$$\sum_{n=1}^{\infty} rac{n^2 2^n}{n^n + 2} (x - 5)^n$$
 $\lim_{n o \infty} \sqrt[n]{rac{n^2 2^n}{n^4 + 2}} = \lim_{n o \infty} rac{\sqrt[n]{n^2 \cdot 2}}{\sqrt[n]{n^4} \sqrt[n]{1 + rac{2}{n^4}}} = 2 \implies R = rac{1}{2}$
 $x = 5.5 \implies \sum_{n=1}^{\infty} rac{n^2}{n^4 + 2} ext{ converges}$
 $x = 4.5 \implies \sum_{n=1}^{\infty} (-1)^n rac{n^2}{n^4 + 2} ext{ converges}$
 $\sum_{n=1}^{\infty} a_n (\sqrt{x} - 5)^n$
 $t = \sqrt{x}$
 $t \in [4.5, 5.5] \implies x \in [4.5^2, 5.5^2]$

$$\operatorname{Let} \ \sum_{n=0}^{\infty} a_n \ \operatorname{diverges}$$
 $a_n \to 0 \ \operatorname{monotonic}$
$$x = 1 \implies \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n \ \operatorname{diverges} \implies R \le 1$$

$$x = -1 \implies \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} (-1)^n a_n \ \operatorname{converges} \ \operatorname{by} \ \operatorname{Leibniz} \implies R \ge 1$$

$$\implies R = 1 \implies x \in [-1,1)$$

Taylor series

Let f be infinitely differentiable

$$f(x)=\sum_{n=0}^{\infty}rac{f^{(n)}(a)}{n!}(x-a)^n$$

For example:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\left(rac{1}{1-x}
ight)^{(n)}(0)=n! \ rac{1}{1-x}=\sum_{n=0}^{\infty}x^n \ rac{\left(rac{1}{1-x}
ight)^{(n)}(0)}{n!}=1 \implies \left(rac{1}{1-x}
ight)^{(n)}(0)=n!$$

$$f(x) = \sin^2(x) \quad x = 0 \ \sin^2(x) = rac{1-\cos 2x}{2} = rac{1}{2} \Biggl(1 - \sum_{n=0}^{\infty} rac{(-1)^n}{(2n)!} (2x)^{2n} \Biggr) = rac{1}{2} \sum_{n=1}^{\infty} rac{(-1)^{n+1}}{(2n)!} (2x)^{2n} = \ = \sum_{n=1}^{\infty} rac{(-1)^{n+1} 2^{2n-1} x^{2n}}{(2n)!}$$

$$f(x) = rac{1}{x+17} = rac{1}{17-(-x)} = rac{1}{17} \cdot rac{1}{1-rac{-x}{17}} \ t = -rac{x}{17} \implies -1 < t < 1 \ \Longrightarrow rac{1}{17} \sum_{n=0}^{\infty} t^n = \sum_{n=0}^{\infty} rac{(-1)^n x^n}{17^{n+1}} \quad x \in (-17, 17)$$

$$rac{1}{(17+x)^2} = -igg(rac{1}{17+x}igg)' = \sum_{n=0}^{\infty} rac{(-1)^{n+1}nx^{n-1}}{17^{n+1}}$$

$$egin{aligned} rac{1}{x^2+x-2} &= rac{1}{(x-1)(x+2)} = rac{A}{x-1} + rac{B}{x+2} = \ &= rac{-1}{3}igg(rac{1}{1-x}igg) - rac{1}{3}\cdotrac{1}{2}\cdotrac{1}{1-rac{-x}{2}} = \ &= -rac{1}{3}igg(\sum_{n=0}^{\infty}x^n + \sum_{n=0}^{\infty}rac{(-1)^nx^n}{2^{n+1}}igg) = \ &= \sum_{n=0}^{\infty}rac{((-1)^{n+1}-2^{n+1})x^n}{3\cdot 2^{n+1}} \quad x\in(-1,1) \end{aligned}$$