Homework W2

Directions: Follow the proof format from class. Explain exactly which assumptions and facts you are using, and explain why each fact implies the next as clearly as possible. Feel free to use any of the facts proved in class or a previous assignment. See textbook section 3.2 for more details about Proofs and 5.2 for Number Theory.

1. Disprove each statement with a counterexample:
   (a) For all $n \in \mathbb{N}$, $2n^2 + 5$ is prime
   (b) Every natural number is either prime or composite
   (c) Let $n, a, b \in \mathbb{N}$. If $n \mid ab$ then $n \mid a$ or $n \mid b$
   (d) If x, y are both irrational, so is xy
2. Show that if p is a prime number, then $p + 7$ is composite.
3. Show that if x, y are both rational, so is xy
4. Let $a, b, n \in \mathbb{Z}$. Suppose that $n \mid a$ and $n \mid b$. Show that $n \mid (a + b)$. Is the converse True?
5. Prove that $\sqrt{5}$ is irrational.
6. Prove that if $a, b, c \in \mathbb{Z}$ and $a^2 + b^2 = c^2$ that a is even or b is even.