (3) in equation (1).

$$\therefore n_e = \int_{E_c}^{\infty} \frac{\pi}{2} \left(\frac{8m_e^*}{h^2} \right)^{\frac{3}{2}} (E - E_F)^{\frac{1}{2}} dE \times e^{\left(\frac{E_F - E}{k_B T}\right)}$$

To solve this equation, let us assume,

$$E - E_c = x k_B T$$

$$\therefore E = E_c + x k_B T$$

On differentiating, $dE = 0 + dx k_B T$

Based on assumption, limits will also change

$$\therefore E \rightarrow E_c \quad \therefore E_c - E_c \rightarrow x = 0$$

$$\therefore E \rightarrow \infty \quad \therefore \infty - E_c \rightarrow x = \infty$$

$$\therefore n_e = \int_0^{\infty} \frac{\pi}{2} \left(\frac{8m_e^*}{h^2} \right)^{\frac{3}{2}} \times (E - E_F)^{\frac{1}{2}} dE \times e^{\left(\frac{E_F - E}{k_B T}\right)}$$

$$\therefore n_e = \int_0^{\infty} \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times (E - E_c)^{\frac{1}{2}} dE \times e^{\left(\frac{E_F}{k_B T} \right)} \times e^{-\left(\frac{E}{k_B T} \right)}$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F}{k_B T} \right)} \int_0^{\infty} (E - E_c)^{\frac{1}{2}} dE \times e^{-\left(\frac{E}{k_B T} \right)}$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F}{k_B T} \right)} \int_0^{\infty} (x k_B T)^{\frac{1}{2}} \times k_B T dx \times e^{-\left(\frac{E_c + x k_B T}{k_B T} \right)}$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F}{k_B T} \right)} \int_0^{\infty} (x)^{\frac{1}{2}} (k_B T)^{\frac{1}{2}} \times k_B T \times e^{-\left(\frac{E_c}{k_B T} \right)} \times e^{-\left(\frac{x k_B T}{k_B T} \right)} dx$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F}{k_B T} \right)} \times e^{-\left(\frac{E_c}{k_B T} \right)} \times (k_B T)^{\frac{3}{2}} \int_0^{\infty} (x)^{\frac{1}{2}} e^{-(x)} dx$$

Gamma Function

$$\int_0^{\infty} (x)^{\frac{1}{2}} e^{-(x)} dx = \frac{\sqrt{\pi}}{2}$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{8 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F - E_c}{k_B T} \right)} \times (k_B T)^{\frac{3}{2}} \times \frac{\sqrt{\pi}}{2}$$

$$\therefore n_e = \frac{\pi}{2} \left(\frac{4 \times 2 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F - E_c}{k_B T} \right)} (k_B T)^{\frac{3}{2}} \frac{(\pi)^{\frac{1}{2}}}{2}$$

$$\therefore n_e = \frac{(\pi)^{\frac{3}{2}}}{4} \left(\frac{(2)^2 \times 2 m_e^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F - E_c}{k_B T} \right)} (k_B T)^{\frac{3}{2}}$$

$$\therefore n_e = \frac{8}{4} \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_F - E_c}{k_B T} \right)}$$

$$\therefore n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} \dots \dots (4)$$

Equation (4) suggests " Density of electrons in conduction band for Intrinsic semiconductors".

2.4.2 Carrier Concentration: Density of holes in valence band

Calculation of hole density:

- Let dp be the number of electrons whose energy lies in the energy interval E and $(E + dE)$ in the conduction band. Then,

$$\therefore dp = N(E)dE \cdot [1 - f(E)]$$

- $N(E) dE$ = density of states
- $1 - f(E)$ = probability function for holes

$$\therefore n_e = \int_{E_c}^{\infty} dp = \int_{E_c}^{\infty} N(E) dE \cdot [1 - f(E)]$$

$$\therefore n_e = \int_{E_c}^{\infty} N(E) dE \cdot [1 - f(E)] \dots \dots \dots (1)$$

$$\therefore N(E)dE = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} E^{\frac{1}{2}} dE$$

$$\therefore N(E)dE = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE \dots \dots \dots (2)$$

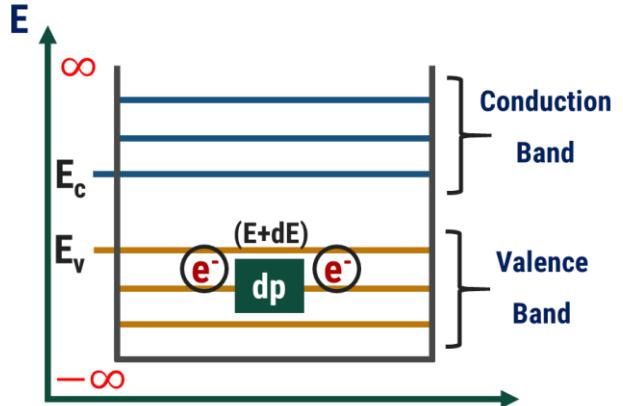
$$f(E) = \frac{1}{1 + e^{\left(\frac{E - E_f}{k_B T}\right)}}$$

Let us consider, $\left(\frac{E - E_F}{k_B T}\right) = x$,

$$\therefore 1 - f(E) = 1 - \frac{1}{1 + e^{(x)}}$$

$$\therefore 1 - f(E) = \frac{1 + e^{(x)} - 1}{1 + e^{(x)}}$$

$$\therefore 1 - f(E) = \frac{1}{\frac{1 + e^{(x)}}{e^{(x)}}}$$



$$\therefore 1 - f(E) = \frac{1}{e^{-(x)} + 1}$$

$$\therefore 1 - f(E) = \frac{1}{e^{-\left(\frac{E-E_F}{k_B T}\right)} + 1}$$

$$\therefore 1 - f(E) = \frac{1}{1 + e^{\left(\frac{E_F - E}{k_B T}\right)}}$$

Here, $E_F - E \gg k_B T = \frac{E_F - E}{k_B T} \gg 1 = e^{\frac{E_F - E}{k_B T}} \ggg 1$

$$\therefore 1 - f(E) = \frac{1}{e^{\left(\frac{E_F - E}{k_B T}\right)}}$$

$$\therefore 1 - f(E) = e^{-\left(\frac{E_F - E}{k_B T}\right)}$$

$$\therefore 1 - f(E) = e^{\left(\frac{E - E_F}{k_B T}\right)} \dots \dots \dots (3)$$

Put the value of equation (2) & (3) In equation (1).

$$\therefore n_p = \int_{-\infty}^{E_v} N(E) dE \cdot [1 - f(E)]$$

$$\therefore n_p = \int_{-\infty}^{E_v} \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2}\right)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE \cdot e^{\left(\frac{E - E_F}{k_B T}\right)}$$

$$\therefore n_p = \int_{\infty}^0 \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2}\right)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE \cdot e^{\left(\frac{E - E_F}{k_B T}\right)}$$

$$\therefore n_p = \int_{\infty}^0 \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2}\right)^{\frac{3}{2}} (E_v - E)^{\frac{1}{2}} dE \cdot e^{\left(\frac{E}{k_B T}\right)} e^{-\left(\frac{E_F}{k_B T}\right)}$$

To solve this equation, let us assume,

$$E_v - E = x k_B T \quad \therefore E = E_v - x k_B T$$

$$\text{On differentiating, } \therefore 0 - dE = dx k_B T$$

Based on assumption, limits will also change

$$\therefore E \rightarrow -\infty \quad \therefore E_v - (-\infty) \rightarrow x = \infty$$

$$\therefore E \rightarrow E_v \quad \therefore E_v - E_v \rightarrow x = 0$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \int_{-\infty}^0 (E_v - E)^{\frac{1}{2}} dE \times e^{\left(\frac{E}{k_B T}\right)}$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \int_{-\infty}^0 (x k_B T)^{\frac{1}{2}} - dx k_B T \times e^{\left(\frac{E_v - x k_B T}{k_B T}\right)}$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \int_{-\infty}^0 (x)^{\frac{1}{2}} (k_B T)^{\frac{1}{2}} k_B T \times e^{\left(\frac{E_v}{k_B T}\right)} \times e^{-\left(\frac{x k_B T}{k_B T}\right)} - dx$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \int_{-\infty}^0 (x)^{\frac{1}{2}} (k_B T)^{\frac{3}{2}} \times e^{\left(\frac{E_v}{k_B T}\right)} \times e^{-(x)} - dx$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \times e^{\left(\frac{E_v}{k_B T}\right)} \times (k_B T)^{\frac{3}{2}} \int_0^\infty (x)^{\frac{1}{2}} \cdot e^{-(x)} dx$$

Gamma Function

$$\int_0^\infty (x)^{\frac{1}{2}} e^{-(x)} dx = \frac{\sqrt{\pi}}{2}$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{8 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{-\left(\frac{E_F}{k_B T}\right)} \times e^{\left(\frac{E_v}{k_B T}\right)} \times (k_B T)^{\frac{3}{2}} \times \frac{\sqrt{\pi}}{2}$$

$$\therefore n_p = \frac{\pi}{2} \left(\frac{4 \times 2 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_v - E_F}{k_B T}\right)} \times (k_B T)^{\frac{3}{2}} \times \frac{(\pi)^{\frac{1}{2}}}{2}$$

$$\therefore n_p = \frac{(\pi)^{\frac{3}{2}}}{4} \left(\frac{(2)^2 \times 2 m_h^*}{h^2} \right)^{\frac{3}{2}} \times e^{\left(\frac{E_v - E_F}{k_B T}\right)} \times (k_B T)^{\frac{3}{2}}$$

$$\therefore n_p = \frac{8}{4} \left(\frac{\pi 2 m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T}\right)}$$

$$\therefore n_p = 2 \left(\frac{\pi 2 m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T}\right)} \dots \dots \dots (4)$$

Equation (4) suggests " Density of holes in valence band for Intrinsic semiconductors."

2.4.3 Fermi level and its variation with temperature

For intrinsic semiconductor:

Density of electrons in conduction band (N_e) = Density of holes in valence band (N_h)

$$\begin{aligned}
 & 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} = 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T} \right)} \\
 \therefore (m_e^*)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} &= (m_h^*)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T} \right)} \dots \dots \dots (1) \\
 \therefore (m_e^*)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} &= (m_h^*)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T} \right)} \\
 \therefore \frac{(m_h^*)^{\frac{3}{2}}}{(m_e^*)^{\frac{3}{2}}} &= \frac{e^{\left(\frac{E_F - E_c}{k_B T} \right)}}{e^{\left(\frac{E_v - E_F}{k_B T} \right)}} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\left(\frac{E_F - E_c}{k_B T} \right)} e^{-\left(\frac{E_v - E_F}{k_B T} \right)} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\left(\frac{E_F - E_c}{k_B T} \right)} e^{\left(\frac{E_F - E_v}{k_B T} \right)} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\left(\frac{E_F - E_c + E_F - E_v}{k_B T} \right)} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\left(\frac{2E_F - (E_c + E_v)}{k_B T} \right)} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= e^{\left(\frac{2E_F}{k_B T} \right)} \times e^{-\left(\frac{E_c + E_v}{k_B T} \right)} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} &= \frac{e^{\left(\frac{2E_F}{k_B T} \right)}}{e^{\left(\frac{E_c + E_v}{k_B T} \right)}} \\
 \therefore \left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} \cdot e^{\left(\frac{E_c + E_v}{k_B T} \right)} &= e^{\left(\frac{2E_F}{k_B T} \right)} \dots \dots \dots (2)
 \end{aligned}$$

Taking log on both sides,

$$\therefore \log e^{\left(\frac{2E_F}{k_B T} \right)} = \log \left[\left(\frac{m_h^*}{m_e^*} \right)^{\frac{3}{2}} \cdot e^{\left(\frac{E_c + E_v}{k_B T} \right)} \right]$$

$$\log [A \cdot B] = \log [A] + \log [B]$$

$$\therefore \log e^{\left(\frac{2 E_F}{k_B T}\right)} = \log \left(\frac{m_h^*}{m_e^*}\right)^{\frac{3}{2}} + \log e^{\left(\frac{E_c + E_v}{k_B T}\right)}$$

$$\therefore \frac{2 E_F}{k_B T} = \left(\frac{3}{2} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c + E_v}{k_B T}\right)$$

$$\therefore E_F = \frac{k_B T}{2} \left(\frac{3}{2} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c + E_v}{k_B T} \right)$$

$$\therefore E_F = \frac{k_B T}{2} \frac{3}{2} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{k_B T}{2} \frac{E_c + E_v}{k_B T}$$

$$\therefore E_F = \frac{3 k_B T}{4} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c + E_v}{2}$$

When $m_h^* = m_e^*$ at $T = 0$ K then,

$$\therefore E_F = \frac{3 k_B T}{4} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c + E_v}{2}$$

$$\therefore E_F = \frac{E_c + E_v}{2}$$

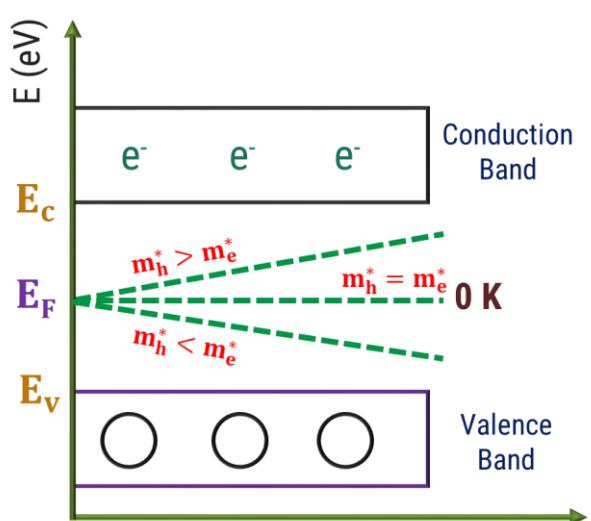
When $m_h^* = m_e^*$ at $T = 0$ K then, this equation shows fermi level (E_F) lies between E_c and E_v at $T = 0$ K temperature.

$$\therefore E_F = \frac{3 k_B T}{4} \log \left(\frac{m_h^*}{m_e^*}\right) + \frac{E_c + E_v}{2}$$

If a small change in temp. occurs then, possibilities are:

(1) Increase in temp. $m_h^* > m_e^*$

(2) decrease in temp. $m_h^* < m_e^*$



Q.1 Determine the position of Fermi level in silicon semiconductor at 300 K. Given that the Bandgap is 1.12 eV, and $m_e^* = 0.12 m_0$ and $m_h^* = 0.28 m_0$ (m_0 = rest mass of an electron).

Ans. $E_g = 1.12 \text{ eV} = 1.12 \times 1.6 \times 10^{-19} \text{ Joule}$

$$T = 300 \text{ K}$$

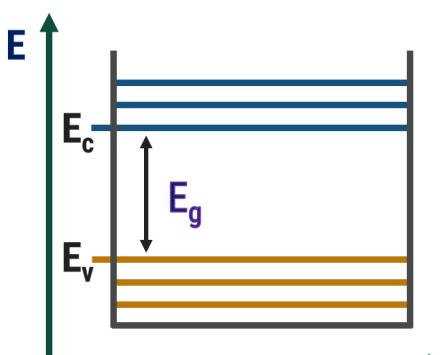
$$m_e^* = 0.12 m_0$$

$$m_h^* = 0.28 m_0$$

$$E_F = ?$$

$$\therefore E_F = \frac{E_c + E_v}{2} + \frac{3 k_B T}{4} \log \left(\frac{m_h^*}{m_e^*}\right)$$

$$\therefore E_F = \frac{E_g}{2} + \frac{3 k_B T}{4} \log \left(\frac{m_h^*}{m_e^*}\right)$$



$$\therefore E_F = \frac{1.12 \times 1.6 \times 10^{-19}}{2} + \frac{3}{4} (1.38 \times 10^{-23} \times 300) \log\left(\frac{0.28 m_0}{0.12 m_0}\right)$$

$$\therefore E_F = \frac{1.12 \times 1.6 \times 10^{-19}}{2} + \frac{3}{4} (1.38 \times 10^{-23} \times 300) \ln\left(\frac{0.28 m_0}{0.12 m_0}\right)$$

$$\therefore E_F = 8.96 \times 10^{-20} + \frac{3}{4} (4.14 \times 10^{-21}) \ln (0.84729)$$

$$\therefore E_F = 9.22 \times 10^{-20} \text{ Joule}$$

$$\therefore E_F = \frac{9.22 \times 10^{-20}}{1.60 \times 10^{-19}}$$

$$\therefore E_F = 0.5764 \text{ eV}$$

2.4.4 Law of Mass action

- ✓ This law states that for a given semiconductor (intrinsic or extrinsic) product of charge carrier concentration remains a constant at any given temperature if doping is varied.
i.e. $n_e \times n_h = n_i^2 = \text{constant}$
- ✓ Where, n_i is the carrier concentration (intrinsic charge carrier density), based on law of mass action.

$$\therefore n_i^2 = n_e \times n_h$$

$$\therefore n_i^2 = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} \times 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T} \right)}$$

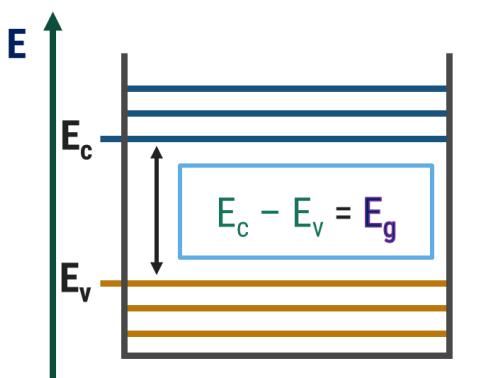
$$\therefore n_i^2 = 2^2 (m_e^*)^{\frac{3}{2}} \left(\frac{2 \pi k_B T}{h^2} \right)^3 e^{\left(\frac{E_F - E_c}{k_B T} \right)} \\ \times (m_h^*)^{\frac{3}{2}} e^{\left(\frac{E_v - E_F}{k_B T} \right)}$$

$$\therefore n_i^2 = 2^2 (m_e^* m_h^*)^{\frac{3}{2}} \left(\frac{2 \pi k_B T}{h^2} \right)^3 e^{\left(\frac{E_F - E_c + E_v - E_F}{k_B T} \right)}$$

$$\therefore n_i^2 = 2^2 (m_e^* m_h^*)^{\frac{3}{2}} \left(\frac{2 \pi k_B T}{h^2} \right)^3 e^{\left(\frac{E_v - E_c}{k_B T} \right)}$$

$$\therefore n_i = 2 (m_e^* m_h^*)^{\frac{3}{4}} \left(\frac{2 \pi k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_v - E_c}{2 k_B T} \right)}$$

$$\therefore n_i = 2 (m_e^* m_h^*)^{\frac{3}{4}} \left(\frac{2 \pi k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{-E_g}{2 k_B T} \right)}$$



- ✓ This equation gives value of intrinsic carrier concentration.
- ✓ For an intrinsic semiconductor, $n_e = n_h = n_i$

2.4.5 Mobility and Conductivity

- ✓ Charge carriers in semiconductor is assumed to be moving freely inside a semiconductor.
In case of intrinsic semiconductor, both electrons and holes contribute to the electrical conductivity.
- ✓ Conductivity due to electrons is given by: $\sigma_e = n_e e \mu_e$
- ✓ Conductivity due to holes is given by: $\sigma_h = n_h e \mu_h$
- ✓ Total conductivity for an intrinsic semiconductor:

$$\sigma_i = \sigma_e + \sigma_h$$

$$\therefore \sigma_i = n_e e \mu_e + n_h e \mu_h \quad \text{we know, } n_e = n_h = n_i$$

$$\therefore \sigma_i = n_i e \mu_e + n_i e \mu_h$$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

- ✓ Substituting the value of n_i ,

$$\therefore \sigma_i = 2 (m_e^* m_h^*)^{\frac{3}{4}} \left(\frac{2 \pi k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{-E_g}{2 k_B T} \right)} e (\mu_e + \mu_h)$$

$$\therefore \sigma_i = 2e (m_e^* m_h^*)^{\frac{3}{4}} \left(\frac{2 \pi k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{-E_g}{2 k_B T} \right)} (\mu_e + \mu_h)$$

$$\therefore \sigma_i = C \times e^{\left(\frac{-E_g}{2 k_B T} \right)} \dots \dots \dots (1)$$

- ✓ From equation (1), it can be seen that σ_i depends on the negative exponential of forbidden energy gap, temperature and on mobility of electrons and holes.
- ✓ Taking logarithm on both sides of equation (1)

$$\therefore \ln \sigma_i = \ln C - \frac{E_g}{2 k_B T}$$

- ✓ From equation we can say that, conductivity increases with temperature.

- Q.1 For an intrinsic silicon, room temperature electrical conductivity is $4 \times 10^{-4} \Omega^{-1}\text{m}^{-1}$. Electron and hole mobilities are $0.14 \text{ m}^2/\text{V sec}$ and $0.040 \text{ m}^2/\text{V sec}$ respectively. Calculate the electron and hole concentration at room temperature.

Ans. $\sigma_i = 4 \times 10^{-4} (\Omega\text{m})^{-1}$

$$\mu_e = 0.14 \text{ m}^2/\text{V sec}$$

$$\mu_h = 0.040 \text{ m}^2/\text{V sec}$$

$$n_i = ?$$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

$$\therefore n_i = \frac{\sigma_i}{e (\mu_e + \mu_h)}$$

$$\therefore n_i = \frac{4 \times 10^{-4}}{(1.6 \times 10^{-19})(0.14 + 0.040)}$$

$$\therefore n_i = \frac{4 \times 10^{-4}}{2.88 \times 10^{-20}}$$

$$\therefore n_i = 1.38 \times 10^{16}/\text{m}^3$$

From law of mass action, we have $n_e = n_h = n_i$

$$\therefore n_e = n_h = 1.38 \times 10^{16}/\text{m}^3$$

- Q.2 Find the resistance of an intrinsic germanium rod 1cm long, 1mm wide and 1mm thick at 300 K. Here, $n_i = 2.5 \times 10^{19} / \text{m}^3$, $\mu_e = 0.39 \text{ m}^2/\text{V sec}$, $\mu_h = 0.19 \text{ m}^2/\text{V sec}$.

Ans. $L = 1\text{cm} = 10^{-2} \text{ m}$

$$b = 1\text{mm} = 10^{-3} \text{ m}$$

$$t = 1\text{mm} = 10^{-3} \text{ m}$$

$$T = 300 \text{ K}$$

$$\mu_e = 0.39 \text{ m}^2/\text{V sec}$$

$$\mu_h = 0.19 \text{ m}^2/\text{V sec}$$

$$R = ?$$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

$$\therefore \sigma_i = (2.5 \times 10^{19}) (1.6 \times 10^{-19}) (0.39 + 0.19)$$

$$\therefore \sigma_i = 2.32 (\Omega m)^{-1}$$

Area = breadth × thickness

$$\text{Area} = (10^{-3} \times 10^{-3} \text{ m}^2)$$

we know, $R = \frac{\rho l}{A}$

$$\therefore R = \frac{10^{-2}}{2.32 (10^{-3} \times 10^{-3})}$$

$$\therefore R = 4310 \Omega$$

- Q.3 In an intrinsic semiconductor, energy gap is 1.2 eV. What is the ratio between its conductivity at 600 K and at 300 K.

Ans. $E_g = 1.2 \text{ eV} = 1.2 \times 1.6 \times 10^{-19} \text{ J}$

$$T_1 = 600 \text{ K}$$

$$T_2 = 300 \text{ K}$$

$$\frac{\sigma_1}{\sigma_2} = ?$$

Let σ_1 be the electrical conductivity at T_1 K and σ_2 be the electrical conductivity at T_2 K.

$$\therefore \frac{\sigma_1}{\sigma_2} = \frac{C \times e^{\left(\frac{-E_g}{2k_B T_1}\right)}}{C \times e^{\left(\frac{-E_g}{2k_B T_2}\right)}}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = e^{\left(\frac{-E_g}{2k_B} \left[\frac{1}{T_1} - \frac{1}{T_2}\right]\right)}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = e^{\left(\frac{-1.2 \times 1.6 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}} \left[\frac{1}{600} - \frac{1}{300}\right]\right)}$$

$$\therefore \frac{\sigma_1}{\sigma_2} = e^{11.59}$$

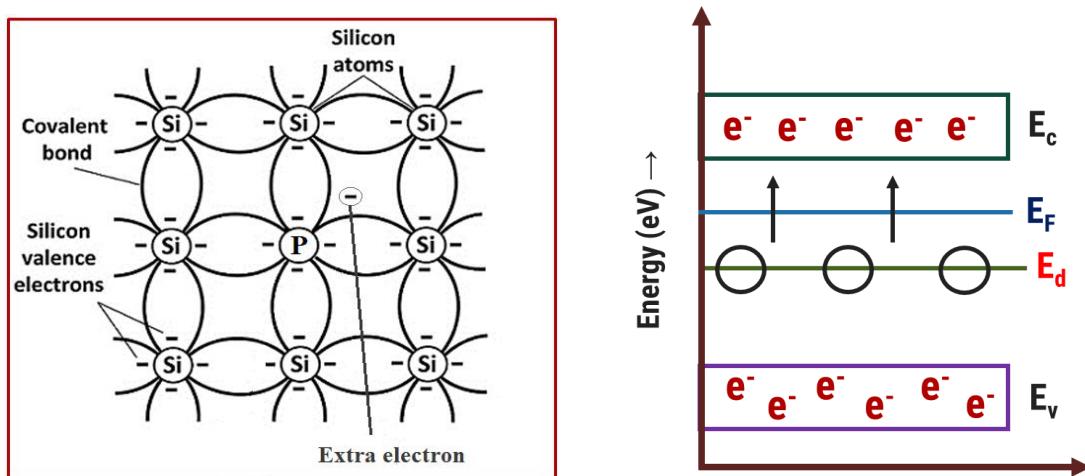
$$\therefore \frac{\sigma_1}{\sigma_2} = 108416.886$$

$$\therefore \frac{\sigma_1}{\sigma_2} = 1.08 \times 10^5$$

2.4.6 Extrinsic semiconductors: Carrier concentration (n-type)

Expression for carrier concentration in n-type semiconductors:

- Before deriving the equation for n-type semiconductor, let us first derive the equation for fermi level.



- Let N_d be the donor concentration i.e. number of atoms per unit volume of material and E_d be the donor energy level.
- Let us assume that $E_c - E_F > k_B T$, so, density of electrons in conduction band is given by

$$\therefore n_e = 2 \left(\frac{2 \pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} \dots \dots (1)$$

- ✓ At $T = 0$ K, the fermi level lies between E_c and E_d and also, all donor levels are filled with electrons (donor atoms).
- ✓ With increase in temperature, more donor atoms moves to conduction band and density of electrons in conduction band increases. Then density of ionized donor atoms must be (N_d^+) .

$$\therefore N_d^+ = N_d [1 - f(E_d)] \dots \dots \dots (2)$$

- ✓ Here N_d is number of donor atoms. (Donors will give it's electrons to conduction band).
 $1 - f(E)$ is the probability of holes.

