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3D involute gear evaluation – Part I: Workpiece coordinates

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ABSTRACT

The definition of 3D workpiece coordinates for cylindrical involute gears, together with their formal relation to conventional cross sections, are prerequisites for an unambiguous and thus reliable evaluation of 3D measurement data. Both will be described in this article, as no corresponding information is currently available in national or international standards and guidelines. The definitions and calculation methods presented here are part of reference algorithms used by the Physikalisch-Technische Bundesanstalt (PTB), the national metrology institute of Germany, to certify gear evaluation software.

1. Introduction

The most important measurands for cylindrical involute gears are traditionally evaluated in two characteristic cross sections: a transverse plane spanned perpendicular to the gear axis and a cylindrical surface that runs concentrically to the gear axis (see Fig. 1). Focus lies on profile, helix, pitch and tooth thickness parameters. Their associated measurands describe dimensional and form errors on gears. They are described in numerous works standards as well as in higher-order national and international standards and guidelines [1–4]. Knowledge of the flank deviations allows fabrication processes to be reliably controlled. In addition, the gears can be assigned to standardized quality classes.

As in the case of the production machines, kinematics of traditional gear-measuring instruments are also exclusively based on the generation principle of the involute. In this way, the measurands searched for can be determined rapidly and with high precision in the given evaluation cross sections. They allow direct conclusions to be drawn on the gearing properties and direct correction parameters to be provided for production processes.

Compared with traditional gear-measuring instruments, modern gear-measuring devices determine the measurement points on involute cylindrical gears as three-dimensional coordinates. Evaluations of the most important measurands, however, are still related to the two conventional evaluation cross sections. This is not a contradiction, but allows further use to be made of the existing evaluations and operational procedures without changes.

Despite a constant increase in the accuracy of measuring instruments, modern CNC-based metrology entails considerable risks compared with mechanical metrology due to the recording and calculation of the measurement points. As special measuring instruments, the mechanical gear-measuring instruments were, for example, designed to record the measurement points as exactly as possible in the given evaluation cross sections. In contrast to this, CNC-measuring devices run in a control loop during the measuring process. As a result, the specified nominal tracks are slightly left. Therefore, it is necessary to determine the measurement points as 3D measurement points and to correct the positional errors computationally. It is at this point that the difficulties start. The existing standards and guidelines [1–4] do not include a comprehensive modeling of the gears. Consequently, definitions and clear instructions required for handling a gear as a 3D object are also lacking.

An elementary approach to 3D descriptions of involute gears was presented in [5]. In 1996, Lotze [6] was the first to introduce a holistic 3D description of involute gears to be treated as regular and freeform geometries in modern coordinate measuring technology. Detailed mathematical formulas can be found in [7]. Independently, Goch proposed an alternative approach that also considered flank modifications. His results have been published in [8–10].

The basis required for the completion of existing standards and guidelines is presented in this two-part paper. In Part I, general definitions and coordinate systems are introduced and calculation methods are stated that allow 3D coordinates to be unambiguously and completely transferred to conventional evaluation cross sections. In addition, it is shown that introducing an involute coordinate system makes evaluations especially easy to perform.

In Part II, the individual evaluations of gear deviations will be described in detail. Among other things, it will be shown that the

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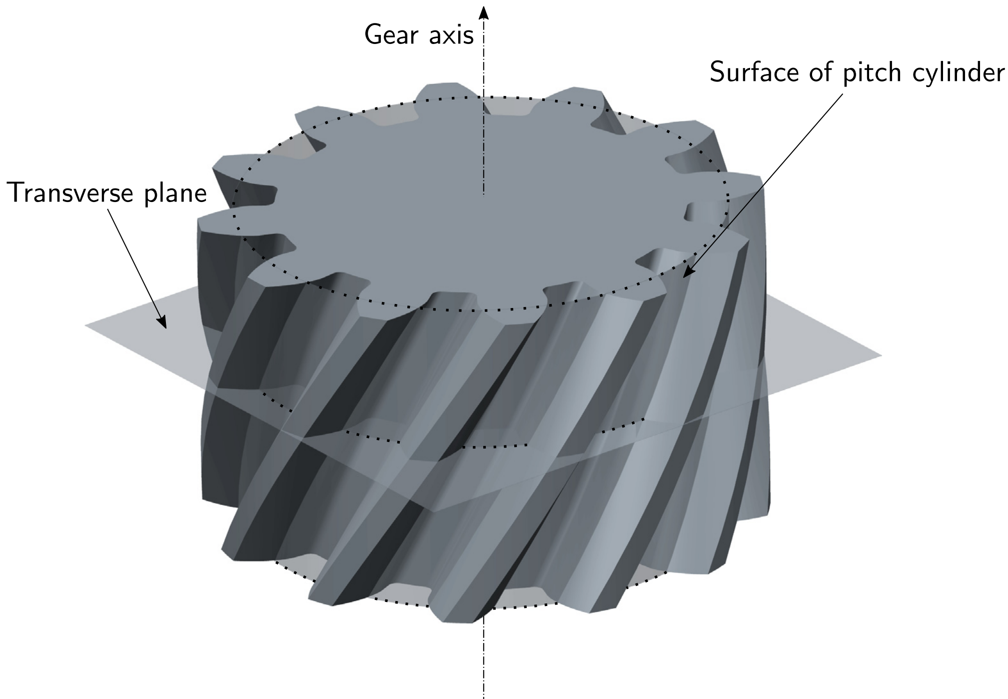

