Coherence and radiometry*

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Recent researches have revealed that there exists an intimate connection between radiometry and the theory of partial coherence. In this paper a review is presented of some of these developments. After a brief discussion of various models for energy transport in optical fields and of some of the basic concepts of the classical theory of optical coherence, the following topics are discussed: the foundations of radiometry, the coherence properties of Lambertian sources, and the relationship between the state of coherence of a source and the directionality of the light that the source generates. Some very recent work is also described which reveals that certain sources that are spatially highly incoherent in a global sense will generate light that is just as directional as a laser beam.

I. INTRODUCTION

During a period of time that spans several centuries various models have evolved relating to the transport of energy by optical radiation. The oldest and conceptually the simplest one is at the heart of radiometry and of the so-called theory of radiative energy transfer. It is based on the simple notion of a light ray, i.e., of a geometrical trajectory along which radiant energy is assumed to be propagated. Vast areas of both theoretical and practical science concerned with quantitative aspects of optical radiation make use of the radiometric model in a fundamental way. It is employed to analyze problems ranging from very practical questions occurring in illumination engineering, relating, for example, to the distribution of radiant energy from various types of sources or to the performance of different kinds of radiation detectors, to sophisticated questions of astrophysics concerning the interior of stars or the nature of the stellar atmosphere. The strong intuitive appeal of the basic radiometric concepts has been largely responsible for a fact that is seldom noted: namely, that the relationship between radiometry and modern theories of radiation (i.e., Maxwell's electromagnetic theory and the quantum theory of radiation) has up to now not been clarified.

In the title of this talk I have coupled radiometry with a modern branch of physical optics, namely, coherence theory which, unlike radiometry, employs rather sophisticated concepts of probability theory and of the theory of random processes. You may well ask: "What could radiometry and coherence theory possibly have in common?" Actually, recent researches on the foundation of radiometry have revealed that there is a very intimate connection between these two fields. Not only have these researches answered some rather puzzling old questions about radiometry, but they have also lead to some interesting developments in coherence theory itself. Moreover, these researches have provided some insight into a question of a considerable practical importance at the present time: they clarify to what an extent radiometric concepts and radiometric laws, which have developed around conventional, rather incoherent thermal sources, also apply to radiation generated by highly coherent sources, such as lasers. Before presenting a review of some of this research let me recall how radiometry describes energy transport and let me also remind you of a few elementary results concerning energy in other theories of radiation.

II. MODELS FOR ENERGY TRANSPORT IN OPTICAL FIELDS

It is a basic assumption of radiometry that energy that is radiated from an element $d\sigma$ of a planar source[1] is distributed according to the elementary law

$$d\mathcal{E}_\nu = B_\nu(\mathbf{r},\mathbf{s}) \cos\theta \, d\sigma \, d\Omega \, dt. \tag{2.1}$$

Here $d\mathcal{E}_\nu$ represents the amount of energy, per unit frequency interval at frequency ν, that is propagated in a short time interval dt from a source element $d\sigma$ at a point Q, specified by position vector $\mathbf{r}$ into an element $d\Omega$ of solid angle around a direction specified by unit vector $\mathbf{s}$. Further, θ denotes the angle that the $\mathbf{s}$ direction makes with the unit normal $\mathbf{n}$ to the source plane (Fig. 1). The proportionality factor $B_\nu(\mathbf{r},\mathbf{s})$, which in general depends on position ($\mathbf{r}$), direction ($\mathbf{s}$), and frequency (ν), is known as the (spectral) *radiance* or *brightness*.

In the theory of radiative energy transfer this simple model for the distribution of radiant energy is somewhat generalized in the sense that the radiance function becomes a field quantity. The surface element $d\sigma$ is no longer restricted to coincide with a radiating surface element of a real source but rather is now an element of any (in general fictitious) surface in a region of space containing radiation, with $\mathbf{n}$ denoting the unit normal to $d\sigma$. Instead of radiance, one now speaks of the specific intensity of radiation, which is often denoted by $K_\nu(\mathbf{r},\mathbf{s})$ or $I_\nu(\mathbf{r},\mathbf{s})$. However, the elementary law (2.1) is assumed to retain its validity with this trivial formal change (radiance → specific intensity). By elementary considerations involving nothing more than simple intuitive quasi-geometrical arguments about energy conservation one is lead

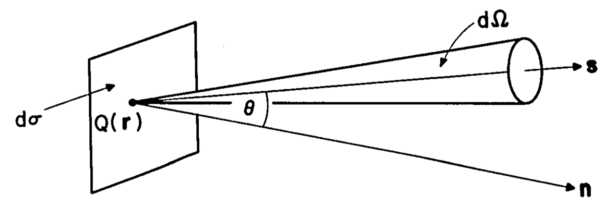

FIG. 1. Illustrating the definition of the radiance function $B(\mathbf{r},\mathbf{s})$.

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to the following integro-differential equation for the propagation of the specific intensity in any isotropic medium[2]:

$$\mathbf{s}\cdot\nabla_r K_\nu(\mathbf{r},\mathbf{s}) = -\alpha_\nu(\mathbf{r},\mathbf{s})K_\nu(\mathbf{r},\mathbf{s}) + \int \beta_\nu(\mathbf{r},\mathbf{s},\mathbf{s}')K_\nu(\mathbf{r},\mathbf{s}')\,d\Omega' + D_\nu(\mathbf{r},\mathbf{s}). \quad (2.2)$$

The left-hand side represents the rate of change of the specific intensity in the direction **s**. The first term on the right represents the rate of decrease of energy due to scattering and absorption, the second term on the right (with the integration extending over the surface of the unit sphere generated by the unit vector **s**′) represents the rate of increase in energy as a result of scattering from all other directions into the **s** direction, and $D_\nu(\mathbf{r},\mathbf{s})$, known as the source function, represents the rate at which energy is emitted due to spontaneous processes. The functions $\alpha_\nu(\mathbf{r},\mathbf{s})$ and $\beta_\nu(\mathbf{r},\mathbf{s},\mathbf{s}')$ are the so-called extinction coefficient and the differential scattering coefficient, respectively.

Equation (2.2) is known as the *equation of radiative transfer* and is one of the basic equations of astrophysics. How well it actually describes energy transport or, even more basically, how the various quantities $(K_\nu,\alpha_\nu,\beta_\nu,D_\nu)$ that appear in this equation are to be interpreted from the standpoint of modern theories of radiation is, to a large extent, an open question. Only in relatively recent times have attempts been made to clarify these questions; however, they have met with rather limited success.

The description of energy transport is quite different within the framework of Maxwell's electromagnetic theory, which is the basis of the whole classical physical optics. Here the quantity that is associated with energy transport is the Poynting vector $\mathbf{S} = (c/4\pi)(\mathbf{E}\times\mathbf{H})$, where **E** and **H** represent the electric and magnetic field vectors and c is the speed of light *in vacuo* (Gaussian system of units being used). It is generally assumed that the optical intensity may be identified with the magnitude of the Poynting vector. We note that there is a qualitative difference between the radiance function of the radiometric model and the optical intensity of physical optics. The radiance and the specific intensity are functions of position (**r**) and of direction (**s**), whereas the optical intensity is independent of direction. Actually, even the dependence of the Poynting vector and hence of the optical intensity on position is somewhat tenuous, because, according to Maxwell's theory, it is not the Poynting vector itself, but rather the integral of its normal component taken over a closed surface that has an unambiguous physical meaning. The integral represents the rate at which energy is transported across the surface. In fact, many paradoxes are known that arise from identifying the Poynting vector too closely with energy transport at a point in the field or because one assumes that the radiant energy "flows" along lines generated by the Poynting vector. Only under special circumstances (e.g., for a plane wave or in the far zone of a radiating source) can such a "hydrodynamic" model be strictly reconciled with electromagnetic theory.

The situation is different yet in the quantum theory of radiation. According to that theory, the radiation field consists of certain nonclassical particles, the photons, each carrying energy of amount $h\nu$, where h is Planck's constant. Although one can also speak of the optical intensity in the quantized field, by defining it via an appropriate operator, it seems more natural to associate energy transport with the photons themselves. However, it is well known that it is not possible to associate a position variable with a photon; in fact, the quantum theory of radiation does not even allow us to speak about the probability of finding a photon at a particular point in space.[3]

We observe that as we proceed from the older models to the more refined ones, less and less can be said about the detailed distribution of energy. According to the radiometric model, energy is localized both in space (**r** dependence of the radiance) and in direction (**s** dependence). Electromagnetic theory says nothing at all about the distribution of energy over directions and what it implies about its distribution throughout space has to be interpreted with caution because of our earlier remarks relating to the physical significance of the Poynting vector. Finally, the quantum theory of radiation tells us nothing at all about the localization of the elementary carriers of energy (the photons) throughout space.

The great physicist H. A. Lorentz in his classic book *The Theory of Electrons*, published in 1909—well before the formulation of the quantum theory of radiation—already warned against treating energy transport by light in a too mechanistic manner. This is what he said (on p. 25): ". . . in general it will not be possible to trace the paths of parts of elements of energy in the same sense in which we can follow in their course the ultimate particles of which matter is made up." And (on p. 26): "It might even be questioned whether in electromagnetic phenomena the transfer of energy really takes place in the way indicated by Poynting's law . . . ."

How is it then possible that the radiometric model, according to which energy is localized both in space and in directions, has been relatively successful—at least in connection with energy transport by light from conventional sources? The usual answer is that this is so because the wavelength of light is very short compared to the linear dimensions of the sources and of the bodies with which the light interacts. In spite of this widely held view no true justification for it has ever been given.