We know, $f(E) = \frac{1}{1 + e^{\left(\frac{E_d - E_f}{k_B T}\right)}}$

$$\therefore 1 - f(E) = 1 - \frac{1}{1 + e^{\left(\frac{E_d - E_f}{k_B T}\right)}}$$

$$\therefore 1 - f(E) = 1 - \left[1 + e^{\left(\frac{E_d - E_f}{k_B T}\right)} \right]^{-1}$$

- ✓ By binominal expansion: $(1 + x)^{-1} = (1 - x)$

$$\therefore 1 - f(E) = 1 - \left[1 - e^{\left(\frac{E_d - E_f}{k_B T}\right)} \right]$$

$$\therefore 1 - f(E) = e^{\left(\frac{E_d - E_f}{k_B T}\right)} \dots \dots \dots (3)$$

- ✓ Substitute equation 3 in equation 2.

$$\therefore N_d^+ = N_d e^{\left(\frac{E_d - E_f}{k_B T}\right)} \dots \dots \dots (4)$$

- ✓ At very low temperature no. electrons or holes pair is generated, where electrons are in conduction band.

$$\therefore n_e = N_d^+$$

$$\therefore n_e = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)}$$

$$\therefore N_d^+ = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)}$$

✓ From equation (4)

$$\therefore N_d e^{\left(\frac{E_d - E_f}{k_B T} \right)} = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)}$$

✓ Taking log on both sides

$$\therefore \ln N_d + \ln e^{\left(\frac{E_d - E_f}{k_B T} \right)} = \ln 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} + \ln e^{\left(\frac{E_F - E_c}{k_B T} \right)}$$

$$\therefore \ln N_d + \frac{E_d - E_f}{k_B T} = \ln \left[2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} \right] + \frac{E_F - E_c}{k_B T}$$

$$\therefore \frac{E_F - E_c}{k_B T} - \frac{E_d - E_f}{k_B T} = \ln N_d - \ln \left[2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} \right]$$

✓ Multiplying by $k_B T$ and simplify above equation

$$\therefore E_F - E_c - E_d + E_f = k_B T \ln N_d - \ln \left[2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} \right]$$

$$\therefore 2E_F - (E_d + E_c) = k_B T \ln \left[\frac{N_d}{2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}}} \right]$$

$$\therefore 2E_F = (E_d + E_c) + k_B T \ln \frac{N_d}{2} \left[\left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right]$$

$$\therefore E_F = \frac{(E_d + E_c)}{2} + \frac{k_B T}{2} \ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}} \dots \dots \dots (5)$$

✓ Above equation gives the value of Fermi energy in N-type semiconductor

- ✓ Expression for carrier concentration in conduction band for N-type semiconductor

$$\therefore n_e = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} \dots \dots \dots (1)$$

- ✓ Let us first simplify the term: $e^{\left(\frac{E_F - E_c}{k_B T} \right)}$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\left(\frac{E_F}{k_B T} \right)} e^{-\left(\frac{E_c}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\left(\frac{\frac{(E_d + E_c)}{2} + \frac{k_B T}{2} \ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}}}{k_B T} \right)} e^{-\left(\frac{E_c}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\left(\frac{\frac{(E_d + E_c)}{2} + \frac{k_B T}{2k_B T} \ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}}}{2k_B T} \right)} e^{-\left(\frac{E_c}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\frac{(E_d + E_c)}{2k_B T}} e^{\frac{1}{2} \ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}}} e^{-\left(\frac{E_c}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\frac{(E_d + E_c - 2E_c)}{2k_B T}} e^{\frac{1}{2} \ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}}}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\frac{(E_d + E_c - 2E_c)}{2k_B T}} e^{\left[\ln \frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right]^{\frac{1}{2}}}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\frac{(E_d - E_c)}{2k_B T}} \left[\frac{N_d}{2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right]^{\frac{1}{2}}$$

$$\therefore e^{\left(\frac{E_F - E_c}{k_B T} \right)} = e^{\frac{(E_d - E_c)}{2k_B T}} \left(\frac{N_d}{2} \right)^{\frac{1}{2}} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{4}} \dots \dots \dots (6)$$

- ✓ Substitute this value in equation (1)

$$\therefore n_e = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_F - E_c}{k_B T} \right)} \dots \dots \dots (1)$$

$$\therefore n_e = 2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\frac{(E_d - E_c)}{2k_B T}} \left(\frac{N_d}{2} \right)^{\frac{1}{2}} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{-\frac{3}{4}}$$

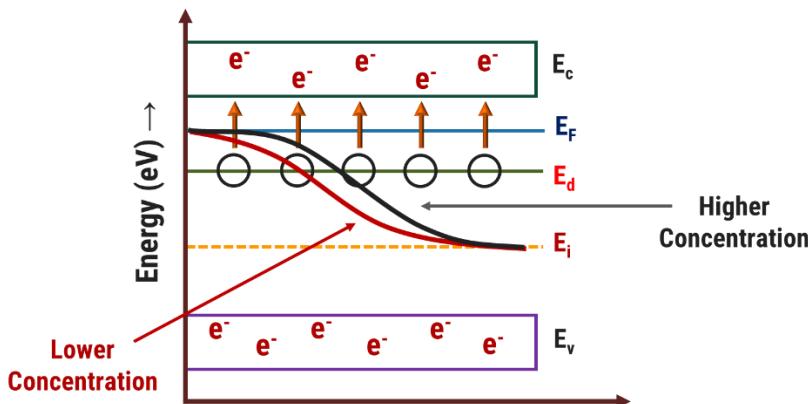
$$\therefore n_e = (2)^{\frac{1}{2}} (2)^{\frac{1}{2}} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{4}} e^{\frac{(E_d - E_c)}{2k_B T}} \left(\frac{N_d}{2} \right)^{\frac{1}{2}}$$

$$\therefore n_e = (2N_d)^{\frac{1}{2}} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{4}} e^{\frac{(E_d - E_c)}{2k_B T}}$$

✓ **Variation of Fermi level with temperature and impurity concentration:**

$$\therefore n_e = (2N_d)^{\frac{1}{2}} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{\frac{3}{4}} e^{\frac{(E_d - E_c)}{2k_B T}}$$

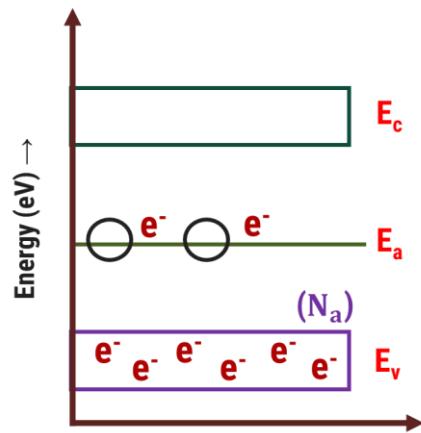
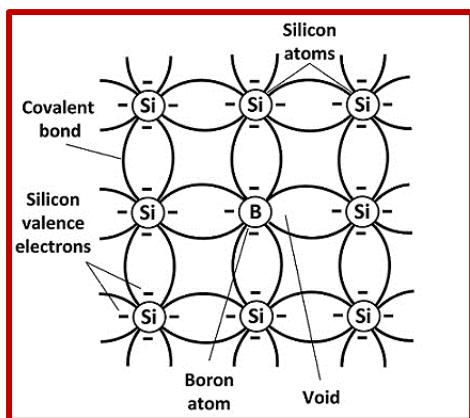
- ✓ We can say, from above equation that fermi level increases with increase in temperature.
- ✓ Now, as temperature increases, more donor atoms gets positively ionized due to donation of electrons in conduction band and the fermi level lies between E_c and E_d .
- ✓ At a particular temperature, when all donor atoms are ionized, electron-hole pairs are generated due to breaking of covalent bonds. Thus we can say that the fermi level gradually shifted towards the intrinsic fermi level E_i .
- ✓ In the figure, the variation of fermi level with high and low donor concentration indicated.
- ✓ From the figure, it is clear that shifting of fermi level with rise of temperature is shown in case of high donor concentration.



2.4.7 Extrinsic semiconductors: Carrier concentration (p-type)

Expression for carrier concentration in n-type semiconductors:

- ✓ Before deriving the equation for p-type semiconductor, let us first derive the equation for fermi level.
- ✓ Let N_a be the acceptor concentration i.e. number of atoms per unit volume of material and E_a be the acceptor energy level.



- ✓ Let us assume that $E_a - E_F > k_B T$, so, density of holes in valence band is given by

$$\therefore n_p = 2 \left(\frac{\pi 2 m_h^* k_B T}{h^2} \right)^{3/2} e^{\left(\frac{E_v - E_F}{k_B T} \right)}$$

- ✓ At $T = 0$ K, the fermi level lies between E_v and E_a and also, all acceptor levels are remains empty.
- ✓ With increase in temperature, more acceptor atoms gets negatively ionized due to transfer of electrons from the valence band.
- ✓ Then density of ionized acceptor atoms must be (N_a^-) .

$$\therefore N_a^- = N_a f(E_a) \dots \dots \dots (2)$$

- ✓ Here N_a is number of acceptor atoms. (Acceptors accepts electrons from valence band and become negatively ionized).
- ✓ $f(E)$ is the probability of electrons.

$$\checkmark \text{ We know, } f(E) = \frac{1}{1 + e^{\left(\frac{E_a - E_f}{k_B T} \right)}}$$

- ✓ Also, fermi level lies between $(E_a - E_F)$ in above equation is positive and grater than $k_B T$.

$$\therefore f(E) = \frac{1}{e^{\left(\frac{E_a - E_f}{k_B T} \right)}}$$

$$\therefore f(E) = e^{\left(\frac{E_F - E_a}{k_B T} \right)} \dots \dots \dots (3)$$

- ✓ Substitute equation 3 in equation 2.

$$\therefore N_a^- = N_a e^{\left(\frac{E_F - E_a}{k_B T}\right)} \dots \dots \dots (4)$$

- ✓ At very low temp. no electron or hole pair is generated. Where, holes are in valence band is equal to the density of ionized acceptors atoms.

$$n_a = N_a^-$$

$$\therefore n_p = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_V - E_F}{k_B T}\right)}$$

$$\therefore N_a^- = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_V - E_F}{k_B T}\right)}$$

From equation (4)

$$\therefore N_a e^{\left(\frac{E_F - E_a}{k_B T}\right)} = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_V - E_F}{k_B T}\right)}$$

Taking log on both sides

$$\therefore \ln N_a + \ln e^{\left(\frac{E_F - E_a}{k_B T}\right)} = \ln 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} + \ln e^{\left(\frac{E_V - E_F}{k_B T}\right)}$$

$$\therefore \ln N_a + \frac{E_F - E_a}{k_B T} = \ln \left[2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} \right] + \frac{E_V - E_F}{k_B T}$$

$$\therefore \frac{E_V - E_F}{k_B T} - \frac{E_F - E_a}{k_B T} = \ln N_a - \ln \left[2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} \right]$$

Multiplying by $k_B T$ and simplify above equation

$$\therefore E_V - E_F - E_F + E_a = k_B T \ln N_a - \ln \left[2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} \right]$$

$$\therefore E_V - E_F - E_F + E_a = k_B T \ln N_a - \ln \left[2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} \right]$$

$$\therefore -2E_F + (E_V + E_a) = k_B T \ln \left[\frac{N_a}{2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}}} \right]$$

$$\therefore 2E_F = (E_V + E_a) - k_B T \ln \frac{N_a}{2} \left[\left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right]$$

$$\therefore E_F = \frac{(E_V + E_a)}{2} - \frac{k_B T}{2} \ln \frac{N_a}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}} \dots \dots \dots (5)$$

$$\therefore n_p = 2 \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\left(\frac{E_V - E_F}{k_B T} \right)} \dots \dots \dots (1)$$

Let us first simplify the term $e^{\left(\frac{E_V - E_F}{k_B T} \right)}$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\left(\frac{E_V}{k_B T} \right)} e^{-\left(\frac{E_F}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\left(\frac{E_V}{k_B T} \right)} e^{-\left(\frac{\frac{(E_V + E_a)}{2} - \frac{k_B T}{2} \ln \frac{N_a}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}}}{k_B T} \right)}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\left(\frac{E_V}{k_B T} \right)} e^{\left(\frac{(-E_V - E_a)}{2k_B T} + \frac{k_B T}{2k_B T} \ln \frac{N_a}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right)}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\left(\frac{E_V}{k_B T} \right)} e^{\frac{(-E_V - E_a)}{2k_B T}} e^{\frac{1}{2} \ln \frac{N_a}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}}}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\frac{(2E_V - E_V - E_a)}{2k_B T}} e^{\frac{1}{2} \ln \frac{N_a}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}}}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\frac{(E_V - E_a)}{2k_B T}} e^{\left[\ln \frac{N_d}{2} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{2}} \right]^{\frac{1}{2}}}$$

$$\therefore e^{\left(\frac{E_V - E_F}{k_B T} \right)} = e^{\frac{(E_V - E_a)}{2k_B T}} \left(\frac{N_a}{2} \right)^{\frac{1}{2}} \left(\frac{2 \pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{4}} \dots \dots \dots (6)$$

Substitute this value in equation (1)

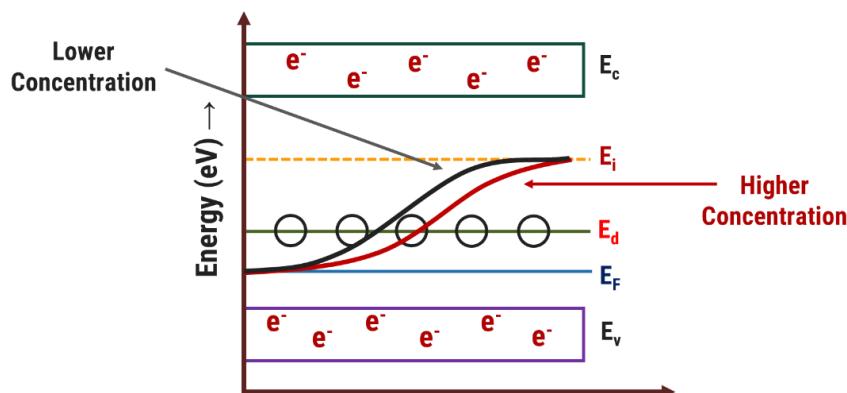
$$\therefore n_p = 2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{\frac{3}{2}} e^{\frac{(E_V - E_a)}{2k_B T}} \left(\frac{N_a}{2} \right)^{\frac{1}{2}} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{-\frac{3}{4}}$$

$$\therefore n_p = (2)^{\frac{1}{2}} (2)^{\frac{1}{2}} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{\frac{3}{4}} e^{\frac{(E_V - E_a)}{2k_B T}} \left(\frac{N_a}{2} \right)^{\frac{1}{2}}$$

$$\therefore n_p = (2N_a)^{\frac{1}{2}} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{\frac{3}{4}} e^{\frac{(E_V - E_a)}{2k_B T}}$$

Variation of Fermi level with temperature and impurity concentration:

- ✓ From the above equation, it is seen that as temperature gets slowly increased, more and more acceptor atoms get negatively ionized due to transfer of electrons from valence band and fermi level lies between E_V and E_a .



2.5 Carrier Generation and Recombination

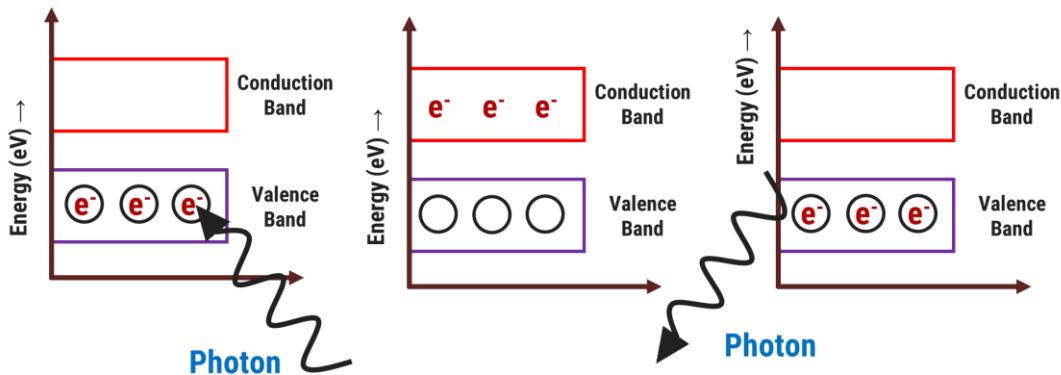
Carrier Generation:

- ✓ “It is a process where electron–hole pairs are created by exciting an electron from valence band to conduction band, thereby creating a hole in valence band.

Recombination:

- ✓ “Recombination is reverse process where electrons and holes from conduction band and valence band respectively recombine and are annihilated (destroyed).”
- ✓ In the above process, both the carriers eventually disappears.

- ✓ The energy difference of initial and final stage of an electron is given off as phonons or photons.



- ✓ This is known as direct recombination or band-to-band transition.
- ✓ An electron from conduction band falls back to valence band and releases energy in the form of photon.
- ✓ The reverse process i.e. generation of electron hole pairs is triggered by sufficient energetic photons, which transfers its energy to a valence band electron, moving it to conduction band and leaving behind a hole in valence band.
- ✓ Energy of incident photon has to be at least of the magnitude of the bandgap.
- ✓ In recombination the transition from excited states to lower energy states, momentum has to be conserved.
- ✓ The energy absorbed or emitted by photon is given by: $E = h\nu$ Here, h = Planck's constant and ν = Frequency of emitted photons
- ✓ As the momentum of photon is very small, no momentum transfer is possible, so direct band to band transition is possible.
- ✓ If $n_e \cdot n_h - n_i^2 > 0$, Carrier recombination dominates
- ✓ If $n_e \cdot n_h - n_i^2 < 0$, Carrier generation dominates
- ✓ **Applications:**
- ✓ Absorption is active process in photodiodes, solar cell and other semiconductor photodetectors, whereas photon emission is the principle of operation in laser diodes, semiconductor lasers.

Phonon Transition (Shockley – Read – Hall (SRH) recombination)

- ✓ Also known as Indirect or Trap-assisted recombination.
- ✓ This process is trap assisted, passing through a lattice defect at energy level E_r within semiconductor bandgap.
- ✓ This trap can be caused by presence of any foreign atom or structural defect.

Generation:

- ✓ Hole emission:

An electron from valence band is trapped, leaving a hole in valence band (hole is emitted from empty trap to valence band.) Fig.(a)

- ✓ Electron emission:

A trapped electron moves from the trap energy level to conduction band. Fig.(b)

Recombination:

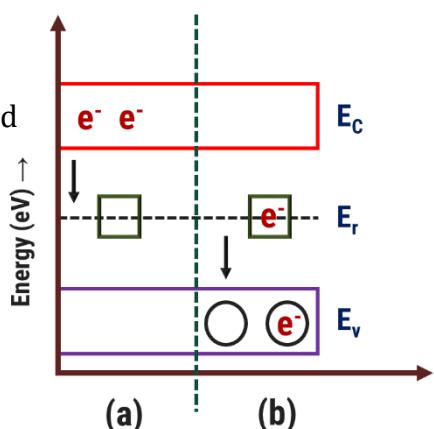
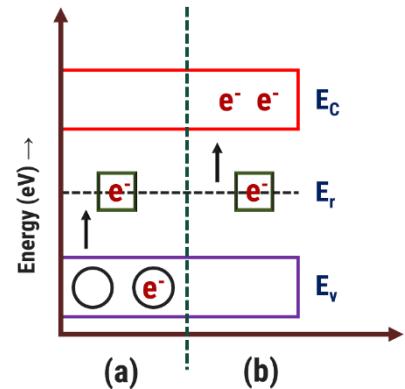
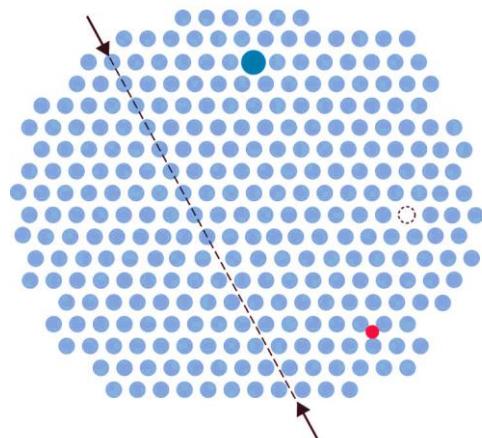
- ✓ Electron Capture:

An electron from conduction band is captured by an empty trap in the band gap. This excess energy ($E_c - E_r$) is transferred to the crystal lattice (phonon transmission). Fig.(a)

- ✓ Hole Captured:

A trapped electron moves to valence band and neutralizes a hole (The hole is captured by trap). A phonon with energy ($E_r - E_v$) is generated. Fig.(b)

- ✓ The electron capture rate is proportional to the electron concentration (n_e) in conduction band. The hole capture rate is proportional to the hole concentration (n_h) in valence band.



- ✓ The hole and electron emission rates are proportional to the concentration of empty traps and filled traps respectively.

Applications:

- ✓ Non radiative/phonon transmission is an unwanted process in optoelectronics, lowering the light generation efficiency and increasing heat loss.

2.6 Carrier transport: Diffusion and Drift

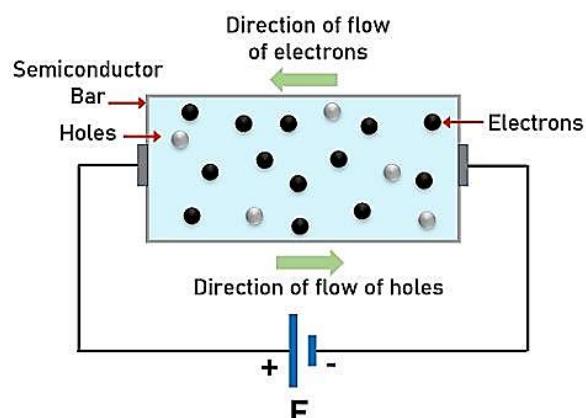
Drift Current:

- ✓ "The flow of charge carriers, which is due to applied voltage or electric field is called drift current."
- ✓ In semiconductors there are two types of charge carriers i.e. holes and electrons.
- ✓ When voltage is applied to semiconductors, free electrons moves towards the positive terminal of battery and holes moves towards the negative terminal.
- ✓ The average velocity that an electron or hole achieves, due to applied voltage or electric field is called "Drift velocity".
- ✓ Drift velocity of electrons is given by: $V_e = \mu_e E$
- ✓ Drift velocity of holes is given by: $V_h = \mu_h E$
- ✓ Drift current density due to free electrons: $J_e = n_e \mu_e E$
- ✓ Drift current density due to free electrons: $J_h = n_h \mu_h E$

$$\therefore n_e = n_h = n_i$$

- ✓ Total drift current density:

$$J = J_e + J_h$$

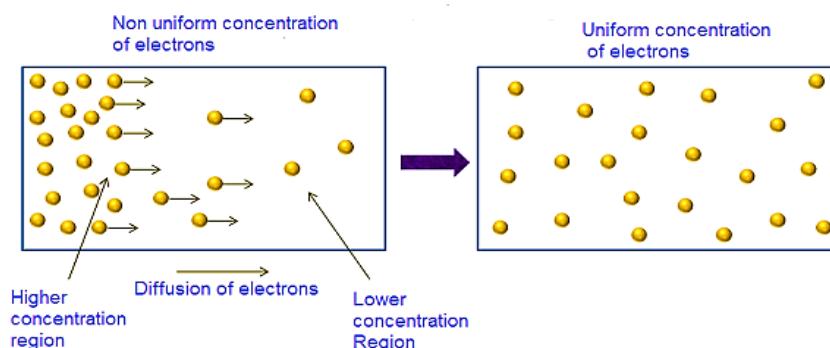


$$J = n_e \mu_e E + n_h \mu_h E$$

$$J = n_e E (\mu_e + \mu_h)$$

Diffusion Current:

- ✓ "Current produced due to the motion of charge carriers from a region of higher concentration to a region of lower concentration region."
- ✓ Regions having more no. of electrons is called higher concentration region and that with less no. of electrons is called lower concentration region.
- ✓ The above process occurs in semiconductors that are non uniformly doped.
- ✓ Let us consider an n-type semiconductor with non uniform doping.
- ✓ Due to non uniform doping, more no. of electrons are present on the left side, whereas lesser no. of electrons are present on the right side.
- ✓ The number of electrons on left side is more, as a result of which they will experience repulsive force from each other.
- ✓ Diffusion current occurs without an external voltage or electric field applied.



Drift current	Diffusion current
Drift current requires external voltage.	Diffusion current does not require external voltage.
It is present in both conductors and semiconductors.	It is present only in semiconductors.
Can be present in intrinsic and extrinsic semiconductors.	Can be present only in extrinsic semiconductors with uneven doping.

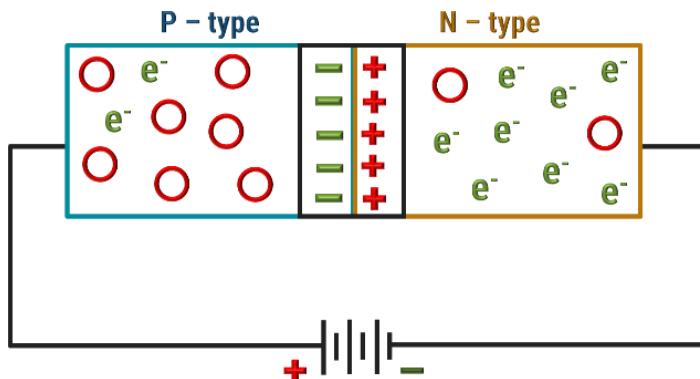
Its value is generally high in reverse bias.	Its value is generally high in forward bias.
Drift current is caused by electric field.	Diffusion current is caused by variation in carrier concentration.

2.7 P – N Junction

- ✓ When a p-type semiconductor is fused (intimately joined) to an n-type semiconductor, a PN junction is formed.
- ✓ A p-n junction diode is a two terminal device which allows electric current only in one direction, while blocks the electric current in opposite or reverse direction.
- ✓ A p-n junction diode is formed when an N-type semiconductor is fused with a P-type semiconductor creating a semiconductor diode. The immobile ions, at the junction creates a zone that is devoid of charge carriers (majority charge carriers).
- ✓ “This zone, depleted or devoid of charge carriers is called ‘Depletion region’.”
- ✓ The thickness of the depletion region is of an order of 10^{-6} m.
- ✓ To change the thickness of depletion barrier width and improve conduction in PN junction diode we apply external voltage as a battery which is known as external biasing.

Forward biasing:

- ✓ When the external voltage applied to the junction is in such a direction that it cancels the potential barrier, thus permitting current flow, the junction is said to be in forward biased condition.



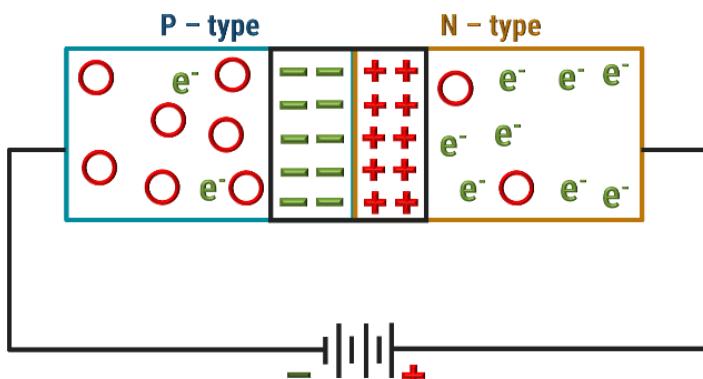
- ✓ To apply forward bias, connect positive terminal of the battery to p-type and negative terminal to n-type as shown in figure.
- ✓ The applied forward potential establishes an electric field which acts against the field due to potential barrier. Therefore, the resultant field is weakened and the barrier height is reduced at the junction as shown in figure. Therefore, the resultant field is weakened and the barrier height is reduced at the junction.
- ✓ As potential barrier voltage is very small (0.1 to 0.3 V), therefore, a small forward voltage is sufficient to completely eliminate the barrier.
- ✓ Once the potential barrier is eliminated by the forward voltage, junction resistance becomes almost zero and a low resistance path is established for the entire circuit. Therefore, current flows in the circuit. This is called forward current.

With forward bias to p-n junction, the following points are worth noting:

- ✓ The potential barrier is reduced and at some forward voltage (0.1 to 0.3 V), it is eliminated altogether.
- ✓ The junction offers low resistance (called forward resistance, R_F) to current flow.
- ✓ Current flows in the circuit due to the establishment of low resistance path. The magnitude of current depends upon the applied forward voltage.

✓ Reverse biasing:

- ✓ When the external voltage applied to the junction is in such a direction that potential barrier is increased, this set-up is said to be in reverse biased condition.



- ✓ To apply reverse bias, connect negative terminal of the battery to p-type and positive terminal to n-type as shown in figure.
- ✓ It is clear that applied reverse voltage establishes an electric field which acts in the same direction as the field due to potential barrier. Therefore, the resultant field at the junction is strengthened and the barrier height is increased as shown in figure.
- ✓ The increased potential barrier prevents the flow of charge carriers across the junction. Thus, a high resistance path is established for the entire circuit and hence the current does not flow.

With reverse bias to p-n junction, the following points are worth noting:

- ✓ The potential barrier is increased.
- ✓ The junction offers very high resistance (called reverse resistance, R_r) to current flow.
- ✓ No current flows in the circuit due to the establishment of high resistance path.

