

Empirical Properties of Asset Returns

Stylized Facts and Statistical Issues

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Abstract

We present a set of stylized empirical facts emerging from the statistical analysis of price variations in various types of financial markets. We first discuss some general issues common to all statistical studies of financial time series. Various statistical properties of asset returns are then described: distributional properties, tail properties and extreme fluctuations, pathwise regularity, linear and nonlinear dependence of returns in time and across stocks. Our description emphasizes properties common to a wide variety of markets and instruments. We then show how these statistical properties invalidate many of the common statistical approaches used to study financial data sets and examine some of the statistical problems encountered in each case.

Keywords

Stylized facts • Heavy tails • Volatility clustering • Long-range dependence • Risk management

Motivation

Although statistical properties of financial time series have been studied for decades, the emergence of high-frequency data poses a critical challenge to researchers in synthetically and meaningfully capturing the properties of this massive data set. To address this challenge, a set of universal properties common across different markets have been identified by independent studies and are classified as **stylized facts**. These stylized facts, such as **heavy tails** and **non-linear dependence**, fundamentally invalidate many of the common statistical approaches used to study financial data. Our goal is to provide a pedagogical overview of these empirical properties, focusing on the data's characteristics and examining the resulting statistical issues encountered during real financial analysis.

Problem Statement

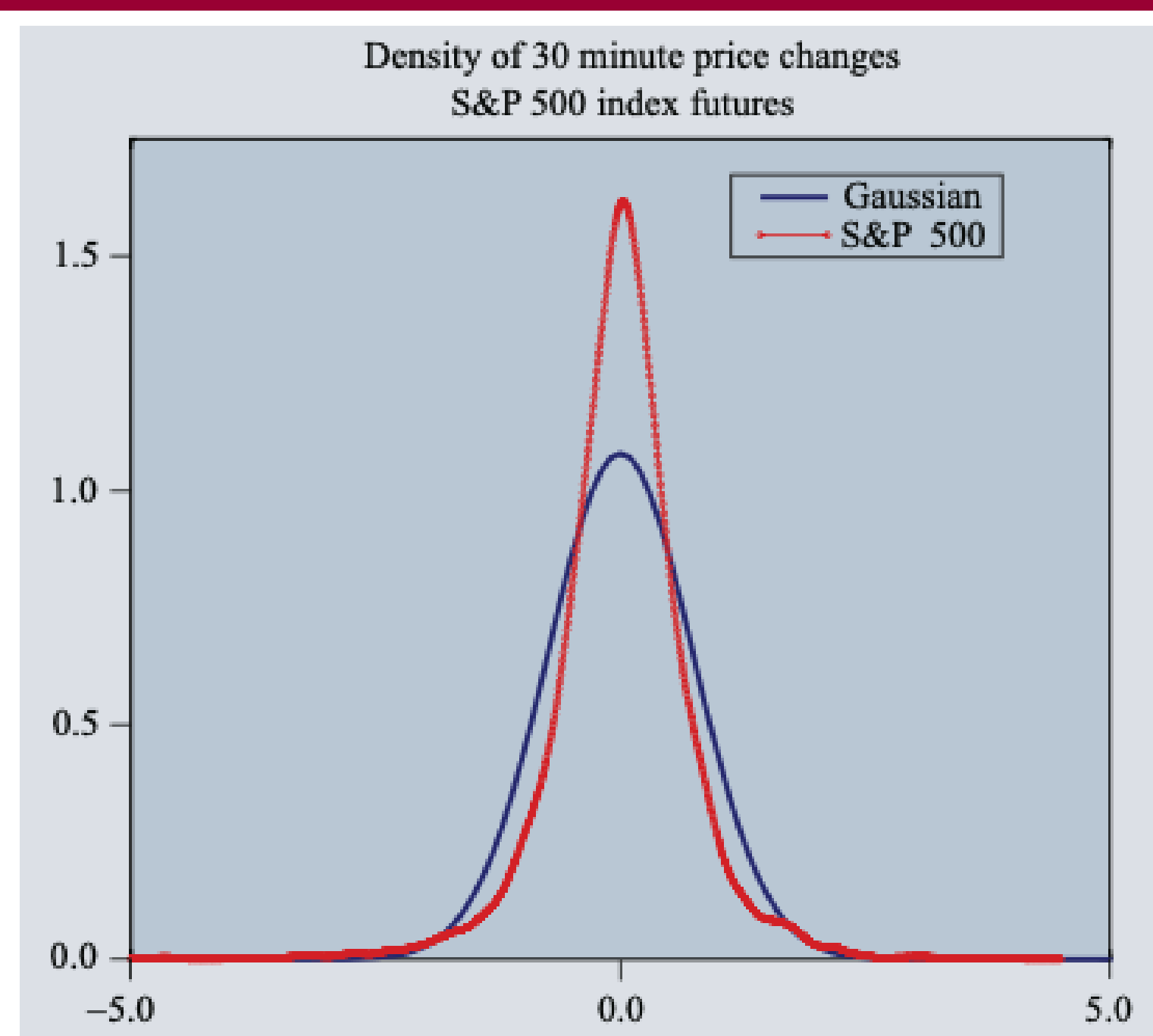
Traditional assumptions:

- ▶ Normal distribution
- ▶ Independent increments
- ▶ Constant volatility

Reality: Data systematically violates these!