Fig. 1. Cross sections for evaluations of cylindrical involute gears.

profile, helix and pitch deviations correlate with one another if the gear is described in one workpiece coordinate system. These correlations are frequently unknown, as they do not have to be taken into account for mechanical gear-measuring instruments, which have a flank-related measuring and evaluation strategy.

A comparison between various gear manufactures of gear-measuring devices has shown that the evaluation of pure 3D data may lead to large deviations in the measurement results [11]. Such deviations may even be far above the measurement accuracy of the gear-measuring instruments.

2. Nomenclature

Table 1
Terms and definitions.

Symbol	Name	Range	Unit
b	Facewidth	$\in \mathbb{R}^+$	mm
c	Helix coefficient	$\in \mathbb{R}_0^+$	–
`flank`	Tooth flank direction	$\begin{cases} -1 & : \text{left} \\ +1 & : \text{right} \end{cases}$	–
`hand`	Slope direction of tooth	$\begin{cases} -1 & : \text{left} \\ 0 & : \text{spur} \\ +1 & : \text{right} \end{cases}$	–
i	Tooth number	$\in \mathbb{N}^+$	–
$\mathrm{inv}\alpha$	Involute α	$\in \mathbb{R}_0^+$	rad
m_n	Normal module	$\in \mathbb{R}^+$	mm
n	Number of teeth	$\in \mathbb{N}^+$	–
$\vec{n}$	Unit normal vector on involute surface	$\in \mathbb{R}^3$	–
$\vec{n}^*$	Normal vector on involute surface	$\in \mathbb{R}^3$	–
r	Radius	$\in \mathbb{R}^+$	mm
r_b	Base radius	$\in \mathbb{R}^+$	mm
r_s	Radius of stylus tip	$\in \mathbb{R}^+$	mm
r_0	Radius of pitch circle	$\in \mathbb{R}^+$	mm
s_{t0}	Tooth thickness at pitch circle in transverse section	$\in \mathbb{R}^+$	mm
`type`	Type of gear	$\begin{cases} -1 & : \text{internal} \\ +1 & : \text{external} \end{cases}$	–
`x`	Profile shift coefficient	$\in \mathbb{R}$	–
$\vec{x}$	Vector on involute surface	$\in \mathbb{R}^3$	–
x	x-coordinate on involute surface	$\in \mathbb{R}$	mm
x_s	x-coordinate of stylus tip centre	$\in \mathbb{R}$	mm
y	y-coordinate on involute surface	$\in \mathbb{R}$	mm
y_s	y-coordinate of stylus tip centre	$\in \mathbb{R}$	mm
z	z-coordinate on involute surface	$\in \mathbb{R}_0^+$	mm
z_s	z-coordinate of stylus tip centre	$\in \mathbb{R}$	mm
α_t	Transverse pressure angle	$\in (0, \pi]$	rad
α_{n0}	Normal pressure angle at pitch circle	$\in (0, \pi]$	rad
α_{t0}	Transverse pressure angle at pitch circle	$\in (0, \pi]$	rad
β_b	Helix angle at base circle	$\in [0, \pi/2)$	rad
β_0	Helix angle at pitch circle	$\in [0, \pi/2)$	rad
φ_b	Position of involute at base circle in reference plane	$\in [0, 2\pi)$	rad

3. Basic equations

The basic geometry of involute cylindrical gears can be completely described in different parameters dependent on their specific application. Designers usually use the normal module m_n, the number of teeth n, the normal pressure angle α_{n0}, the helix angle β_0, the face width b and the profile shift coefficient x, if applied. However, the physical behavior of the gear kinematics can be better described by its generative parameters, namely, the base radius r_b, the position of the involute at the base circle in the reference plane φ_b and the helix coefficient c. The most important relations between the parameters listed above are described by Eqs. (1)–(6). For the evaluation of measurement data, generative parameters are preferred.

