Boolean algebra

Properties

AND

(1) Idempotency :- $x \cdot x = x$, $x + x = x$

(2) Identity → $x + 1 = 1$, $x \cdot 0 = 0$; $x + 0 = x$, $x \cdot 1 = x$

A op B op C

A + B + C

$((A+B)+C)$

$(A+(B+C))$

(3) Commutativity :→ $x + y = y + x$

→ $xy = yx$

(4) Associativity :- $(x+y)+z = x+(y+z)$

→ $(x \cdot y) \cdot z = x \cdot (y \cdot z)$

1 + 0

(5) Complementation → $x + \bar{x} = 1$

$x \cdot \bar{x} = 0$

(6) Distributivity →

→ $x(y+z) = (x \cdot y) + (x \cdot z)$

→ $x + y \cdot z = (x+y)(x+z)$

Proof → $x + x = x$

L.H.S

$(x+x) \cdot 1$

→ $(x+x) \cdot (x+\bar{x})$

$x + y$, $x + z$ → $x + y \cdot z$

→ $x + x \cdot \bar{x}$

→ $x + 0 = x$

Proof $x \cdot x = x$

$(x \cdot x) + 0$

$(x \cdot x) + (x\bar{x})$

$x(x + \bar{x})$

$x \cdot 1 = x$

$x\bar{x} = 0$

$x + \bar{x} = 1$

Duality Principal

OR → $\cdot$ ↓ +, + ↓ $\cdot$, 1 ↓ 0

AND

Proof $x + 1 = 1$

Proof $x+1=1$

AND +

$(x+1)\cdot 1$

$\Rightarrow (x+1)(x+\bar{x})$

$\Rightarrow 1(x+\bar{x})$

$\Rightarrow 1$

Absorption:

$\hookrightarrow x + x\cdot y \Rightarrow x$

$A + A\cdot B = A$

$C + C\cdot A \Rightarrow C$

$x\cdot 1 + x\cdot y$

$\Rightarrow x(1+y)$

$\Rightarrow x\cdot 1$

$\Rightarrow x$

$\Rightarrow x\cdot(x+y) = x$

$A\cdot(A+B) \Rightarrow A$

$C\cdot(C+B) = C$

$\Rightarrow x\cdot x + xy \Rightarrow x$

$\Rightarrow x + xy \Rightarrow x$

De Morgan's Law

$(\overline{x+y}) \Rightarrow \bar{x}\cdot\bar{y}$

$(\overline{x\cdot y}) \Rightarrow \bar{x}+\bar{y}$

Variable $\Rightarrow$ $A B C$, $x y z$

Literals $\Rightarrow$

$x + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

$\Rightarrow 7$

$(xy)(\bar{x}+\bar{y}) \Rightarrow 0$

$(xy)(\overline{xy}) \Rightarrow 0$

$(xy)+(\bar{x}+\bar{y}) = 1$

$(xy)+(\overline{xy})$

$\Rightarrow 1$

Consensus theorem

$xy + \bar{x}z$

$xy + \bar{x}z + yz = \ldots$

$xyp + \bar{x}\bar{y}q + pq \Rightarrow xyp + \overline{xy}q$

$\Rightarrow xyp + (\bar{x} + \bar{y})q$

$xy + \bar{x}z + yz = xy + \bar{x}z$

$(x + \bar{x}) = 1$

$\Rightarrow xy + \bar{x}z + yz(1)$

$\Rightarrow xy + \bar{x}z + yzx + yz\bar{x}$

$(1 + z) = 1$

$(1 + y) \Rightarrow 1$

$\Rightarrow xy(1 + z) + \bar{x}z(1 + y)$

$\Rightarrow xy + \bar{x}z$ Ans

duality Principal

→ literal Same

→ AND OR ↔ interchange

→ Constant (1, 0) ⇒ interchange

$(a + b) = (b + a) \Rightarrow a \cdot b = b \cdot a$

$a + \bar{a} \Rightarrow 1 \Rightarrow a \cdot \bar{a} \Rightarrow 0$

duality ↔ Complement

→ literal Same

→ Interchangable NOT, OR, 1, 0

→

$\bar{x} + \bar{y} \Rightarrow$ Complement $\Rightarrow \overline{\bar{x} + \bar{y}} \Rightarrow \bar{\bar{x}} \cdot \bar{\bar{y}} \Rightarrow x \cdot y$ Dual

Dual $\Rightarrow \bar{x} \cdot \bar{y}$

Complement $\Rightarrow \overline{\bar{x} \cdot \bar{y}} \Rightarrow \bar{\bar{x}} + \bar{\bar{y}}$

$\Rightarrow x + y$

$x + y$

Same

Q → $\overline{xz} + x\bar{z} + yz$

$\Rightarrow (\bar{x}+\bar{z}) + x\bar{z} + yz$

$\Rightarrow \bar{x}+\bar{z}+x\bar{z}+yz$

$\Rightarrow \bar{z}(1+x)+\bar{x}+yz$

$\Rightarrow \bar{z}+\bar{x}+yz$

$\Rightarrow (\bar{z}+y)(\bar{z}+z)+\bar{x}$

$\Rightarrow \bar{z}+y+\bar{x}$ Ans

Q $x(\overline{\bar{y}+x\bar{z}}) \Rightarrow x[\bar{\bar{y}}\cdot\overline{x\bar{z}}]$

$\Rightarrow x[y\cdot(\bar{x}+\bar{\bar{z}})]$

$\Rightarrow x[y\cdot(\bar{x}+z)]$

$\Rightarrow x[y\bar{x}+yz]$

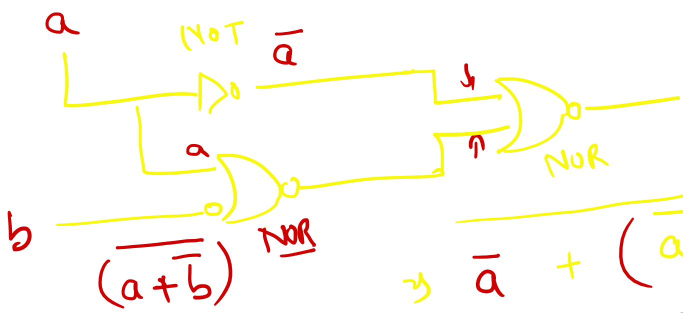
$\bar{x}\cdot x = 0$ $\Rightarrow xyz$ Ans

Q $\overline{xy}(x+y) \Rightarrow (\bar{x}+\bar{y})[x+y]$

$\Rightarrow \bar{x}\cdot x+\bar{x}y+\bar{y}x+\bar{y}y$ $\bar{x}\cdot x=0$

$\Rightarrow \bar{x}y+\bar{y}x$ Ans $\bar{y}\cdot y=0$

Q



$\Rightarrow \overline{\bar{a}+\overline{(a+\bar{b})}}$

$\Rightarrow \bar{\bar{a}}\cdot\overline{\overline{(a+\bar{b})}}$

$\Rightarrow a\cdot(a+\bar{b})$

$\Rightarrow a+a\bar{b}$

$\Rightarrow a$ Ans

Q

$\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}c + abc$

$\bar{c} + c = 1$

$\Rightarrow \bar{a}\bar{b}(\bar{c} + c) + a\bar{b}c + abc$

$\Rightarrow \bar{a}\bar{b} + ac(\bar{b} + b)$

$\Rightarrow \bar{a}\bar{b} + ac(1)$

$\Rightarrow \bar{a}\bar{b} + ac$ Ans