$$\int_0^1 x\,dx = \begin{Bmatrix} \Delta x_i = \frac{1}{m} \\ x_i = \frac{i}{m} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \frac{f\left(\frac{i}{m}\right)}{m} = \lim_{m\to\infty} \sum_{i=1}^{m} \frac{i}{m^2} = \lim_{m\to\infty} \frac{1}{m^2} \sum_{i=1}^{m} i =$$
$$= \lim_{m\to\infty} \frac{1}{m^2} \cdot \frac{m(m+1)}{2} = \lim_{m\to\infty} \frac{m^2+m}{2m^2} = \frac{1}{2}$$

$$\int_0^5 (5-x)\,d = \begin{Bmatrix} \delta x_i = \frac{5}{m} \\ x_i = \frac{5i}{m} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \left(5 - \frac{5i}{m}\right) \cdot \frac{5}{m} =$$
$$= \lim_{m\to\infty} \sum_{i=1}^{m} \frac{25}{m} - \sum_{i=1}^{m} \frac{25i}{m^2} = 25 - \lim_{m\to\infty} \frac{25}{m^2} \sum_{i=1}^{m} i = \frac{25}{2}$$

$$\int_0^1 x^2\,dx = \begin{Bmatrix} \Delta x_i = \frac{1}{m} \\ x_i = \frac{i}{m} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \frac{i^2}{m^3} = \lim_{m\to\infty} \frac{1}{m^3} \sum_{i=1}^{m} i^2 =$$
$$= \lim_{m\to\infty} \frac{m(m+1)(2m+1)}{6m^3} = \lim_{m\to\infty} \frac{2m^3+3m^2+m}{m^3} = \frac{1}{3}$$

$$\int_3^5 x\,d = \begin{Bmatrix} \Delta x_i = 3 + \frac{2}{m} \\ x_i = \frac{2i}{m} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \frac{6}{m} + \frac{4i}{m^2} = \lim_{m\to\infty} 6 \sum_{i=1}^{m} \frac{1}{m} + \frac{4m^2+4m}{2m^2} =$$
$$= 6 + 2 = 8$$

$$\int_0^1 \sqrt{x}\,dx = \begin{Bmatrix} \Delta x_i = \frac{1}{m} \\ x_i = \frac{i}{m} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \sqrt{\frac{i}{m}} \frac{1}{m} = \lim_{m\to\infty} \frac{1}{m\sqrt{m}} \sum_{i=1}^{m} \sqrt{i} = ???$$
$$\int_0^1 \sqrt{x}\,dx = \begin{Bmatrix} \Delta x_i = \frac{2i-1}{m^2} \\ x_i = \frac{i^2}{m^2} \end{Bmatrix} = \lim_{m\to\infty} \sum_{i=1}^{m} \frac{i(2i-1)}{m^3} = \lim_{m\to\infty} \frac{2}{m^3} \sum_{i=1}^{m} i^2 - \frac{1}{m^3} \sum_{i=1}^{m} i =$$
$$= \lim_{m\to\infty} \frac{2(2m^3+3m^2+m)}{6m^3} - \frac{m^2+m}{2m^3} = \lim_{m\to\infty} \frac{4m^3+3m^2-m}{6m^3} = \frac{2}{3}$$

$$D(x) = \begin{cases} 0 & x \in \mathbb{Q} \\ 1 & x \notin \mathbb{Q} \end{cases}$$
$$x_i \in \mathbb{Q} \implies \sum_{i=1}^{m} \frac{1}{m} \cdot 0 \to 0$$
$$x_i \notin \mathbb{Q} \implies \sum_{i=1}^{m} \frac{1}{m} \cdot 1 \to 1$$
$$\implies D(x) \text{ is not Riemann-integrable}$$

$$f(x) = \begin{cases} \frac{1}{x^2} & x > 0 \\ 0 & x = 0 \end{cases} \text{ on } [0,1]$$
$$f \text{ is not bounded at } 0 \implies f \text{ is not integrable on } [0,1]$$

$$\int_1^4 \frac{1}{x^3}\,dx = \left\{ \begin{matrix} \Delta x_i = 4^{i/n}\left(1-\frac{1}{4^{1/n}}\right) \\ x_i = 4^{i/n} \end{matrix} \right\} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{4^{3i/n}} \cdot 4^{i/n}\left(1-\frac{1}{4^{1/n}}\right) =$$

$$= \lim_{n\to\infty} \left(1-\frac{1}{4^{1/n}}\right) \cdot \sum_{i=1}^{n} \frac{1}{4^{2i/n}} = \lim_{n\to\infty} \left(1-\frac{1}{4^{1/n}}\right) \cdot \frac{1}{4^{2/n}} \cdot \frac{\left(\frac{1}{4^2}-1\right)}{\frac{1}{4^{2/n}}-1} =$$

$$\lim_{n\to\infty} \frac{\frac{1}{4^{2/n}}\left(1-\frac{1}{16}\right)}{1+4^{1/n}} = \frac{15}{32}$$