The shape of the involute profile is described by the radius r_b of the base circle.

$$r_b = \frac{1}{2} \cdot \frac{n \cdot m_n}{\sqrt{\tan^2 \alpha_{n0} + \cos^2 \beta_0}} \tag{1}$$

Often, the involute is described by the following involute function inv α_t:

$$\mathrm{inv}\ \alpha_t = \tan \alpha_t - \alpha_t \tag{2}$$

In this equation α_t is called the transverse pressure angle. It represents the gear radius r and is calculated by

$$\alpha_t = \arccos\left(\frac{r_b}{r}\right). \tag{3}$$

The value of the helical slope is described by its radius-independent helix coefficient c:

$$c = \frac{\tan \beta_b}{r_b}. \tag{4}$$

Here, β_b is the helix angle at the base circle. It is given by

$$\beta_b = \arccos\left(\cos \alpha_{n0} \cdot \sqrt{\tan^2 \alpha_{n0} + \cos^2 \beta_0}\right). \tag{5}$$

The pitch radius r_0 is calculated as follows:

$$r_0 = \frac{1}{2} \cdot \frac{n \cdot m_n}{\cos \beta_0} \tag{6}$$

4. Gear coordinate systems

Depending on the application, it is useful to describe involute gears in either Cartesian or involute coordinates. Both systems allow involute gears to be represented as 3D objects. The Cartesian