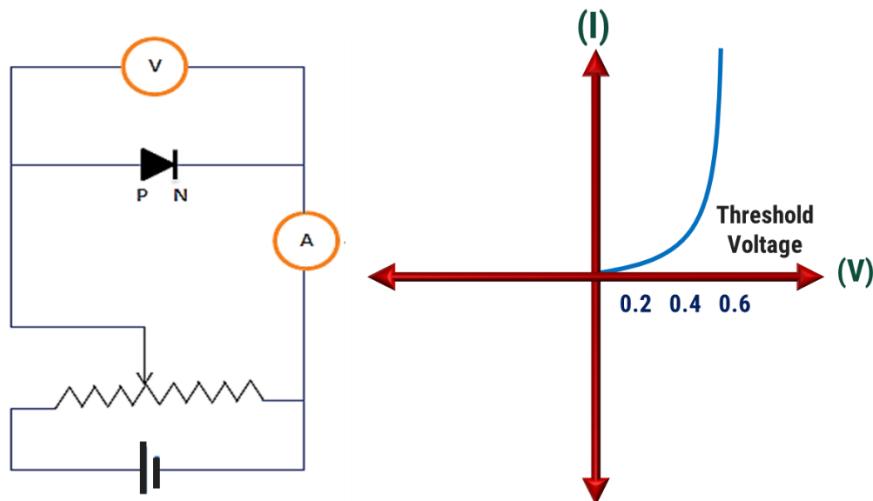
2.7.1 Current – Voltage (I-V) characteristics of a P – N junction Diode:

- ✓ The behaviour of a diode can be obtained by means of graph known as volt-ampere or I – V characteristics.
- ✓ “It is a graph between voltage across the terminals of a pn junction diode and the current flowing through it.”
- ✓ Characteristics of a diode can be studied under forward biasing and reverse biasing.

(a) Forward biasing of a diode:

- ✓ The circuit diagram for obtaining forward characteristics of a diode is shown in fig.
- ✓ When P-N junction diode is forward biased and if the applied voltage is gradually increases in steps, at some forward voltage (V_F), the potential barrier is altogether eliminated and current starts flowing.
- ✓ It is 0.3 V for Ge and 0.7 V for Si.

- ✓ “The voltage is known as threshold voltage (V_{th})” (Also called knee voltage or cut-in voltage).



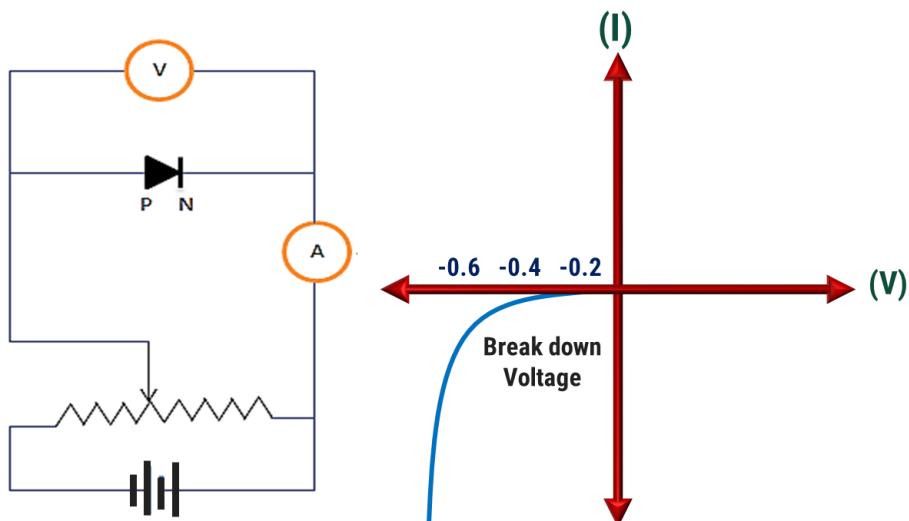
- ✓ Once the external voltage exceeds the barrier potential, the current increases exponentially as shown in graph. This is called the linear operating region of diode.
- ✓ $V_F = V_{th} = V_B$ (V_B = Barrier potential)
- ✓ Forward resistance can be calculated as:

$$R_F = \frac{\Delta V_F}{\Delta I_F}$$

- ✓ If the forward voltage is increased beyond the safe limit, damage of diode is likely to occur due to overheating.

(b) Reverse biasing of a diode:

- ✓ The circuit diagram for plotting reverse characteristics of a diode is as shown in fig.
- ✓ When a p-n junction is reverse biased, majority carriers are blocked and only a small current due to minority carriers flows through the diode.
- ✓ As the reverse voltage is increased in suitable steps, reverse current reaches its maximum or saturation value. This is called reverse saturation current or leakage current.
- ✓ The diode current is recorded at each step and graph is plotted as shown in fig.
- ✓ When the breakdown voltage is more than the applied voltage, diode current is very small and almost constant.

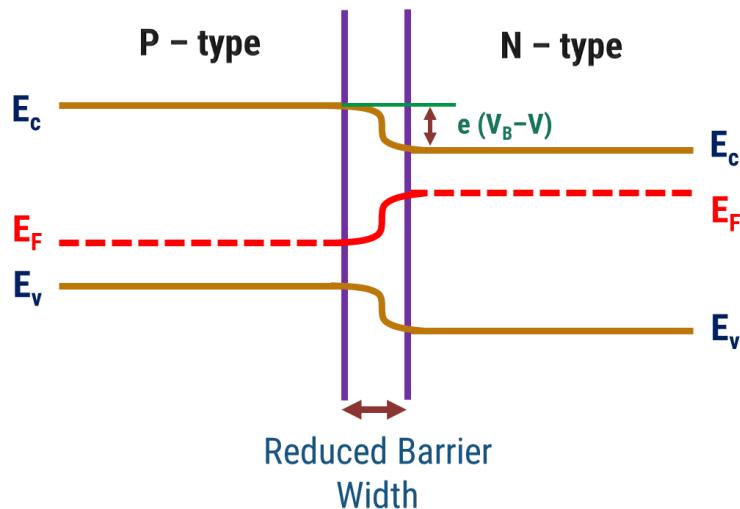


- ✓ When external voltage exceeds the breakdown voltage, current sharply exceeds, this curve is called the zero resistance path.
- ✓ Reverse current is of an order of (μA) for Ge and (nA) for Si.
- ✓ Resistance of diode from the curve is:

$$R_R = \frac{\Delta V_R}{\Delta I_R}$$

2.7.2 Energy band diagram of P – N junction:

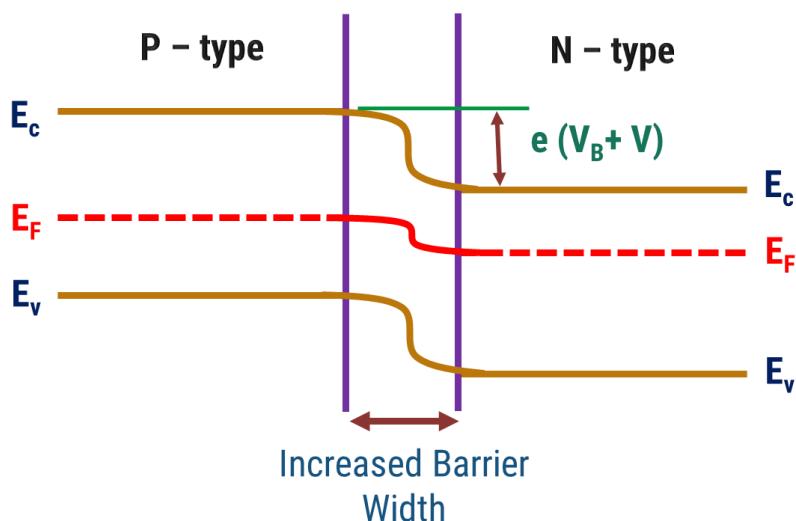
(a) Forward biasing of a diode:



- ✓ An external battery of voltage V , with its positive terminal connected to p-type and negative terminal is connected to n-type.

- ✓ The energy of electrons in N – region, increases by eV (As N-type is connected to -ve terminal).
- ✓ Now the fermi level is rises by a factor eV and hence potential barrier is reduced to $e(V_B - V)$ and the barrier width is reduced.
- ✓ V_B is the potential barrier across junction
- ✓ The electron thus face a reduced potential and can cross the junction.
- ✓ For the current flow, the applied potential should be grater than barrier potential.

(b) Reverse biasing of a diode:



- ✓ An external battery with voltage V is connected, with its positive terminal connected to N – region and negative terminal is connected to P – region.
- ✓ The energy of electrons is now reduced by eV . So the fermi level shifted down by a factor eV and potential barrier increases to $e(V_B + V)$, thereby increasing barrier width.
- ✓ It now becomes difficult for electrons to cross the junction, so there is no current flow.
- ✓ However, a very small current can flow due to the minority charge carriers (μA for Ge and nA for Si).

Application:

- ✓ P-N junction diodes are used in clamping circuits for dc restoration.
- ✓ They are used in clamping circuits for wave shaping.
- ✓ They are used in voltage multipliers.
- ✓ They are used as switch in digital logic circuits.
- ✓ They are used in demodulation circuits and optical communications.

2.7.3 Zener Diode (P – N Junction)

- ✓ It is named after Clarence Zener who discovered Zener effect.
- ✓ “A Zener diode is a type of p-n junction diode that allows current flow not only from its anode to cathode, but also in reverse direction when Zener voltage is achieved.”
- ✓ The Zener diode’s operation depends on the heavy doping of its p-n junction.
- ✓ The Zener diode is designed to operate in the break down region without damage.

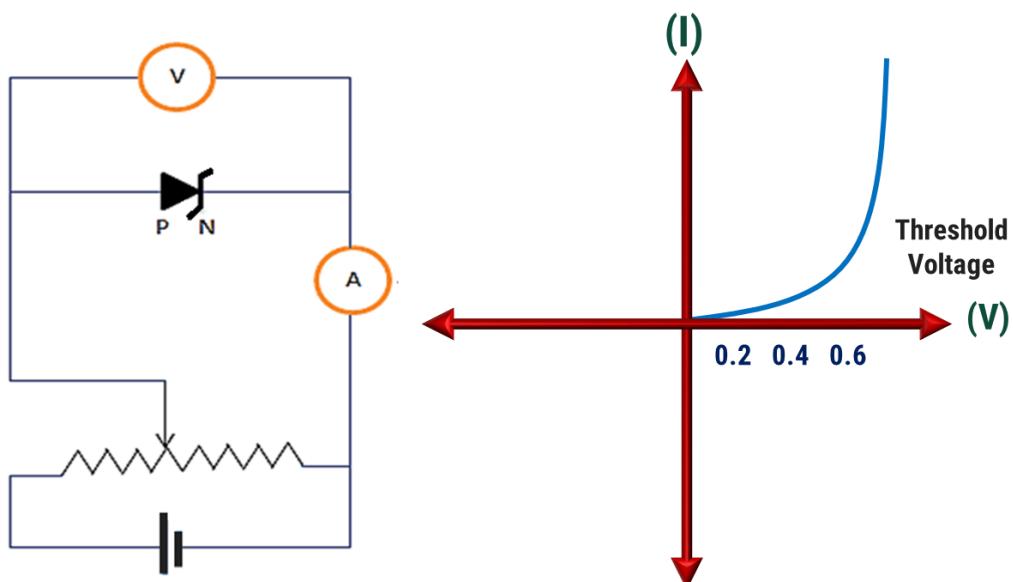
Working Principle:

- ✓ “It is a type of electrical breakdown that occurs in reversed biased p-n junctions when the electrical field enables tunneling of electrons from the valence band to conduction band of a semiconductor leading to a large number of free minority charge carriers, which suddenly increases the reverse current.”

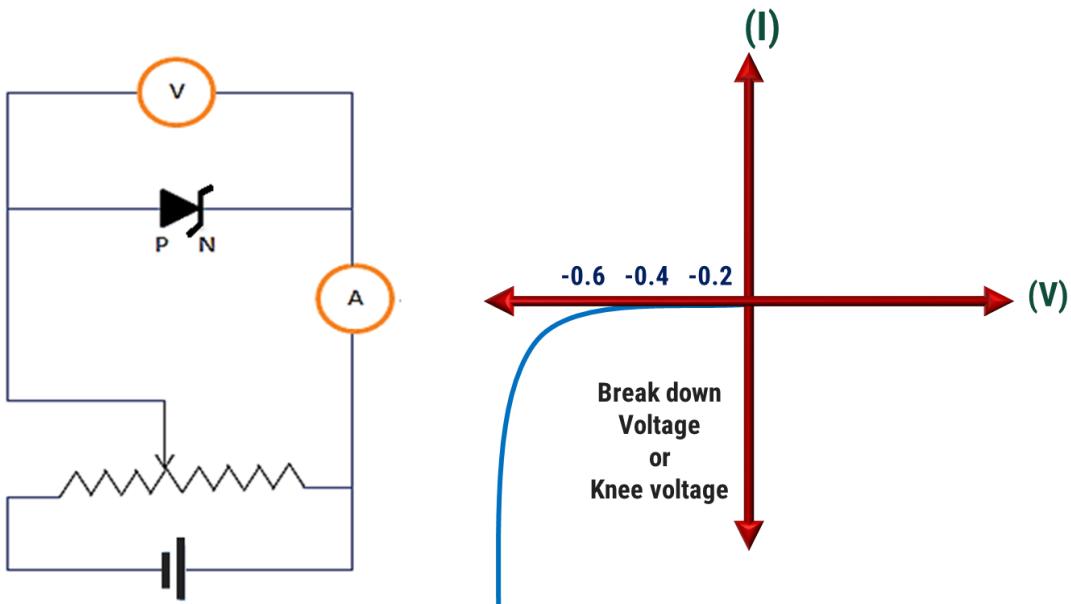
Working of Zener diode:

(a) Forward biasing of a diode:

- ✓ When Zener diode is connected in forward bias, it will act like normal p-n junction diode, having a voltage drop of around 0.7 V.



(b) Reverse biasing of a diode:



- ✓ Under the reverse bias condition, the breakdown of a Zener diode occurs. The breakdown and hence the Zener voltage depends on the amount of doping.
- ✓ As the reverse voltage is increased, the reverse current remains negligible up to a point called the knee point.
- ✓ At this knee point voltage is very sharp as compared to the normal p-n junction diode.
- ✓ The reverse current increases sharply to a very high value after this point.
- ✓ The Zener diode will not burn as diode has entered breakdown. The external resistance connected to the circuit prevents the Zener diode from burning.
- ✓ The maximum permissible value of current is denoted by I_{\max} and the minimum current sustain breakdown is called I_{\min} .

Application:

- ✓ Zener diodes are highly used as voltage regulator.
- ✓ They are used in wave shaping circuits as peak limiters or clippers.
- ✓ They are used for meter protection to prevent against damage from accidental overload.

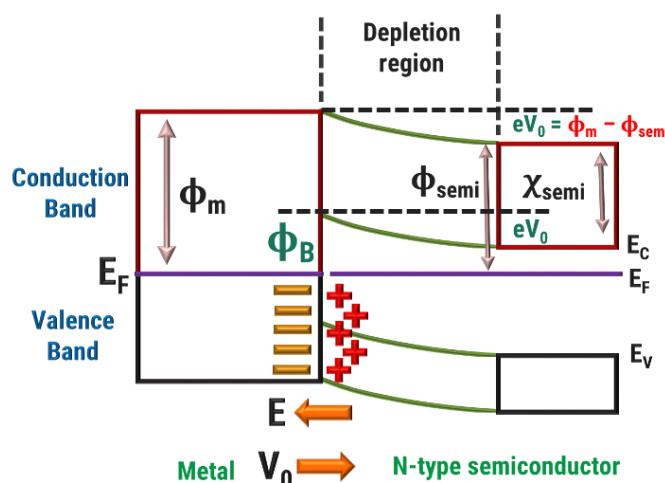
2.8 Metal – Semiconductor Junction (Schottky and Ohmic)

- ✓ When metal and semiconductor are brought into contact, there are two types of junctions formed, depending on the work function (ϕ) of semiconductors and its relation with metal.
- ✓ Generally n-type semiconductor and metals like platinum, molybdenum, chromium, and tungsten are used.
- ✓ **Work function (Φ)** is the minimum energy required to transfer an electron from a point within a solid to a point just outside its surface.
- ✓ **χ is the electron affinity** means amount of energy released or spent when an electron is added to any place.

1. $\Phi_m > \Phi_{semi} \rightarrow$ Schottky junction

2. $\Phi_m > \Phi_{semi} \rightarrow$ Ohmic junction

1. Schottky Contact ($\Phi_m > \Phi_{semi}$):



- ✓ When contact is made the fermi level should line up at equilibrium.
- ✓ The fermi level lines up and a positive potential is created on semiconductor side and a negative potential on metal side.

- ✓ When the contact is formed, due to low charge carrier density on semiconductor side, electrons are removed not only from the surface, but also from certain depth of semiconductor.
- ✓ This leads to formation of depletion region on the semiconductor side.
- ✓ The fermi level lines up and a positive potential is created on semiconductor side and a negative potential on metal side.
- ✓ So, the bends (V. B. and C.B.) bends up in the direction of electric field.
- ✓ There is built in potential in Schottky junction, given as difference of work function.

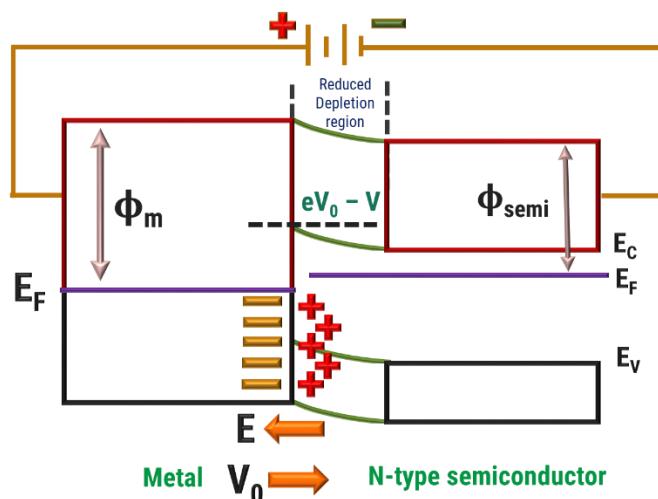
$$\phi_m - \phi_{\text{semi}} = eV_0$$

- ✓ This contact potential acts as a barrier for electrons to move from semiconductor to metal. When the contact was made, electrons moved to metal side and formed a depletion region on semiconductor side which prevents further motion of electrons.
- ✓ This is the Schottky barrier, denoted by

$$\Phi_B = (\phi_m - \phi_{\text{semi}}) + (E_c - E_F)$$

$\Phi_B = (\phi_m - \chi_{\text{semi}})$ (χ_{semi} is the electron affinity of semiconductor).

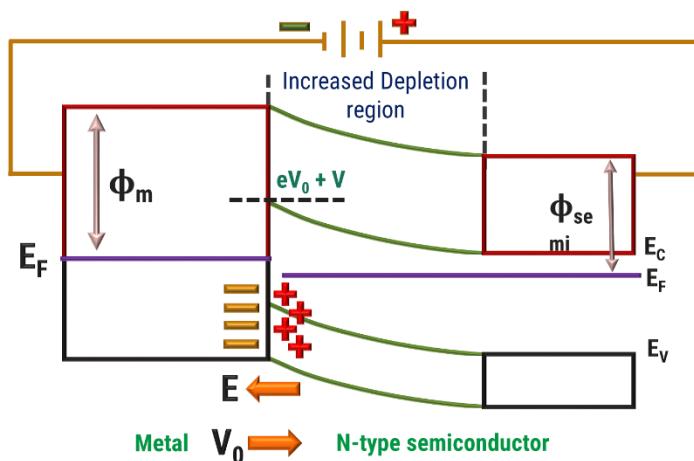
(a) Forward Bias



- ✓ The external voltage is applied in such a way that it opposes the built-in potential.

- ✓ The fermi levels no longer line up, but are shifted. The magnitude of the shift depends on the applied voltage. The depletion layer is thus narrowed and electrons move from semiconductor to metal.
- ✓ A large current, exponentially related to 'V' now starts flowing
- ✓ $I = I_0 [e^{(eV/kT)} - 1]$, I_0 is a constant and depends on ϕ_B (Schottky barrier).

(b) Reverse Bias



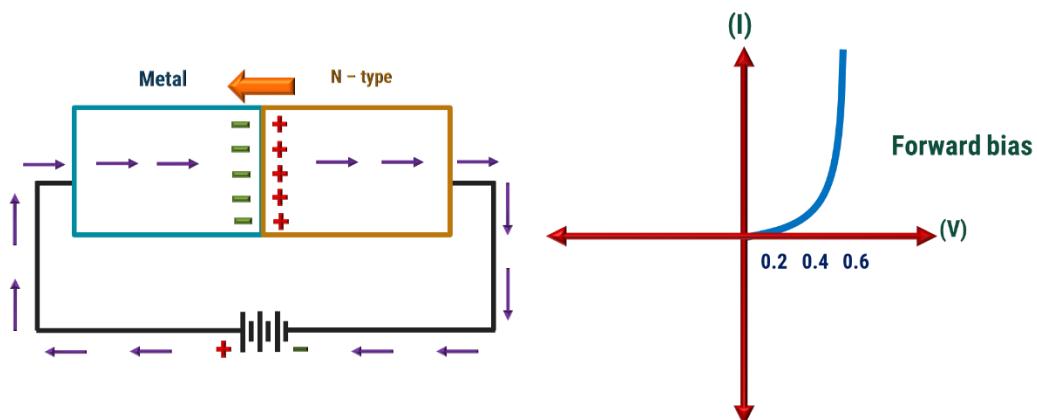
- ✓ In this case, the external potential, applied is in the same direction as built-in potential. Again, the fermi levels do not line up, but the barrier for electron motion from n-type to metal becomes higher.
- ✓ The applied voltage adds on to the built-in potential and depletion region gets wider.

Schottky Diode:

- ✓ The Schottky diode named after Walter H Schottky is also called "Schottky barrier diode".
- ✓ It is a semiconductor diode formed by the junction between an n-type semiconductor and metals like molybdenum, chromium, platinum, tungsten, etc.
- ✓ It has a low forward voltage drop and a very fast switching action.
- ✓ Schottky barrier is a depletion layer formed at the junction of metal and n-type semiconductor.

- ✓ It is the potential energy barrier that electrons have to overcome in order to flow across the diode.
- ✓ One of the most important characteristics of a Schottky barrier is its height.
- ✓ As shown in the diagram, the atoms that lose electrons at the n-side become positive ions whereas the atoms that gain extra electrons at the metal side become negative ions. These positive and negative ions together from a depletion layer.
- ✓ The depletion layer formed is more on the n-side, so the free electrons need a great energy to overcome this barrier. Hence there is no conduction in an unbiased diode.

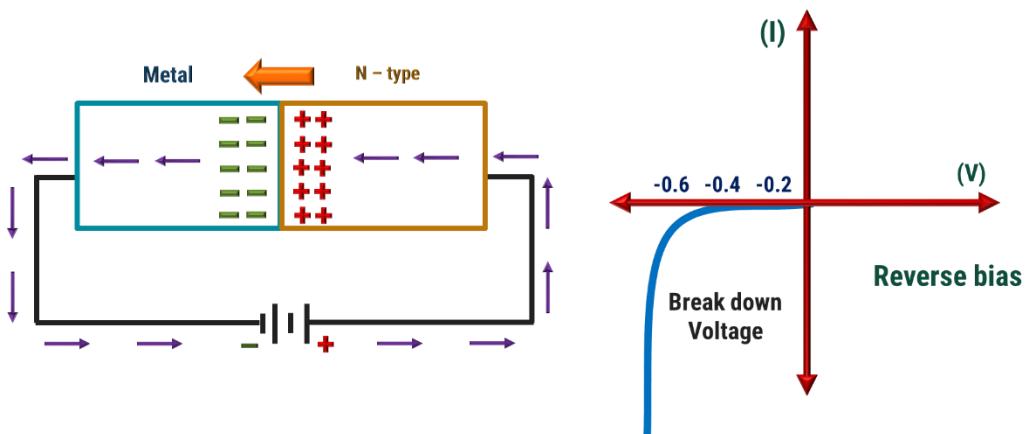
(a) Forward Bias



- ✓ If the positive terminal of battery is connected to the metal and negative terminal of battery is connected to the n-type the Schottky diode is said to be forward biased.
- ✓ When forward voltage is applied to Schottky diode, a large no. of free electrons are generated. When the barrier voltage or built-in voltage is overcome, these electrons cross the junction and a current starts flowing metal.
- ✓ The I-V characteristics are almost similar to PN junction diode.
- ✓ The forward voltage drop is around 0.2 to 0.3 V.

(b) Reverse Bias

- ✓ In the negative terminal of battery is connected to metal and positive terminal of battery is connected to n-type, the Schottky diode is said to be reversed biased.



- ✓ When reverse bias is applied, the depletion width increases and the electric current stops. A small leakage current flows due to thermally excited electrons in metal.
- ✓ Also, the reverse saturation current occurs at very low voltage as compared to silicon diode.

Advantages

- ✓ Low junction capacitance
- ✓ Fast recovery time
- ✓ High current density
- ✓ High efficiency

Disadvantages

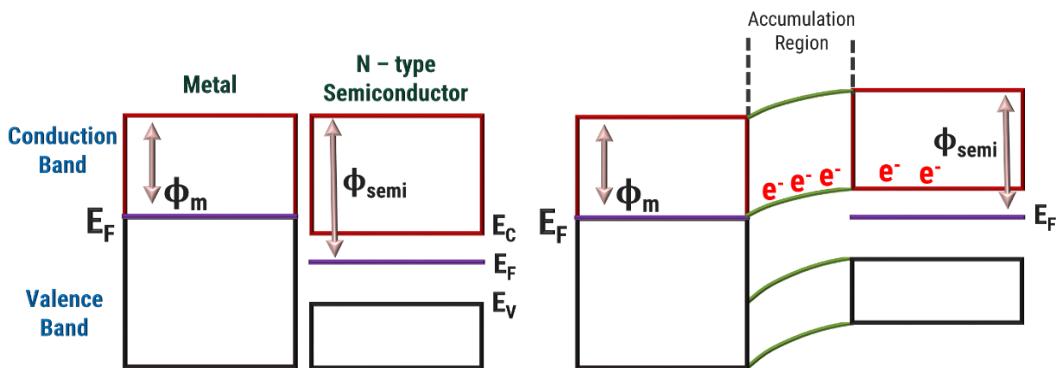
- ✓ Reverse saturation current is large

Applications

- ✓ Rectifiers
- ✓ Radio frequency applications
- ✓ Power supplies
- ✓ Logic circuits

2. Ohmic Contact ($\phi_m > \phi_{semi}$):

- When the semiconductor has a higher work function than metal, an Ohmic junction is formed.



- An Ohmic contact is defined as a metal – semiconductor contact that has a negligible contact resistance relative to the bulk or series resistance of the semiconductors.
- At equilibrium, the fermi levels line up. The electrons move from the metal to the semiconductor energy states of C.B, so that there is accumulation region near the interface of semiconductor.
- The accumulation region has a higher conductivity than bulk of the semi-conductor.
- Thus Ohmic junction behaves as a resistor conducting in both forward and reverse bias.
- For Ohmic junction, depending on the direction of current flow, heat can be generated or absorbed.
- This can be used as a practical cooling device.

2.9 Semiconductor materials of interest for optoelectronics

- When photons of energy equal to or greater than the band gap energy are incident on a semiconductor, electrons from the valence band are excited to conduction band, thereby creating electron-hole pair.

(a) Photoconductivity

"The increase in conductivity of a material due to EHP (electron hole pair) arising from optical excitation is called photoconductivity."

(b) Luminescence

"The property of light emission is called luminescence."

(c) Photoluminescence

When electrons are excited by the absorption of photons of suitable frequency and energy equal to or greater than bandgap the resulting radiation due to recombination of electron - hole pairs is called photoluminescence.

2.10 LED's (Light emitting Diodes)

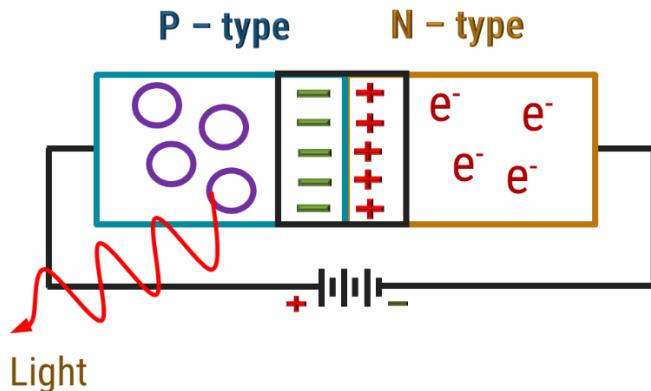
- ✓ "It is a two-lead semiconductor device which emits light, when electrons (from conduction band) recombine with holes (in valence band)."
- ✓ LED's are basically p-n junctions that are made from a very thin layer of fairly heavily doped semiconductor material and depending on the semiconductor material used, and the amount of doping, when forward biased, an LED emits light of a specific wavelength.

