

one

**Example 0.1.**  $y = \#$  of the egg laid,  $x = \#$  of eggs that survive  $y \sim \text{poisson}(\lambda)$ ,  
 $x|y \sim \text{binomial}(Y, P)$

1. pmf directly:  $p(X = x) \rightarrow X \sim \text{poission}(\lambda P)$
2. mgfs:

$$M_x(t) = E(e_{tx}) = E(E(e^{tx}Y)) = E(pe_t + (1-p)^y) = \sum_{y=0}^{\infty} \frac{s^y \lambda^y e^{-y}}{y!} = \frac{e^{-\lambda}}{e^{-s\lambda}} \cdot \sum_{y=0}^{\infty} \frac{(sy)^\lambda}{y!} = e^{s-1} \lambda$$

recall that mgf of  $\text{poisson}(\lambda)$  is  $M_x(t)e^{\lambda(e^t-1)}$ ,  $E(x) = E_y(E(X|Y)) = E(Yp) = \lambda p$

**Definition 0.2.** A r.v.  $X$  is said to have a mixture distribution if the dist of  $x$  depends on a quality that has a dist.

\* In general, hierarchical models led to mixture dist

**Theorem 0.3.** For any two r.v.s  $X$  and  $Y$ ,  $V(Y) = E(V(Y|x)) + V(E(Y|X))$

*Proof.*

$$\begin{aligned} V(Y) &= E(Y^2) - (EY)^2 \\ &= E(E(Y^2|X)) - E(m(x))^2 \\ &= E(E(Y^2|X) - m(x)^2) + E(m(x)^2 - E(m(x)^2)) \\ &= E(V(Y|X)) + V(m(x)) \\ &= E(V(Y|X) + V(E(Y|X))) \end{aligned}$$

□

**Example 0.4.**

$$X|Y \sim \text{Binomial}(Y, P), Y \sim \text{Poisson}$$

$$V(x) = \lambda p$$

$$\begin{aligned} V(x) &= V(E(X|Y)) + E(V(X|Y)) \\ &= V(YP) + E(YP(1-P)) \\ &= P^2\lambda + P(1-P)\lambda \\ &= P^2\lambda + P\lambda - P^2\lambda \\ &= P\lambda \end{aligned}$$

## Covariance and correction

**Definition 0.5.**  $cov(X, Y) = E((x - \mu_x)(Y - \mu_y))$ ,  $corr(X, Y) = \frac{cov(X, Y)}{\sigma_x \sigma_y}$

Basic Fact:

1.  $cov(X, Y) = E(XY) - E(X)E(Y)$
2.  $cov(X, Y) = 0$  if X,Y is i.d.

*Proof.*

$$\begin{aligned} cov(X, Y) &= E((X - \mu_x)(Y - \mu_y)) \\ &= E(XY - \mu_x Y - X\mu_y + \mu_x \mu_y) \\ &= E(XY) - 2\mu_x \mu_y - \mu_x \mu_y \\ &= E(XY) - \mu_x \mu_y \end{aligned}$$

□