Recent researches have revealed that appreciably more is involved in the foundations of radiometry and of the theory of radiative energy transfer than a short wavelength limit. That this must be so can be seen from the following simple example: Suppose we compare the angular distribution of radiation from a conventional thermal source and from a well-stabilized laser. Under ordinary circumstances the thermal source will radiate in accordance with Lambert's law,

$$J(\theta) = J(0)\cos\theta, \quad (2.3)$$

where $J(\theta)$ denotes the radiant intensity in a direction making an angle θ with the normal **n** to the radiating surface [Fig. 2(a)]. This rather broad angular dependence is exhibited on a polar diagram in Fig. 2(c). On the other hand, the light generated by a laser [Fig. 2(b)] will be extremely directional: Practically all the radiated energy will be concentrated in a very narrow solid angle around the forward direction; typically for a laser with a cross-section diameter 2 mm say, 99% of the radiated energy will be concentrated within a cone of angular divergence of the order of 10^{-4} rad. Thus the polar diagram of the radiant intensity has a needle-like form [Fig. 2(d)]

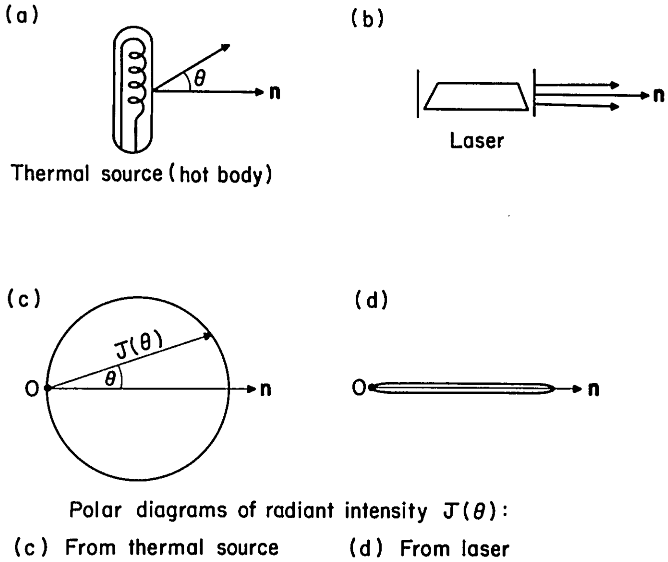

FIG. 2. Comparison of the angular distribution of radiant intensity from a thermal source and from a laser.

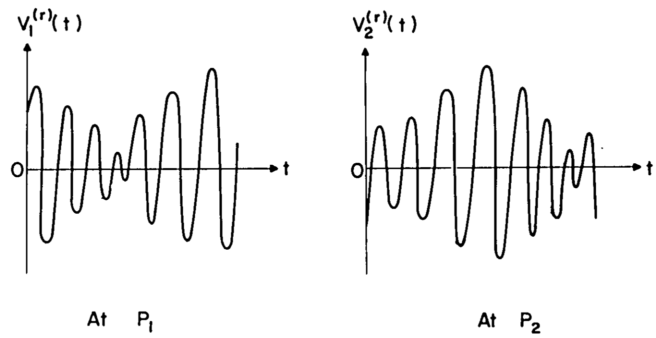

FIG. 3. Illustrating the temporal behavior of the field variable $V^{(r)}(t)$ at two points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$ in space.

which, roughly speaking, resembles the behavior of the Dirac delta function with its singularity at $\theta = 0$.

Now one of the chief differences between the two sources is that the thermal source is spatially almost completely incoherent, whereas the laser source is, of course, spatially highly coherent. Hence the comparison of the angular distribution of radiation from the two sources seems to indicate that there must be a close connection between the state of coherence of a source and the distribution of the radiant intensity of the light that the source generates. In turn, this result implies that there must be an intimate relationship between radiometry and the theory of partial coherence, a supposition that has been fully justified by recent researches. To appreciate this development it might be helpful to review at this point some of the basic concepts of optical coherence theory.

III. SOME BASIC CONCEPTS OF OPTICAL COHERENCE THEORY[4]

Every optical field has some fluctuations associated with it. The fluctuations are usually too rapid to be directly perceived by the eye or to be readily revealed by ordinary laboratory experiments. However, the existence of the fluctuations can be deduced indirectly, for example, from experiments involving interference effects. The concept of partial coherence is intimately related to a measure of statistical similarity—known more technically as correlation—between light fluctuations.

Imagine that we have two very good detectors that allow us to record the detailed time behavior of the optical field at two points P_1 and P_2, specified by position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. The result may be curves such as those indicated in Fig. 3, which show how the real field variable $V^{(r)}(\mathbf{r},t)$ varies at these points in the course of time. For simplicity we consider $V^{(r)}(\mathbf{r},t)$ to be a scalar, e.g., one of the Cartesian components of the electric field at the space-time point $(\mathbf{r},t)$. In the analysis of coherence effects it is convenient to associate with this real field variable a complex one, $V(\mathbf{r},t)$, known as the complex analytic signal. We will not discuss this step here; it is explained in many texts.[5]

The basic quantity used for the analysis of the simplest coherence effects in optical fields is the so-called *mutual coherence function*[6]

$$\Gamma_{12}(\tau) = \langle V_1(t+\tau)V_2^*(t)\rangle, \tag{3.1}$$

where the asterisk denotes the complex conjugate and the sharp brackets denote either a time average or an ensemble average. For the sake of simplicity we use abbreviated notations throughout this section: thus, for example, $V_1(t+\tau)$, is an abbreviation for $V(\mathbf{r}_1,t+\tau)$, etc. In terms of the mutual coherence function $\Gamma_{12}(\tau)$ one defines the so-called *complex degree of coherence* of the light vibrations at the two points P_1 and P_2 as

$$\gamma_{12}(\tau) = \Gamma_{12}(\tau)/\sqrt{I_1}\,\sqrt{I_2}, \tag{3.2}$$

where

$$I_j = \Gamma_{jj}(0) = \langle V_j(t)V_j^*(t)\rangle \qquad (j = 1,2) \tag{3.3}$$

is the (averaged) optical intensity at the two points, as customarily defined in scalar wave theory.[7] It may be shown that the absolute value of the complex degree of coherence obeys the inequality

$$0 \leqslant |\gamma_{12}(\tau)| \leqslant 1 \tag{3.4}$$

for all values of its arguments. The limiting cases $\gamma_{12}(\tau) = 0$ and $|\gamma_{12}(\tau)| = 1$ represent complete (second-order) incoherence and complete (second-order) coherence, respectively, of the vibrations at the two points P_1 and P_2, at times $t+\tau$ and t, respectively. When the two points coincide, the dependence of γ on τ reveals so-called *temporal coherence;* when τ is kept fixed (usually at zero value) the dependence of γ on the location of the two points P_1 and P_2 reveals what is known as *spatial coherence.*

Two well-known interference experiments, illustrated in Figs. 4 and 5, provide quantitative measures of temporal and spatial coherence. The sharpness and the location of the interference fringes in the plane $\mathcal{B}$ of the Michelson interferometer (Fig. 4) give information about the degree of temporal coherence of the light at the dividing mirror M_0 for the time delay τ associated with the relative path difference between the two beams that reach M_0 via the two other mirrors

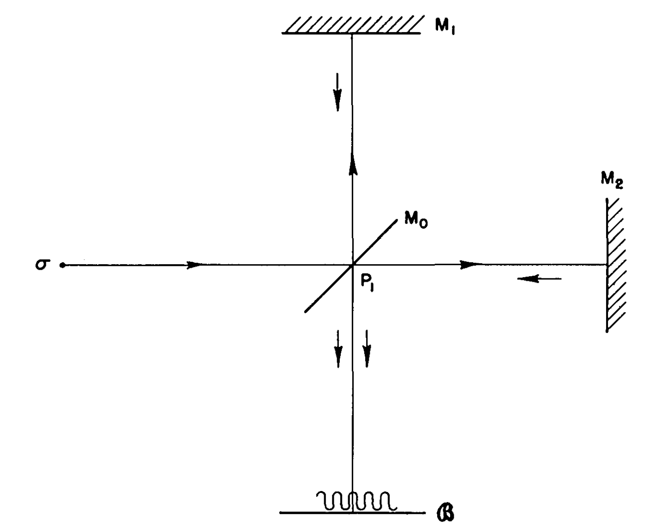

FIG. 4. Temporal coherence illustrated by means of a Michelson interferometer.

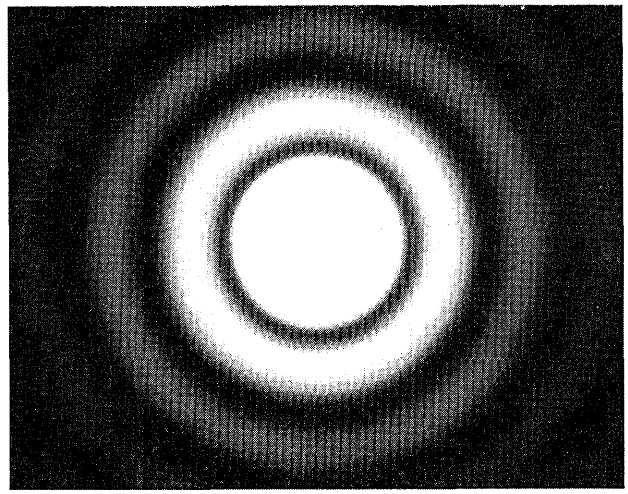

FIG. 6. The diffraction image of a star formed by a well-corrected telescope (courtesy of B. J. Thompson).

M_1 and M_2. The sharpness and the location of the fringes around the symmetrically situated point Q in the plane $\mathscr{B}$ of Young's interference experiment (Fig. 5) provide information about the degree of spatial coherence, for zero time delay ($\tau = 0$) of the light at the pinholes P_1 and P_2.

