Find $\sqrt{10}$ with some accuracy

$$\text{Let } f(x) = \sqrt{x}$$
$$\text{Let } a = 9$$
$$\text{Let } k = 3$$
$$f(a) = a^{1/2} = 3$$
$$f'(a) = \frac{1}{2}a^{-1/2} = \frac{1}{6}$$
$$f''(a) = -\frac{1}{4}a^{-3/2} = -\frac{1}{108}$$
$$f'''(a) = \frac{3}{8}a^{-5/2} = \frac{1}{648}$$
$$f^{(4)}(a) = -\frac{15}{16}a^{-7/2}$$
$$\sqrt{10} \approx \sum_{n=0}^{3} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a) + \frac{f''(a)}{2} + \frac{f'''(a)}{6} =$$
$$= 3 + \frac{1}{6} - \frac{1}{216} + \frac{1}{3888} \approx 3.162294$$
$$|R_3(x)| = \frac{f^{(4)}(c)}{4!}(x-9)^4$$
$$\implies c \in (9,10) : |R_3(10)| = \left|\frac{f^{(4)}(c)}{24}\right| = \left|-\frac{\frac{15}{16}c^{(-7/2)}}{24}\right| \le \frac{15}{128 \cdot 3^7} \approx 0.00001786 < 10^{-4}$$
$$\implies \sqrt{10} \approx 3 + \frac{1}{6} - \frac{1}{216} + \frac{1}{3888} \pm \frac{15}{128 \cdot 3^7}$$

Find $\int_0^1 e^{-t^2}\,dt$ with accuracy 10^{-4}

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$
$$e^{-t^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}t^{2n}$$
$$\int_0^1 e^{-t^2}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^1 t^{2n}\,dt = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)}$$
$$|R_k| \le |a_{k+1}| = \frac{1}{(k+1)!(2k+3)} < 10^{-4}$$
$$\implies (k+1)!(2k+3) > 10000$$
$$\implies k \ge 6$$
$$\implies \int_0^1 e^{-t^2}\,dt \approx \sum_{n=0}^{6} \frac{(-1)^n}{n!(2n+1)}$$

$$\sin^2\left(\frac{1}{10}\right), 10^{-5}$$

$$\sin^2(x) = \frac{1-\cos(2x)}{2} = \frac{1}{2} - \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(2x)^{2n} =$$

$$= \frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n)!}(2x)^{2n} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}2^{2n-1}}{(2n)!}x^{2n} =$$

$$= \{k = n-1\} = \sum_{k=0}^{\infty}\frac{(-1)^{k+2}2^{2k+1}}{(2k+2)!}x^{2k+2}$$

$$x = \frac{1}{10}$$

$$a = 0$$

$$\implies \left|R_k\left(\frac{1}{10}\right)\right| \leq |a_{k+1}| = \left|\frac{2^{2k+3}}{(2k+4)!}\cdot\frac{1}{10^{2k+4}}\right| < 10^{-5}$$
