Base b = (b-1)'s Complement ← diminished Complement (r-1)'s
" b's " ← radix Complement (r's)

Base 2
- 1's Complement → $(2^n - 1) - x$
- 2's Complement → $(2^n - x)$ ← $(2^n - 1) - x + 1$

2^n → 1111
$-x$ → 1001
1's Comp → 0110
+1
2's Comp → 0111

Base 3
- 2's Complement → $(3^n - 1) - x$
- 3's Complement → $3^n - x$

Base 10
- 9's Complement → $(10^n - 1) - x$
- 10's Complement → $(10^n - x)$

Base 16
- 15's Complement → $(16^n - 1) - x$
- 16's Complement → $(16^n - x)$

(146)
9's
999
−146
853 dm
+1
10's
854 dm

A→10
B→11
C→12
D→13
E→14
F→15

(AB2)
15's → FFF − AB2 = 54D dm
16's → 54D + 1 = 54E dm

dm

example base 2 $(2^n - 1) - x$

1111
→ 1001
1's → 0110

example base 3
$x \rightarrow (121)_3$ (26)
2's → $(3^n - 1) - x$
→ 222 − 121 = 101 dm
2's → 101 + 1 = 102 dm

$x \rightarrow 10100100$
2's → 01011100

2's $x \rightarrow 101011$ ← LSB
1's → 010100 + 1
2's → 010101
2's → 010101
← 1's 1

Subtraction in (r-1)'s Complement

example Base 10 → $x - y$ → $72 - 23$ → 49 (9's)

→ $72 + (-23)$

9's: 99 − 23 = 76

Step 1 → 72 + 76

Carry discard → 148

Add +1 → 148 +1 (carry)

→ 49 ans

148 +1

Base 10: $x - y$ → $24 - 97$ → -73

→ $x + (-y)$

→ $24 + (-97)$

→ 24 + 02 → Carry X → 9's Complement

9's: 99 − 97 = 02

→ 26 ans (9's)

→ -73

99 − 26 = 73

Base 10: $x - y$ → $82 - 32$

→ $x + (-y)$ → 9's complement

→ $x + \bar{y}$ → $(10^2 - 1) - y$

→ $x + 10^2 - 1 - y$

→ $99 + (x - y) - 100$

→ $-1 + (x - y) + 1$

$82 - 32$ → $82 + (-32)$

$82 + 67$

149 (9's: 99 − 32 = 67)

+1

50 ans

1 → 100

$(x-y)$

$x - y$

$46 - 92$

$\Rightarrow x + (-92)$

$\Rightarrow x + \bar{y}$

$\Rightarrow x + [(10^n - 1) - y]$

$\Rightarrow x + (99 - y)$

$\Rightarrow 99 + (x - y)$

$\Rightarrow (10^n - 1) - [99 + (x - y)]$

$\Rightarrow 99 - 99 - (x - y)$

$\Rightarrow (x - y) \Rightarrow -53$ Ans

53

$(10^n - 1) - 53$

$46 - 92 \Rightarrow 46 + (-92)$

$46 + 07$

$\Rightarrow 46 + 07 = 53$

99
92
07

1

Carry

Example

base 2

Solve 1's

(r-1)'s

$101011 \leftarrow x$

$010110 \leftarrow y$

$\bar{y} \Rightarrow 101001$

$(x - y)$

$x + (-y)$

$x + \bar{y}$

binary

$x + \bar{y} =$ $\overset{1\,1}{1\,0\,1\,0\,1\,1}$ + $1\,0\,1\,0\,0\,1$ = $\overset{1}{0}\,1\,0\,1\,0\,0$

Carry discard

$+1$

010101 Ans

$\bar{y} = (2^n - 1) - y$

$x + (2^n - 1) - y$

From

$\Rightarrow (2^n - 1) + (x - y)$

$\Rightarrow (2^6 - 1) + (x - y) - 2^6$

$\Rightarrow x + (x - y) + 1$

$\Rightarrow (x - y)$

Without Carry Example

unsigned No

$x \Rightarrow 010110$

$y \Rightarrow 101011$

$x - y \Rightarrow x + (-y) \Rightarrow x + \bar{y}$

$\overset{1\,1}{0\,1\,0\,1\,1\,0} \leftarrow x \rightarrow x$

$+ \; 0\,1\,0\,1\,0\,0 \leftarrow \bar{y} \rightarrow (2^6 - 1) - y$

$1\,0\,1\,0\,1\,0$

$\Rightarrow x + (2^6 - 1) - y$

$(x - y)$

$\bar{y} = (2^6-1) - y$

$= x + (2^6-1) - y$

$= (2^6-1) + (x-y)$

$= A$

$\bar{A} = (2^6-1) - [A]$

$= 2^6 - 1 - 2^6 + 1 - (x-y)$

$= x + y$

Example base 3 (0,1,2)