It seems that phenomena involving temporal coherence of light are generally reasonably well understood, largely, I believe, because they can usually be interpreted in an alternative way, in terms of the spectral properties of the light. Spatial coherence effects, on the other hand, do not appear to be so well understood, and because they play a basic role in some of the recent developments on the foundation of radiometry I will say a few words about them.

It is often asserted that thermal sources are spatially completely incoherent in the sense that no correlations exist between the light vibrations at two distinct points in the source. Actually, as we will soon see, this statement is not correct; in any radiating source the field is always correlated (i.e., spatially coherent), at least over distances of the order of the mean wavelength of the emitted light. However, the usual thermal sources may be regarded as *"globally" incoherent* in the sense that their linear dimensions are very large compared to the linear dimensions of the source region over which appreciable spatial coherence exists. Even though such sources are spatially incoherent in this sense, they may produce fields that will have a high degree of spatial coherence over arbitrarily large regions of space. Before explaining why this is so let me illustrate this fact by a simple example that I am sure is familiar to most of you, though you have probably not interpreted it before from the standpoint of coherence theory.

Suppose that we view a star on a good observing night through a well-corrected telescope. We will then observe in the focal plane of the telescope the diffraction image of the star formed by the telescope. It consists of a bright central disk surrounded by rings of rapidly diminishing intensity (Fig. 6). From the standpoint of coherence theory this image tells us something important about the spatial coherence of the starlight entering the telescope. According to elementary diffraction theory, the image may be considered as being formed by the combined effects of the so-called Huygens' secondary wavelets proceeding to the focal plane from the primary wave falling on the aperture of the telescope (Fig. 7). Now the fact that the diffraction image has pronounced intensity minima and maxima implies that the Huygens' wavelets are capable of strongly interfering with each other—in fact, their interference produces zero field along the dark rings of the diffraction image. This is only possible if the light entering the telescope is strongly spatially coherent across the whole aperture of the telescope. Yet, as we know, the light originated in very many atoms in the star, atoms that to a very high degree of approximation may be considered to have ra-

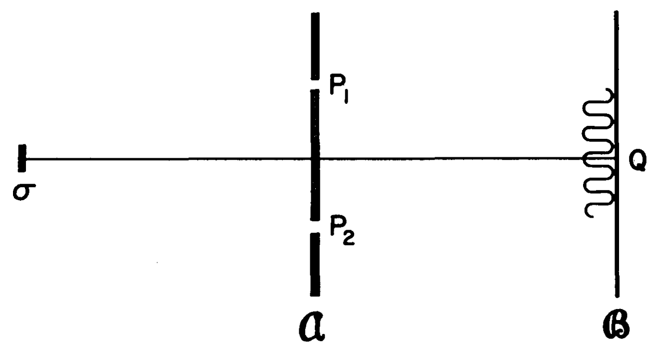

FIG. 5. Spatial coherence illustrated by means of Young's interference experiment.

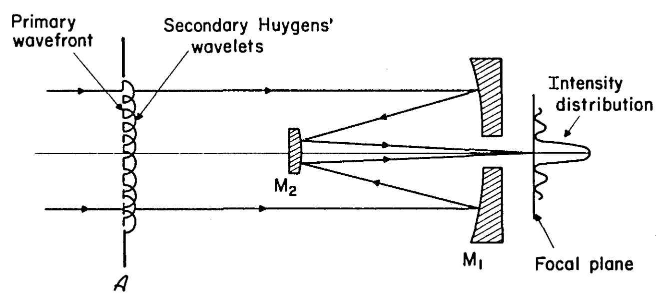

FIG. 7. Formation of the diffraction image in the focal plane of a telescope.

diated independently of each other. Thus the spatial coherence of the light entering the telescope is in this case not a manifestation of any coherence properties of the source itself, but rather—unlike in the case of laser light—it somehow must have been *generated in the process of propagation* of the light from the star to the telescope. It is not difficult to explain in mathematical language how this comes about. For our purposes it will be sufficient to give a rather simple, though necessarily less rigorous explanation, which, however, provides a good intuitive understanding of the underlying physics.

Consider two *independently* radiating point sources S_1 and S_2 and let us examine the behavior of the light at two points P_1 and P_2 in the field. We denote by A_1 and A_2 the wave trains arriving at P_1 and P_2 from the point source S_1 and by B_1 and B_2 the wave trains arriving at these points from the other point source, S_2 (Fig. 8). Since the point sources S_1 and S_2 are assumed to radiate independently, the light that they emit will be uncorrelated. We may express this fact symbolically by writing

$$\langle A_i B_j \rangle = 0 \qquad (i,j = 1,2). \tag{3.5}$$

Now for the sake of simplicity let us also assume that the distances S_1P_1 and S_1P_2 are approximately equal, as are the distances S_2P_1 and S_2P_2. Then we obviously have

$$A_2 \approx A_1, \tag{3.6a}$$

$$B_2 \approx B_1. \tag{3.6b}$$

Now the light vibrations at the points P_1 and P_2 are generated by superposition of the A and B wave trains, so that the fields V_1 and V_2 at the two points are

$$\text{at } P_1: \qquad V_1 = A_1 + B_1, \tag{3.7a}$$

$$\text{at } P_2: \qquad V_2 = A_2 + B_2. \tag{3.7b}$$

In view of the relations (3.6) we obviously have

$$V_1 \approx V_2. \tag{3.8}$$

This simple formula tells us that the light fluctuations at the points P_1 and P_2 will be very similar to each other, i.e., the field at these two points will have a high degree of spatial coherence, in spite of the fact that the two point sources S_1 and S_2 are completely uncorrelated. We see that this spatial coherence has been generated by superposition in the process of propagation.

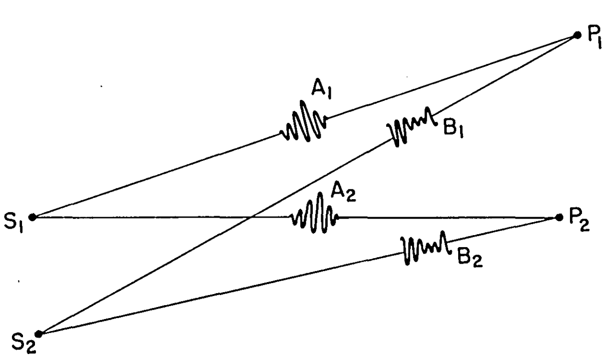

FIG. 8. Illustrating the generation of spatial coherence from two uncorrelated point sources.

One can easily extend this argument to radiation from any number of independent point sources and hence, in an appropriate limit, to radiation from an extended "globally" incoherent source, such as a star. The whole argument can be made mathematically quite rigorous and leads to the so-called van Cittert-Zernike theorem (Ref. 5, Sec. 10.4.2), which is one of the basic results of the classical theory of partial coherence. However, the physical origin of spatial coherence in a field from a highly uncorrelated source is quite clearly evident from the simple argument that I just gave.

There is another important conclusion that readily follows from the same type of argument. If we take the two field points P_1 and P_2 to be situated in the source region itself one finds that the light vibrations in that region must necessarily be correlated over distances of at least the order of magnitude of the mean wavelength of the light. This conclusion has a close bearing on some questions that arise with regard to the foundations of radiometry, to which we will now turn our attention.

IV. FOUNDATIONS OF RADIOMETRY

We saw earlier from our comparison of the angular distribution of radiation generated by a thermal source and by a laser, that there must be a close relationship between the radiometric properties of a source and its coherence properties. Considerable progress has been made in recent years to clarify this relationship, which turned out to be a rather subtle one, as we will see.

According to radiometry the total power radiated by a planar source per unit frequency interval at frequency ν is obtained at once from the elementary law (2.1) as

$$\mathcal{P}_\nu = \int_{(2\pi)} d\Omega \int_\sigma d\sigma B_\nu(\mathbf{r},\mathbf{s}) \cos\theta, \tag{4.1}$$

where the integrations extend over the source σ and over the 2π-solid angle formed by all the $\mathbf{s}$ directions that point into the half-space into which the source is radiating. Equation (4.1) may be rewritten in two equivalent forms

$$\mathcal{P}_\nu = \int_{(2\pi)} J_\nu(\mathbf{s})\, d\Omega = \int_\sigma E_\nu(\mathbf{r})\, d\sigma, \tag{4.2}$$

where

$$E_\nu(\mathbf{r}) = \int_{(2\pi)} B_\nu(\mathbf{r},\mathbf{s}) \cos\theta\, d\Omega, \tag{4.3}$$

and

$$J_\nu(\mathbf{s}) = \cos\theta \int_\sigma B_\nu(\mathbf{r},\mathbf{s})\, d\sigma. \tag{4.4}$$

The formulas (4.3) and (4.4) define two important radiometric quantities in terms of the radiance $B_\nu(\mathbf{r},\mathbf{s})$, namely the *radiant emittance* $E_\nu(\mathbf{r})$ and the *radiant intensity* $J_\nu(\mathbf{s})$.

Before proceeding any further we note that there is an implicit assumption built into Eq. (4.1), namely, that the total radiated power is obtained by simply adding the contributions from all elements of the source and from all directions. In other words, the various contributions are treated as if they were uncorrelated. From what we have learned earlier about coherence properties of sources and of optical fields, it is clear that this assumption is suspect.

The basic question then is: Can this quasi-geometrical radiometric model be reconciled with physical optics? And if so, what is the relation between the fundamental radiometric quantity, namely, the radiance, and the fundamental quantity of coherence theory, the mutual coherence function?