1	Ga As	Infrared (850 – 940 nm)
2	Ga As P	Red (630 – 660 nm)
3	Ga P	Yellow (585 – 595 nm)
4	Al Ga P	Green (550 – 570 nm)
5	Si C	Blue (430 – 505 nm)

Working of LED

- ✓ As seen in the diagram, the fermi levels line up in equilibrium. There is built in potential due to which, electrons from n-side are not able to cross the junction.
- ✓ When an extra forward voltage is applied to an LED, the width of the depletion layer decreases on increasing applied voltage.

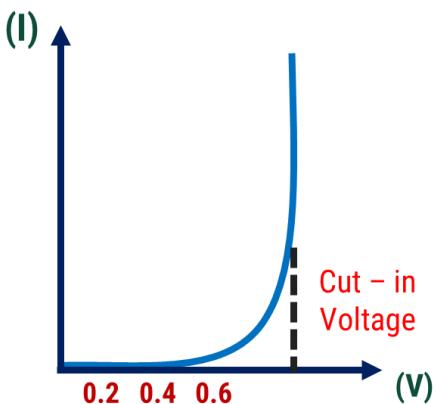
- ✓ The electrons now have sufficient energy to overcome the potential barrier. These electrons, on crossing the barrier, recombines with holes and releases the difference of energy ($E_c - E_v$) in the form of photon.



- ✓ Each recombination of carriers, emits some light.
- ✓ The energy of photons depends on the forbidden energy gap.
- ✓ When the forward bias is applied to the LED, the intensity of emitted light is small. As the forward current increases, the emitted light also increases.

Characteristics of LED

- ✓ From the graph, it is seen that current increases exponentially, after a certain voltage.
- ✓ Till then, due to potential barrier, current is almost zero.

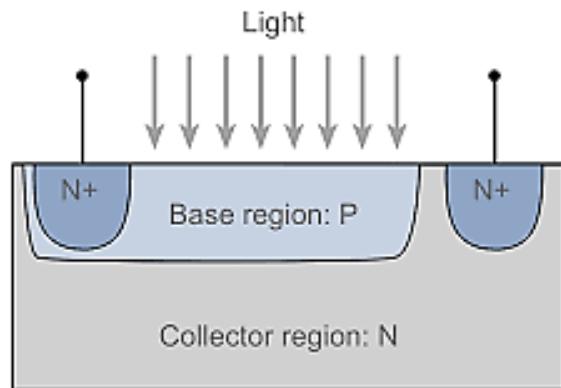


Application

- ✓ Camera flashes
- ✓ Traffic signals
- ✓ General lighting
- ✓ Medical device

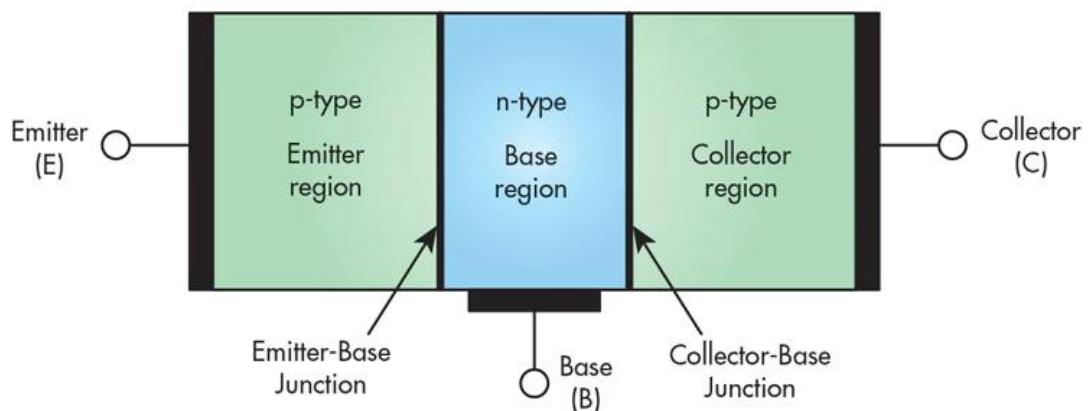
2.10.1 Photodiodes

- ✓ “A photodiodes is a semiconductor device that converts light into electric current.”
- ✓ The working principle of photodiodes is “Photoelectric effect”.



2.10.2 Phototransistor

- ✓ A phototransistor is a device that is able to sense light levels and alter the current flowing between emitter and collector according to the level of light it receives.



IMP questions:

- 1) What are intrinsic and extrinsic semiconductors?
- 2) Difference between P-type and N-type semiconductor.
- 3) What is drift and diffusion current?
- 4) Define carrier generation and recombination.
- 5) Define work function and electron affinity.
- 6) Write a short note on Ohmic contacts.
- 7) Write a short note on Zener diode.

Descriptive questions:

- 1) Derive an expression for density of holes in valence band of an intrinsic semiconductor.
- 2) Derive an expression for density of electrons in conduction band of an intrinsic semiconductor.
- 3) Write a short note on direct and indirect recombination?
- 4) What changes takes place when a P-N junction is (1) Forward bias (2) Reverse bias.

Numericals:

- 1) The intrinsic carrier density at room temperature in Germanium is $2.37 \times 10^{19}/\text{m}^3$. If electrons and hole mobility are 0.38 and $0.18 \text{ m}^2/\text{V}\cdot\text{sec}$ respectively, find out its resistivity.

[Ans. $\rho = 0.471 \Omega\text{m}$]

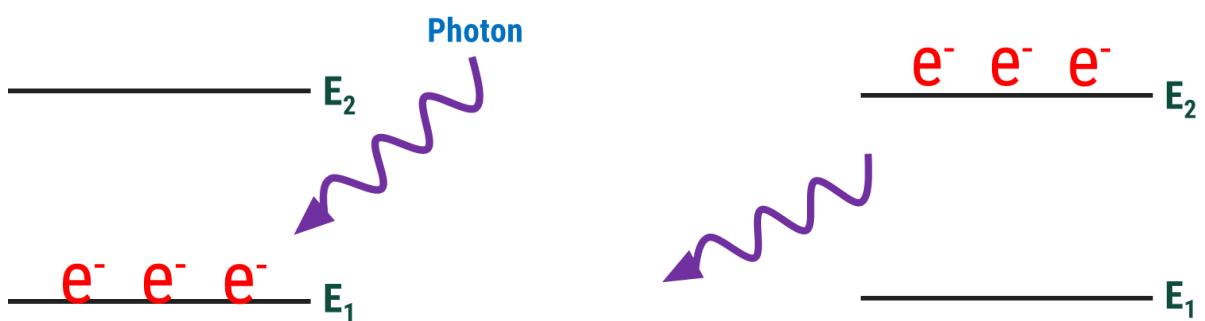
- 2) The electron and hole mobilities in intrinsic antimony are 6 and $0.2 \text{ m}^2/\text{V}\cdot\text{sec}$ respectively. At room temperature, resistivity is $2 \times 10^{-4} \Omega\text{m}$. Assuming the material is intrinsic, determine its intrinsic carrier density at room temperature.

[Ans. $n_i = 5.04 \times 10^{21}/\text{m}^3$]

UNIT – 3 (Light Semiconductor Interaction)

3.1 Introduction:

- ✓ Electrons in semiconductors tend to stay in lowest energy state as it is most stable. If additional energy is supplied to electrons by heating, light, or electron beams, it absorbs these energies and jump (transit) to high energy state.
- ✓ Such transitions of electrons from lower to higher energy states by absorbing external energy are called excitations.
- ✓ But higher energy states are unstable therefore to gain stability the electrons in higher energy states transit to lower energy states in certain lifetime. Such transitions of excited electrons from higher energy states to lower energy states are called relaxations.



- ✓ In semiconductors, transitions of electrons from higher energy states to lower energy states is also called electron hole recombination.
- ✓ During relaxation electrons release additional energy in form of light or heat. Therefore electron hole recombinations are of two types : radiative recombination's and non-radiative recombinations.
- ✓ Radiative recombinations emit photons and energies of photons correspond to a difference in the energies between initial and final energy states related to the transitions.
- ✓ In contrast, in the non-radiative recombination, phonons are emitted to crystal lattices or electrons are trapped in the defects, and the transition energy is transformed into forms other than light.

3.2 Optical transitions in bulk semiconductor:

- ✓ According to Einstein, whenever there is interaction between radiation and matter, there can be three main processes than can occur. One is absorption and other two are emissions.

3.2.1 Induced Absorption:

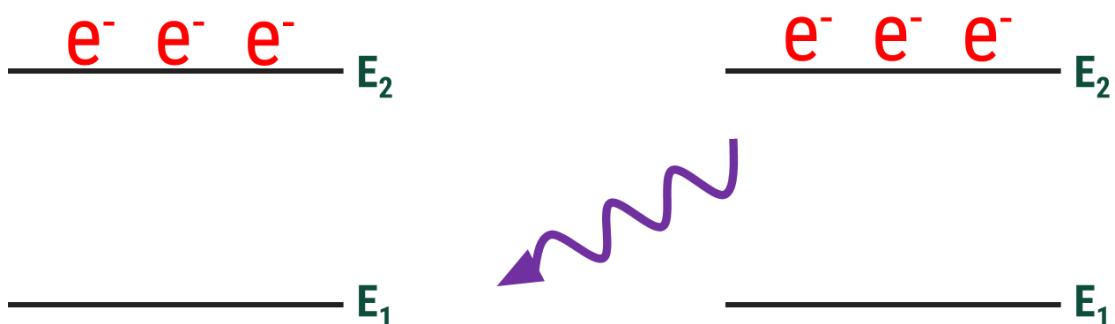
- ✓ Initially an atom is in the ground state (E_1), i.e. all of its electrons possess the lowest possible energy states.
- ✓ If an atom transits from ground state (E_1) to higher energy state (E_2) by absorbing energy of incident photons then the process is called absorption (induced absorption or stimulated absorption).
- ✓ Energy of photon (E) = $\hbar\nu = (E_2 - E_1)$
- ✓ For absorption: **Atom + Photon → Atom***



- ✓ The rate of absorption (R_{12}) is directly proportional to the radiation (ρ) and population of lower energy level (N_1 = No. of atoms in lower energy level).
- ✓ i.e. $R_{12} \propto N_1 \rho$
- ✓ i.e. $R_{12} = B_{12} N_1 \rho$
- ✓ where B_{12} is called the Einstein's coefficient (constant) for absorption of radiation.

3.2.2 Spontaneous Emission:

- ✓ "The process of emission of photons when an atom transits from higher energy level to lower energy level without use of external energy (source) is called spontaneous emission."
- ✓ Suppose an atom is in higher energy level, As this level is unstable, the atom spontaneously returns to lower energy level on its own.
- ✓ It will emit energy, equal to the difference of two energy levels i.e. $\hbar\nu = (E_2 - E_1)$
- ✓ For spontaneous emission: **Atom* → Atom + Photon**

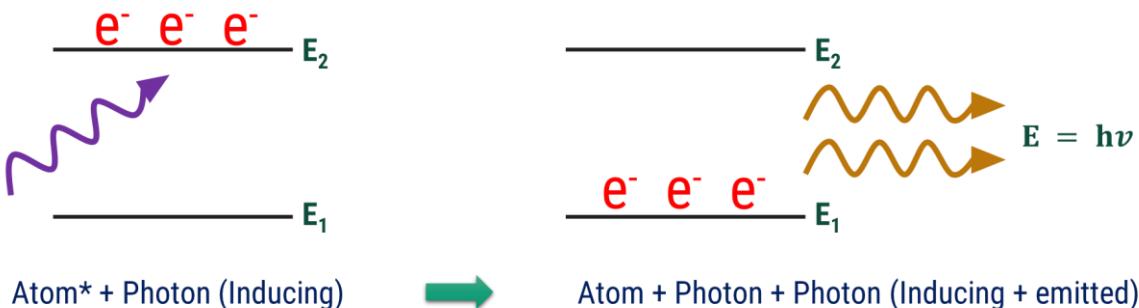


- ✓ The rate of spontaneous emission (R_{21}) is directly proportional to the population of higher energy level (N_2 = No. of atoms in higher energy level).
- ✓ i.e. $R_{21(Sp)} \propto N_2$
- ✓ i.e. $R_{21(Sp)} = A_{21(Sp)} N_2$
- ✓ where A_{21} is called the Einstein's coefficient (constant) for spontaneous emission of radiation.

3.2.3 Stimulated Emission:

- ✓ "The process of emission of photons when an atom transits from higher energy level to lower energy level, with use of external energy (source) is called stimulated emission."
- ✓ For this process, the atom has to be in higher energy state.

- ✓ Suppose a photon (with energy $E_2 - E_1$) is incident on atom, this incident photon will stimulate the excited atom to transit to lower energy E_1 .



- ✓ Both the inducing and emitted photons will have the same phase, energy and direction.
- ✓ i.e. they will be coherent.
- ✓ This is the process responsible for LASER.
 - ✓ The rate of spontaneous emission $R_{21(\text{St})}$ is directly proportional to the radiation (ρ) and
 - ✓ population of higher energy level (N_2 = No. of atoms in higher energy level).
 - ✓ i.e. $R_{21(\text{St})} \propto N_2 \rho$
 - ✓ i.e. $R_{21(\text{St})} = B_{21(\text{St})} N_2 \rho$
 - ✓ where B_{21} is called the Einstein's coefficient (constant) for stimulated emission of radiation.

3.2.4 Difference between spontaneous and stimulated emission

Spontaneous emission	Stimulated emission
Emission of photon take place without inducement.	Emission of photon takes place by inducement.
It is random process.	It is not a random process.
Photons do not get multiplied by chain reaction.	Photons get multiplied by chain reaction.
It is an uncontrollable process.	It is a controllable process.

Intensity is low.	Intensity is high.
Emission takes place in all directions.	Emission takes place in specific direction.
Polychromatic light is emitted.	Monochromatic light is emitted.

3.2.5 Relation between Einstein's coefficient:

- ✓ Einstein obtained a mathematical expression to establish the existence of the two different kinds of emission processes i.e. stimulated and spontaneous.
- ✓ We can't predict which particular atom will make a transition from one state to another at a particular instance, but we can calculate the rate of transmission between the states.
- ✓ Let us assume that an atomic system is in thermal equilibrium i.e. no. of upward transitions is equal to the no. of downward transitions per unit volume per second.
- ✓ Rate of absorption = Rate of emission

$$B_{12} N_1 \rho = A_{21} N_2 + B_{21} N_2 \rho \dots \dots \dots (1)$$

$$\therefore (B_{12} N_1 - B_{21} N_2) \rho = A_{21} N_2$$

$$\therefore \rho = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

Dividing by N_2

$$\therefore \rho = \frac{A_{21}}{(B_{12} \frac{N_1}{N_2} - B_{21})} \dots \dots \dots (2)$$

- ✓ N_1 and N_2 is the no. of atoms in ground and excited state, in thermal equilibrium
- ✓ So as per Boltzmann distribution law,

$$N = N_0 \cdot e^{\left(\frac{-E}{k_B T}\right)}$$

- ✓ We can write,

$$N_1 = N_0 \cdot e^{\left(\frac{-E_1}{k_B T}\right)} \text{ and } N_2 = N_0 \cdot e^{\left(\frac{-E_2}{k_B T}\right)} \dots \dots \dots (3)$$

- ✓ Where N_0 is the total no. of atoms

- ✓ k_B is the Boltzmann constant
- ✓ From equation no. (3)

$$\therefore \frac{N_1}{N_2} = \frac{N_0 \cdot e^{\left(\frac{-E_1}{k_B T}\right)}}{N_0 \cdot e^{\left(\frac{-E_2}{k_B T}\right)}}$$

$$\therefore \frac{N_1}{N_2} = e^{-\left(\frac{E_2 - E_1}{k_B T}\right)}$$

- ✓ We know $E_2 - E_1 = hv$

$$\therefore \frac{N_1}{N_2} = e^{-\left(\frac{hv}{k_B T}\right)}$$

$$\therefore \frac{N_2}{N_1} = e^{\left(\frac{hv}{k_B T}\right)} \dots \dots \dots (4)$$

$$\therefore \rho = \frac{A_{21}}{(B_{12} e^{\left(\frac{hv}{k_B T}\right)} - B_{21})}$$

$$\therefore \rho = \frac{A_{21}}{B_{21}} \left[\frac{1}{\left(\frac{B_{12}}{B_{21}} e^{\left(\frac{hv}{k_B T}\right)} - 1 \right)} \right] \dots \dots \dots (5)$$

- ✓ From Planck's theory of radiation,

$$\therefore \rho = \frac{8\pi h v^3}{c^3} \left[\frac{1}{e^{\left(\frac{hv}{k_B T}\right)} - 1} \right] \dots \dots \dots (6)$$

- ✓ Comparing equation (5) & (6)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h v^3}{c^3} \text{ and } B_{12} = B_{21}$$

- ✓ From equation (7), we can say that probability of absorption is equal to the probability of stimulated emission.
- ✓ Also, from equation (7), the ratio of spontaneous and stimulated emission is proportional to v^3 .
- i.e. Spontaneous emission dominates over stimulated emission.

3.2.6 Ratio of spontaneous and stimulated emission:

$$\frac{R_{21}(\text{St})}{R_{21}(\text{Sp})} = \frac{B_{21}N_2 \rho}{A_{21}N_2}$$

$$\frac{R_{21}(\text{St})}{R_{21}(\text{Sp})} = \frac{B_{21} \rho}{A_{21}} \dots \dots \dots (1)$$

$$\frac{B_{21} \rho}{A_{21}} = \frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{\left(\frac{h\nu}{k_B T}\right)} - 1}$$

But $B_{12} = B_{21}$

$$\frac{B_{21} \rho}{A_{21}} = \frac{1}{\left(\frac{B_{12}}{B_{12}}\right) e^{\left(\frac{h\nu}{k_B T}\right)} - 1} \dots \dots \dots (2)$$

Excited state
(N_2) 

ground state
(N_1) 

- ✓ From equation (1) & (2) we can write

$$\frac{R_{21}(\text{Sp})}{R_{21}(\text{St})} = e^{\left(\frac{h\nu}{k_B T}\right)} - 1 \dots \dots \dots (3)$$

- ✓ From above equation, we can say that Spontaneous emission far exceeds the no. of stimulated emission. So, laser action is not possible.
- ✓ In order to achieve more stimulated emission as compared to spontaneous emission, the population of excited state (N_2) should be made larger as compared to the population of ground state (N_1). This is what we call "Population inversion".

3.3 LASER:

Basic used terms in Laser:

- ✓ Population Inversion:

The state of achieving more no. of atoms in excited state as compared to ground state is called population inversion.

$E_2 > E_1$ and $N_2 > N_1$

✓ Pumping:

The mechanism of exciting atoms from lower energy state to higher energy state by applying energy from external source is called pumping.

✓ Active medium:

The medium in which population inversion is achieved for Laser.

✓ Optical resonator:

A pair of reflecting surfaces (mirrors), of which one is perfect and other is a partial reflector is called optical resonator.

Principal of Laser:

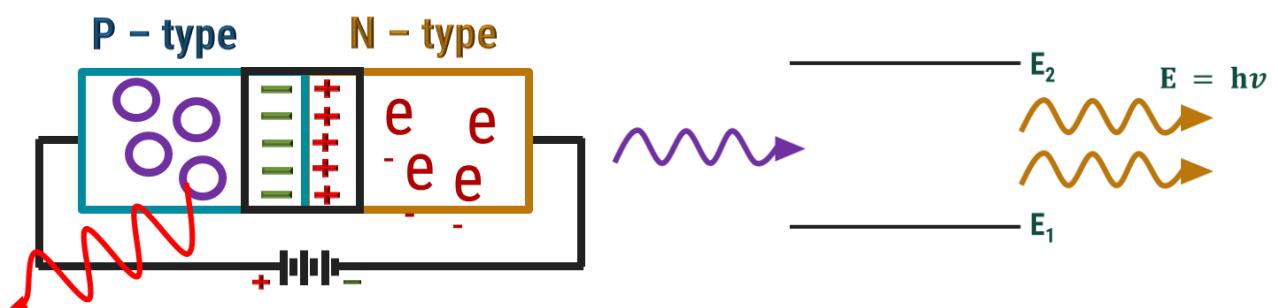
- ✓ The working of Laser is based on the principle of stimulated emission along with light amplification.
- ✓ For stimulated emission to take place, population of higher energy atoms should be more than population of lower energy atoms.
- ✓ This is achieved by pumping. Depending on the active medium, the pumping types can be optical pumping or electrical pumping or direct conversion pumping.
- ✓ Light amplification is achieved by photon multiplication within optical resonator cavity.
- ✓ The first step is to achieve population inversion in the active medium.
- ✓ A spontaneous emitted photon of an excited atom stimulates another atom in its path to release a second photon. These two coherent photons in turn stimulate two other atoms to release two more photons. So, there are four coherent photons.
- ✓ The photons thus get multiplied like a chain reaction, thereby producing an amplified light, resulting in an intense Laser beam.

Semiconductor Laser:

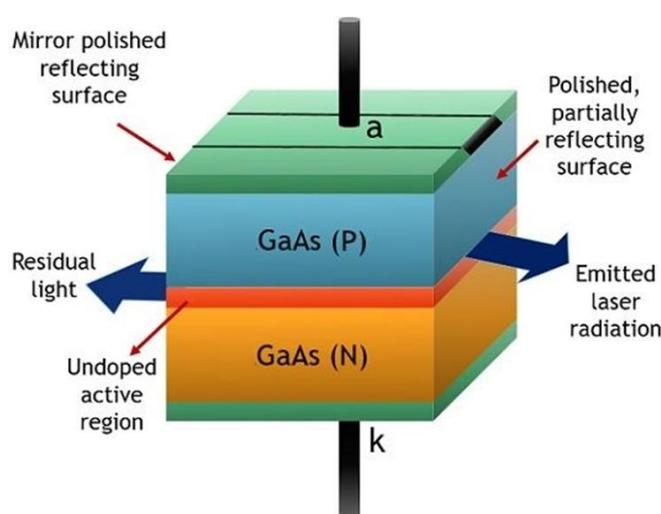
- ✓ It is a specially fabricated p-n junction diode that emits Laser light when it is forward biased.

Working Principle:

- ✓ When a p-n junction is forward biased, the electrons from n-region and holes from p-region cross the junction and recombine with each other.
- ✓ During this recombination, the energy is released in the form of photons.
- ✓ The photons emitted during recombination stimulated other electrons and holes to recombine as a result of which stimulated emission takes place.
- ✓ The basic principle of semiconductor Laser is Electroluminescence.



- ✓ Based on the materials used for fabricating the p-n junctions, the semiconductor Laser can be classified into 'Homo-junctions' (having same semiconductor materials with equal bandgaps on both sides, but different doping) and 'Hetero-junctions' (having different semiconductor materials with different bandgaps on both sides).
- ✓ Semiconducting Laser is also called Laser diode or Injection laser diode.



- ✓ A homo-junction GaAs semiconductor diode is given in Figure. The p-n junction is fabricated using a single semiconductor material.

- ✓ The active medium here, is a p-n junction made from GaAs. The p-type and N-type are doped by Germanium and Tellurium respectively.
- ✓ The thickness of p-n junction layer is made very thin, of an order of few microns and the end faces are made reflecting, that forms the optical resonator cavity.
- ✓ Current is supplied to the p-n junction through metal electrodes, fixed at the upper and lower faces.

Working:

- ✓ Population inversion here is achieved by injecting electrons across the junction from n-type to p-type semiconductor, by means of forward biasing voltage applied across it.
- ✓ The recombination of electrons and holes in the p-n junction region, results in emission of photons.
- ✓ The process is spontaneous and the Laser radiation is random and incoherent.
- ✓ The photons trigger the stimulated emission of photons to get generated by increasing the rate of recombination of electrons and holes.
- ✓ If the current density is increased, the emission becomes more and more coherent and radiation intensity increases.
- ✓ The wavelength of the emitted radiation, depends on the concentration of donor and acceptor atoms and energy gap of semiconductor.
- ✓ Band gap of GaAs is 1.44 eV.

$$E_g = hv = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{E_g}$$

$$\lambda = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.44 \times 1.6 \times 10^{-19}}$$

$\lambda = 8628 \text{ \AA}$ Wave-length of emitted radiation (IR).

Advantages:

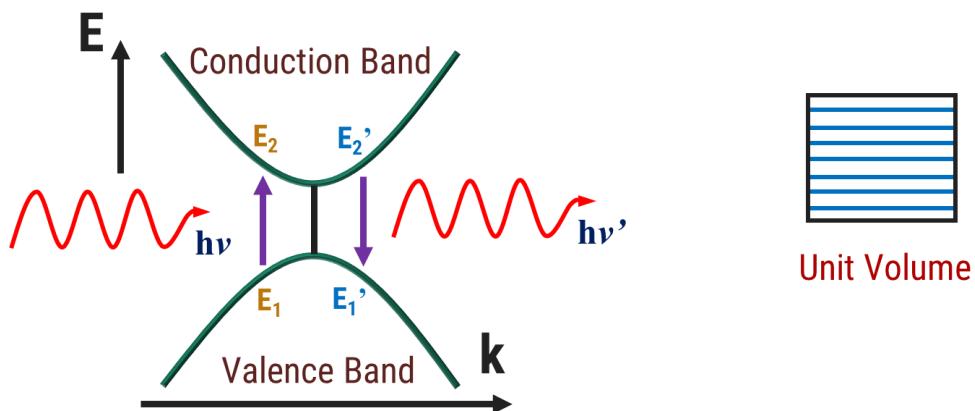
- ✓ Modulation of output is possible by controlling the junction current.
- ✓ It is small compact and has higher efficiency.
- ✓ The fabrication is simple and the output is continuous.
- ✓ Low power consumption and less costly.

Disadvantages:

- ✓ Output is in the form of wide beam.
- ✓ Threshold current density is large.
- ✓ Monochromaticity and coherence is poor as compared to other Laser.
- ✓ Temperature affects output of Laser.

Applications:

- ✓ They are used in long haul communication.
- ✓ Used in optical communication in PC or mobile phones.
- ✓ Used in Laser printers.
- ✓ Used in Laser microscopes, line Laser.

3.4 Joint density of states:

- ✓ We know that density of states defines the number of allowed energy states per unit volume.
- ✓ To know or find the number of transitions or emissions (because that gives the no. of photons emitted) we need to know the emissions per unit volume.
- ✓ This can also give an idea of the power emitted.
- ✓ The number of emissions per unit volume can be found by multiplying density of states and probability of occupations.

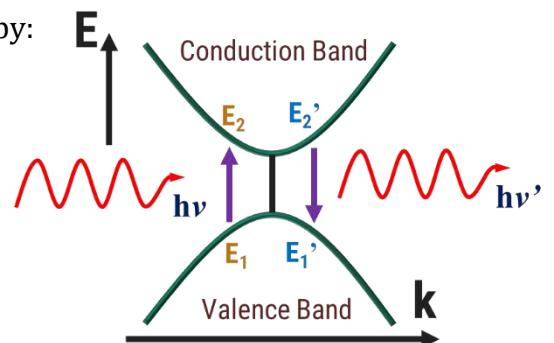
i.e. no. of emission/unit volume = density of states × probability of occupation.

- ✓ Concentration of electrons in conduction band is given by:

$$n_e = \int Z_c(E) \cdot dE \cdot f(E)$$

- ✓ Concentration of holes in valence band is given by:

$$n_h = \int Z_v(E) \cdot dE \cdot [1 - f(E)]$$



- ✓ Suppose a radiation of energy $h\nu$ is incident on an electron sitting at certain level in valence band (E_1), it makes a transition to a vacant state in conduction band (E_2).
- ✓ This means that absorption involves a certain energy level in valence and a corresponding vacant level in conduction band.
- ✓ Similarly, if there is an electron at certain energy level (E_2') in conduction band and it makes a transition at a vacant level (E_1') in valence band, a photon with energy $h\nu'$ is emitted.
- ✓ We can thus say that absorption and emission involve a state in valence band and conduction band.
- ✓ For a photon interaction, we have one state in valence band and one state in conduction band and so we define joint density of states that takes care of number of states for a given energy $h\nu$.
- ✓ For a fixed incident energy $h\nu$, if E_2 is fixed then E_1 will also be fixed.