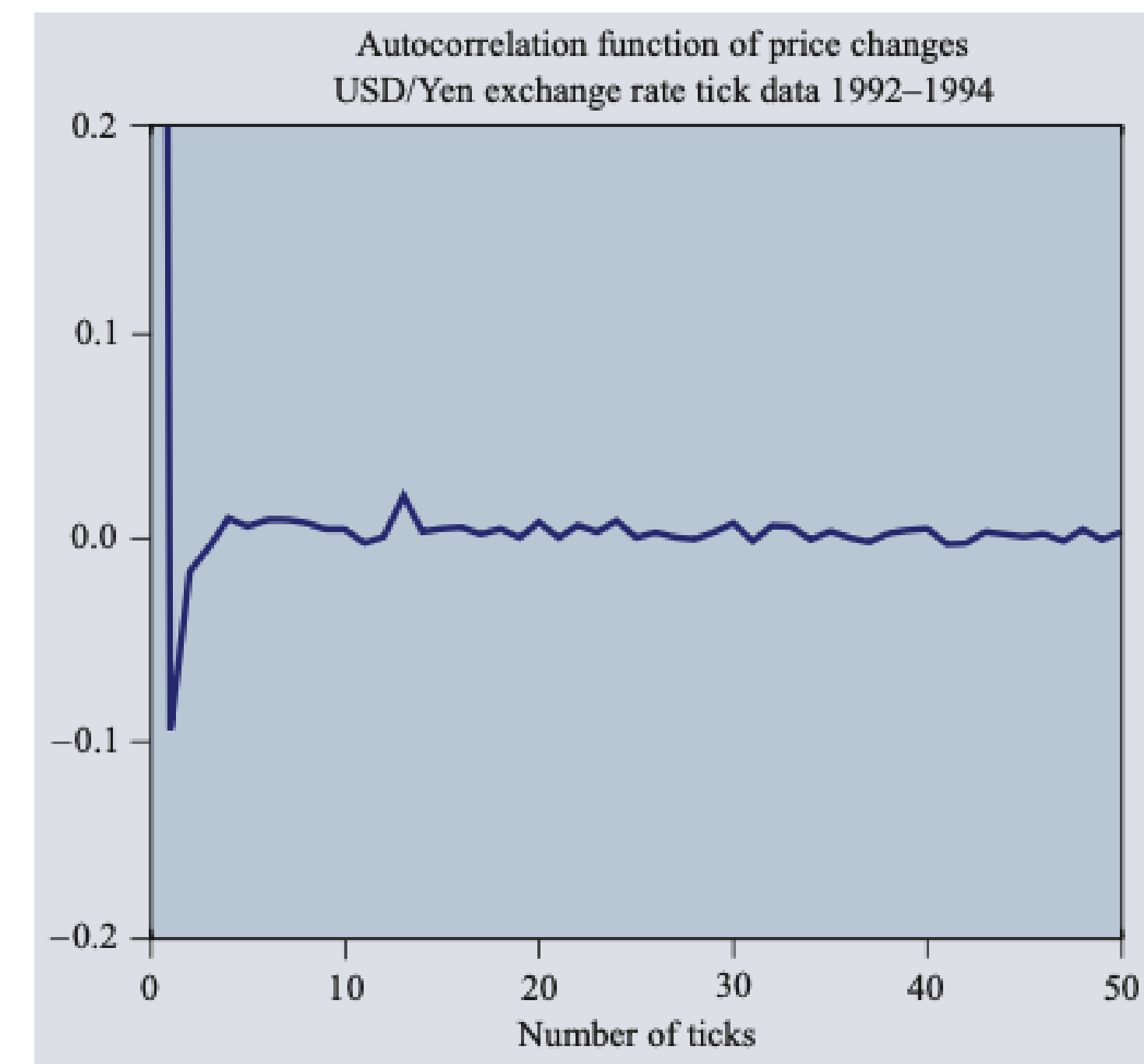
Objective: Document model-free properties common across markets, robust across time, and constraining enough to rule out inadequate models.