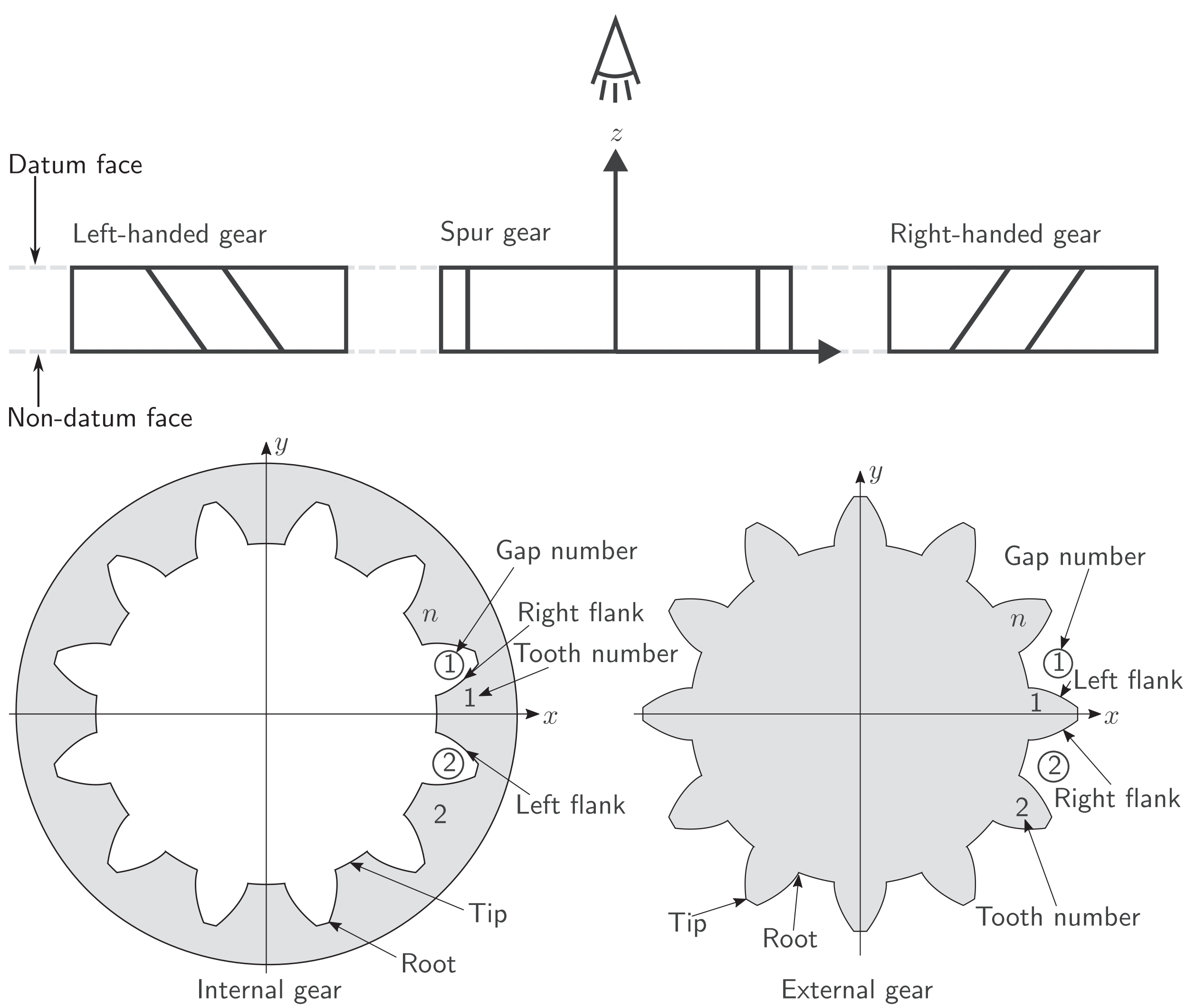

Fig. 2. Workpiece coordinates of internal and external gears.

coordinates are preferred when numerical data representation of the volumetric object is required – for example during the design phase and for generating software for production and measuring machines. In contrast to this, involute coordinates are preferred when analyzing measurement data. Deviations of the involute surface can easily be handled in linear relations, which will be discussed in Part II of this paper. Moreover, information on the kinematic behavior of the gear can be derived directly.

Although these two alternative coordinate systems have been used for several years, a unique convention for the definition of the position of the coordinate origin does currently not exist. This section aims to address this shortcoming.

4.1. Definition of coordinate systems

The proposed definition of the workpiece coordinate system can be seen in Fig. 2. It considers common standards and guidelines [1–4] that traditionally represent gear data in specific cross sections (Fig. 1). Important and so far not defined requirements that allow a gear to be described completely as a 3D object are added in this paper. The location of the origin of the coordinate systems is one of these new basic conventions. Fundamentally, all gear z-coordinates are described in positive numbers. Consequently, the x-y plane is located in the non-datum face of the gear; the z-axis and the gear axis are the same. The x-axis is located in the middle of the first tooth.

4.2. Cartesian coordinates

The Cartesian coordinates are described by their x-, y- and z-components. Coordinate systems that represent the gears in polar, cylindrical or other coordinates are not discussed in this paper. They can easily be transformed into Cartesian coordinates.

4.3. Involute coordinates

Involute gears are described by the coordinates φ_b, r and z. The angle φ_b represents the position of a flank; it is directed at the involute origin on the base circle in the x-y plane at $z = 0$. r is the radius that points from the gear axis to a point on the flank surface, c is a radius-independent parameter of the helix angle and z is the coordinate along the gear axis.

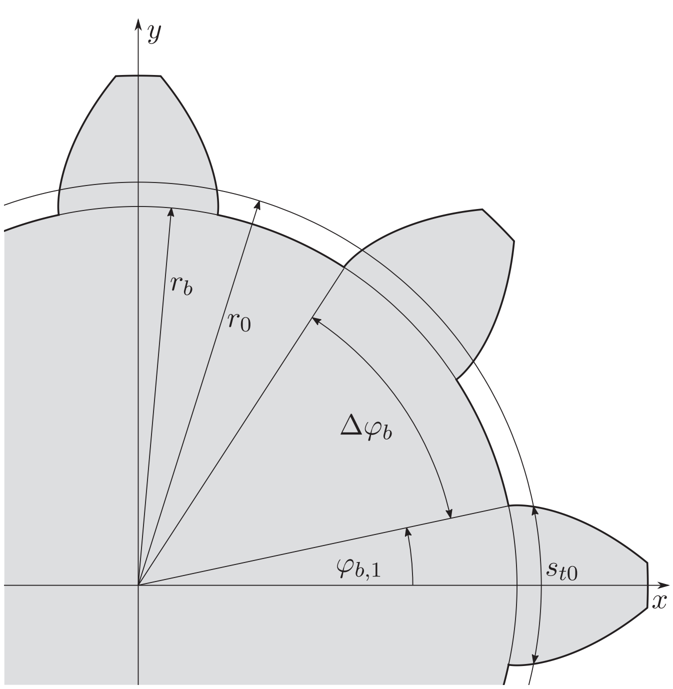

Fig. 3. Definition of tooth position.