- ✓ For a given energy hv there are several pairs of $E_2 - E_1$ and so there are number of pairs of states available for a photon of energy hv to interact and this is given by density of states.
- ✓ Since the absorption or emission takes place from the top of valence band and bottom of conduction band respectively, we can have a parabolic approximation as,

$$\therefore E_2 = E_c + \frac{\hbar^2 k^2}{2m_c} \dots \dots \dots (1) \text{ for any level in conduction band}$$

$$\therefore E_1 = E_v - \frac{\hbar^2 k^2}{2m_v} \dots \dots \dots (2) \text{ for any level in valence band}$$

- ✓ The energy of photon hv is given as:

$$\therefore hv = E_2 - E_1$$

$$\therefore hv = E_c + \frac{\hbar^2 k^2}{2m_c} - \left(E_v - \frac{\hbar^2 k^2}{2m_v} \right)$$

$$\therefore hv = (E_c - E_v) + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_c} + \frac{1}{m_v} \right)$$

$$\therefore hv = E_g + \frac{\hbar^2 k^2}{2} \left(\frac{1}{m_r} \right)$$

- ✓ Where, E_g = forbidden gap & m_r = reduced mass

$$\therefore hv = E_g + \frac{\hbar^2 k^2}{2m_r}$$

$$\therefore k^2 = (hv - E_g) \frac{2m_r}{\hbar^2} \dots \dots \dots (3)$$

- ✓ Substituting this value of k^2 in equation (1) and (2)

$$E_2 = E_c + \frac{\hbar^2}{2m_c} (hv - E_g) \frac{2m_r}{\hbar^2}$$

$$\therefore E_2 = E_c + \frac{m_r}{m_c} (hv - E_g) \dots \dots \dots (4)$$

- ✓ In the above equation E_c , m_r , m_c , E_g are constants, only E_2 and v are variables.

$$E_1 = E_v - \frac{\hbar^2}{2m_v} (hv - E_g) \frac{2m_r}{\hbar^2}$$

$$\therefore E_1 = E_v - \frac{m_r}{m_v} (hv - E_g) \dots \dots \dots (5)$$

- ✓ i.e. there is 1-1 correspondence between (E_2 & E_1) and v , Hence we have,

$$Z_c(E_2) dE_2 = Z(v) dv \dots \dots \dots (6)$$

- ✓ Where $Z(v) dv$ is the number of state per unit volume available for photons of energy between hv and $h(v + dv)$ to interact (either absorb or emit).
- ✓ Once we found out $Z(v) dv$, i.e. density of states available for interaction and multiply it by probability of emission or absorption, we can obtain total number of emission or absorption per unit volume.

- ✓ From equation (6)

$$Z(v) = Z_c(E_2) \frac{dE_2}{dv} \dots \dots \dots (7)$$

- ✓ From equation (4)

$$E_2 = E_c + \frac{m_r}{m_c} (hv - E_g)$$

$$\therefore (E_2 - E_c)^{\frac{1}{2}} = \left(\frac{m_r}{m_c} \right)^{\frac{1}{2}} (hv - E_g)^{\frac{1}{2}} \dots \dots \dots (8)$$

- ✓ Optical joint density of states gives the number of states available for photons to interact with from equation (7), we have

$$Z(v) = Z_c(E_2) \frac{dE_2}{dv} \dots \dots \dots (7)$$

$$\therefore Z(v) = \frac{\pi}{2 h^3} (8 m_c)^{\frac{3}{2}} (E_2 - E_c)^{\frac{1}{2}} \frac{m_r}{m_c} h \dots \dots \dots (9)$$

- ✓ From equation (8)

$$\therefore Z(v) = \frac{\pi}{2 h^3} (8 m_c)^{\frac{3}{2}} \left(\frac{m_r}{m_c} \right)^{\frac{1}{2}} (hv - E_g)^{\frac{1}{2}} \frac{m_r}{m_c} h \dots \dots \dots (9)$$

$$\therefore Z(v) = \frac{\pi}{2 h^2} (8m_c)^{\frac{3}{2}} \left(\frac{m_r}{m_c}\right)^{\frac{3}{2}} (hv - E_g)^{\frac{1}{2}}$$

$$\therefore Z(v) = \frac{\pi}{2 h^2} (8m_r)^{\frac{3}{2}} (hv - E_g)^{\frac{1}{2}}$$

$$\therefore Z(v) = \frac{\pi}{2 h^2} (2 \times 2^2 m_r)^{\frac{3}{2}} (hv - E_g)^{\frac{1}{2}}$$

$$\therefore Z(v) = \frac{8\pi}{2 h^2} (2m_r)^{\frac{3}{2}} (hv - E_g)^{\frac{1}{2}}$$

$$\therefore Z(v) = \frac{4\pi}{h^2} (2m_r)^{\frac{3}{2}} (hv - E_g)^{\frac{1}{2}}$$

- ✓ This is expression for optical joint density of states.

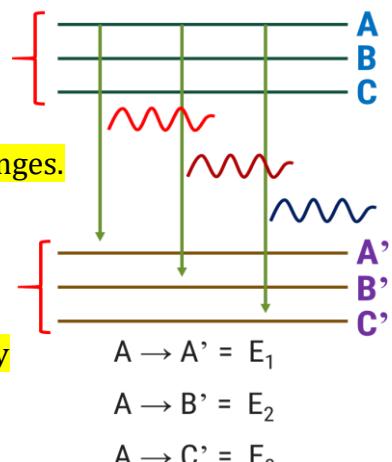
3.5 Transition Rate (Fermi's Golden rule)

Just for information:

- ✓ In each transition the $E = hv$ (energy of photon) will be different. So, if frequency v changes then wavelength λ changes.

$$v = \frac{1}{\lambda}$$

- ✓ In this transition Bohr successfully explained the energy spectra. But couldn't explain the intensity or brightness of spectra.



- ✓ So it was explained by the use of Fermi's Golden rule. By the use of transition rate.

- ✓ In quantum physics, Fermi's golden rule is used to calculate transition rates. The transition rate depends upon the strength of coupling between the initial and final state of a system and upon the number of ways the transition can happen (joint density of states).
- ✓ The transition probability is given by:

$$\lambda_{if} = \frac{2\pi}{\hbar} |M_{if}|^2 Z_f$$

- ✓ Where λ_{if} is transition probability, $|M_{if}|^2$ is matrix element for interaction and Z_f is joint density of final state.
- ✓ The above equation is known as fermi's golden rule.
- ✓ The transition probability λ is called the decay probability and is related to mean lifetime τ of the state.

$$\lambda = \frac{1}{\tau}$$

- ✓ The general form of fermi's golden rule can be applied to atomic transitions. Nuclear decay and scattering.
- ✓ The transition can proceed more rapidly if coupling between initial and final states is stronger.
- ✓ This coupling term is traditionally called the matrix element for the transition.
- ✓ This matrix element can be placed in the form of an integral, where the interaction (that causes transition) is expressed as a potential V that operates on initial state wave function.
- ✓ The transition probability is proportional to the square of integral of interaction over all of the space appropriate to the problem.

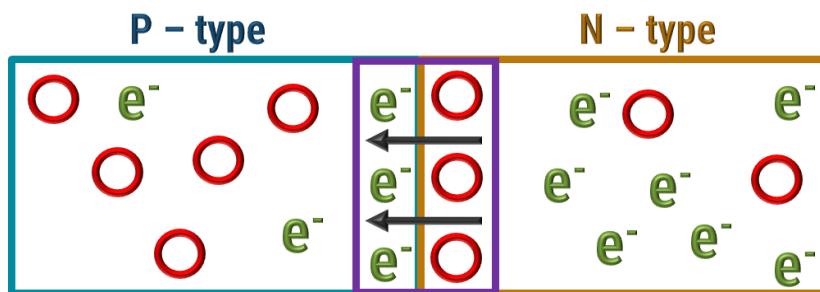
$$M_{if} = \int \psi_f^* V \cdot \psi_i d\nu$$

- ✓ Where V is operator for physical interaction that couples initial and final states,
- ✓ ψ_f^* is wave function for final state and
- ✓ ψ_i is wave function for initial state.

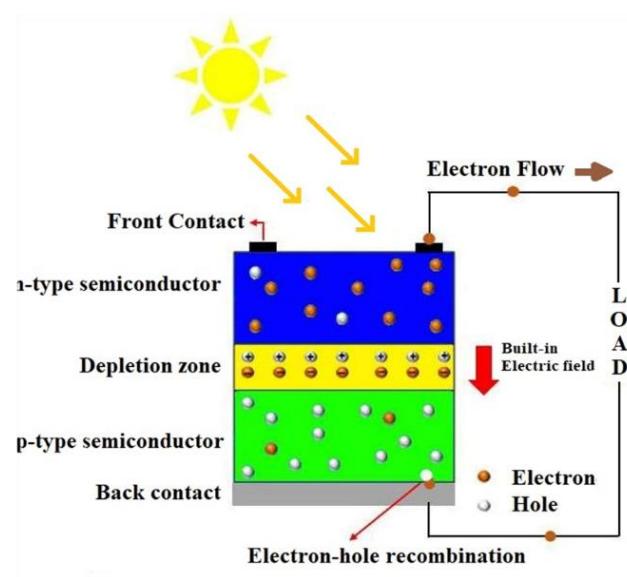
3.6 Photovoltaic effect

- ✓ “The effect due to which light energy is converted into electrical energy is called photovoltaic effect.”
- ✓ It was first discovered by Edmund Becquerel in 1839.
- ✓ The photovoltaic effect occurs in solar cells that are composed of two different types of semiconductors,

- ✓ p-type and n-type joined together to form a p-n junction.
- ✓ Whenever these semiconductors are joined, an electric field is formed in the region as electrons move to p-side and holes move to n-side.
- ✓ This movement of electron creates an electron hole pair so, free electrons from n-type region try to diffuse to p-type region and holes in p-type region try to diffuse to n-type region in the crystal.



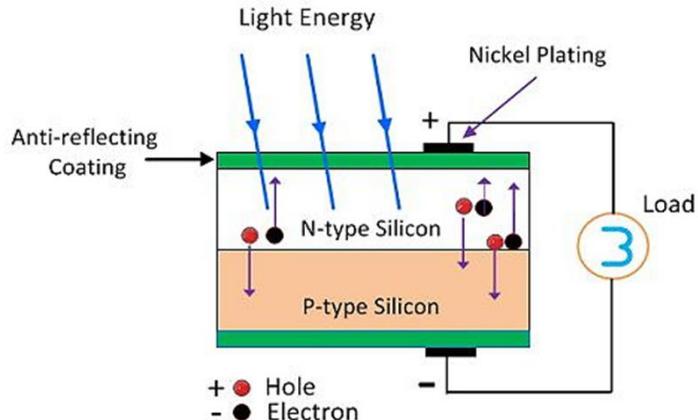
- ✓ Each free electron of n-type region while comes to the p-type region due to diffusion, it leaves a positive donor ion behind it in the n-type region.
- ✓ These positive and negative ions concentration layer creates an electric field across the junction which is directed from positive to negative that is from n-type side to p-type side.
- ✓ Due to this electric field (E) electrons experience a drift in the direction of field and generates drift current.
- ✓ In normal thermal equilibrium condition, the diffusion of charge carrier is equal and opposite of drift of charge carriers, hence the thickness of potential barrier remains fixed.
- ✓ Now the n-type surface of the silicon crystal is exposed to the sunlight. So, when light of suitable wavelength is incident on these cells, energy from the photons is transferred to the semiconducting materials in the p-n junction.
- ✓ This energy causes electrons to jump to a higher energy level (conduction band), leaving behind a hole in the valence band.



- ✓ So, it is this movement of electrons creates an electron hole pair, leading to a flow of current in the cell.

Construction of solar cell:

- ✓ P-N junction is one of the essential requirement in the construction of solar cell.
- ✓ This P-N junction is constructed by using semiconducting materials such as germanium silicon, gallium arsenide, cadmium arsenide, etc.
- ✓ The P-N junction is packed in a cylindrical metal container with a glass window on top so that light may fall on both P-and N-type materials uniformly.
- ✓ In order to avoid recombination of generated electrons in the P-and N-regions, their thickness is kept very small.
- ✓ For the generation of more number of charge carriers under the influence of indenting light, heavily doped P-and N-type of materials are used.
- ✓ Thus, the use of heavily doped P-and N-regions gives a large photo voltage.
- ✓ Nickel plated metal ring around P-type layer act as the positive output terminal and the metal contact at the bottom serves as the negative output terminal.



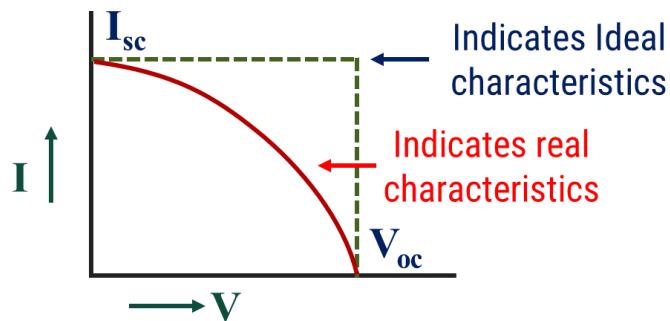
Working of solar cell:

- ✓ When the photons of suitable energy allowed to incident on P-N junction, photons impart sufficient energy to the valence electrons and remove them from their parent atoms.
- ✓ Hence, electron-hole pairs are generated in both P-region and N-region of the junction (as shown in Figure).
- ✓ These newly generated charge carriers reach the depletion region by diffusion where they are separated by a strong potential barrier existing there.

- ✓ At the same time, the minority charge carriers (electrons in P-region, holes in N-region) are supported by the existing potential barrier at the junction for their flow across the junction.
- ✓ Flow of minority charge carriers across the junction constitute the minority current, which is directly proportional to the illumination and also depends on the surface area being exposed to the light.

Parameters of solar cell:

- ✓ The I – V characteristics can be studied with the help of below diagram.



- ✓ The output voltage from solar cell when load impedance is very high, $R_L \rightarrow \infty$ is called open circuit voltage (V_{oc}).
- ✓ The current output when the load impedance is very small i.e. $R_L \rightarrow 0$ is called the short circuit current (I_{sc}).
- ✓ The product of open circuit voltages (V_{oc}) and short circuit (I_{sc}) gives the ideal power output from the solar cell, i.e. $P_{ideal} = V_{oc} \times I_{sc}$.
- ✓ But the actual power output is less than the ideal output, i.e. $P_{max} = I_m \times V_m$.
- ✓ The fill factor (FF) of solar cell is defined as the maximum useful power w.r.t. ideal power.

$$\text{i.e. } FF = \frac{P_{max}}{P_{ideal}} = \frac{I_m \times V_m}{I_{sc} \times V_{oc}}$$

- ✓ Efficiency ' η ' of solar cell is given by:

$$\eta = \frac{I_m \times V_m}{\text{Incident light power}}$$

- ✓ The voltage developed by every cell is of an order of 0.5 to 0.6 V and efficiency is around 15 – 20 % for silicon cells.

Merits:

1. For this type of conversion, the input energy i.e. solar energy is available at no cost.
2. Such a process is free from pollution like chemical, thermal and radioactive.

Demerits:

1. The input solar energy is not always constant. It changes with time of day and also with seasons. So, output is not constant energy.
3. Solar cells have low efficiency: The energy generated during daytime can be stored and used, whenever required. But the storage mechanisms are very expensive.
4. The modules require a large area to be installed.

Application of solar cell:

1. Communication
2. Electricity for Remote areas
3. Water pumping
4. Refrigeration
5. Medical Application
6. Signal System
7. Charging Vehicle batteries
8. Public Utilities
9. Science and Research

3.6.1 Optical loss:

- ✓ Optical losses in solar cell, mainly affects the power output, by lowering the short circuit current (I_{sc}), thus reducing the efficiency.

The efficiency of solar cell can be reduced by following reasons:

- ✓ Mismatch of band gap of materials with the solar spectrum.
- ✓ Total spectrum of solar energy is not absorbed.
- ✓ Reflection loss of light.
- ✓ Less intensity of light.

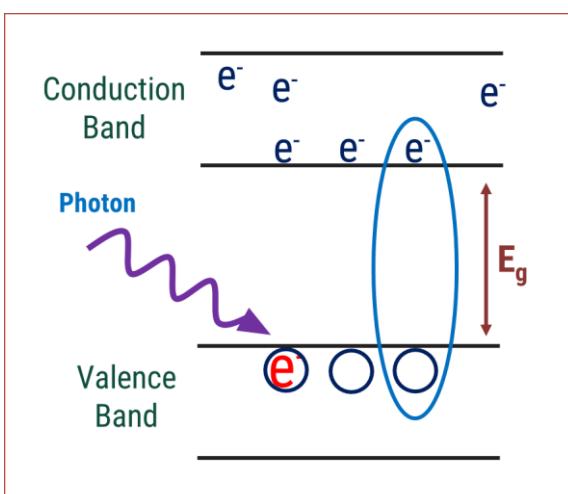
Remedies:

1. Proper selection of materials, increases the efficiency of solar cell.
2. Solar cell can be made thicker for more absorption of light.
3. Top contact coverage of the solar cell surface can be minimized.
4. Anti – reflection coating can be used on the top surface to increase absorption.
5. The optical path length can be increased by a combination of surface texturing and light trapping.

3.7 Exciton:

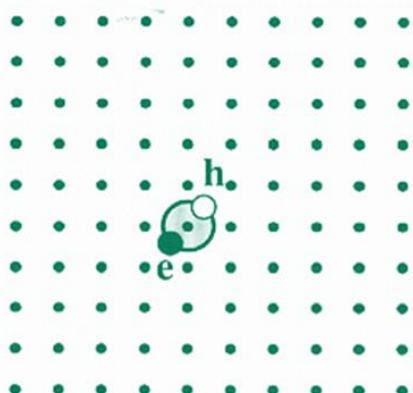
- ✓ The concept of excitons was first proposed by Yakov Frenkel in 1931.
- ✓ The absorption of a photon by an inter band transition in a semiconductor creates an electron in conduction band and a hole in valence band.
- ✓ These oppositely charged particles attract each other through Coulomb interaction and there may be a probability of formation of a neutral electron hole pair called exciton.
- ✓ “An exciton is a bound state of an electron and hole which are attracted to each other by an electrostatic force.”

- ✓ When the photon is absorbed by a semiconductor, an electron hole pair is created.
- ✓ The electron in the conduction band is now attracted to the localized hole by repulsive Coulomb forces from the large number of electrons surrounding the hole and excited electrons.
- ✓ This attraction provides a stabilizing energy balance.



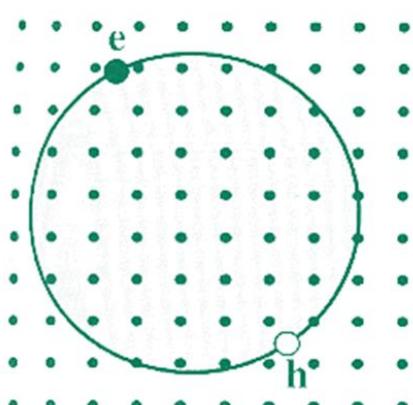
3.7.1 Frenkel exciton:

- ✓ "When there is a strong electron hole attraction, like in ionic crystals, the electrons and holes are tightly bound to each other, this type of exciton is called Frenkel exciton."
- ✓ They are also called tight bound excitons. They are found in insulators and molecular crystal.
- ✓ They are bound to specific atoms or molecules and they move by hopping from one atom to another. Their binding energy is of a range 0.1 eV to 1 eV. They are stable at room temperature.



3.7.2 Wannier - Mott excitons :

- ✓ "When the electron hole separation is much larger to the lattice constant, then the exciton is called Wannier-Mott exciton."
- ✓ In this, the electron hole pair is weakly bound. They are also called free excitons. They mainly exist in semiconductor. They have large radius.
- ✓ They are delocalized state that can move freely throughout the crystal. They have a binding energy of around 0.01 eV. They are stable at cryogenic temperature.



3.8 Drude model:

- ✓ Electrons in a metal behave like particles in an ideal gas. (i.e. No coulombian interaction and collision between particles). This is called independent electron approximation.
- ✓ The effect of electron-electron and electron-ion interaction is ignored.
- ✓ Mean free time between collisions is τ . It is independent of electrons position and velocity.
- ✓ Electrons achieve thermal equilibrium by collisions with ions and their mean kinetic energy can be given by: $\frac{1}{2}mV^2 = 3K_B T$.
- ✓ Mean free time between collisions is τ .
- ✓ It is independent of electrons position and velocity.
- ✓ After the collisions, they move in random directions with a speed that depends on the temperature of the region where collision occurred i.e., hotter the region higher is the speed of emerging electrons.

Application of Drude model:

1. Electrical conductivity and Drude model:

- ✓ To apply the Drude model the density of gas formed by free electrons must be known. This parameter is called the 'conduction electron density'. i.e. no. of electrons per unit volume. The conduction electron density can be computed by assuming that each atom contributes 'z' electrons for conduction, where 'z' is the no. of outer shell electrons for metal atoms.
- ✓ The average drift velocity of an electron in metal can be computed by following equation.

$$\therefore V_d = -\frac{eE}{m}\tau$$

- ✓ The current density can be computed as

$$\therefore J = neV_d$$

$$\therefore J = -ne\frac{-eE}{m}\tau = \frac{ne^2 E \tau}{m}$$

- ✓ From, electrical conductivity $J = \sigma E$

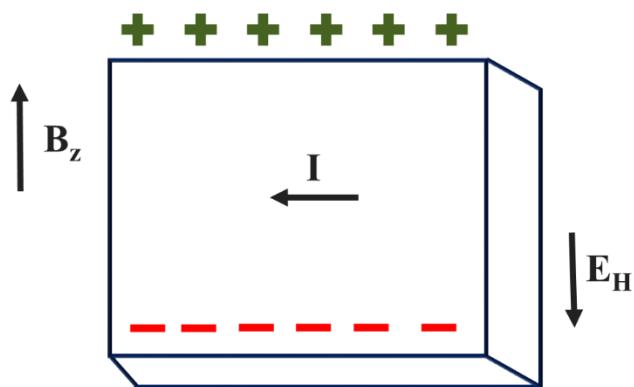
$$\therefore \sigma E = \frac{ne^2 E \tau}{m}$$

$$\therefore \sigma = \frac{ne^2 \tau}{m}$$

- ✓ This is the expression for DC conductivity in Drude Model.

2. Hall effect and Drude model:

- ✓ The Drude model also explains the Hall effect. It is a phenomenon in which an electric field "E_H" arises perpendicular to both the current density 'J_x' (The point in the direction of electron movement) and magnetic field 'B_z'.
- ✓ The Hall effect occurs when a current flows through a conductor which is already under the influence of a magnetic field. As a result of the magnetic field, the positive charges accumulate on one side of the conductor.



- ✓ For electrons to pass through the given region, the field 'E_H' must cancel the Lorentz force that acts in opposite direction.

$$F_L = qE + qV \times B$$

- ✓ The magnitude of Hall field can be calculated as below:

$$E_H = R_H J_x B_z \quad \text{where, } R_H \text{ is the Hall coefficient}$$

$$R_H = -\frac{1}{ne}$$

- ✓ J_x is the current density
- ✓ B_z is the magnetic field
- ✓ As the Hall coefficient, approaches unity, The Hall field "EH" more likely cancels the Lorentz force.

3. Wiedemann – Franz law and Drude model:

- ✓ The Wiedemann – Franz law states that for any metal at a given temperature, the ratio of thermal conductivity to electrical conductivity is a constant 'L' (L is called the Lorentz number).

$$\therefore \frac{K}{\sigma} = LT$$

- ✓ K = Thermal conductivity
- ✓ σ = electrical conductivity
- ✓ L = Lorentz number
- ✓ T = absolute temperature
- ✓ The ideal gas related equations derived from the Drude model can be used to generate an equivalent formula for "LT".
- ✓ Now,

$$\therefore \frac{K}{\sigma} = \frac{\frac{1}{2}nV K_B \lambda}{\frac{ne^2 \lambda V}{3K_B T}}$$

$$\text{i.e. } \frac{K}{\sigma} = \frac{3}{2} \left(\frac{K_B}{e} \right)^2 T$$

$$\therefore \frac{K}{\sigma} = LT$$

- ✓ Despite some minor inconsistency, the theoretical calculation often gives accurate results.

Drawbacks of Drude model:

1. Drude model does not consider the collision between electron – electron. It also does not consider the interaction between electrons and ions.
2. De – Broglie wavelength of electrons with some thermal energy are in nanometer scale. So, electrons cannot be treated as classical particle under the conditions of Drude Model.
3. The Drude model explains the conductivity of metals at low temperatures as the assumption of constant mean free path based on atomic spacing is incorrect.
4. The Drude model can't explain the conductivity of alloys. Even small impurities can drastically decreases the conductivity of metals in a way that can't be predicated.
5. From the view of classical mechanics, the electrons should contribute greatly to the heat capacity of metals. But this result does not agree with the experimental data.

UNIT – 4 (Measurements)

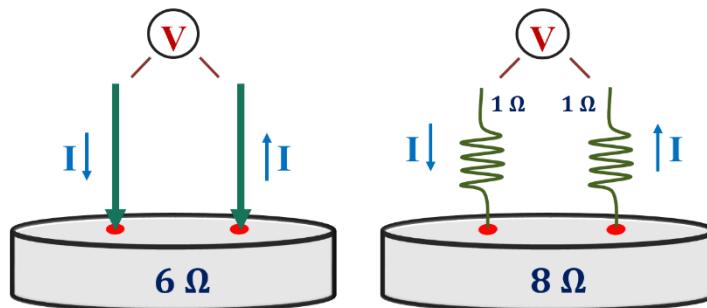
4.1 Introduction:

- ✓ Semiconductor electronics plays vital role in every aspect of life. Semiconductor materials is basis of modern electronic industry and it plays important role in the foundation of electronic circuitry with components such as integrated circuits, diodes, solar cells and transistors.
- ✓ Therefore, precise measurement of electrical and optical properties of given semiconductor is an important step for fabrication of semiconductor device.
- ✓ The efficiency of an opto-electronic device depends on how charge carriers move inside semiconducting material.
- ✓ The semiconductor electrical properties such as electrical resistivity, mobility, the charge carrier concentration, etc. allows us to characterize electrical properties of semiconductor.
- ✓ The electrical characterization of semiconductor is helpful to improve functionality of device.
- ✓ In this chapter we will also give brief overview about UV-Visible spectroscopy, which give information about transmission, absorption and energy band gap of given semiconductor.

4.2 Four – Probe Method

- ✓ Any type of material will have some amount of resistance (R), and by using the equation of ohm's law we can measure the resistance of any materials.
- ✓ There are few techniques to measure the value of resistance for any type of materials.
 - 1) Two Probe Method
 - 2) Four Probe Method

❖ **Two Probe Method:**



- ✓ The major problem in such method is error due to contact resistance of measuring leads.
- ✓ The above method cannot be used for materials having random shapes.
- ✓ For some type of materials soldering the test leads would be difficult.
- ✓ In case of semiconductors, the heating of samples due to soldering results in injection of impurities into the materials thereby affecting intrinsic electrical resistivity.
- ✓ In case of semiconductors, contacts between metallic probes and semiconducting sample are not Ohmic in nature (rather they are of Schottky nature) works as barrier.

❖ **Four – Probe Method:**

- ✓ Four-probe method, also known as four terminal sensing (4T sensing) or 4-wire sensing is an electrical impedance measuring technique, that uses separate pairs of current carrying and voltage sensing electrodes to make more accurate measurements.
- ✓ It is used to measure the sheet resistance of either a bulk or a thin film specimen.
- ✓ It uses a set of four probes, distance between each probe is 's'.
- ✓ A current source is connected between the outer two probes and a voltmeter is connected between the inner two probes.
- ✓ To measure the resistivity of the sample, a constant current I goes through the two outer probes and voltage drop between the inner two probes is measured.
- ✓ A very high impedance voltmeter is used, so current does not flow through the voltmeter, hence the contact resistance are eliminated.