Heavy Tails Distribution



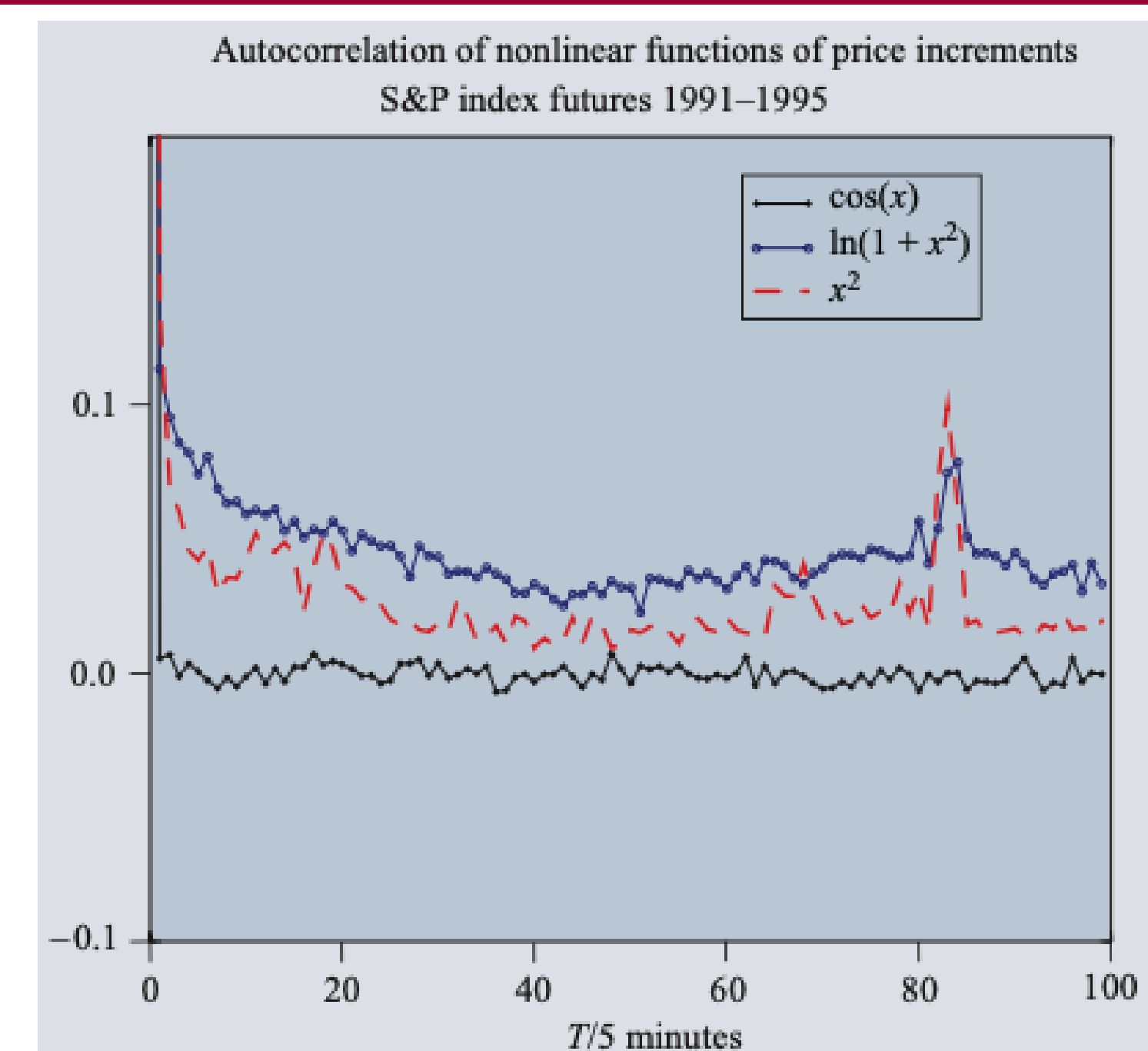
Returns display **power-law tails** with $\alpha \in [2, 5]$. Excludes both stable laws and normal distribution. Consistently heavier than Gaussian.

Linear Autocorrelations



Returns are **insignificant** except at very short scales (<20 min). Supports efficient markets but does NOT imply independence.

Nonlinear Correlations



Absolute/squared returns show strong persistence over days/weeks.

The 11 Stylized Facts

1. Absence of autocorrelations
2. Heavy tails
3. Gain/loss asymmetry
4. Aggregational Gaussianity
5. Intermittency
6. Volatility clustering
7. Conditional heavy tails
8. Slow decay of volatility correlations
9. Leverage effect
10. Volume/volatility correlation
11. Asymmetry in time scales

Results & Conclusions

- ▶ Facts emerge **consistently** across markets, instruments, time periods
- ▶ Heavy tails and clustering require **fundamental departure** from classical assumptions
- ▶ Properties qualitatively stable; quantitative estimates challenging
- ▶ Critical for **risk management** and VaR calculations

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