$$\sum_{n=1}^{\infty} \frac{n^2 2^n}{n^n + 2}(x-5)^n$$

$$\lim_{n\to\infty} \sqrt[n]{\frac{n^2 2^n}{n^4+2}} = \lim_{n\to\infty} \frac{\sqrt[n]{n}^2 \cdot 2}{\sqrt[n]{n}^4 \sqrt[n]{1+\frac{2}{n^4}}} = 2 \implies R = \frac{1}{2}$$

$$x = 5.5 \implies \sum_{n=1}^{\infty} \frac{n^2}{n^4+2} \text{ converges}$$

$$x = 4.5 \implies \sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^4+2} \text{ converges}$$

$$\sum_{n=1}^{\infty} a_n(\sqrt{x}-5)^n$$

$$t = \sqrt{x}$$

$$t \in [4.5, 5.5] \implies x \in [4.5^2, 5.5^2]$$

$$\text{Let } \sum_{n=0}^{\infty} a_n \text{ diverges}$$

$$a_n \to 0 \text{ monotonic}$$

$$x = 1 \implies \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} a_n \text{ diverges} \implies R \leq 1$$

$$x = -1 \implies \sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} (-1)^n a_n \text{ converges by Leibniz} \implies R \geq 1$$

$$\implies R = 1 \implies x \in [-1, 1)$$

Taylor series

$$\text{Let } f \text{ be infinitely differentiable}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

$$\text{For example:}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\left(\frac{1}{1-x}\right)^{(n)}(0) = n!$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$\frac{\left(\frac{1}{1-x}\right)^{(n)}(0)}{n!} = 1 \implies \left(\frac{1}{1-x}\right)^{(n)}(0) = n!$$

$$f(x) = \sin^2(x) \quad x = 0$$

$$\sin^2(x) = \frac{1-\cos 2x}{2} = \frac{1}{2}\left(1 - \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(2x)^{2n}\right) = \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n)!}(2x)^{2n} =$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{2n-1}x^{2n}}{(2n)!}$$

$$f(x) = \frac{1}{x+17} = \frac{1}{17-(-x)} = \frac{1}{17}\cdot\frac{1}{1-\frac{-x}{17}}$$

$$t = -\frac{x}{17} \implies -1 < t < 1$$

$$\implies \frac{1}{17}\sum_{n=0}^{\infty} t^n = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{17^{n+1}} \quad x \in (-17, 17)$$

$$\frac{1}{(17+x)^2} = -\left(\frac{1}{17+x}\right)' = \sum_{n=0}^{\infty}\frac{(-1)^{n+1}nx^{n-1}}{17^{n+1}}$$

$$\frac{1}{x^2+x-2} = \frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} =$$

$$= \frac{-1}{3}\left(\frac{1}{1-x}\right) - \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{1}{1-\frac{-x}{2}} =$$

$$= -\frac{1}{3}\left(\sum_{n=0}^{\infty}x^n + \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{2^{n+1}}\right) =$$

$$= \sum_{n=0}^{\infty}\frac{((-1)^{n+1} - 2^{n+1})x^n}{3\cdot 2^{n+1}} \quad x \in (-1, 1)$$