A complete description of a gear coordinate system comprises the definition of the gear axis and its coordinate origin as well as an unambiguous definition of the position of each single tooth (Fig. 3). Both are estimated in different ways. Usually, the definition of the gear axis (z-axis) or the position of the x-y plane is given from the design data or has to be estimated by measuring the reference surfaces of the gear. Alternative references that lead to similar results exist as well. None of these shall be discussed in this paper, as they are already well known. However, it is important to consider the definition provided in Section 4.1 regardless of the method chosen.

Of great importance for handling involute coordinates is the estimation of the position of the teeth, which depends only on the parameter φ_b. Attention must be paid on the calculation of φ_b, as two different cases have to be distinguished: the estimation of the position of the first tooth and the estimation of the position of all the other teeth.

The calculation for the first tooth position $\varphi_{b,1}$ considers the definition of the coordinate system's origin, the tooth thickness at pitch circle s_{t0} as well as `flank` and `type` of the gear. $\varphi_{b,1}$ is calculated by means of the Eqs. (7), (9) and (10):

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = r_0 \begin{pmatrix} \cos\left(\mathtt{type} \cdot \mathtt{flank} \cdot s_{t0}/(2r_0)\right) \\ \sin\left(\mathtt{type} \cdot \mathtt{flank} \cdot s_{t0}/(2r_0)\right) \\ 0 \end{pmatrix} \tag{7}$$

The profile shift of the gear influences only the tooth thickness, which is completely considered in Eq. (8).

$$s_{t0} = \frac{r_0 \pi}{n} + 2 \cdot \mathrm{x} \cdot m_n \cdot \tan \alpha_{t0}. \tag{8}$$

For $y \geqslant 0$, we have

$$\varphi_{b,1} = -\mathtt{flank} \cdot \frac{r_0\sqrt{1-(r_b/r_0)^2}}{r_b} + \mathtt{flank} \cdot \arccos\left(\frac{r_b}{r_0}\right) + \arccos\left(\frac{x}{r_0}\right), \tag{9}$$

whereas, for $y < 0$, we obtain

$$\varphi_{b,1} = -\mathtt{flank} \cdot \frac{r_0\sqrt{1-(r_b/r_0)^2}}{r_b} + \mathtt{flank} \cdot \arccos\left(\frac{r_b}{r_0}\right) - \arccos\left(\frac{x}{r_0}\right). \tag{10}$$

The calculation of nominal positions of the involute at the base circle $\varphi_{b,i}$ for all the other teeth depends on the nominal position $\varphi_{b,1}, \mathtt{type}, \mathtt{flank}$, number of teeth n and the gear's nominal pitch. It is described by

$$\varphi_{b,i} = -\left(\mathtt{type} \cdot \mathtt{flank} \cdot \varphi_{b,1} + (i-1)\Delta\varphi_b\right), \tag{11}$$

where $\Delta\varphi_b$ is the angle of one nominal pitch, and is calculated by

$$\Delta\varphi_b = \frac{2\pi}{n}. \tag{12}$$

The values of φ_b are defined to be within the interval $[0, 2\pi)$. If the results of Eqs. (9), (10) or (11) become negative, φ_b has to be corrected according to

$$\varphi_b \mapsto \varphi_b \bmod 2\pi. \tag{13}$$

4.4. Cartesian coordinates expressed by means of involute coordinates

Cartesian coordinates and the normal vector of the involute gear's surface can be expressed by means of involute coordinates, as described in Eqs. (14)–(17):

$$\vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} r \cdot \cos\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \mathrm{inv}\alpha_t\right) \\ r \cdot \sin\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \mathrm{inv}\alpha_t\right) \\ z \end{pmatrix} \tag{14}$$

Note that $r = r_b / \cos(\alpha_t)$ depends on the variable parameter α_t.