Since we are now considering the transport of energy at one particular frequency ν, it will be convenient to employ instead of the mutual coherence function its corresponding spectral component, known as the *cross-spectral density function,*

$$W_{12}(\nu) = \int_{-\infty}^{\infty} \Gamma_{12}(\tau) e^{2\pi i\nu\tau} d\tau. \tag{4.5}$$

It may be shown [Ref. 8, Eqs. (2.4) and (2.5a)] that this function is a measure of the correlation between the Fourier components $\hat{V}_1(\nu)$ and $\hat{V}_2(\nu)$ (interpreted in the sense of the theory of generalized functions) of the light vibrations at the two field points $P_1(\mathbf{r}_1)$ and $P_2(\mathbf{r}_2)$. In place of the complex degree of coherence $\gamma_{12}(\tau)$, defined by Eq. (3.2), we will now employ the spectral degree of spatial coherence[8]

$$\mu_{12}(\nu) = W_{12}(\nu)/[I_1(\nu)]^{1/2}[I_2(\nu)]^{1/2}, \tag{4.6}$$

where

$$I_j(\nu) = W_{jj}(\nu) \qquad (j = 1,2) \tag{4.7}$$

is a measure of the optical intensity at frequency ν at the point $P(\mathbf{r}_j)$. The absolute value of the spectral degree of spatial coherence may be shown to be bounded by zero and unity,

$$0 \leqslant |\mu_{12}(\nu)| \leqslant 1, \tag{4.8}$$

with the extreme limit zero representing complete spatial incoherence and the other extreme limit, unity, representing complete spatial coherence.

In an important paper[9] published in 1968, A. Walther proposed an expression for the radiance in terms of the cross-spectral density. In our notation this expression is

$$B(\mathbf{r},\mathbf{s}) = \left(\frac{k}{2\pi}\right)^2 \cos\theta \times \int W^{(0)}(\mathbf{r} + \mathbf{r}'/2, \mathbf{r} - \mathbf{r}'/2) e^{-ik\mathbf{s}_\perp \cdot \mathbf{r}'} d^2\mathbf{r}', \tag{4.9}$$

where

$$k = 2\pi\nu/c \tag{4.10}$$

is the free-space wave number associated with the frequency component ν. In (4.9) $W^{(0)}(\mathbf{r} + \mathbf{r}'/2, \mathbf{r} - \mathbf{r}'/2)$ is the cross-spectral density function of the field at points $\mathbf{r}_1 = \mathbf{r} + \mathbf{r}'/2$, $\mathbf{r}_2 = \mathbf{r} - \mathbf{r}'/2$ in the source plane at frequency ν (which we do not show explicitly from now on), and $\mathbf{s}_\perp$ is the two-dimensional vector obtained by projecting the three-dimensional unit vector $\mathbf{s}$ onto the source plane.

Walther's expression (4.9) for the radiance in terms of the cross-spectral density function of the field in the source plane is consistent with the radiometric formula (4.1) in the sense that when used in the integrand of Eq. (4.1) (but with the σ integration extending formally over the whole plane containing the planar source), it leads to the correct value for the total radiated power $\mathcal{P}$ when $\mathcal{P}$ is calculated on the basis of physical optics. Thus Walther's formula (4.9) appears at first sight to provide the missing link between radiometry and physical optics. However, a closer scrutiny shows that this cannot be the complete answer. For in the first place it is not difficult to find other (nonequivalent) expressions for a quantity $B(\mathbf{r},\mathbf{s})$ that lead to the correct value for the radiant power $\mathcal{P}$, (again calculated from physical optics), when substituted in the radiometric formula (4.1). In fact Walther himself introduced a few years later one such alternative expression.[10] Moreover, Marchand and I showed[11,12] that there are sources for which both the expressions for the radiance proposed by Walther will become negative for some values of their arguments, a result that is in contradiction with the physical significance attributed to the radiance. However, I wish to add that in spite of these conclusions, Walther's paper represents a major contribution to the foundation of radiometry and has become the nucleus of practically all subsequent research on this subject.[13]

Because one can find many different radiance functions that will lead to the correct radiant power $\mathcal{P}$, the question arises whether there is one amongst them that will be consistent with other (explicit or implicit) postulates of radiometry. Recently, one of my students (Friberg, Ref. 14) showed that this cannot be done. More specifically, if $\mathcal{L}$ denotes any linear transform, $J(\mathbf{s})$ denotes the radiant intensity as calculated on the basis of physical optics [Eq. (5.4) below], and $\mathbf{r}$ represents any point in the source plane, Friberg's theorem asserts that *there is no radiance function that satisfies the following four requirements for a planar source of any state of coherence:*

$$\text{(I)} \quad B(\mathbf{r},\mathbf{s}) = \mathcal{L}\{W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)\},$$

$$\text{(II)} \quad B(\mathbf{r},\mathbf{s}) \geqslant 0,$$

$$\text{(III)} \quad B(\mathbf{r},\mathbf{s}) = 0 \text{ when } \mathbf{r} \notin \sigma,$$

$$\text{(IV)} \quad \cos\theta \int_\sigma B(\mathbf{r},\mathbf{s})\, d\sigma = [J(\mathbf{s})]_{\text{phys. opt.}}.$$

The first requirement expresses the fact that seems physically quite natural, namely, that the radiance should depend linearly on the cross-spectral density function of the light distribution across the source. The second and third requirements express two facts implied by the radiometric interpretation of the physical significance of the radiance; namely, that it should be non-negative and that it should have zero values at all points $\mathbf{r}$ in the source plane which lie outside the area occupied by the source. The fourth requirement expresses the condition that the radiometric expression (4.4) on the left-hand side should correctly represent the radiant intensity $[J(\mathbf{s})]_{\text{phys.opt.}}$ when the radiant intensity is calculated on the basis of physical optics[15] [Eq. (5.4) below].

It would thus seem that the radiometric model cannot be fully reconciled with physical optics. Although this result may perhaps appear rather surprising at first sight, it is consistent with the warning of Lorentz that I quoted earlier, about the impossibility of tracing, in general, the path of parts of energy through an optical field in too detailed a manner.

Friberg's theorem also contains a hint that the radiance is not a quantity that in the context of the theory of the quantized field would represent an observable. In fact there is a strong formal resemblance between Friberg's theorem and a theorem due to Wigner[16,17] relating to the so-called phase space representation of quantum mechanics.[18] Because this analogy seems to suggest the true significance of the radiance function, let me say a few words about Wigner's theorem.

For simplicity let us consider a quantum-mechanical system with only one degree of freedom. Let[19] $\hat{q}$ and $\hat{p}$ be the position and the momentum operators, $\hat{\rho}(\hat{q},\hat{p})$ the density operator that represents the state of the system, and $\hat{G}(\hat{q},\hat{p})$ an observable. The expectation value of the observable $\hat{G}$ is then given by

$$\langle \hat{G} \rangle = \mathrm{Tr}(\hat{\rho}\hat{G}), \tag{4.11}$$

where Tr denotes the trace. In the phase-space representation of quantum mechanics one maps all the operators onto c-number functions,

$$\begin{aligned} \hat{q} &\rightarrow q, \qquad \hat{p} \rightarrow p, \\ \hat{\rho}(\hat{q},\hat{p}) &\rightarrow \Phi(q,p), \\ G(\hat{q},\hat{p}) &\rightarrow g(q,p), \end{aligned} \tag{4.12}$$

according to some prescribed rule that depends on the choice in which the noncommuting operators $\hat{q}$ and $\hat{p}$ are ordered in expressions involving their products.[20] The following question now arises: Can one find, among all such mappings (assumed to be linear) one that makes it possible to express the quantum-mechanical expectation value (4.11) of any observable $\hat{G}$ in the form of a classical c-number average with respect to the "phase-space" distribution function $\Phi(q,p)$, i.e., such that

$$\langle \hat{G} \rangle = \int\int g(q,p)\Phi(q,p)\, dq\, dp? \tag{4.13}$$

It is known that formally this can be done, in fact via many different mappings. However, *Wigner's theorem* asserts that *there is no phase space distribution function $\Phi(q,p)$ that will have all the properties of a true probability density.* In particular $\Phi(q,p)$ may become negative for some values of its arguments.

In spite of this fact, the phase-space representation of quantum mechanics has proved a very powerful technique for solving quantum-mechanical problems by methods of classical statistical mechanics. For example during the past fifteen years or so this technique has played an important role in various theories of the laser and in investigations concerning the statistical properties of light.

In its mathematical structure, radiometry has much in common with the phase-space representation of quantum mechanics. In particular both the phase-space distribution function $\Phi(q,p)$ and the radiance function $B(\mathbf{r},\mathbf{s})$ are functions of variables whose quantum-mechanical counterparts do not commute. Moreover, Walther's expression (4.9) for the radiance has a close formal similarity with the expression for the phase-space distribution function originally introduced by Wigner[16] and which in modern notation reads

$$\Phi(q,p) = \frac{1}{2\pi\hbar} \times \int \langle q - q'/2|\hat{\rho}|q + q'/2\rangle e^{ipq'/\hbar}\, dq', \tag{4.14}$$

where $\hbar$ is Planck's constant divided by 2π. It may well be that future work on the foundation of radiometry and the theory of radiative energy transfer will reveal that this correspondence is not a purely formal one, but that it is rooted in common physical principles. Moreover, Friberg's theorem and the analogy that I just spoke about suggest that although the radiance may not have the simple intuitive physical meaning that one traditionally attributes to it, it may nevertheless be used in calculating values of quantities that are truly measurable.

