$u_1, \ldots u_n \sim$ uniform $(0,1)$

$$f_{u_{(k)}}(t)=\begin{pmatrix} n \\ k-1,1,n-k \end{pmatrix} t^{k-1}(1-t)^{n-k}, 0 \leq t \leq 1$$

$$f_{u(i)}, u_{(k)}(s,t)=\begin{pmatrix} n \\ i-1,1,k-i-1,n-k \end{pmatrix} s^{i-1}(t-s)^{k-i-1}(1-t)^{n-k}, 0 \leq s \leq t \leq 1$$

(c) $f_{u(1) \ldots u(n)}\left(s_1, s_2, s_n\right)=\begin{pmatrix} n \\ 1,1 \ldots, 1 \end{pmatrix} f\left(s_1\right) \cdots f\left(s_n\right)$

$$=n! \cdot 1 \cdot 1 \cdot 1=n! \quad 0 \leqslant S_1 \leq S_2 \quad S_n \leqslant 1$$

Theorem 0.1. *for any random sample x_1, x_n of cont .r.v.S having cdf F and density f, the order statistics $X_{(k)}, 1 \leq k \leq n$, has density .$f_{x(k)}(t)=\begin{pmatrix} n \\ k-1,1,n-k \end{pmatrix} F(t)^{k-1} f(t)[1-F(t)]^{n-k}$ for all t. and the joint density of $\left(X_{(i)}, X_{(k)}\right)$ is*

Finally, the joint density of $\left(X_{(1)}, X_{(2)} \cdots X_{(0)}\right)$

$$f_{x_{(1)} \ldots x_{(n)}}\left(S_1, S_2, \cdots S_n\right)=n! f\left(S_1\right) \cdot f\left(S_2\right) \cdots f\left(S_n\right)=n! \prod_{i=1}^n f\left(S_i\right), S_1 \leq S_2 \leq \ldots S_k$$

Comment: the prob of any ties is zero, whenever F is continuum dist.

Example 0.2. *How would you find the list of the sample Range?*

$$x_1 \ldots x_n \sim F \quad R_n=x_{(n)}-x_{(1)}$$

(1) $f_{X(1), X_{(n)}}$
(2) $\left(X_{(1)}, X_{(n)}\right) \rightarrow\left(X_{(1)}, R_n\right)$
(3) marginal R_n.

Example 0.3. *(by stems reliability)*
$Y_1, Y_2 \ldots=$ life time of sample electric components (Suppisise indepenonont)
$\sim \operatorname{Exp}($ *mean* $=\theta)$

suppose we organize these in a more complicated system.
$T=$ life time of the system.
(a) Simple series circuit. w/ m component.

$$T=\min \left(Y_1, Y_m\right)$$

$$F_T(t)=P(T \leq t)=1-P(T>t)=1-\left(1-F_y(t)\right)^m=1-e^{-m t / \theta}, t>0$$

(b) Simple parallel circus w/ m component.

$$T=\max \left(Y_1, \ldots Y_m\right)$$

$$F_T(t)=P(T \leq t)=F_Y(t)^m=\left(1-e^{-t / \theta}\right)^m, t>0.$$

(C) Parallel systems in series

m units are in series and each unit has k components in parallel. $Y_{ij} =$ j^{th} comport of i^{th} unit. $T = \min_{1 \leq i \leq m} \left(\max_{1 \leq j \leq n} \left(Y_{ij}\right)\right)$

$$
\begin{aligned}
F_T(t) &= 1 - \left[1 - F_{u(t)}(t)\right]^m \\
&= 1 - \left[1 - \left(1 - e^{-t/\theta}\right)^k\right]^m, t > 0
\end{aligned}
$$