- ✓ Differential resistance can be measured as:

$$dR = \rho \left(\frac{dx}{A} \right)$$

- ✓ dR is the resistance between two points with the distance dx .
- ✓ A is the area that current goes through.
- ✓ ρ is the resistivity of the sample.

❖ Case: 1 Bulk material (when $s \ll t$):

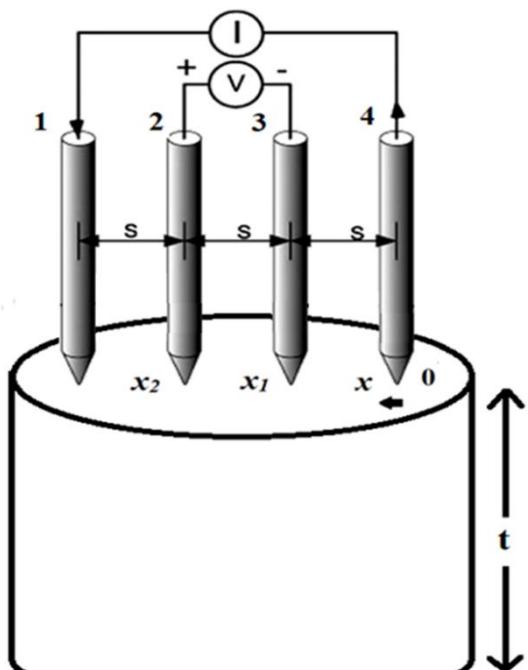
$$dR = \rho \left(\frac{dx}{A} \right)$$

- ✓ Integrating both the sides,

$$R = \int_{x_1}^{x_2} \rho \left(\frac{dx}{A} \right)$$

$$R = \int_{x_1}^{x_2} \rho \left(\frac{dx}{2\pi x^2} \right)$$

- ✓ Area of half hemispherical shell $= 2\pi x^2$.



Explanation:

- ✓ Current is injected through the outer probes it travels outward from the point of contact through bulk area.

$$\therefore R = \frac{\rho}{2\pi} \int_{x_1}^{x_2} \frac{1}{x^2} dx$$

$$\therefore R = \frac{\rho}{2\pi} \left[-\frac{1}{x} \right]_s^{2s}$$

$$\therefore R = \frac{\rho}{2\pi} \left[-\frac{1}{2s} + \frac{1}{s} \right]$$

$$\therefore R = \frac{\rho}{2\pi} \left[\frac{1}{2s} \right]$$

$$\therefore R = \frac{\rho}{4\pi s}$$

- ✓ Due to super position of current, at the outer two probes, we have

$$R = \frac{V}{2l}$$

$$\frac{\rho}{4\pi s} = \frac{V}{2l}$$

$$\therefore \rho = 2\pi s \left(\frac{V}{I} \right)$$

- ✓ Where, V = potential difference between inner probes
 - ✓ I = current through outer probes
 - ✓ s = spacing between probes
 - ✓ ρ = Resistivity of sample
- ❖ Case: 2 For thin sheet (when $t \ll s$):

- ✓ For a very thin sample, we get current rings instead of hemispheres. i.e., current travels in short cylindrical shell of equipotential.
- ✓ So the area will be the area of the cylinder i.e., $A = 2\pi xt$.

$$dR = \rho \left(\frac{dx}{A} \right) = \rho \left(\frac{dx}{2\pi xt} \right)$$

- ✓ Integrating both sides,

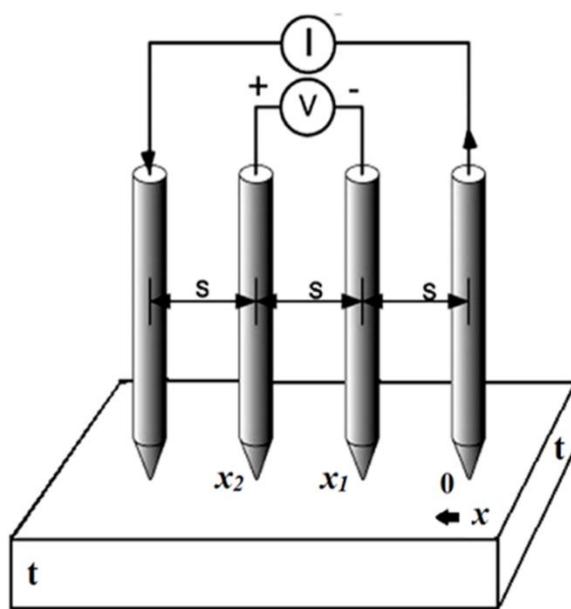
$$R = \int_{x_1}^{x_2} \rho \left(\frac{dx}{2\pi xt} \right)$$

$$R = \int_{x_1}^{x_2} \rho \left(\frac{dx}{2\pi xt} \right)$$

$$\therefore R = \frac{\rho}{2\pi t} \int_{x_1}^{x_2} \frac{1}{x} dx$$

$$\therefore R = \frac{\rho}{2\pi t} [\ln(x)]_1^{x_2}$$

$$\therefore R = \frac{\rho}{2\pi t} [\ln 2]$$



- ✓ Due to super position of current, at the outer two probes, we have

$$R = \frac{V}{2l}$$

- ✓ By Comparing above two equation we get,

$$\frac{V}{2I} = \frac{\rho}{2\pi t} [\ln 2]$$

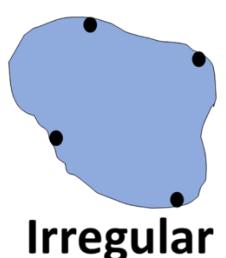
$$\therefore \rho = \frac{\pi t}{\ln 2} \left(\frac{V}{I} \right)$$

$$\therefore \rho = 4.53 t \left(\frac{V}{I} \right)$$

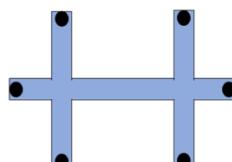
- ✓ Above expression gives the value of resistivity for thin film.

4.3 Vander Pauw measurement

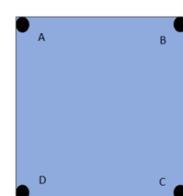
- ✓ The van der Pauw method involves applying a current and measuring voltage using four small contacts on the circumference of a flat, arbitrarily shaped sample of uniform thickness.
- ✓ This method is particularly useful for measuring very small samples because geometric spacing of the contacts is unimportant. Effects due to a sample's size, which is the approximate probe spacing, are irrelevant.



Irregular



Bridge



Square

- ✓ From the measurement made, the following properties of a material can be calculated.
 - 1) Resistivity of the material
 - 2) Doping type
 - 3) Carrier density of majority charge carriers
 - 4) Mobility of charges carriers.

✓ The basic requirements of the method are as under:

1. All the contacts should be at the periphery of the sample.
2. Size of the contacts should be negligibly small as compared to that of the sample.
3. All the contacts should be Ohmic.
4. Sample should be very thin as compared to its area.
5. Surface of the sample should be singly connected i.e. the sample should be free from steps and discontinuities.
6. Sample should be flat so that the sample surface and contacts lie in the sample plane.

Resistance Measurement:

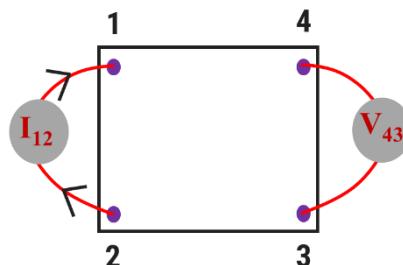
(1)

$$R_{12,43} = \frac{V_{43}}{I_{12}}$$

$$R_{21,34} = \frac{V_{34}}{I_{21}}$$

$$R_{43,12} = \frac{V_{12}}{I_{43}}$$

$$R_{34,21} = \frac{V_{21}}{I_{34}}$$



Vertical Resistance Measurement

$$R_{\text{vertical}} = \frac{R_{12,43} + R_{21,34} + R_{43,12} + R_{34,21}}{4}$$

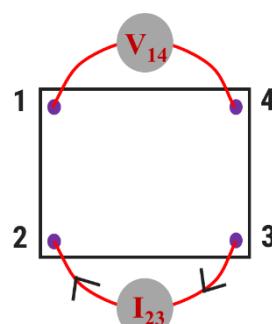
(2)

$$R_{23,14} = \frac{V_{14}}{I_{23}}$$

$$R_{32,41} = \frac{V_{41}}{I_{32}}$$

$$R_{14,23} = \frac{V_{23}}{I_{14}}$$

$$R_{41,32} = \frac{V_{32}}{I_{41}}$$



Horizontal Resistance Measurement

$$R_{\text{horizontal}} = \frac{R_{23,14} + R_{32,41} + R_{14,23} + R_{41,32}}{4}$$

- ✓ The Vander Pauw formula becomes:

$$e^{\frac{-\pi R_{\text{vertical}}}{R_s}} + e^{\frac{-\pi R_{\text{horizontal}}}{R_s}} = 1$$

$$R_{\text{vertical}} = R_{\text{horizontal}} = R$$

$$e^{\frac{-\pi R}{R_s}} + e^{\frac{-\pi R}{R_s}} = 1$$

$$2 e^{\frac{-\pi R}{R_s}} = 1$$

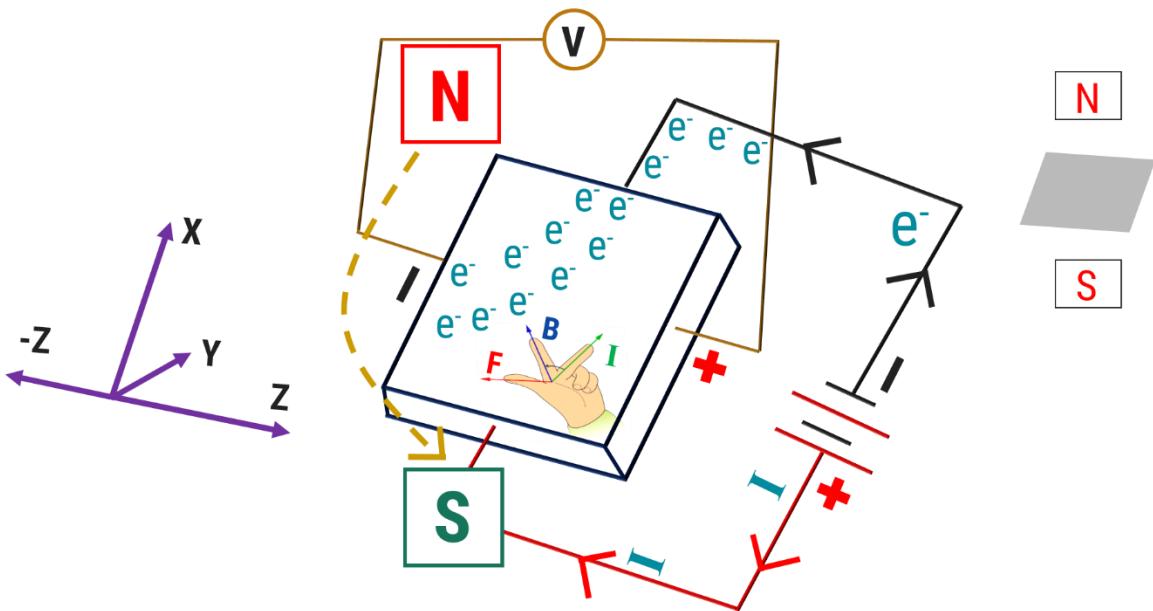
$$\frac{\pi R}{R_s} = \ln 2$$

$$R_s = \frac{\ln 2}{\pi R}$$

$$\rho = R_s d$$

4.4 Hall effect:

- ✓ **Hall Effect:** If a sample of conductor or semiconductor carrying current I is placed in a transverse magnetic field B, an electric field E induced in a direction perpendicular to both current and magnetic field. This phenomena is known as Hall effect and generated voltage is known as Hall voltage V_H .



- ✓ In order to derive the equation of Hall mobility first we will derive the equation of Hall coefficient.
- ✓ Magnetic field vector 'B' is applied on this sample along y-axis. Now if a current 'I' is passed through the sample in the direction of x-axis, charge carriers (in this case we assume electrons) will drift with drift velocity ' v_d ' in the opposite direction '-z'. Magnetic force ' F_B ' acts on each drifting electron, pushing it toward the left edge of the sample.

$$F_B = - e (v \times B) = - B e v \sin (90^\circ) = - B e v$$

$$F_B = - B e v \dots \dots \dots (1)$$

- ✓ The Lorentz force is exerted on electrons in the negative -Z direction and causes a negative charge to accumulate at the left edge of the sample. Moreover, the right edge of the sample becomes positively charged due to loss of electrons. Therefore, a potential difference causes a field E_H in negative -Z direction.
- ✓ In the equilibrium condition,

$$E_H = B v \dots \dots \dots (2)$$

- ✓ The current density is given by,

$$J = n e v \dots \dots \dots (3)$$

- ✓ Where n is number of carrier concentration, comparing equation (2) & (3),

$$E_H = \frac{B J_x}{n e} \dots \dots \dots (4)$$

- ✓ Hall effect is explained by Hall coefficient,

$$E_H = R_H B J_x \dots \dots \dots (5)$$

$$\text{where, } R_H = \frac{1}{n e}$$

- ✓ As the electric field in n-type semiconductor is established in negative $-z$ direction value of Hall coefficient becomes.

$$R_H = -\frac{E}{B J_x} = -\frac{1}{n e} \dots \dots \dots (6)$$

- ✓ Value of Hall coefficient (R_H) is negative for n-type semiconductor.
- ✓ In case of p-type semiconductor current is due to majority charge carriers holes, in that case,

$$R_H = \frac{E}{B J_x} = \frac{1}{p e} \dots \dots \dots (7)$$

- ✓ In case of p – type semiconductor value of Hall coefficient (R_H) is positive.

❖ Determination of value of Hall coefficient:

- ✓ Hall voltage generates Hall electric field (E_H),

$$E_H = \frac{V_H}{t}$$

$$V_H = E_H t \dots \dots \dots (8)$$

- ✓ Substituting value of E_H from equation (5)

$$V_H = R_H B J_x t$$

- ✓ If w is the width of the semiconductor, then its cross-section area (A) will be wt and the current density,

$$J_x = \frac{I}{w t}$$

$$V_H = \frac{R_H B t I}{w t}$$

$$V_H = \frac{R_H B I}{w}$$

Therefore, $R_H = \frac{V_H w}{B I} \dots \dots \dots (9)$

- ✓ The value of hall voltage is opposite for n-type and p-type semiconductor.

❖ Determination of Hall mobility:

- ✓ In case of n-type semiconductor value of conductivity is given by,

$$\sigma_e = n_e e \mu_e$$

Therefore, $\mu_e = \frac{\sigma_e}{n_e e}$

- ✓ We know for the n-type semiconductor,

$$R_H = -\frac{1}{n e}$$

Hence, $\mu_e = -\sigma_e R_H \dots \dots \dots (10)$

- ✓ Similarly, for p-type semiconductor

$$R_H = \frac{1}{n e}$$

$$\mu_h = \sigma_h R_H \dots \dots \dots (11)$$

- ✓ The Hall mobility μ_h is defined as the product of Hall coefficient (R_H) and conductivity (σ).

Q.1 A n-type semiconductor material has Hall coefficient and the conductivity $1.15 \times 10^{-3} \text{ m}^3/\text{C}$ and $115 (\Omega \cdot \text{m})^{-1}$ respectively. Calculate charge carrier density and electron mobility.

Ans. $R_H = 1.15 \times 10^{-3} \text{ m}^3/\text{C}$

$$\sigma_e = 115 (\Omega \cdot \text{m})^{-1}$$

$$n_e = ?$$

$$\mu_e = ?$$

$$\mu_e = \sigma_e \times R_H$$

$$\mu_e = 115 \times 1.15 \times 10^{-3}$$

$$\mu_e = 0.13 \text{ m}^2/\text{V} \cdot \text{sec}$$

$$\mu_e = \frac{\sigma_e}{n_e e}$$

$$n_e = \frac{\sigma_e}{\mu_e e}$$

$$n_e = \frac{115}{0.13 \times 1.6 \times 10^{-19}}$$

$$\mu_e = 5.53 \times 10^{21}$$

Q.2 A semiconductor having the Hall coefficient $3.75 \times 10^{-4} \text{ m}^3/\text{C}$. The resistivity of the sample is $7.21 \times 10^{-3} \Omega \cdot \text{m}$. Calculate the mobility and density of charge carriers.

Ans. $R_H = 3.75 \times 10^{-4} \text{ m}^3/\text{C}$

$$\rho = 7.21 \times 10^{-3} \Omega \cdot \text{m}$$

$$n = ?$$

$$\mu = ?$$

$$R_H = \frac{1}{n e}$$

$$n = \frac{1}{R_H e}$$

$$n = \frac{1}{3.75 \times 10^{-4} \times 1.6 \times 10^{-19}}$$

$$n = 3.75 \times 10^{22} \text{ charge carrier/m}^3$$

Mobility of charge carrier

$$\mu = \sigma R_H$$

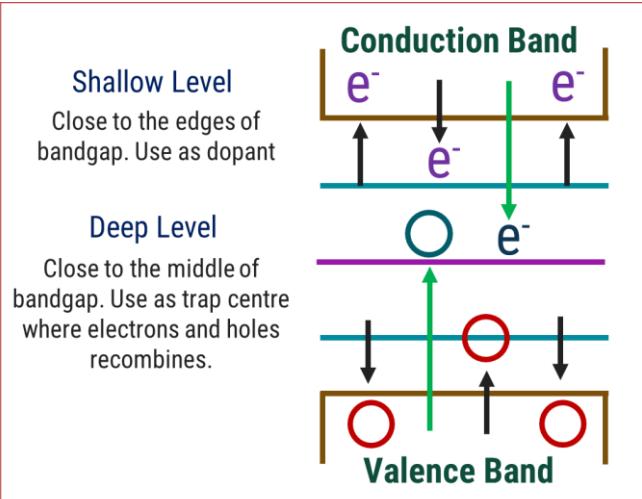
$$\mu = \frac{R_H}{\rho}$$

$$\mu = \frac{5.75 \times 10^{-4}}{7.21 \times 10^{-3}}$$

$$\mu = 0.0520 \text{ m}^2 \text{V}^{-2} \text{s}^{-1}$$

4.5 Deep level Transient Spectroscopy (DLTS):

- ✓ 'Deep level', here is used to denote the crystal defects or traps in mainly p-n junction and Schottky diodes. 'Transient Spectroscopy' means a powerful set of techniques, used to identify electrons and structural properties of short-lived excited states.
- ✓ Deep Level Transient Spectroscopy 'DLTS' was initially introduced by D. V. Lang in the year 1974.
- ✓ It is an efficient method to observe and identify deep level impurities in the semiconductors.
- ✓ It is a capacitance transient thermal scanning technique which uses the capacitance of P-N junction to monitor and find out the defects.
- ✓ DLTS helps to distinguish between the majority and minority carrier traps.



- ✓ It also gives the concentrations, energy and capture rates of both types of traps.
- ✓ When the voltage across a p-n junction is charged, there is a corresponding change in the depletion layer width. This change in width causes a change in no. of free charge carriers on both sides, resulting in the change in junction capacitance.
- ✓ Consider a p-n junction with the deep level (i.e. defects) present, having its energy as E_T .
- ✓ In steady state, there is no net flow of charge carriers across the trap.
- ✓ The total density of deep level state N_T can be given by,

$$N_T = \frac{e_p n_T}{e_n + e_p}$$

Where, e_p = hole emission rate

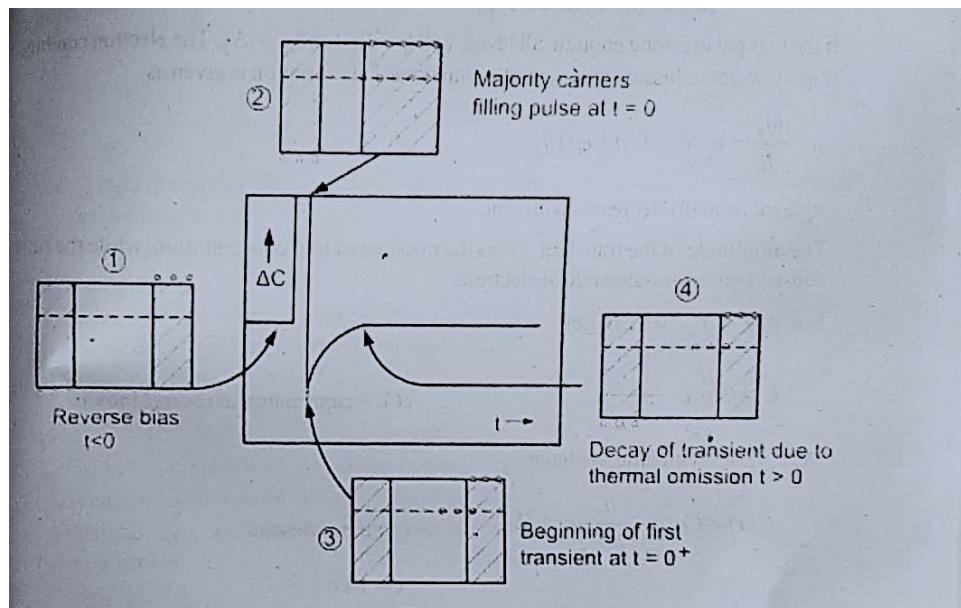
e_n = electron emission rate

n_T = density of filled traps under steady states

- ✓ If the system is distributed from steady state, there is a change in no. of ' n_T' . Leading to a change in total charge in depletion layer, thus changing the capacitance.

❖ Basic Analysis:

- ✓ Let us consider, an asymmetric diode i.e. the one in which one side is heavily doped as compared to the other.
- ✓ We are assuming a diode in which p-side is heavily doped as compared to n-side, so the width of depletion layer is more on the n-side. I.e. space charge region (SCR) is more on n-side.
- ✓ The following figure, shows the four processes of generating capacitance transient, due to majority carrier levels.



- ✓ From figure, Process (1) shows that traps in the space charge region are empty because no free carriers are available for capture($t < 0$).
- ✓ Process (2) Reverse bias is reduced and the electrons are captured in traps($t = 0$).
- ✓ Process (3) When the reverse bias is restored, the capacitance drops to a minimum value as electrons are trapped($t = 0^+$).
- ✓ Process (4) Decay of transient due to thermal emission of trapped electrons($t > 0$).
- ✓ Suppose we have a reverse bias V_R applied to the sample and decrease it to zero for a short time. The electrons will flow into what was previously the depletion region and the levels in this volume will capture electrons.
- ✓ So we get,

$$\frac{dn_T}{dt} = c_n(N_T - n_T)$$

- ✓ Where, c_n = capture time const. of $e^- \rho$.
 - ✓ If the bias pulse is long enough, all levels will be filled and $N_T = n_T$. The electron emitting traps now start emission and n_T varies with time. The variation is given as,
- $$\frac{dn_T}{dt} = e_p N_T - (e_n + e_p)n_T$$

Where, n_T = exponentially decrease with time.

- ✓ The amplitude of the transient, gives the measure of trap concentration, while the time constant gives emission rate of electrons.
- ✓ For, $n_T \ll N_d$, we can get

$$C = C_0 \left(1 - \frac{n_T}{2N_D} \right)$$

Where, C_0 = capacitance at reverse bias.

- ✓ As n_i varies with time, we have

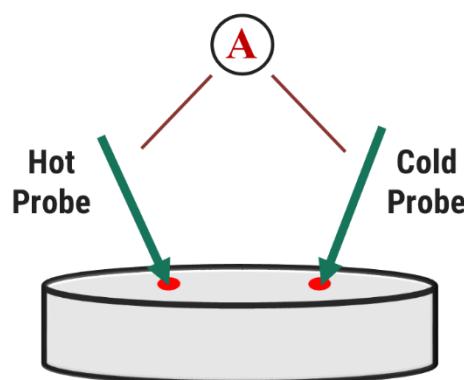
$$C(t) = C_0 \left(1 - \frac{n_T}{2N_D} e^{-t/\tau} \right)$$

Where, $\tau = \text{Time constant} = \frac{1}{e_n}$

- ✓ Thus the emission rates and trap concentrations can be determined from the changes in capacitance of p-n junction. These changes are in the form of capacitance transients.

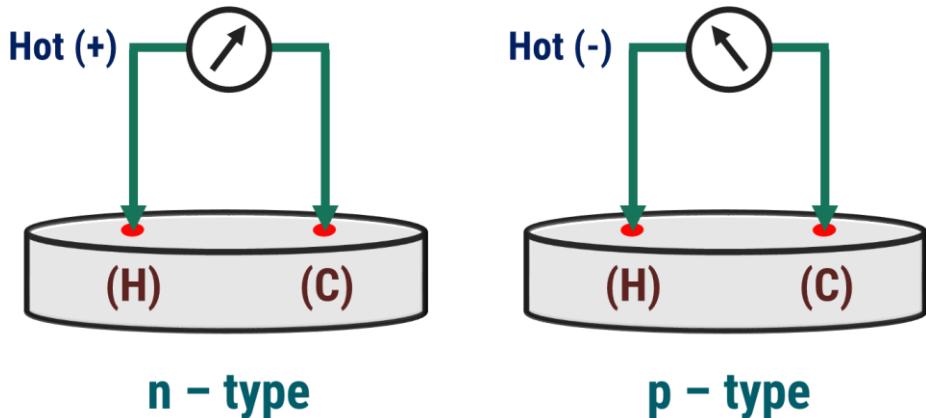
4.6 Hot Point Probe measurement:

- ✓ A hot point probe is a method of determining quickly whether a semiconductor sample is n-type or p-type.
- ✓ Basic principle lying behind the hot probe method is Seebeck Effect.



- ✓ A micro ammeter is attached to the sample and a heat source, such as soldering iron is placed on one of the leads.
- ✓ The heat source will cause the charge carriers to move to the cold probe.

- ✓ This diffusion of charge carriers causes a current flow.
- ✓ The net current will depend on the majority charge carriers i.e. electrons for n-type semiconductor and holes for p-type semiconductor.
- ✓ If the carriers are positive, the current flow will be in same direction and if the charge carriers are negative, the current will flow in opposite direction.



- ✓ If the hot side is positive with respect to the cold side, the sample is n-type (as the majority charge carriers i.e. electrons move to cold side, leaving the hot side positive).
- ✓ If the hot side is negative, the sample is P-type.
- ✓ The sample type can be determined by the direction of deflection in current meter.

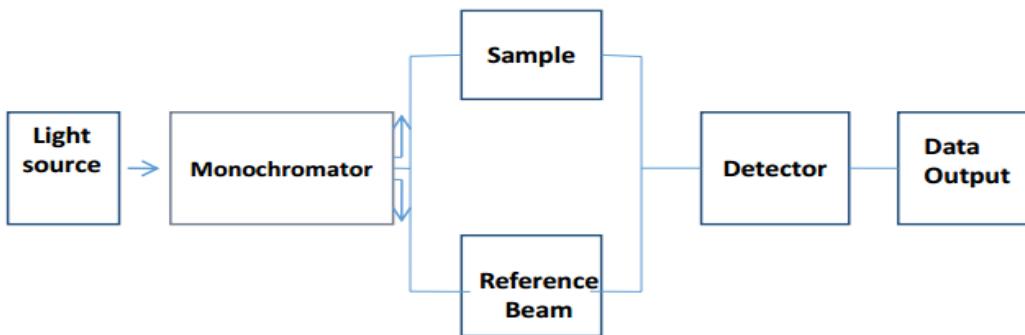
4.7 Capacitance Voltage Measurements:

- ✓ Capacitance-voltage profiling (or C-V profiling, sometimes CV profiling) is a technique for characterizing semiconductor materials and devices.
- ✓ Hillibrand and Gold first described the use of capacitance -voltage (C-V) methods to determine the majority carrier concentration in semiconductors. C-V measurements are capable of yielding quantitative information about the diffusion potential and doping concentration in semiconductor materials.
- ✓ The technique employs p-n junctions, metal-semiconductor (MS) junctions (Schottky barriers), electrolyte-semiconductor junctions, metal-insulator-semiconductor (MIS) capacitors, and MIS field effect transistors (MISFETs).

- ✓ The applied voltage is varied, and the capacitance is measured and plotted as a function of voltage.
- ✓ The technique uses a depletion region, a region which is empty of conducting electrons and holes, but may contain ionized donors and electrically active defects or traps.
- ✓ The depletion region with its ionized charges inside behaves like a capacitor. By varying the voltage applied to the junction it is possible to vary the depletion width.
- ✓ The dependence of the depletion width upon the applied voltage provides information on the semiconductor's internal characteristics, such as its doping profile and electrically active defect densities.
- ✓ Measurements may be done at DC, or using both DC and a small-signal AC signal (the conductance method), or using a large-signal transient voltage.