V. THE RADIANT INTENSITY FROM A SOURCE OF ANY STATE OF COHERENCE

In the preceding section we put forward evidence which indicates that the radiance $B(\mathbf{r},\mathbf{s})$ as customarily defined in radiometry does not represent a measurable physical quantity. One can show that the same is also true about the radiant emittance $E(\mathbf{r})$. However, the third basic radiometric quantity, the radiant intensity $J(\mathbf{s})$, acquires an unambiguous physical meaning as a measurable quantity if it is defined not via the radiometric formula (4.4) but more directly as representing the (averaged) rate at which energy is radiated by the source per unit solid angle around the $\mathbf{s}$ direction. The radiant intensity defined in this way, (denoted $[J(\mathbf{s})]_{\text{phys.opt.}}$ in Sec. IV), may readily be expressed in terms of the cross-spectral density function of the light in the source plane, irrespective of the state of coherence of the source. I will briefly indicate how this expression may be derived.

One can readily show that the radiant intensity as just defined may be expressed in terms of the energy flux vector $\mathbf{F}(\mathbf{r})$ of the field by the formula

$$J(\mathbf{s}) = \lim_{R\to\infty} [R^2|\langle \mathbf{F}(R\mathbf{s})\rangle|]. \tag{5.1}$$

On expressing the flux vector in terms of the complex scalar field variable $V(\mathbf{r})$ (see Footnote 7), one finds that (with a suitable choice of units)

$$\begin{aligned} J(\mathbf{s}) &= \lim_{R\to\infty} [R^2\langle V(R\mathbf{s})V^*(R\mathbf{s})\rangle] \\ &= \lim_{R\to\infty} [R^2 W(R\mathbf{s},R\mathbf{s})], \end{aligned} \tag{5.2}$$

where $W(R\mathbf{s},R\mathbf{s})$ is, of course, a "diagonal" element of the cross-spectral density function $W(\mathbf{r}_1,\mathbf{r}_2)$ of the field. Now because the field obeys the wave equation one can show that in free space the cross-spectral density function obeys the two Helmholtz' equations[21]

$$\begin{aligned} \nabla_1^2 W(\mathbf{r}_1,\mathbf{r}_2) + k^2 W(\mathbf{r}_1,\mathbf{r}_2) &= 0, \\ \nabla_2^2 W(\mathbf{r}_1,\mathbf{r}_2) + k^2 W(\mathbf{r}_1,\mathbf{r}_2) &= 0, \end{aligned} \tag{5.3}$$

where ∇_1^2 is the Laplacian operator acting on the coordinates of $\mathbf{r}_1$, with similar interpretation of ∇_2^2. Using standard mathematical techniques for solving Helmholtz' equation one can readily express the cross-spectral density function at any pair of points in the far zone in terms of its values at all pairs of points in the source plane. If we make use of that relation on the right-hand-side of Eq. (5.2) we finally obtain the following expression for the radiant intensity:

$$J(\mathbf{s}) = (2\pi k)^2 \cos^2\theta\, \tilde{W}^{(0)}(k\mathbf{s}_\perp, -k\mathbf{s}_\perp). \tag{5.4}$$

In (5.4) $\tilde{W}^{(0)}(\mathbf{f}_1,\mathbf{f}_2)$ is the spatial Fourier transform of the

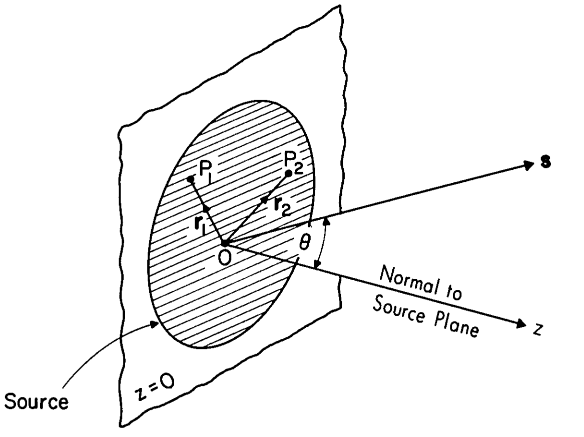

FIG. 9. Illustrating the notation relating to the formulas (5.4) and (5.5).

cross-spectral density function $W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)$ of the field in the source plane $z = 0$ (see Fig. 9), viz.

$$\tilde{W}^{(0)}(\mathbf{f}_1,\mathbf{f}_2) = \frac{1}{(2\pi)^4} \times \int\int W^{(0)}(\mathbf{r}_1,\mathbf{r}_2) e^{-i(\mathbf{f}_1\cdot\mathbf{r}_1+\mathbf{f}_2\cdot\mathbf{r}_2)}\, d^2r_1\, d^2r_2\,, \quad (5.5)$$

the integration extending twice independently over the source and $\mathbf{s}_\perp$ denotes, as before, the two-dimensional vector obtained by projecting the three-dimensional unit vector $\mathbf{s}$ onto the source plane.

Equation (5.4) is an important formula. It expresses the radiant intensity in terms of the spatial Fourier transform of the cross-spectral density function of the light at the source and thus establishes a relation between a measurable radiometric quantity and a basic quantity of coherence theory. We note that according to Eq. (5.4) only those (two-dimensional) spatial frequency components $\mathbf{f}_1$ and $\mathbf{f}_2$ contribute to the radiant intensity for which

$$\mathbf{f}_1 = k\mathbf{s}_\perp, \qquad \mathbf{f}_2 = -k\mathbf{s}_\perp. \quad (5.6)$$

Thus we may say that only certain *"anti-diagonal" elements* of the spatial Fourier transform of the cross-spectral density function $W^{(0)}$ contribute to the radiant intensity. Moreover, since

$$|k\mathbf{s}_\perp| \leqslant |k\mathbf{s}| = k, \quad (5.7)$$

the spatial frequencies that contribute, all have magnitudes that do not exceed the wave number of the light. We call such frequencies *low frequencies.* Thus the formula (5.4) implies that *the radiant intensity is uniquely specified by the low-frequency anti-diagonal elements of the Fourier transform of the cross-spectral density function of the light at the source.*

With the help of the formula (5.4) one can obtain a great deal of insight into a variety of problems that involve radiation from planar sources of any state of coherence. I will conclude this talk by giving a few examples of this kind.

VI. COHERENCE AND THE DIRECTIONALITY OF LIGHT BEAMS

It is generally believed that in order to generate a highly directional light beam such as a laser beam it is necessary to have a source that is spatially completely coherent. Actually, as I remarked earlier, the relation between directionality of light and the state of coherence of the source[22] turns out to be a rather subtle one. I will now show that the formula (5.4) for the radiant intensity provides a clarification of this question and also that it leads to the rather surprising prediction that certain sources that are far from fully coherent will produce beams that are just as directional as laser beams.

Let

$$\mu^{(0)}(\mathbf{r}_1,\mathbf{r}_2) = W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)/[I^{(0)}(\mathbf{r}_1)]^{1/2}[I^{(0)}(\mathbf{r}_2)]^{1/2} \quad (6.1)$$

be the spectral degree of spatial coherence of the light in the source plane.[23] For the sake of simplicity we will restrict our discussion to radiation from planar sources for which $\mu^{(0)}(\mathbf{r}_1,\mathbf{r}_2)$ depends on the position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$ of points in the source plane only through the difference $\mathbf{r}_1 - \mathbf{r}_2$, i.e., $\mu^{(0)}(\mathbf{r}_1,\mathbf{r}_2)$ is of the form

$$\mu^{(0)}(\mathbf{r}_1,\mathbf{r}_2) = g^{(0)}(\mathbf{r}_1 - \mathbf{r}_2), \quad (6.2)$$

where $g^{(0)}(\mathbf{r}')$ is some function of a (two-dimensional) vector $\mathbf{r}'$. We will also assume that the (averaged) intensity $I^{(0)}(\mathbf{r})$ changes much more slowly with $\mathbf{r}$ than $g^{(0)}(\mathbf{r}')$ changes with $\mathbf{r}'$, remaining approximately constant over distances of the order of the correlation length Δ [the effective width Δ of $g^{(0)}$], as indicated for a one-dimensional source in Fig. 10. In addition we assume that the linear dimensions of the source are large compared both with the wavelength of the light and with the correlation length Δ. A source with these properties is said to be a *quasi-homogeneous source.*[24–26]

Let us now consider a quasi-homogeneous source with a Gaussian correlation function

$$g^{(0)}(\mathbf{r}_1 - \mathbf{r}_2) = e^{-|\mathbf{r}_1-\mathbf{r}_2|^2/2\sigma^2}. \quad (6.3)$$

According to Eqs. (6.1), (6.2), and (6.3) the cross-spectral density function of the light in the source plane is of the form

$$W^{(0)}(\mathbf{r}_1,\mathbf{r}_2) = [I^{(0)}(\mathbf{r}_1)]^{1/2}[I^{(0)}(\mathbf{r}_2)]^{1/2} e^{-|\mathbf{r}_1-\mathbf{r}_2|^2/2\sigma^2}. \quad (6.4)$$

On substituting from (6.4) into our general expression (5.4) for the radiant intensity one readily finds that the angular distribution of the radiant intensity is to a high degree of accuracy independent of the exact form of the optical intensity distribution $I^{(0)}(\mathbf{r})$ across the source, provided only that $I^{(0)}(\mathbf{r})$ varies sufficiently slowly with $\mathbf{r}$ as we assumed. The resulting expression for the radiant intensity is[27]

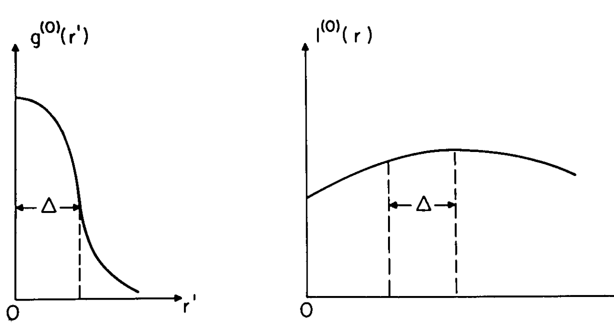

FIG. 10. Illustrating the concept of a quasi-homogeneous source. The degree of coherence $g^{(0)}(\mathbf{r}_1 - \mathbf{r}_2)$ of the light across the source changes much more rapidly with $r' = |\mathbf{r}_1 - \mathbf{r}_2|$ than the intensity $I^{(0)}(\mathbf{r})$ changes with $r = |\mathbf{r}|$.