2's Complement

$x \to (212)_3$ $\quad x + (-y)$

$y \to (121)_3$ $\quad x + \bar{y}$

1

$x \to 212 \to x$

$\bar{y} \to 101 \to (3^3-1) - y$

$x + \bar{y} \to 1020 \to x + (3^3-1) - y = (3^3-1) + (x-y)$

$= (3^3-1) + (x-y) - 3^3$

$= -1 + (x-y) + 1$

$= (x-y)$

Carry discard $\quad +1$

$21 = (x-y)$

$\bar{y} = 222 - 121 = 101$

Without Carry

$222 - 212 = 010$

$222 - 201 = 021$

Q $\quad x = (121)_3 \quad y = (212)_3$

1

$x \to 121 \leftarrow x$

$\bar{y} \to 010 \leftarrow (3^3-1) - y$

$x + \bar{y} \to 201 \leftarrow x + (3^3-1) - y = \overbrace{(3^3-1) + (x-y)}^{B}$

$(\overline{x+\bar{y}}) = (21)_3$

$= -(21)_3$

$\bar{B} = (3^3-1) - B$

$= (3^3-1) - 3^3 + 1 - (x-y)$

$= -(x+y)$

Substraction using 9's Complement

Example base 10

$246 - 129$

$x + (-y) = x + (\bar{y})$

9's | 1 1

10's | $x \to 246 \to x$

$\bar{y} = 877 \to (10^3) - y$

$= 10^3 + (x-y)$

n's
10's
999
123
876
+1
Add 1" 877 ← 10's

$x \to 246$
$\bar{y} \to 877$
$x + \bar{y} \to 1123$
Carry discard
$\to (123)$ Ans

$\to x + (10^3) - y$
$\to x + 10^3 - y \to 10^3 + (x - y)$
$\to 10^3 + (x-y) - 10^3$
$\to (x-y)$

Example without Carry

$x \quad y$
$\to 123 - 246$

10's
999
246
753
+1
10's → 754

$x \to 123 \leftarrow x$
$\bar{y} \to +754 \leftarrow 10^3 - y \to 10^3 + (x - y)$
$x + \bar{y} \to 877 \leftarrow x + 10^3 - y \to P$
$(x + \bar{y}) \to (123)$

$\bar{P} \to 10^3 - P$
$\to 10^3 - 10^3 - (x-y)$
$\to -(x-y)$

$(x-y) \to -(123)$ Ans

999
877
122
+1
10's → 123

Example base 3
10's complement

Q $(210) - (120)$

$x \quad -y$
$\to x + (-y) \to x + (\bar{y})$

$\bar{y} \to 222$
120
+102
+1
110

$x \to 210$
$\bar{y} \to 110$
$x + \bar{y} \to 1020$
Carry
$\to (20)_3$ Ans

$\to x$
$\to 3^n - y \to$
$\to x + 3^3 - y \to 3^3 + (x-y) - 3^3$
$\to (x-y)$

Without Carry
base 3

$\to 120 - 210$
$\leftarrow x$

Q

Wir... base 3

$\Rightarrow 120 - 210$

$x \Rightarrow 120 \leftarrow x$

$\bar{y} \Rightarrow 20 \leftarrow 3^3 - y$

$(x+\bar{y}) \quad 210 \leftarrow x + 3^3 - y \Rightarrow \overline{3^3 + (x-y)}$

$\bar{a} \Rightarrow 3^3 - a$

$\Rightarrow 3^3 - 3^3 - (x-y)$

$\Rightarrow -(x-y)$

222
2 10
012
+1
20

$(\overline{x+\bar{y}}) \Rightarrow (20) \quad (4) \Rightarrow (11)$

$(x-y) \Rightarrow (-20)$ Ans

222
− 210
012
+1
20

Base 2

Carry

2's

$x \Rightarrow 1010110 \quad y \Rightarrow 0101010$

1 11

$x \Rightarrow 1010110 \leftarrow x$

$\bar{y} \Rightarrow 1010110 \leftarrow 2^7 - y$

$x + \bar{y} \quad 10101100 \leftarrow x + (2^7 - y) \Rightarrow 2^7 + (x-y)$

$\Rightarrow 2^7 + (x-y) - 2^7$

$\Rightarrow (x-y)$

Carry discard

(0101100) Ans

Carry bar 2

Example

$x \Rightarrow 0101010 \quad y \Rightarrow 1010110 \Rightarrow 2's$

$x = 0101010 \leftarrow x$

$\bar{y} \Rightarrow 0101010 \leftarrow 2^7 - y$

$(x+\bar{y}) \quad 1010100 \Rightarrow x + 2^7 - y \Rightarrow \overline{2^7 + (x-y)} = c$

$(\overline{x+\bar{y}}) \Rightarrow 0101100$ Ans

$c \Rightarrow 2^7 - c$

$\Rightarrow 2^7 - 2^7 - (x-y)$

$\Rightarrow -(x-y)$

$x - y \Rightarrow (-0101100)$