4.8 UV – Vis Spectroscopy:

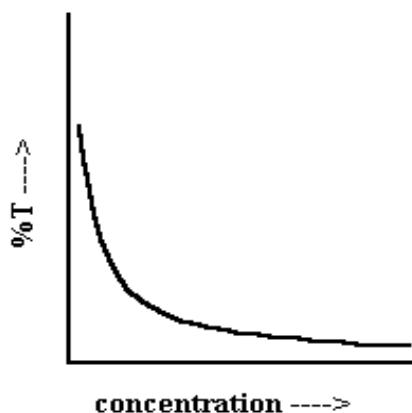
- ✓ UV- Vis Spectroscopy is related to the interaction of light with matter.
- ✓ As light is absorbed by matter, the result is an increase in the energy content of the atoms or molecules.
- ✓ This method is used to measure the energy band gap of different materials, by measuring absorption spectrum.
- ✓ UV – Vis (λ) = 2000 – 8000 Å
- ✓ Bandgap refers to the energy difference between bottom of conduction band and top of valence band, through which electrons are able to jump from one band to another.
- ✓ The instrument used in UV-Vis spectroscopy is called UV-Vis spectro-photometer.
- ✓ It measures the intensity of light after passing through a sample (I), and compares it to the intensity of light before it passes through the sample (I_0).
- ✓ A light of wavelength (λ) and energy ($h\nu$) is made to pass through monochromator to get a parallel beam.



- ✓ It then passes through a beam splitter and is incident on two cells, a reference cell and a sample cell.
- ✓ The intensities of light from reference and sample cells is collected by the detector.
- ✓ The ratio of intensities is called transmittance (T).

$$\text{i.e. } T = \frac{I}{I_0}$$

- ✓ The transmittance (T) is usually expressed in percentage (%T).
- ✓ If we plot %T w.r.t. to concentration of sample, we get an exponential decrease in transmittance, with an increase in concentration.
- ✓ Higher the concentration, more is the absorption so less is the value of I, through the sample cell.



- ✓ As it is difficult to extract characteristics from the exponential relation, we define a relation absorption (A).

$$\text{Absorption (A)} = \log\left(\frac{I_0}{I}\right)$$

- ✓ The Tauc's relation is used to determine the bandgap (E_g) in semiconductor.

$$\alpha h\nu = A (h\nu - E_g)^n$$

$$\alpha = \frac{A (h\nu - E_g)^n}{h\nu}$$

- ✓ Rearranging above relation, we can have

$$(\alpha h\nu)^{\frac{1}{n}} = A^{\frac{1}{n}} h\nu - A^{\frac{1}{n}} E_g$$

Where, α = absorption coefficient and is given by $\alpha = \frac{\ln\left(\frac{1}{T}\right)}{x}$

$$T = \text{Transmittance} = \frac{I}{I_0}$$

- ✓ $n = 1/2, 2, 3/2$, and 3 for direct allowed, indirect allowed, direct forbidden, and indirect forbidden transitions respectively.
- ✓ Now, we can plot a graph of $(\alpha h\nu)^{1/n}$ along x-axis vs $h\nu$ (along y axis), we will get slop as $A^{1/n}$ and y intercept as $A^{1/n} E_g$. Dividing y intercept by A^n we can estimate the band gap.

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SYLLABUS OF PHYSICS – II.....***

GTU PREVIOUS YEAR PAPERS.....***

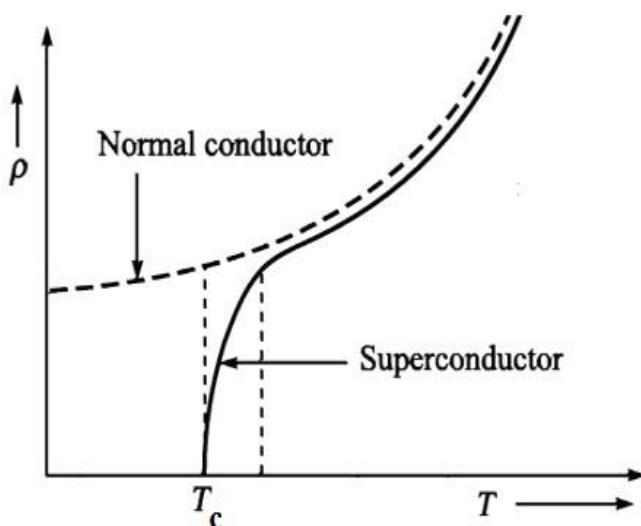
I N D E X

UNIT 5

5.1: INTRODUCTION TO SUPERCONDUCTIVITY

- ✓ Helium gas—was liquefied at 4.2 K by Dutch Physicist Heike Kamerlingh Onnes in 1908. Superconductivity was discovered by K. Onnes in 1911. Liquid Helium has a temperature of 4.2 K. It was observed that the resistance of mercury dropped from 0.08Ω at about 4.3 K to less than $3 \times 10^{-6} \Omega$ at 4.2 K. A large number and wide variety of metals, alloys, binary and ternary chemical compounds have been found to show superconductivity at various temperatures.

- ✓ Superconductivity is a state in which quantum mechanics operates on a macroscopic scale of the order of many atomic distances rather than the usual atomic and subatomic scale. The superconducting state is influenced by the temperature, magnetic field and current. All these three parameters have critical values, above which material enters into normal state. Every superconductor has its own transition temperature (T_c).



- ✓ Good electrical conductors such as silver, gold, and copper are not good superconductors because the resistivity of these conductors at low temperatures is limited to low resistivity i.e. residual resistivity, value due to scattering of electrons from crystal defects and impurities. Similarly, good superconducting materials like zinc and lead are not good electrical conductors.

5.2: DEFINITIONS

❖ **SUPERCONDUCTOR**

- ✓ It is a material that loses all its resistance (offers zero resistance) to the flow of electric current when it is cooled below a certain temperature called the critical temperature or transition temperature T_c .
- ✓ e.g. Mercury, Zinc, Niobium, etc...

❖ **CRITICAL TEMPERATURE (T_c)**

- ✓ The temperature at which a material's electrical resistivity drops to absolute zero is called the critical temperature or transition temperature. It is denoted by T_c .

❖ **SUPERCONDUCTIVITY**

- ✓ It is a phenomenon in which certain metals, alloys and ceramics conduct electricity without resistance when it is cooled below a certain temperature called the critical temperature.

5.3: PROPERTIES OF SUPERCONDUCTORS

5.3.1: ELECTRICAL RESISTANCE

- ✓ The electrical resistance of superconducting material is very low and is of the order of $10^{-7} \Omega$.

5.3.2: EFFECT OF IMPURITIES

- ✓ When impurities are added to superconducting elements, the superconducting property is not lost but the T_c value is lowered.

5.3.3: ISOTOPE EFFECT

- ✓ The critical temperature of a superconductor is found to vary with its isotopic mass. The atomic mass of Hg varies from 199.5 to 203.4. Due to this variation in atomic mass, the transition

temperature of isotopes of Hg varies from 4.185 to 4.146 K. They are related as $T_c \propto \frac{1}{\sqrt{M}}$ where M is the isotopic mass.

5.3.4: MAGNETIC FIELD EFFECT

- ✓ If a sufficiently strong magnetic field is applied to a superconductor at any temperature below critical temperature T_c , the superconductor is found to undergo a transition from the superconducting state to the normal state. OR This minimum magnetic field required to destroy the superconducting state is called critical magnetic field H_c . $H_c = H_0 \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$ where H_0 is the applied magnetic field at 0 K.

5.3.5: EFFECT OF PRESSURE AND STRESS

- ✓ Certain materials are found to exhibit the superconductivity phenomena on increasing the pressure over them. For e.g. Cs shows superconductivity at $T_c = 1.5$ K and 110 k bar Also, in Superconductors, an increase in stress results in increase of the T_c value.

5.3.6: CRITICAL CURRENT DENSITY

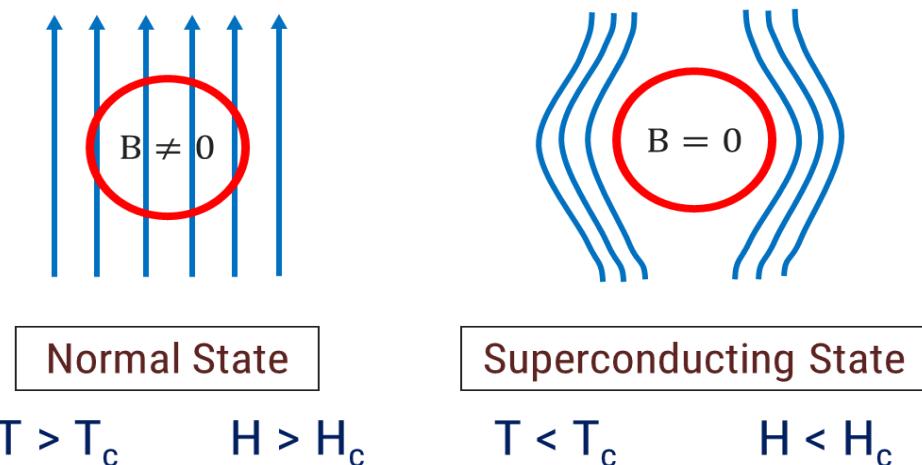
- ✓ It is defined as the maximum current that can be permitted in a superconducting material without destroying its superconductivity state. OR Minimum current required to destroy the superconducting state is called critical current density (J_c) and this current is called critical current (I_c). The equation relating $J_c = \frac{I_c}{A}$ and $I_c = 2 \pi r H_c$.

5.3.7: PERSISTENT CURRENT

- ✓ If current is made to flow through a superconducting ring then it is observed that the current flows through the material without any significant loss. This steady flow of current in a superconducting ring without any potential deriving is called the persistent current.

5.3.8: MEISSNER EFFECT

- The complete expulsion of all the magnetic field lines by a superconductor material is called Meissner effect. The Meissner effect is a distinct characteristic of a superconductor from a normal perfect conductor. In addition, this effect is exhibited by the superconducting materials only when the applied field is less than the critical field H_c .



- To Prove that $\chi_m = -1$ for superconductors
- We know that for a magnetic material the magnetic induction or magnetic flux density B is given by the equation $B = \mu_0 (M + H)$ where μ_0 is the permeability of free space; M is the intensity of magnetization; H is the applied magnetic field.
- But for the superconductors, we know that $B = 0$, thus the above equation can be written as

$$\therefore \text{i. e. } 0 = \mu_0 (M + H)$$

$$\therefore \text{i. e. } 0 = M + H \text{ since } \mu_0 \neq 0$$

$$\therefore \text{i. e. } M = -H$$

$$\text{OR } \frac{M}{H} = -1 = \chi_m$$

- ✓ Where χ_m is called as the magnetic susceptibility. This means that for a superconductor, the susceptibility is negative and maximum, i.e. a superconductor exhibits perfect diamagnetism. For all other magnetic materials, the susceptibility values are positive.

EXAMPLES

- Q.1 For mercury of mass 202, value of α is 0.5 and T_c is 4.2 K. Find transition temperature for isotope of mercury of mass 200.

Ans. $M_1 = 202, T_{c1} = 4.2 \text{ K}, M_2 = 200, \alpha = 0.5, T_{c2} = ?$

$$T_c M^\alpha = \text{constant}$$

$$\therefore T_{c1} M_1^\alpha = T_{c2} M_2^\alpha$$

$$\therefore T_{c2} = \left(\frac{M_1}{M_2} \right)^\alpha T_{c1}$$

$$\therefore T_{c2} = \left(\frac{202}{200} \right)^{0.5} 4.2$$

$$\therefore T_{c2} = 1.004987 \times 4.2$$

$$\therefore T_{c2} = 4.2209 \text{ K}$$

- Q.2 The critical temperature of a metal with isotopic mass 199.5 is 4.185 K. Calculate the isotopic mass if critical temperature falls to 4.133 K.

Ans. $M_1 = 199.5, T_{c1} = 4.185 \text{ K}, T_{c2} = 4.133 \text{ K}, M_2 = ?$

$$T_c M^\alpha = \text{constant}$$

$$\therefore T_{c1} M_1^\alpha = T_{c2} M_2^\alpha$$

$$\therefore M_2^\alpha = \frac{T_{c1}}{T_{c2}} (M_1^\alpha) \quad \text{But } \alpha = 0.5$$

$$\therefore \sqrt{M_2} = \frac{T_{c1}}{T_{c2}} \sqrt{M_1}$$

$$\therefore \sqrt{M_2} = \frac{4.185}{4.133} \sqrt{199.5}$$

$$\therefore \sqrt{M_2} = 1.01258 \sqrt{199.5}$$

$$\therefore \sqrt{M_2} = 14.301$$

$$\therefore M_2 = 204.55$$

Q.3 Two isotopes of lead of mass 206 and 210 have T_c values of 7.193 K and 7.125 K respectively. Calculate the isotopes constant for lead (Pb).

Ans. $M_1 = 206, M_2 = 210, T_{c1} = 7.193 \text{ K}, T_{c2} = 7.125 \text{ K}, \alpha = ?$

$$\therefore T_{c1} M_1^\alpha = T_{c2} M_2^\alpha$$

$$\therefore \left(\frac{T_{c1}}{T_{c2}} \right) = \left(\frac{M_2}{M_1} \right)^\alpha$$

$$\therefore \left(\frac{7.193}{7.125} \right) = \left(\frac{210}{206} \right)^\alpha$$

$$\therefore 1.00954 = (1.0941)^\alpha$$

$$\therefore \log (1.00954) = \alpha \log (1.0941)$$

$$\therefore 4.12519 \times 10^{-3} = \alpha (1.0941 \times 10^{-3})$$

$$\therefore \alpha = \frac{1.0941 \times 10^{-3}}{4.12519 \times 10^{-3}}$$

$$\therefore \alpha = 0.4941$$

- Q.4 A superconductor tin has a critical temperature of 3.7 K in zero magnetic field and a critical field of 0.0306 T at 0 K. Find the critical field at 2 K.

Ans. $H_c(0) = 0.0306 \text{ T}$, $T_c = 3.7 \text{ K}$, $T = 2 \text{ K}$, $H_c = ?$

Now, the critical field at any temperature T K is given as: $H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$

$$\therefore H_c = 0.0306 \left[1 - \left(\frac{2}{3.7} \right)^2 \right]$$

$$\therefore H_c = 0.0306 \times 0.708$$

$$\therefore H_c = 0.0216 \text{ T}$$

- Q.5 The transition temperature for lead is 7.2 K. However, at 5 K, it loses the superconducting property when subjected to a magnetic field of $3.3 \times 10^4 \text{ A/m}$. Find the value of the magnetic field that will allow the metal to remain its superconductivity at 0 K.

Ans. $T_c = 7.2 \text{ K}$, $T = 5 \text{ K}$, $H_c = 3.3 \times 10^4 \text{ A/m}$, $H_c(0) = ?$

$$\therefore H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore H_c(0) = H_c \left[1 - \left(\frac{T}{T_c} \right)^2 \right]^{-1}$$

$$\therefore H_c(0) = 3.3 \times 10^4 \left[1 - \left(\frac{5}{7.2} \right)^2 \right]^{-1}$$

$$\therefore H_c(0) = 6.37 \times 10^4 \text{ A/m}$$

Q.6 Calculate the transition temperature of niobium for which the critical field is $1 \times 10^5 \text{ A/m}$ at 8 K and $2 \times 10^5 \text{ A/m}$ at 0 K.

Ans. $H_c = 1 \times 10^5 \text{ A/m}$, $T = 8 \text{ K}$, $H_c(0) = 1 \times 10^5 \text{ A/m}$, $T_c = ?$

$$\therefore H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore \frac{H_c}{H_c(0)} = 1 - \left(\frac{T}{T_c} \right)^2$$

$$\therefore \sqrt{1 - \frac{H_c}{H_c(0)}} = \sqrt{\left(\frac{T}{T_c} \right)^2}$$

$$\therefore \frac{T}{T_c} = \left[1 - \left(\frac{H_c}{H_c(0)} \right) \right]^{\frac{1}{2}}$$

$$\therefore T_c = \frac{T}{\left[1 - \left(\frac{H_c}{H_c(0)} \right) \right]^{\frac{1}{2}}}$$

$$\therefore T_c = \frac{8}{\left[1 - \left(\frac{1 \times 10^5}{2 \times 10^5} \right) \right]^{\frac{1}{2}}}$$

$$\therefore T_c = 8\sqrt{2} \text{ K}$$

$$\therefore T_c = 11.3 \text{ K}$$

Q.7 Critical temperature of a superconductor is 78 K and critical field is 0.518 T at 0 K. Find critical field at 25 K and 58 K.

Ans. $T = 25 \text{ K}$ $T = 58 \text{ K}$

$$H_c(0) = 0.518 \text{ T}$$

$$H_c(0) = 0.518 \text{ T}$$

$$T_c = 78 \text{ K}$$

$$T_c = 78 \text{ K}$$

$$\therefore H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore H_c = 0.518 \left[1 - \left(\frac{25}{78} \right)^2 \right]$$

$$\therefore H_c = 0.518 \left[1 - \left(\frac{58}{78} \right)^2 \right]$$

$$\therefore H_c = 0.46514 \text{ T}$$

$$\therefore H_c = 0.2314 \text{ T}$$

Q.8 Calculate the critical current through a long thin superconducting wire of radius 0.5 mm. The critical field is $7.2 \times 10^3 \text{ A/m}$.

Ans. $H_c = 7.2 \times 10^3 \text{ A/m}$, $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$, $I_c = ?$

According to Silsbee's law: $I_c = 2\pi r H_c$

$$\therefore I_c = (2\pi)(0.5 \times 10^{-3})(7.2 \times 10^3)$$

$$\therefore I_c = 22.608 \text{ A}$$

Q.9 Calculate the critical current for a superconducting wire of lead (Pb) having a diameter of 1 mm at 4.2 K. Critical temperature of lead is 7.18 K and $H_c(0)$ is $6.5 \times 10^4 \text{ A/m}$.

Ans. $H_c(0) = 6.5 \times 10^4 \text{ A/m}$, $T_c = 7.18 \text{ K}$, $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$, $T = 4.2 \text{ K}$, $I_c = ?$, $H_c = ?$

$$\text{We know, } H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore H_c = 6.5 \times 10^4 \left[1 - \left(\frac{4.2}{7.18} \right)^2 \right]$$

$$\therefore H_c = 6.5 \times 10^4 (1 - 0.342)$$

$$\therefore H_c = 6.5 \times 10^4 (0.0657)$$

$$\therefore H_c = 42.75 \times 10^3 \text{ A/m}$$

According to Silsbee's law: $I_c = 2\pi r H_c$

$$\therefore I_c = (2\pi) (0.5 \times 10^{-3}) (4.2 \times 10^3)$$

$$\therefore I_c = 134.26 \text{ A}$$

- Q.10 Calculate critical current density for a superconducting wire of lead having diameter of 1.5 mm at 5.3 K. The value of critical temperature of lead is 7.8 K and critical magnetic field at 0 K is $6.5 \times 10^4 \text{ A/m}$.

Ans. $H_c(0) = 6.5 \times 10^4 \text{ A/m}$, $T_c = 7.8 \text{ K}$, $r = 0.75 \text{ mm} = 0.75 \times 10^{-3} \text{ m}$, $T = 5.3 \text{ K}$, $J_c = ?$, $I_c = ?$, $H_c = ?$

$$\text{We know, } H_c = H_c(0) \left[1 - \left(\frac{T}{T_c} \right)^2 \right]$$

$$\therefore H_c = 6.5 \times 10^4 \left[1 - \left(\frac{5.3}{7.8} \right)^2 \right]$$

$$\therefore H_c = 3.498 \times 10^4 \text{ A/m}$$

Now, $I_c = 2\pi r H_c$

$$\therefore I_c = (2\pi) (0.75 \times 10^{-3}) (3.498 \times 10^4)$$

$$\therefore I_c = 164.75 \text{ A}$$

Current density: $J_c = \frac{I_c}{A}$

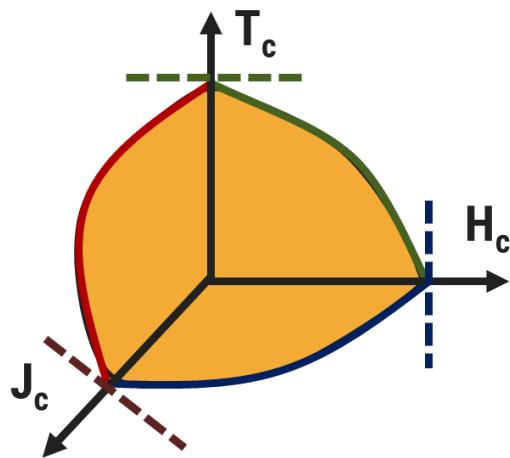
$$\therefore J_c = \frac{I_c}{\pi r^2}$$

$$\therefore J_c = \frac{164.75}{\pi (0.75)^2}$$

$$\therefore J_c = 93.28 \times 10^6 \text{ A/m}$$

5.3: THREE IMPORTANT FACTORS TO DEFINE A SUPERCONDUCTING STATE

- ✓ Critical temperature (T_c)
- ✓ Critical current density (J_c)
- ✓ Critical magnetic field (H_c)



- ✓ Each of the above three parameters is very dependent on the other two properties. The highest values for H_c and J_c occur at 0 K; while the highest value for T_c occurs when H and J are zero. Thus the plot of all these three parameters represents a critical surface. Within the surface, the material is superconducting and outside the surface, the material is said to be in the normal state.

5.4: TYPES OF SUPERCONDUCTORS

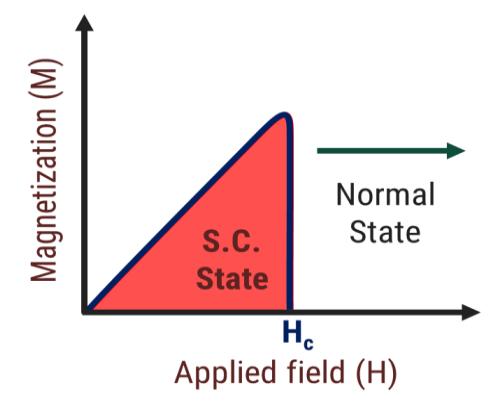
- ✓ There are two types of super conductors based on their variation in magnetization, due to external magnetic field applied.

Type I superconductor or soft super conductor

Type II superconductor or hard superconductor

❖ TYPE I SUPERCONDUCTORS:

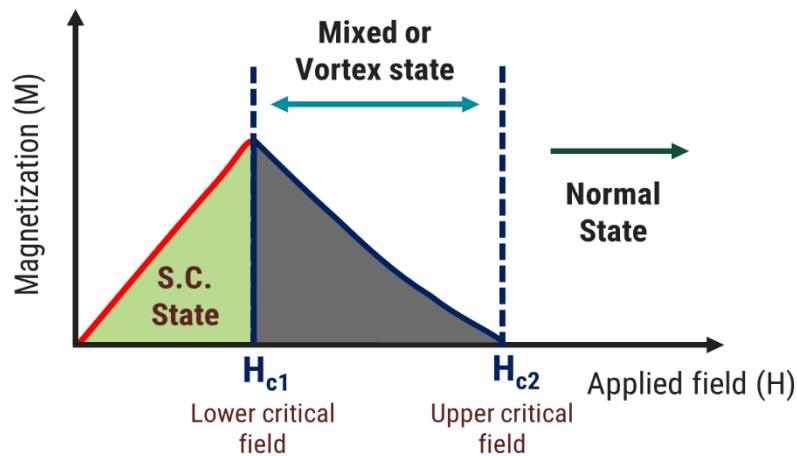
- ✓ When the super conductor is kept in the magnetic field and if the field is increased the superconductor becomes normal conductor abruptly at critical magnetic field as shown in fig. These types of materials are termed as Type I superconductors.
- ✓ Below critical field, the specimen excludes all the magnetic lines of force and exhibit perfect Meissner effect. Hence, Type I superconductors are perfect diamagnetic, represented by negative sign in magnetization.



❖ TYPE II SUPERCONDUCTORS:

- ✓ When the super conductor kept in the magnetic field and if the field is increased, below the lower critical field H_{c1} , the material exhibit perfect diamagnetism (i.e.) it behaves as a super conductor and above H_{c1} , the magnetization decreases and hence the magnetic flux starts penetrating

through the material. The specimen is said to be in a mixed state between H_{c1} and H_{c2} . above H_{c2} (upper critical field) it becomes normal conductor as shown in fig.



- ✓ The materials which lose its superconducting property gradually due to increase on the magnetic field are called Type II superconductor. Value of H_c for type-II superconductor is 100 times or even more as compared to type-I superconductor Because of relatively large magnetic field requirement for type-II superconductor.

I.e. YBCO ($Y_1Ba_2Cu_3O_7$), Nb - Ti (Niobium titanium)

5.4.3: DIFFERENCE BETWEEN TYPE I AND TYPE II SUPERCONDUCTORS

Type-I superconductors	Type-II superconductors
They are called soft superconductors	They are known as hard superconductors
They exhibit complete Meissner effect	They do not exhibit complete Meissner effect
They show perfect diamagnetic behavior	They do not show perfect diamagnetic behavior

It requires low magnetic field to destroy the superconductivity	It requires large magnetic field to destroy the superconductivity
They have only one critical magnetic field (H_c)	They have two critical magnetic fields, lower critical magnetic field (H_{c1}) and upper critical magnetic field (H_{c2})
There is no mixed or intermediate state in case of these materials	Mixed or intermediate state is present in these materials
They have limited applications because of low field strength. e.g. Pb, Hg, Zn, Nb	They have wider technical applications because of high field strength. e.g. Nb-Sn, V-Ga, YBCO

5.4.4: LOW TEMPERATURE AND HIGH TEMPERATURE SUPERCONDUCTORS

❖ **LOW TEMPERATURE SUPERCONDUCTORS (LTSC):**

- ✓ Superconductors that require liquid helium as a coolant are called low temperature Superconductors (LTSC).

For LTSC superconductors the temperature is usually well below 20 K (-253 °C).

Temperature of liquid helium is 4.2 K above absolute zero (0 K).

❖ **HIGH TEMPERATURE SUPERCONDUCTORS (HTSC):**

- ✓ Superconductors having their critical temperature (T_c) value above the temperature of liquid nitrogen are called high temperature superconductors.
- ✓ In a superconductor if the transition temperature is high i.e. greater than 20K, then it is also called high temperature superconductor. Earlier it was believed that the superconductivity was only in metals.

- ✓ Surprisingly in 1986, Muller and Bednorz discovered high temperature superconductor in ceramics. They made a particular type of ceramic material from a compound of barium, lanthanum, copper and oxygen (Ba-La-Cu-O).
- ✓ This compound superconductor showed superconductivity even at a temperature as high as 30 K. Similar materials with higher transition temperature soon followed when $\text{Y}_1\text{Ba}_2\text{Cu}_3\text{O}_7$ (YBCO) or the so called 1-2-3 compound was discovered.
- ✓ The onset of superconductivity for 1-2-3 occurs at 93 K. The highest critical temperature at ambient pressure discovered so far (2009) is 134 K for a doped $\text{HgBa}_2\text{Ca}_2\text{Cu}_3\text{O}_{8+\delta}$.

5.5: BCS THEORY

- ✓ The microscopic theory of superconductivity developed by J. Bardeen, L.N. Cooper and J.R. Schriener in 1957, successfully explained the effect like zero resistivity, Meissner effect etc. this theory is known as BCS theory.

❖ **PRINCIPLE:**

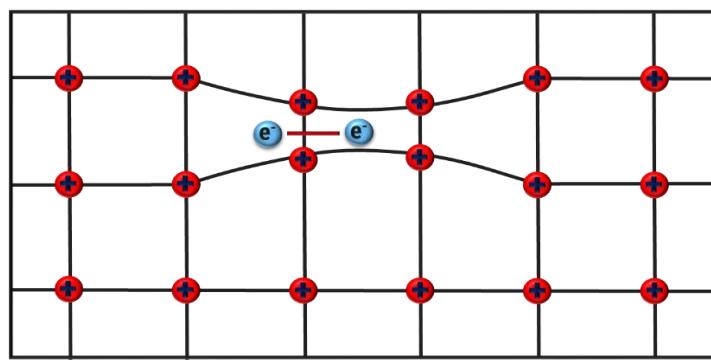
- ✓ This theory states that the electrons experience a special kind of attractive interaction, overcoming the coulomb forces of repulsion between them; as a result cooper pairs (i.e) electron pair are formed. At low temperature, these pairs move without any restriction through the lattice points and the material becomes superconductor. Here the electron-lattice-electron interaction should be stronger than electron-electron interaction.