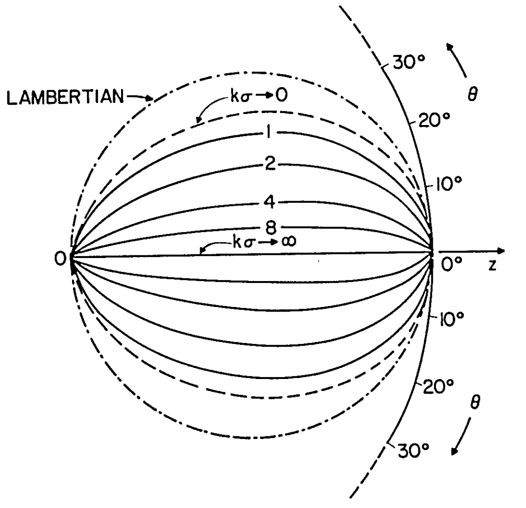

FIG. 11. Polar diagrams of the normalized radiant intensity $J(\mathbf{s})/J^{(0)}$ from a Gaussian-correlated quasi-homogeneous source [Eq. (6.5)] for different values of the rms width σ of the degree of coherence [Eq. (6.3)]. [After W. H. Carter and E. Wolf, J. Opt. Soc. Am. **67**, 790 (1977).]

$$\frac{J(\mathbf{s})}{J^{(0)}} \approx \cos^2\theta e^{-(1/2)(k\sigma)^2\sin^2\theta}, \tag{6.5}$$

where $J^{(0)}$ is the radiant intensity in the forward direction $\theta = 0$, which may readily be shown to be given by

$$J^{(0)} = \frac{(k\sigma)^2}{2\pi} \int I^{(0)}(\mathbf{r})\, d^2r. \tag{6.6}$$

In Fig. 11 polar diagrams are shown of the normalized radiant intensity from such Gaussian-correlated quasi-homogeneous sources, as determined from Eq. (6.5). The curves in Fig. 11 indicate that with increasing values of σ, i.e., with increasing correlation length of the light across the source, the radiant intensity becomes more and more peaked around the forward direction $\theta = 0$. Hence the light generated by a quasi-homogeneous Gaussian-correlated source becomes more and more directional as the area of coherence of the light in the source plane becomes larger and larger. More specifically the polar diagrams show that in order to generate a beam that is highly directional the linear dimensions of the regions of the source over which the light is highly correlated must be large compared with the wavelength—a fact that may be expressed by saying that the light must be *locally* coherent over distances of many wavelengths. However, because of our assumption of quasi-homogeneity, the linear dimensions of the source must be large compared to the correlation distance, i.e., such a source may be said to be spatially rather incoherent in the *global* sense. Thus we see that *complete spatial coherence of the source is not necessary to obtain a highly directional beam.*

A question that we may well ask at this point is the following one: is there a source of this class, i.e., one that is spatially highly incoherent in the global sense that would generate light whose radiant intensity distribution is identical with that produced by a laser? E. Collett and I have recently shown[28] that this is indeed so.

To indicate why this is possible and what the characteristics of such a source are let us return to the formula (5.4) for the radiant intensity from a source of any state of coherence, viz.

$$J(\mathbf{s}) = (2\pi k)^2 \cos^2\theta \tilde{W}^{(0)}(k\mathbf{s}_\perp, -k\mathbf{s}_\perp). \tag{6.7}$$

This formula implies that *any two sources with cross-spectral density functions whose four-dimensional spatial Fourier transforms have the same spatial low-frequency anti-diagonal elements will generate fields that have identical distributions of the radiant intensity.* (The two fields will have, in general, entirely different coherence properties, because these are determined by all the low-frequency elements,[29] not just by the anti-diagonal ones.) Our problem now is to try to determine a cross-spectral density function, say $W_Q^{(0)}(\mathbf{r}_1, \mathbf{r}_2)$, of a quasi-homogeneous source, which has a Fourier transform whose low-frequency anti-diagonal elements are identical with those of a laser source. It turns out that many such functions $W_Q^{(0)}$, representing different "equivalent" quasi-homogeneous sources, can be found.[30] I will only consider the one discussed in Ref. 28, which is particularly simple.

We assume that the laser generates an intensity distribution $I_L^{(0)}(\mathbf{r})$ across the output mirror (taken to be plane) that is Gaussian,

$$I_L^{(0)}(\mathbf{r}) = A_L e^{-r^2/2\sigma_L^2}, \tag{6.8}$$

where A_L and σ_L are positive constants and $r = |\mathbf{r}|$ denotes the radial distance from the axis of the laser. Since the laser light is spatially coherent and co-phasal over the output mirror, its degree of coherence is simply

$$g_L^{(0)}(\mathbf{r}_1 - \mathbf{r}_2) \equiv 1 \tag{6.9}$$

for all points $\mathbf{r}_1$ and $\mathbf{r}_2$ on the mirror. If, for the sake of simplicity, diffraction at the edge of the mirror is ignored, one finds that a quasi-homogeneous source with the following characteristics will generate light that has a distribution of radiant intensity which is identical with that of the laser beam:

$$g_Q^{(0)}(\mathbf{r}_1 - \mathbf{r}_2)\, e^{-|\mathbf{r}_1-\mathbf{r}_2|^2/8\sigma_L^2}, \tag{6.10}$$

$$I_Q^{(0)}(\mathbf{r}) = (\sigma_L/\sigma_Q)^2 A_L e^{-r^2/2\sigma_Q^2}, \tag{6.11}$$

where

$$\sigma_Q \gg 2\sigma_L. \tag{6.12}$$

We see from Eq. (6.10) that this quasi-homogeneous source is Gaussian correlated, with a root-mean-square (rms) width that is precisely twice the rms width σ_L of the Gaussian intensity distribution of the laser output. The intensity distribution of our quasi-homogeneous source is according to Eq. (6.11) also Gaussian, but we see from Eq. (6.12) that its rms width is very much larger than that of the laser output; hence our "equivalent" quasi-homogeneous source is much larger than the laser cross section. According to Eqs. (6.11), (6.12), and (6.8) the intensity distribution at the center of the quasi-homogeneous source is much smaller than at the center of the laser output mirror. These results are illustrated in Figs. 12 and 13. It seems remarkable that a source that is spatially so highly incoherent in the global sense should generate light that is just as directional as a laser beam.

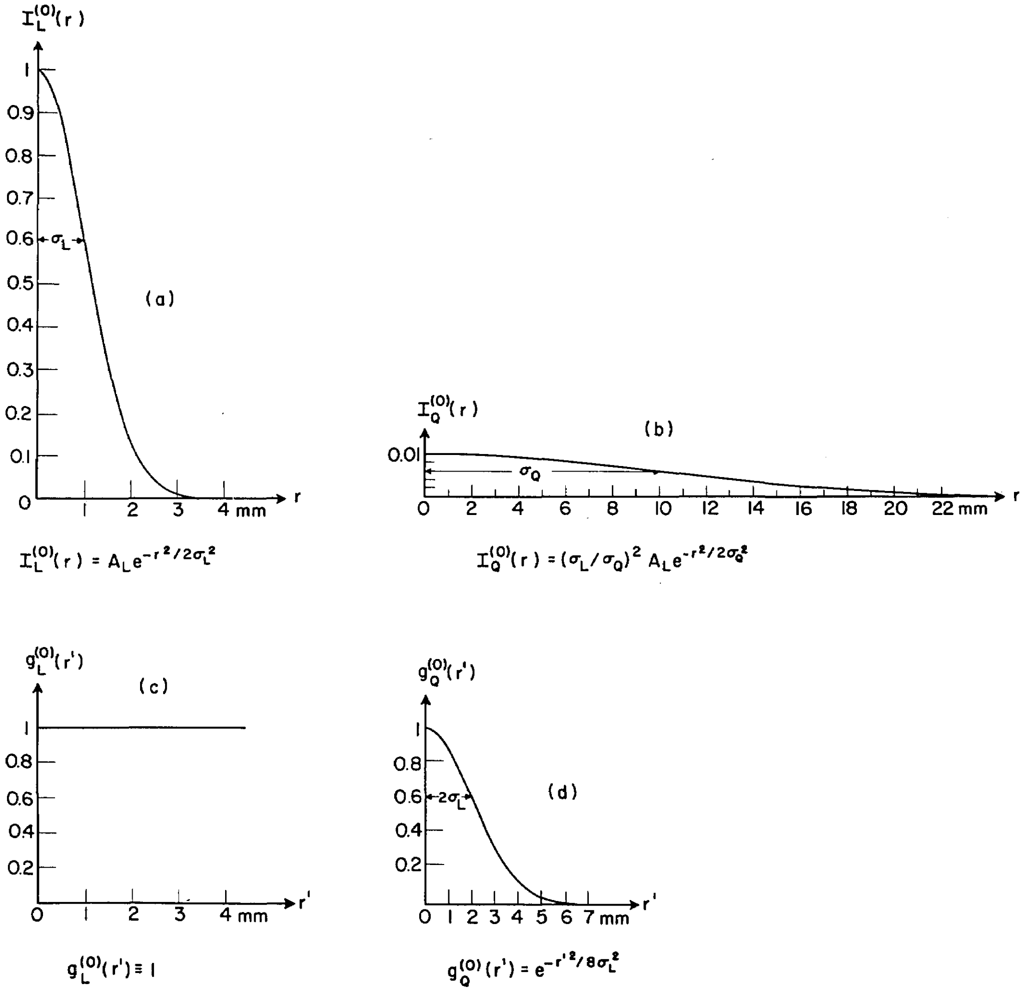

FIG. 12. Optical intensity distribution [(a)] and the spectral degree of spatial coherence [(c)] of a laser source and of a quasi-homogeneous source [(b) and (d)] which produce fields with identical distributions of the radiant intensity. The curves pertain to sources with $\sigma_L = 1$ mm, $\sigma_Q = 10$ mm. The optical intensity of the laser output is normalized to unity at the center of the output mirror. The vertical scale in (b) is ten times that of (a). [After E. Collett and E. Wolf, Opt. Lett. **2**, 27–29 (1978)].