The normal vector $\vec{n}^*$ at this point $\vec{x}$ is calculated by

$$\vec{n}^* = \mathtt{type} \cdot \mathtt{flank} \cdot \frac{\partial \vec{x}}{\partial \alpha_t} \times \frac{\partial \vec{x}}{\partial z}. \tag{15}$$

Using Eqs. (3) and (4) results in

$$\vec{n}^* = \mathtt{type} \cdot \mathtt{flank} \cdot r_b \cdot \frac{\tan\alpha_t}{\cos^2\alpha_t} \begin{pmatrix} \sin\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \tan\alpha_t\right) \\ -\cos\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \tan\alpha_t\right) \\ \tan(\mathtt{hand} \cdot \beta_b) \end{pmatrix}. \tag{16}$$

Hence, the unit normal vector $\vec{n} = \vec{n}^* / |\vec{n}^*|$ becomes

$$\vec{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix} = \mathtt{type} \cdot \mathtt{flank} \cdot \cos\beta_b \begin{pmatrix} \sin\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \tan\alpha_t\right) \\ -\cos\left(\varphi_{b,i} + \mathtt{hand} \cdot c \cdot z + \mathtt{flank} \cdot \tan\alpha_t\right) \\ \tan(\mathtt{hand} \cdot \beta_b) \end{pmatrix}. \tag{17}$$

4.5. Involute coordinates expressed by means of Cartesian coordinates

Involute coordinates can be expressed by means of Cartesian coordinates, as described in Eqs. (18)–(20).

The radius r is given by the coordinates of the transversal plane of any z-level:

$$r = \sqrt{x^2 + y^2} \tag{18}$$

The value $\varphi_{b,i}$, which describes the position of the involute origin at the base circle at $z = 0$, is calculated by means of the Cartesian coordinate triple. If the coordinates result from actual data (e.g. measurement data), $\varphi_{b,i}$ may differ for each coordinate triple.

Depending on the quadrant where the x and y coordinates are located, $\varphi_{b,i}$ is expressed by one of the following equations.

For $y \geqslant 0$, we have

$$\varphi_{b,i} = -\mathtt{flank} \cdot \frac{r\sqrt{1 - (r_b/r)^2}}{r_b} + \mathtt{flank} \cdot \arccos\left(\frac{r_b}{r}\right) + \arccos\left(\frac{x}{r}\right) - \mathtt{hand} \cdot c \cdot z, \tag{19}$$

while for $y < 0$, we obtain

$$\varphi_{b,i} = -\mathtt{flank} \cdot \frac{r\sqrt{1 - (r_b/r)^2}}{r_b} + \mathtt{flank} \cdot \arccos\left(\frac{r_b}{r}\right) - \arccos\left(\frac{x}{r}\right) - \mathtt{hand} \cdot c \cdot z. \tag{20}$$

If $\varphi_{b,i}$ becomes negative, it has to be corrected according to

$$\varphi_{b,i} \mapsto \varphi_{b,i} \bmod 2\pi. \tag{21}$$

The z-coordinates in the involute and in the Cartesian coordinate system are identical.

5. Calculation from sphere center point to flank contact point

For the measurement of gears, styli with spherical tips are often used. In this case, the gear-measuring device records the center position of the probe tip instead of the contact point. Tip compensation is required in order to obtain the position of the contact point. The contact point is calculated by means of Eq. (22).

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x_s - r_s \cdot n_x \\ y_s - r_s \cdot n_y \\ z_s - r_s \cdot n_z \end{pmatrix} \tag{22}$$

In this equation, the vector $(x_s, y_s, z_s)^T$ is the position of the sphere center and r_s is the radius of the stylus sphere. The vector $(n_x, n_y, n_z)^T$ is the unit normal vector at the contact point (according to the Eqs. (16) and (17)). Finally, the vector $(x, y, z)^T$ represents the position of the contacting point.

6. Conclusions and outlook

This paper describes the entire unambiguous 3D coordinate system for cylindrical involute gears. This includes various gear types such as internal and external, left- and right-handed helical and spur gears, considering also profile shift. Calculation rules are presented for the evaluation of cylindrical involute gears that allow Cartesian 3D measurement points to be transferred to conventional evaluation cross sections. The definitions presented remedy a lack of information in existing international standards and guidelines. They form an important basis for uniform, and thus reliable, gear evaluation.

The treatment of gear deviations will be discussed in Part II of this article. Both parts are of fundamental importance for cylindrical involute gear evaluation software testing, which will soon be offered via an online validation service provided by the Physikalisch-Technische Bundesanstalt.

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