$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

$$egin{aligned} \lim_{n o\infty}rac{rac{2^{n+1}x^{n+1}}{(n+1)!}}{rac{2^nx^n}{n!}}&=\lim_{n o\infty}rac{2x}{n+1}=0 \ orall x\in\mathbb{R}:\sum_{n=0}^{\infty}rac{2^nx^n}{n!} o S(x) \end{aligned}$$

$$egin{aligned} \sum_{n=1}^{\infty} rac{\sin(n!x)}{n^3+n+1}, x \in [-2\pi,2] \ rac{\sin(n!x)}{n^3+n+1} & \leq rac{1}{n^3+n+1} \leq rac{1}{n^3} \ \sum_{n=1}^{\infty} rac{1}{n}
ightarrow M \implies \sum_{n=1}^{\infty} rac{\sin(n!x)}{n^3+n+1}
ightrightarrow S(x) \end{aligned}$$

$$\sum_{n=2}^{\infty} rac{x^4+x^2}{n \ln^2(n)}, x \in [-7,2]$$
 $rac{x^4+x^2}{n \ln^2(n)} \leq rac{7^4+7^2}{n \ln^2(n)}$ $\sum_{n=2}^{\infty} rac{1}{n \ln^2(n)} o M$ by the Cauchy's condensation test $\implies \sum_{n=2}^{\infty} rac{x^4+x^2}{n \ln^2(n)}
ightrightarrows S(x)$

$$\sum_{n=0}^{\infty} rac{\sin(x)}{(1+x)^n} \ \sum_{n=0}^{\infty} rac{\sin(x)}{(1+x)^n} = \sin(x) \cdot \sum_{n=0}^{\infty} t^n = rac{1}{1-rac{1}{1+x}} = \sin(x) \cdot rac{1+x}{x} \ ext{Let } f(x) = egin{cases} \sin(x) \cdot rac{1+x}{x} & x
eq 0 \\ 0 & x = 0 \end{cases} \ f ext{ is not continuous } \implies x \in [0,\pi) \implies \sum_{n=0}^{\infty} rac{\sin(x)}{(1+x)^n}
ot \Rightarrow f(x) \ ext{Let } d_N = \sup_{x \in [0,\pi)} |S_N(x) - f(x)| \ d'_N = \sup_{x \in [0,\pi)} |S_N(x) - f(x)| = \max\{d_N, |S_N(0) - f(0)|\} = d_N \ d'_n
ot o 0 \implies d_n
ot o 0 \implies x \in (0,\pi) \implies \sum_{n=0}^{\infty} rac{\sin(x)}{(1+x)^n}
ot o f(x) \ d'_n
ot o 0 \implies d_n
ot o 0 \implies x \in (0,\pi) \implies \sum_{n=0}^{\infty} rac{\sin(x)}{(1+x)^n}
ot o f(x) \ d'_n
ot o 0 \implies d_n
ot o 0 \implies x \in (0,\pi) \implies \sum_{n=0}^{\infty} \frac{\sin(x)}{(1+x)^n}
ot o f(x) \ d'_n
ot o 0 \implies d_n
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ot o 0 \implies x \in (0,\pi)
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ot$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$$

$$\text{Let } x = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \cdot x^n = \sum_{n=1}^{\infty} \int_0^x t^{n-1} dt \stackrel{t \in [-\frac{1}{2}, \frac{1}{2}]}{= \sum_{n=1}^{\infty} t^{n-1} \Rightarrow S(x)} \int_0^x \sum_{n=1}^{\infty} t^{n-1} dt = \int_0^x \frac{1}{1-t} dt = -\ln|1-x|$$

$$\implies \sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n} = \ln\left(\frac{3}{2}\right)$$

$$\sum_{n=1}^{\infty}nx^n \ \sum_{n=1}^{\infty}nx^{n-1} = rac{x}{(1-x)^2}$$