One may well ask whether it is possible to produce a quasi-homogeneous source for testing these predictions. Up to the present time little work has been done on constructing sources with prescribed coherence properties, but some pioneering research in this direction has been carried out in the last few years by Bertolotti and his collaborators.[31] They constructed new types of secondary sources by scattering laser light on liquid crystals, under the application of a dc electric field. By varying the strength of the field, secondary sources are obtained whose coherence properties can to some extent be controlled by the strength of the applied fields. It seems plausable that some of our theoretical predictions could be tested with sources of this kind.

VII. COHERENCE PROPERTIES OF LAMBERTIAN SOURCES

Another situation that may readily be clarified with the help of the general formula (5.4) for the radiant intensity concerns Lambertian sources. Such sources will give rise to a radiant intensity that has the directional dependence

$$J(\mathbf{s}) = C \cos\theta, \tag{7.1}$$

(Lambert's law) where, as before, θ denotes the angle between the **s** direction and the normal to the source plane and C is a constant.

It is often asserted that sources of this type are spatially completely incoherent. This belief has presumably its origin in the fact that the usual Lambertian sources are thermal sources and that thermal sources at laboratory temperatures generate light chiefly by the highly random and uncorrelated process of spontaneous emission. However, we have seen earlier that even independent radiators produce field correlations in the source plane, and hence the question as to the kind of correlation that exists in Lambertian sources deserves attention.[32]

We will only consider planar Lambertian sources which are quasi-homogeneous, as is usually the case in practice. If we substitute from Eq. (7.1) into the left-hand side of Eq. (5.4) and make use of the relation

$$\cos\theta = (1 - s_x^2 - s_y^2)^{1/2} = (1 - \mathbf{s}_\perp^2)^{1/2}, \tag{7.2}$$

it follows after a straightforward though somewhat lengthy calculation that the cross-spectral density function of the light in the source plane must necessarily be of the form[33]

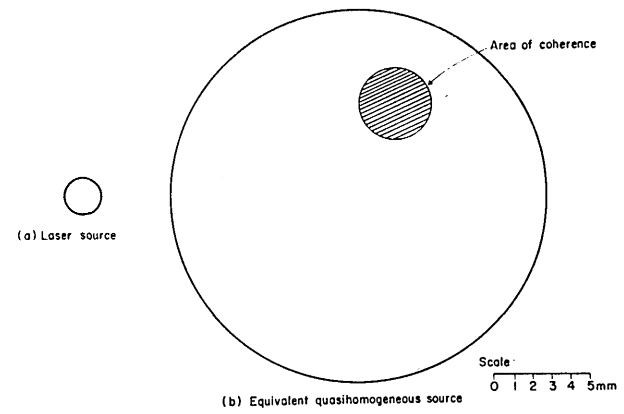

FIG. 13. Illustrating the effective sizes of the laser source (a) and of the "equivalent" quasi-homogeneous source (b) of Fig. 12. The effective area of coherence of the quasi-homogeneous source is shown shaded in Fig. (b).

$$W^{(0)}(\mathbf{r}_1,\mathbf{r}_2) = I^{(0)}[(\mathbf{r}_1+\mathbf{r}_2)/2] \times \frac{\sin k|\mathbf{r}_1-\mathbf{r}_2|}{k|\mathbf{r}_1-\mathbf{r}_2|} + [W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)]^{HF}, \quad (7.3)$$

where $I^{(0)}(\mathbf{r})$ denotes the intensity distribution across the source and $[W^{(0)}(\mathbf{r}_1,\mathbf{r}_2)]^{HF}$ represents a high-frequency (nonradiating) part. If we ignore this nonradiating part we readily obtain from (7.3) the following expression for the spectral degree of spatial coherence [cf. Eqs. (6.2) and (6.1)]:

$$g^{(0)}(\mathbf{r}_1-\mathbf{r}_2) = \sin k|\mathbf{r}_1-\mathbf{r}_2|/k|\mathbf{r}_1-\mathbf{r}_2|. \quad (7.4)$$

[In deriving the expression (7.4) an obvious approximation was made that is implied by the quasi-homogeneity of the source.] Thus we see that *all quasi-homogeneous planar Lambertian sources have identical spatial coherence properties in the sense that their spectral degree of spatial coherence is given by Eq. (7.4)* (except possibly for a nonradiating high-frequency contribution that we ignored).

The behavior of the degree of spatial coherence given by Eq. (7.4) is shown in Fig. 14 as a function of the separation of the points $\mathbf{r}_1$ and $\mathbf{r}_2$ (in units of $1/k = \lambda/2\pi$, where λ is the wavelength of the light). Since its first zero occurs when $r' \equiv |\mathbf{r}'| = \pi/k = \lambda/2$ we see that light across a Lambertian source is not completely spatially incoherent but is correlated over distances of the order of the wavelength of the light. These results are found to be in agreement with known correlation properties of blackbody radiation.[34]

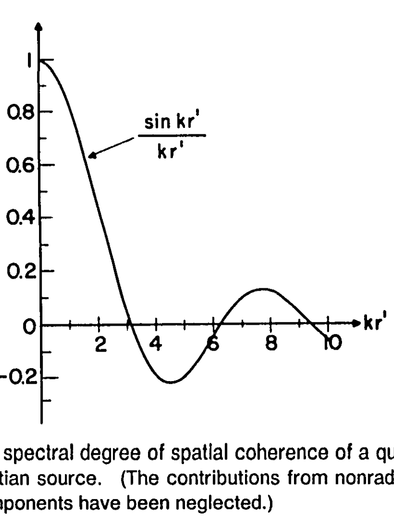

FIG. 14. The spectral degree of spatial coherence of a quasi-homogeneous Lambertian source. (The contributions from nonradiating spatial frequency components have been neglected.)

VIII. CONCLUDING REMARKS

It had originally been my intention to also review in this address recent researches on the foundations of the theory of radiative energy transfer which, as we noted earlier, may be regarded as a natural extension of radiometry. However, in preparing this talk it soon became apparent that it is not possible to do so in the available time. I will only mention, in this connection, that in spite of a good deal of research[35] much remains to be done in this area. Only in some special cases such as, for example, for transport in free space, has the relationship between the phenomenological theory of radiative energy transfer and the modern theories of radiation been clarified to a large extent.[36,37]

Returning to the foundations of radiometry I wish to stress a fact that should perhaps be obvious from our review: namely, that the connection between radiometry and physical optics is a rather subtle one and that no simple direct correspondence exists between all the quantities that describe energy transport in radiometry and in physical optics. In particular it is incorrect to identify irradiance with optical intensity, as has become rather customary in recent years in some publications.

I will conclude this talk by noting that although radiometry is one of the oldest branches of optics—predating Maxwell's electromagnetic theory and the quantum theory of radiation—there are many new and valuable results that have recently been obtained in this field, and undoubtedly more will follow. This development illustrates a recurring lesson that one can learn from the history of physics: namely, that subjects that often appear to be well understood and perhaps even a little old-fashioned have frequently some surprises in store for us.

[1]Throughout this talk we shall only be concerned with sources that generate radiation which is steady in the macroscopic sense. Such sources need not be, however, in thermal equilibrium with its surroundings.

[2]See, for example, E. Hopf, *Mathematical Problems of Radiative Equilibrium* (Cambridge U. P., Cambridge, 1934), Sec. 2.

[3]See, for example, A. I. Akhiezer and V. B. Berestetskii, *Quantum Electrodynamics* (Interscience, New York, 1965), Sec. 2.2, or W. Pauli, "Die Allgemeinen Principien der Wellenmechanik," in *Handbuch der Physik*, 2 Aufl., Band 24, 1 Teil, edited by H. Geiger and K. Scheel (Springer, Berlin, 1933), pp. 92 and 260.

[4]For a fuller account of optical coherence theory see, for example, L. Mandel and E. Wolf "Coherence properties of optical fields," Rev. Mod. Phys. 37, 231–287 (1965) or Ref. 5 quoted below.

[5]See, for example, M. Born and E. Wolf, *Principles of Optics*, 5th ed. (Pergamon, New York, 1975), Sec. 10.2.

[6]The mutual coherence function is independent of t because of our earlier assumption (cf. Footnote 1) that the radiation is steady in the macroscopic sense. In the language of the theory of random processes this assumption is tantamount to the statement that the fluctuations can be described as a stationary random process.

