E.数学专题签到处

问题

给定常数N,共Q次询问。 每次询问给定x,求

$$\sum_{i=1}^{N} \binom{3i}{x}$$

答案对109+7取模

分析

 $1 \le N \le 10^6$ 用阶乘求组合数

$$\binom{n}{m} = \frac{n!}{m!(n-m)!}$$

对于每个询问如果暴力算值,时间复杂度为 O(NQ),直接爆,所以可以预见的是需要推式子

分析

$$\begin{pmatrix} 3i \\ x \end{pmatrix} = \begin{pmatrix} 3i-1 \\ x \end{pmatrix} + \begin{pmatrix} 3i-1 \\ x-1 \end{pmatrix} \\
= \begin{pmatrix} 3i-2 \\ x \end{pmatrix} + 2 \begin{pmatrix} 3i-2 \\ x-1 \end{pmatrix} + \begin{pmatrix} 3i-2 \\ x-2 \end{pmatrix} \\
= \begin{pmatrix} 3i-3 \\ x \end{pmatrix} + 3 \begin{pmatrix} 3i-3 \\ x-1 \end{pmatrix} + 3 \begin{pmatrix} 3i-3 \\ x-2 \end{pmatrix} + \begin{pmatrix} 3i-3 \\ x-3 \end{pmatrix} \quad (x > 3)$$

$$\begin{split} &\sum_{1}^{N} \binom{3i}{x} \\ &= \sum_{1}^{N} \binom{3i-3}{x} + 3\sum_{1}^{N} \binom{3i-3}{x-1} + 3\sum_{1}^{N} \binom{3i-3}{x-2} + \sum_{1}^{N} \binom{3i-3}{x-3} \\ &= \sum_{1}^{N} \binom{3i}{x} + \binom{0}{x} - \binom{3N}{x} \\ &+ 3\left[\sum_{1}^{N} \binom{3i}{x-1} + \binom{0}{x-1} - \binom{3N}{x-1}\right] \\ &+ 3\left[\sum_{1}^{N} \binom{3i}{x-2} + \binom{0}{x-2} - \binom{3N}{x-2}\right] \\ &+ \left[\sum_{1}^{N} \binom{3i}{x-3} + \binom{0}{x-3} - \binom{3N}{x-3}\right] \\ &= \sum_{1}^{N} \binom{3i}{x} + 3\sum_{1}^{N} \binom{3i}{x-1} + 3\sum_{1}^{N} \binom{3i}{x-2} + \sum_{1}^{N} \binom{3i}{x-3} - \binom{3N+3}{x} \quad (x > 3) \end{split}$$

$$\binom{3N+3}{x} = 3\sum_{1}^{N} \binom{3i}{x-1} + 3\sum_{1}^{N} \binom{3i}{x-2} + \sum_{1}^{N} \binom{3i}{x-3}$$
 (1)

$$\binom{3N+3}{x+1} = 3\sum_{1}^{N} \binom{3i}{x} + 3\sum_{1}^{N} \binom{3i}{x-1} + \sum_{1}^{N} \binom{3i}{x-2}$$
 (2)

(2)-(1)得

$$\sum_{1}^{N} {3i \choose x} = \frac{2\sum_{1}^{N} {3i \choose x-2} + \sum_{1}^{N} {3i \choose x-3} + {3N+3 \choose x+1} - {3N+3 \choose x}}{3} \qquad (x > 3)$$

分析

- 根据递推式我们首先求出x=1,2,3的值,就可以一直推下去
- •注:式子会出现负值,记得加上模
- 时间复杂度O(nlogn)