❖ **IMPORTANT FEATURES OF BCS THEORY:**

- ✓ Electrons form pairs (called cooper pair) which propagate throughout the lattice.
- ✓ The propagation of cooper pairs is without resistance because the electrons move in resonance with phonons.

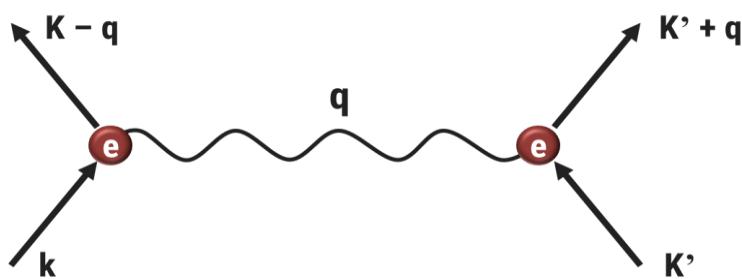
❖ **ELECTRON-LATTICE-ELECTRON INTERACTION:**

- ✓ When an electron (1st) moves through the lattice, it will be attracted by the core (+ve charge) of the lattice. Due to this attraction, ion core is disturbed and it is called as lattice distortion. The lattice vibrations are quantized in terms of phonons.
- ✓ The deformation produces a region of increased positive charge. Thus if another electron (2nd) moves through this region as shown in figure. It will be attracted by the greater concentration of positive charge and hence the energy of the 2nd electron is lowered.
- ✓ Hence two electrons interact through the lattice or the phonons field resulting in lowering the energy of electrons. This lowering of energy implies that the force between the two electrons is attractive. This type of interaction is called electrons - lattice electron interaction. The interaction is strong only when the two electrons have equal and opposite momenta and spins.



❖ EXPLANATION:

- ✓ Consider the 1st electron with wave vector k distorts the lattice, here by emitting phonons of wave vector q . This results in the wave vector $k - q$ for the 1st electron. Now if the 2nd electron with wave vector k' , seeks the lattice it takes up the energy from the lattice and its wave vector changes $k' + q$ as shown in figure. Two electrons with wave vectors $k - q$ and $k' + q$ form a pair known as cooper pair.

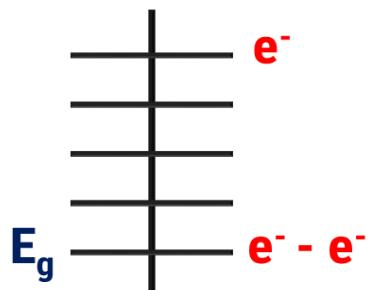


5.5.1: COOPER PAIRS

- ✓ The fundamental of BCS theory is that superconductivity occurs when an attractive interaction between two electrons, by mean of phonon exchange takes place. This dominates the usual repulsive Coulomb interaction.
- ✓ “Two such electrons which interacts alternatively in phonon field are called Cooper Pairs.”
- ✓ The energy of cooper pairs of electrons in bound state is less than the energy of electron in Free State.
- ✓ This difference in energy is due to binding energy, which is supplied if the pair is broken. At lower temperature (lower than T_c), the electron-electron lattice interaction is stronger than electron-electron Coulomb repulsive interaction, which leads to pairing up of valence electrons.
- ✓ This pairing is complete at $T = 0$ K and completely broken at critical temperature (T_c).

5.5.2: ENERGY GAP

- ✓ “The energy difference between the free state of an electron (i.e. energy of individual electron) and the paired state (i.e. energy of paired electron) appears as the energy gap at the fermi surface.”
- ✓ The normal electron states are above the energy gap and superconducting electron states are below the energy gap (as shown in figure).
- ✓ The energy gap in superconductors is a function of temperature. As the pairing is complete at $T = 0$ K, the difference in energy of free electron states is maximum.
- ✓ BCS theory predicts that the energy gap in superconductors at $T = 0$ K is $E_g = -3.53 K_B T_c$.



- ✓ This equation can be verified by the absorption of electromagnetic radiation. Photons with energies equal to or greater than energy gaps can be absorbed.

5.5.3: COHERENCE LENGTH

- ✓ The paired electrons are not scattered because of their peculiar property of smoothly riding over the lattice imperfections without exchanging energy with them.
- ✓ The concept of coherence can be explained by the fact that superconductivity is due to mutual interaction and correlation of electrons that extend upto a considerable distance.
- ✓ "The maximum distance upto which the state of paired electrons are correlated to produce superconductivity is called Coherence length (ε_0)."
- ✓ The paired electrons can be thousands of atomic spacing approx. (0.001 nm) apart.
- ✓ The properties of a superconductor depends on the correlation of electrons within a volume (ε_0)³ (Called the 'coherence volume').
- ✓ This is mainly because, a large no. of electrons in such a volume act together to produce superconductivity with an extremely sharp transition.

5.6: PENETRATION DEPTH

- ✓ It is a concept proposed by London brothers in the year 1935 that described the Meissner effect.
- ✓ In a superconductor the London penetration depth (λ_L) characterizes the distance to which a magnetic field penetrates into a superconductor.
- ✓ The exponential decay of magnetic field at the interior surface of superconductor.

$$\therefore B(x) = B_0 e^{\left(\frac{-x}{\lambda}\right)}$$

X = distance inside the superconductor from surface

B_0 = Magnetic field at the surface

λ_L = Penetration depth

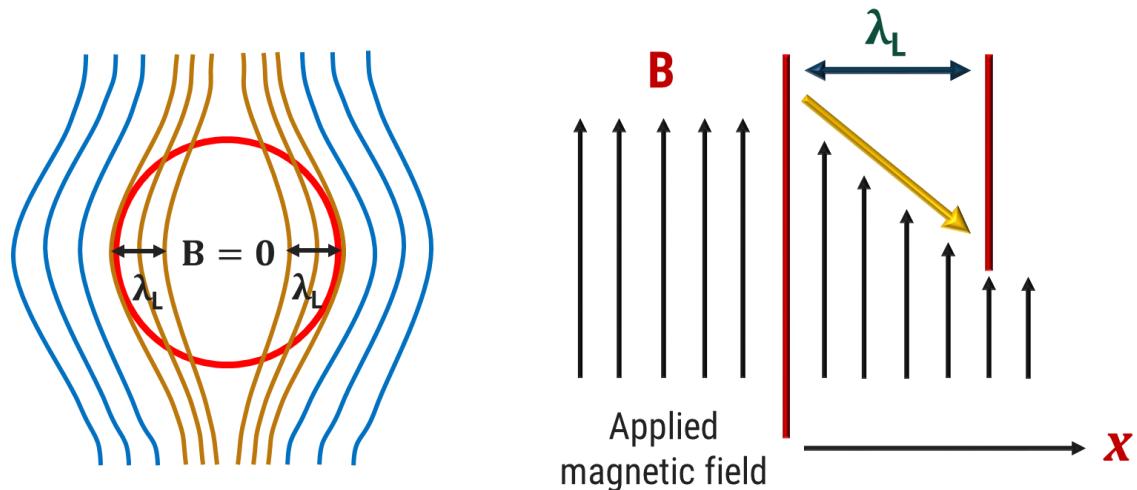
λ_L is the distance across which the magnetic field becomes 'e' times weaker.

$$\lambda_L \text{ can be defined as: } \lambda_L = \sqrt{\frac{m}{\mu_0 n_s q^2}}$$

m = mass of the charge carriers

n_s = number super-electron density

μ_0 = absolute permeability



❖ LONDON EQUATIONS:

- ✓ Maxwell's equations could not explain the zero resistance and perfect diamagnetism observed as main characteristics of superconductivity.

- ✓ F. London and H. London (London brothers) suggested that motion of superconducting electrons in the presence of applied electric field E. If a magnetic field is applied to a superconductor which is initially in zero field, the magnetic field is a function of time.

- ✓ London brothers assume there are two types of electrons:

Normal electrons: $T > T_c$

Super electrons: $T < T_c$

- ✓ When electric field is applied there will be force exerted on electrons: $F = q E$

$$\therefore m a = q E$$

$$\therefore m \frac{\partial v}{\partial t} = q E \dots \dots \dots (1)$$

Due to force there will be motion of charge, so super-current density (J): $J = n q v$

Where, n = concentration of super-electrons, v = drift velocity, q = electric charge

By taking derivation w.r.t. time: $\frac{\partial J}{\partial t} = n q \frac{\partial v}{\partial t} \dots \dots \dots (2)$

$$\therefore \frac{\partial J}{\partial t} = n q \frac{q E}{m} \quad \text{From equation (1)}$$

$$\therefore \frac{\partial J}{\partial t} = \frac{n q^2 E}{m} \dots \dots \dots (3)$$

- ✓ This equation known as first London equation.

Now, when electric field (E) becomes zero means current density (J) is constant

By taking curl of both sides of equation (3), we get

$$\therefore \nabla \times \frac{\partial J}{\partial t} = \frac{n q^2}{m} (\nabla \times E)$$

$$\therefore \frac{\partial}{\partial t} (\nabla \times J) = \frac{n q^2}{m} (\nabla \times E)$$

$$\therefore \frac{\partial}{\partial t} (\nabla \times J) = \frac{n q^2}{m} - \frac{\partial B}{\partial t}$$

By taking integration on both sides

$$\therefore \int \frac{\partial}{\partial t} (\nabla \times J) = \frac{n q^2}{m} \int -\frac{\partial B}{\partial t}$$

$$\therefore (\nabla \times J) = -\frac{n q^2}{m} B \dots \dots \dots (4)$$

- ✓ This equation known as second London equation.

The super current density is related to magnetic field 'B' by another Maxwell's equation as,

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

But for electrostatic current the curl of E becomes zero.

$$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\therefore \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) = \vec{J} \dots \dots \dots (5)$$

Substituting equation (5) in equation (4), we get

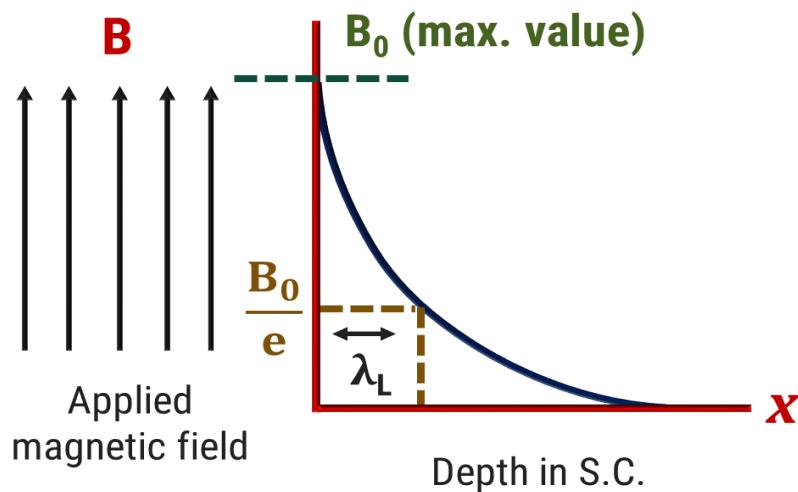
$$\therefore \left(\nabla \times \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) \right) = -\frac{n q^2}{m} B$$

$$\therefore \frac{m}{\mu_0 n q^2} (\nabla \times (\vec{\nabla} \times \vec{B})) = -B$$

$$\therefore \frac{m}{\mu_0 n q^2} (\nabla \times (\vec{\nabla} \times \vec{B})) + B = 0$$

$$\text{Where, } \therefore \lambda_L^2 = \frac{m}{\mu_0 n_s q^2}$$

- ✓ Above equation defines: London penetration depth (λ_L).



For $x = \lambda_L$,

$$\therefore B(x) = B_0 e^{\left(\frac{-x}{\lambda}\right)}$$

$$\therefore B(x) = B_0 e^{(-1)}$$

$$\therefore B(x) = \frac{B_0}{e}$$

- ✓ The field decays to B_0/e at a distance $x = \lambda_L$ inside the superconducting material.

Typical values of λ range from 50 to 500 nm.

- ✓ The penetration depth (λ_L) is determined by the superfluid density, which is an important quantity that determines T_c in high-temperature superconductors. At a temperature T , penetration depth is given by:

$$\therefore \lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{-\frac{1}{2}}$$

Where, λ_0 is penetration depth at 0 K. It is given by:

$$\therefore \lambda_L = \left[\frac{m}{\mu_0 n_s q^2} \right]^{\frac{1}{2}}$$

EXAMPLES

Q.1 The number of super-electrons in a superconductor is $10^{28}/m^3$ at critical temperature $T_c = 3$ K. Calculate the penetration depth at 0 K and 1 K.

Ans. $m = 3.1 \times 10^{-31}$ kg, $\mu_0 = 12.56 \times 10^{-7}$, $q = 1.6 \times 10^{-19}$ C, $n_s = 10^{28}/m^3$, $\lambda_0 = ?$, $\lambda_T = ?$

At $T = 0$ K, the penetration depth is

$$\therefore \lambda_0 = \left[\frac{m}{\mu_0 n_s q^2} \right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = \left[\frac{9.1 \times 10^{-31}}{12.56 \times 10^{-7} \times 10^{28} \times (1.6 \times 10^{-19})^2} \right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 0.530 \times 10^{-7} \text{ m}$$

$$\therefore \lambda_0 = 530 \text{ \AA}$$

$$\text{As, we know that } \lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{-\frac{1}{2}}$$

$$\therefore \lambda_T = 530 \left[1 - \left(\frac{1}{3} \right)^4 \right]^{-\frac{1}{2}}$$

$$\therefore \lambda_T = 530 \left[1 - \frac{1}{81} \right]^{-\frac{1}{2}}$$

$$\therefore \lambda_T = \frac{530 \times 9}{0.94}$$

$$\therefore \lambda_T = 533.30 \text{ \AA}$$

- Q.2 The penetration depth λ of Hg at 3.5 K is about 750 \AA. Find the penetration depth at 0 K. Given T_c for Hg = 4.153 K.

Ans. $\lambda_T = 750 \text{ \AA}$, $T = 3.5 \text{ K}$, $T_c = 4.153 \text{ K}$, $\lambda_0 = ?$

The penetration depth at temperature (T) is given by

$$\therefore \lambda_T = \lambda_0 \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{-\frac{1}{2}}$$

$$\therefore \lambda_0 = \frac{\lambda_T}{\left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{\frac{1}{2}}}$$

$$\therefore \lambda_0 = \lambda_T \left[1 - \left(\frac{T}{T_c} \right)^4 \right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 \left[1 - \left(\frac{3.5}{4.153} \right)^4 \right]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 [1 - (0.84)^4]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 [1 - 0.50]^{\frac{1}{2}}$$

$$\therefore \lambda_0 = 750 [0.50]^{\frac{1}{2}}$$

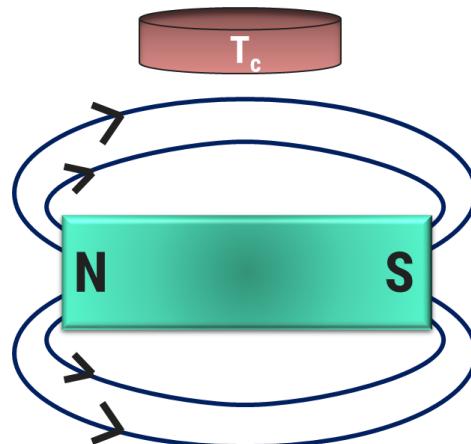
$$\therefore \lambda_0 = 750 \times 0.71$$

$$\therefore \lambda_0 = 530 \text{ \AA}$$

5.7: APPLICATION OF SUPERCONDUCTORS

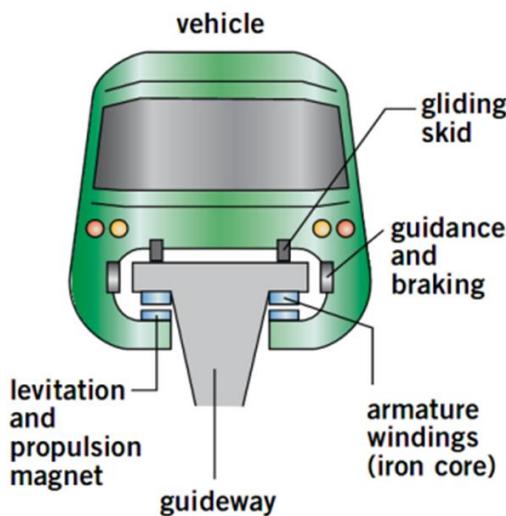
5.7.1: MAGNETIC LEVITATION

- ✓ It is the process by which an object is suspended above another object with no other support, but magnetic field.
- ✓ This phenomena is used in Maglev train. In Maglev trains, the levitation is brought due to presence of enormous repulsion among two highly powerful magnetic field when a small magnet is brought close to a superconducting magnet gets repelled.



- ✓ This repulsion takes place due to the induced currents in the superconductor which is being generated by the magnetic field of the magnet. Because of zero resistance property of the superconductor this current persists, and thus the field due to this induced current repels the field due to the magnet.
- ✓ As a result, the magnet floats freely above the superconductor.

- ✓ In attractive Electromagnetic suspension (EMS), the electromagnets installed on the train bogies attract the iron rails. The vehicle magnets wrap around the iron guide-ways and the attractive upward force lifts the train.
- ✓ Electrodynamic suspension uses repulsive force (magnet of same polarity) to levitate the train.



- ✓ Magnet of same polarity creates a repulsive force between levitation magnet and guideway. Because of superconductor is used, conduction is possible even when there is no power. Maglev trains travel at speed of about 500 km/h. A similar magnetic propulsion system is being used to launch the satellite into orbits directly from the earth without the use of rockets.

5.7.2: JOSEPHSON EFFECT

- ✓ Josephson effect was defined by Brian Josephson, in the year 1962.
- ✓ This is called Josephson junction or SIS junction.
- ✓ There are two type of Josephson effects: (1) D.C. Josephson effect & (2) A.C. Josephson effect



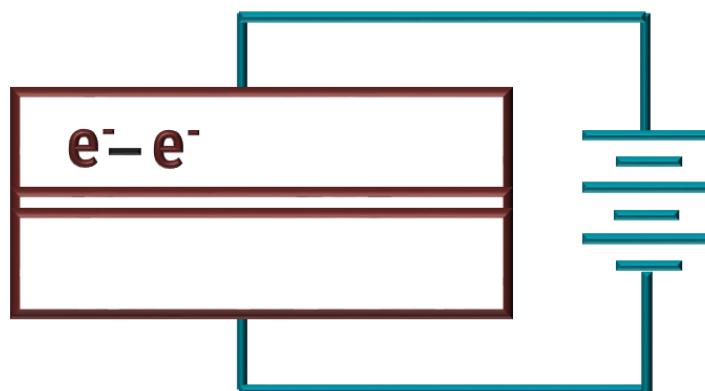
❖ **D.C. JOSEPHSON EFFECT:**

- ✓ When superconductors are separated by a thin insulating layer, the tunneling of electron pairs takes place from one superconductor to another.
- ✓ The cooper pairs of electrons tunnel through the insulating layer, where their thickness is very small, of an order of 1nm. The supercurrent (J) which is developed across the insulating junction depending on phase difference.
- ✓ A dc current is noted when there is no potential difference is applied across the junction. The expression for the current for this effect is

$$\therefore I_s = I_{\max} \sin \theta$$

Where theta is the phase difference between the two electrons of cooper pairs.

❖ **A.C. JOSEPHSON EFFECT:**



- ✓ When an external magnetic or electric field of potential 'v' is applied across the junction, the zero - voltage current developed across the junction depending on applied field, which is the "AC Josephson effect".
- ✓ When a potential 'v' is applied across the junction, cooper pairs on both the sides of the junction differs by an energy equals to $2eV$, where $2e$ is the charge on a cooper pair of electrons.

- ✓ Since in superconductors, current is carried by cooper pairs of electrons, if a copper pair passes across the gap, then it emits a photon of energy ($h\nu$) equal to 2eV.
- ✓ Then, the frequency of emitted radiation is $\nu = 2\text{eV}/h$.
- ✓ This is the oscillating frequency of sinusoidal current across the gap. Suppose the P.D. across the gap is 1mV, then the frequency will be of the order of 483.6 GHz.
- ✓ Applied voltage changes the phase difference:

$$\frac{\partial \theta}{\partial t} = \frac{2\text{eV}}{h} \text{ Due to this change we call it A. C. Josephson effect}$$

The value of current is given as: $I_s = I_{\max} \sin \left[\frac{2\pi\text{eV}}{h} t + \theta \right]$

EXAMPLES

- Q.1 A Josephson junction having a voltage of $8.50 \mu\text{V}$ across its terminals, then calculate the frequency of the alternating current. [Plank's constant = $6.625 \times 10^{-34} \text{ J} \cdot \text{sec}$]

Ans. $V = 8.5 \times 10^{-6} \text{ V}$, $h = 6.625 \times 10^{-34} \text{ J} \cdot \text{sec}$, $e = 1.6 \times 10^{-19}$, $V=?$

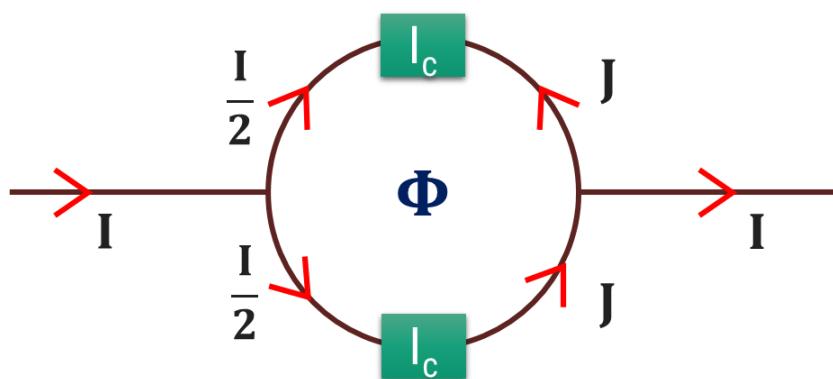
Frequency of alternating current, $\nu = \frac{2\text{eV}}{h}$

$$\therefore \nu = \frac{2 \times (1.6 \times 10^{-19}) (8.5 \times 10^{-6})}{6.625 \times 10^{-34}}$$

$$\therefore \nu = 4.1 \times 10^9 \text{ Hz}$$

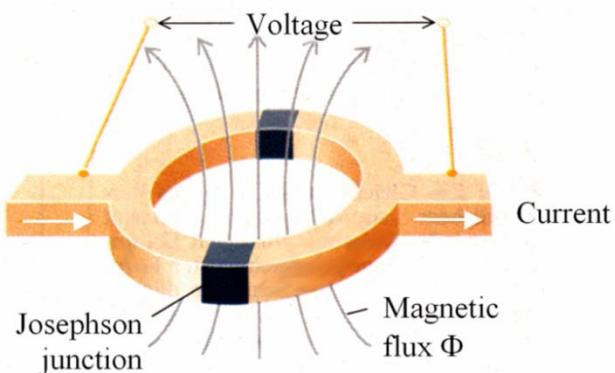
5.7.3: SQUID (SUPERCONDUCTING QUANTUM INTERFACE DEVICES)

- ✓ It works on the principle of Josephson effect.
- ✓ When current is passed through one side of superconductor, it flows equally through the Josephson junction.



❖ CONSTRUCTION:

- ✓ As shown in the diagram, it is seen that SQUID is formed when two superconductors and insulators are arranged in such a way that they form two parallel Josephson junction.



- ✓ It is connected to measuring device which can measure nominal changes in voltage and magnetic field across junction.

❖ **WORKING:**

- ✓ Current through the SQUID is highly sensitive to magnetic flux through the closed circuit. Even an extremely small magnetic flux can be detected with the device.
- ✓ The current through the circuit will have a periodicity which is very sensitive to the magnetic flux passing normally through the closed circuit. As a result, extremely small magnetic flux can be detected with this device.
- ✓ This device can also be used to detect voltages as small as 10 – 15 V. Magnetic field changes as small as 10^{-21} T can be detected.

❖ **APPLICATION:**

- ✓ Used to measure earth's magnetic field.
- ✓ It can detect weak magnetic field produced by biological currents like human brain.
- ✓ Based on these behavior, today we have (1) Magneto-Cardio Graphy (MCG)
(2) Magneto- Plethismo Graphy
- ✓ Detection of iron deposition in human liver.
- ✓ They are used in NMR imaging systems.
- ✓ They are used to detect brain tumors and clots using superconducting-solenoids.

5.7.4: OTHER APPLICATIONS OF SUPERCONDUCTORS

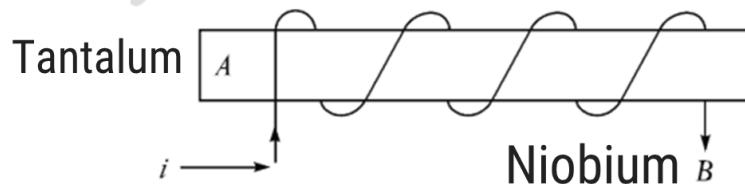
❖ **SUPERCONDUCTING MAGNETS:**

- ✓ Generally, when current flows through coil, it generates magnetic field. If the coil is replaced by a superconducting material, it generates a very large magnetic field.
- ✓ In this case, there is no effect of joule heating or resistive loss. Superconducting magnets are used in medical diagnosis, spectroscopy, magnetic levitation, etc.

- ✓ Superconducting electrical generators are small in size compared to conventional electric generators and produces more power as compared to ordinary generators.
- ✓ Superconducting materials are used in no-loss transmission lines and as relays on switching circuit.

❖ **CRYOTRON:**

- ✓ Two different superconducting materials are taken A= Tantalum Core and B= Niobium as winding wire.



- ✓ Taken them as such that: $H_{cA} < H_{cB}$
- ✓ If the current passes through winding B, it induces magnetic field.
- ✓ If this induced magnetic field is greater than H_{cA} , its corresponding state will be destroyed, thereby increasing resistivity and hence decreasing the current.
- ✓ Thus we can say that current through the winding controls superconducting state of core.
- ✓ It is useful in circuit breaker, relays etc...

5.8: ASSIGNMENT

1. Define superconductivity and critical temperature. (GTU: Jan-2019)
2. Explain mechanism of superconductivity. (GTU: Jan-2019)
3. Discuss the properties of superconductors. (GTU: Jan-2019, Jan-2020)
4. Distinguish between Type-I and Type-II superconductors. (GTU: Jan-2020)
5. Explain the BCS theory for superconductivity. (GTU: May-2019)

6. Differentiate between soft and hard superconductors. (GTU: May-2019)
 7. What is London penetration depth? Derive its equations. (GTU: May-2019, Jan-2020)
 8. Distinguish between dc and ac Josephson effect.
 9. What is a SQUID? Explain the construction, working and applications of SQUID. (GTU: May-2019, Jan-2020)
 10. Write a note on cryotron. (GTU: Jan-2020)
 11. The critical temperature of a metal with isotopic mass 199.5 is 4.185 K. Calculate its critical temperature when its isotopic mass is changed to 203.4.
- [Ans. 4.145 K]
12. A lead super conductor with $T_c = 7.2$ K has a critical magnetic field of 6.5×10^3 Am $^{-1}$ at absolute zero. What would be the value of critical field at 5 K temperature?
- [Ans. 3.36×10^3 Am $^{-1}$]
13. The critical current density equal to 1.71×10^8 A/m 2 is required to change a superconducting wire of radius 0.5 mm at 4.2 K. If the critical temperature of the material is 7.18 K, calculate the maximum value of the critical magnetic field. [Hint: $H_c(0) = ?$]
(GTU: May-2019)

[Ans. 64.986×10^3 Am $^{-1}$]

14. The critical temperature of Nb is 9.15 K. At zero kelvin, the critical field is 0.196 T. Calculate the critical field at 6 K. (GTU: Jan-2019)

[Ans. 0.1117 T]

15. What is the frequency of the electromagnetic waves radiate from a Josephson junction, if the voltage drop at the junction is 650 μ V.

[Ans. 313.95×10^9 Hz]