[7]We mentioned earlier that within the framework of Maxwell's electromagnetic theory the optical intensity is usually identified with the magnitude of the energy flux vector (the Poynting vector). It would therefore seem more appropriate to identify the optical intensity in a complex scalar wavefield $V(\mathbf{r},t)$ with the magnitude of

the flux vector $\mathbf{F} = \alpha(\dot{V}\nabla V^* + \dot{V}^*\nabla V)$ associated with that field. (Here $\dot{V} = \partial V/\partial t$ and α is a constant, depending on the choice of units.) However, under experimental conditions frequently encountered in practice (e.g., when measurements are made in the far zone of a radiating system and the field is quasi-monochromatic), $\langle|\mathbf{F}|\rangle$ may be shown to be proportional to $\langle VV^*\rangle$ (at least to a high degree of accuracy).

[8]L. Mandel and E. Wolf, "Spectral coherence and the concept of cross-spectral purity," J. Opt. Soc. Am. **66**, 529–535 (1976).

[9]A. Walther, "Radiometry and coherence," J. Opt. Soc. Am. **58**, 1256–1259 (1968).

[10]A. Walther, "Radiometry and coherence," J. Opt. Soc. Am. **63**, 1622–1623 (1973).

[11]E. W. Marchand and E. Wolf, "Radiometry with sources of any state of coherence," J. Opt. Soc. Am. **64**, 1219–1226 (1974).

[12]E. W. Marchand and E. Wolf, "Walther's definition of generalized radiance," J. Opt. Soc. Am. **64**, 1273–1274 (1974); see also A. Walther, "Reply to Marchand and Wolf", J. Opt. Soc. Am. **64**, 1275 (1974).

[13]A somewhat different approach was described by A. S. Marathay, "Radiometry of partially coherent fields I," Opt. Acta **23**, 785–794 (1976); II, *ibid.* **23**, 795–798 (1976).

[14]A. T. Friberg, "On the existence of a radiance function for a partially coherent planar source," in *Proceedings of the Fourth Rochester Conference on Coherence and Quantum Optics,* edited by L. Mandel and E. Wolf (Plenum, New York, in press).

[15]The radiometric definition of the radiant intensity $J(\mathbf{s})$, via the formula (4.4), implies that $J(\mathbf{s})$ represents the average radiated power per unit solid angle around the $\mathbf{s}$ direction. One can calculate this radiated power directly from physical optics, without introducing any hypothetical radiance function $B(\mathbf{r},\mathbf{s})$, as will be discussed in Sec. 5. $[J(\mathbf{s})]_{\text{phys.opt.}}$ denotes here the radiant intensity when calculated in this more direct manner.

[16]E. Wigner, "On the quantum correction for thermodynamic equilibrium," Phys. Rev. **40**, 749–759 (1932).

[17]E. P. Wigner, "Quantum mechanical distribution functions revisited," in *Perspectives in Quantum Theory,* edited by W. Yourgrau and A. van der Merwe (M.I.T. Press, Cambridge, Mass., 1971), pp. 25–36.

[18]For a discussion of Wigner's theorem and of related researches, see M. D. Srinivas and E. Wolf, "Some nonclassical features of phase-space representations of quantum mechanics," Phys. Rev. D **11**, 1477–1485 (1975).

[19]Carets denote operators.

[20]A general theory of such mappings was formulated by G. S. Agarwal and E. Wolf, "Calculus of functions of noncommuting operators and general phase-space methods in quantum mechanics. I. Mapping theorems and ordering of functions of noncommuting operators," Phys. Rev. D **2**, 2161–2186 (1970); "II. Quantum mechanics in phase space," Phys. Rev. D **2**, 2187–2205 (1970); "III. A generalized Wick theorem and multitime mapping," Phys. Rev. D **2**, 2206–2225 (1970). These papers also contain an extensive bibliography of earlier publications on this subject.

[21]The two Helmholtz equations (5.3) for the cross-spectral density function may be obtained, for example, by taking the Fourier transform of the two wave equations that the mutual coherence function is known to satisfy (cf. Ref. 5, Sec. 10.7.1).

[22]This relationship appears to have been first considered, for the special case of radiation from large statistically homogeneous sources, by E. Wolf and W. H. Carter, "Angular distribution of radiant intensity from sources of different degrees of spatial coherence," Opt. Commun. **13**, 205–209 (1975).

[23]As in the previous sections we describe the coherence properties of a source in terms of correlation functions involving the field distribution in the source plane. Such a description may be employed whether the source is a primary or a secondary one. However, when the source is a primary one, one may characterize its coherence properties in an alternative way, by means of correlation functions involving the true source variable (e.g., the charge-current density distribution). For a primary scalar source this alternative approach is discussed in a forthcoming paper by W. H. Carter and E. Wolf, "Coherence and radiant intensity in scalar wavefields generated by fluctuating primary planar sources (submitted to J. Opt. Soc. Am.).

[24]W. H. Carter and E. Wolf "Coherence and radiometry with quasihomogeneous planar sources," J. Opt. Soc. Am. **67**, 785–796 (1977).

[25]E. Wolf and W. H. Carter, "On the radiation efficiency of quasihomogeneous sources of different degrees of spatial coherence," in *Proceedings of the Fourth Rochester Conference on Coherence and Quantum Optics,* edited by L. Mandel and E. Wolf (Plenum, New York, in press).

[26]Formally a slightly different but essentially equivalent class of sources has been considered by H. A. Ferwerda and M. G. van Heel "On the coherence properties of thermionic emission sources," Optik **47**, 357–362 (1977). See also H. A. Ferwerda and M. G. van Heel, "Determination of Coherence Length from Directionality," in *Proceedings of the Fourth Rochester Conference on Coherence and Quantum Optics,* edited by L. Mandel and E. Wolf (Plenum, New York, in press).

[27]The angular distribution of radiant intensity from some other types of model sources is discussed in the following papers: H. P. Baltes, B. Steinle, and G. Antes, "Spectral coherence and the radiant intensity from statistically homogeneous and isotropic planar sources," Opt. Commun. **18**, 242–246 (1976); B. Steinle and H. P. Baltes, "Radiant intensity and spatial coherence for finite planar sources," J. Opt. Soc. Am. **67**, 241–247 (1977); H. P. Baltes, B. Steinle, and G. Antes "Radiometric and correlation properties of bounded planar sources," in *Proceedings of the Fourth Rochester Conference on Coherence and Quantum Optics,* edited by L. Mandel and E. Wolf (Plenum, New York, in press); W. H. Carter and M. Bertolotti, "An analysis of the far-field coherence and radiant intensity of light scattered from liquid crystals" (J. Opt. Soc. Am., in press).

[28]E. Collett and E. Wolf, "Is complete spatial coherence necessary for the generation of highly directional light beams?," Opt. Lett. **2**, 27–29 (1978).

[29]E. W. Marchand and E. Wolf, "Angular correlation and the far-zone behavior of partially coherent fields," J. Opt. Soc. Am. **62**, 379–385 (1972), Eq. (34).

[30]Actually all of them have the same degree of spatial coherence $g_Q(\mathbf{r}_1 - \mathbf{r}_2)$. They can only differ by their intensity distributions $I_Q(\mathbf{r})$.

[31]See, for example, F. Scudieri, M. Bertolotti, and R. Bartolino, "Light scattered by a liquid crystal: A new quasi-thermal source," Appl. Opt. **13**, 181–185 (1974); M. Bertolotti, F. Scudieri, and S. Verginelli, "Spatial coherence of light scattered by media with large correlation length of refractive index fluctuations," Appl. Opt. **15**, 1842–1844 (1976).

[32]It has been shown that a spatially completely incoherent source would give rise to radiant intensity that falls off with θ in proportion to $\cos^2\theta$ rather than $\cos\theta$. [T. J. Skinner, Ph.D. Thesis (Boston University, 1965), p. 46; E. W. Marchand and E. Wolf, Ref. 11, Sec. V; W. H. Carter and E. Wolf, Ref. 24, Sec. III].

[33]The formula (7.3) is a generalization to quasi-homogeneous sources of an expression derived for large homogeneous sources by M. Beran and G. Parrent, "The mutual coherence function of incoherent radiation," Nuovo Cimento **27**, 1049–1063 (1963), Sec. 8; A. Walther, Ref. 9, Sec. 4; W. H. Carter and E. Wolf, Ref. 24, Sec. II.

[34]Ref. 24, Sec. II, especially Eqs. (31) and (32).

[35]See, for example, the following publications and the references quoted therein: (a) Yu. N. Barabanenkov, Yu. A. Kravtsov, S. M. Rytov, and V. I. Tatarskii, "Status of the theory of propagation of waves in randomly inhomogeneous medium," Sov. Phys.—Usp. **13**, 551–575 (1971); (b) V. I. Tatarskii, *The Effects of Turbulent Atmosphere on Wave Propagation* (U.S. Department of Commerce, National Technical Service, Springfield, Va., 1971), Sec. 63; (c) Yu. A. Kravtsov, C. M. Rytov, and V. I. Tatarskii, "Statistical problems in diffraction theory," Sov. Phys.—Usp. **18**, 118–130 (1975); (d) Yu. N. Barabanenkov, "Multiple scattering of waves by ensembles of particles and the theory of radiation transport," Sov. Phys.—Usp. **18**, 673–689 (1976); (e) A. Ishimaru, "Theory and application of wave propagation and scattering in random media," Proc. IEEE **65**, 1030–1061 (1977).

[36]E. Wolf, "New theory of radiative energy transfer in free electromagnetic fields," Phys. Rev. D **13**, 869–886 (1976).

[37]M. S. Zubairy and E. Wolf, "Exact equations for radiative transfer of energy and momentum in free electromagnetic fields," Opt. Commun. **20**, 321–324 (1977).