Let
$$f(x) = \cosh x$$

Calculate revolution surface area on [0, 1]

$$A(f) = 2\pi \int_0^1 f(x) \sqrt{1 + (f'(x))^2} \, dx = 2\pi \int_0^1 f^2(x) \, dx = 2\pi \int_0^1 rac{e^{2x} + 2 + e^{-2x}}{4} \, dx = \ = rac{\pi}{2} igg(rac{e^{2x}}{2} + 2x - rac{e^{-2x}}{2} igg) igg)_0^1 = rac{\pi}{2} igg(rac{e^2}{2} + 2 - rac{e^{-2}}{2} igg)$$

$$egin{aligned} \lim_{n o\infty}\sum_{k=1}^nrac{n}{n^2+k^2}&=\lim_{n o\infty}rac{1}{n}\cdotrac{n^2}{n^2+k^2}=\lim_{\lambda(P) o0}S(f,P,C)\ f(c_k)&=f\left(rac{k}{n}
ight)=rac{n^2}{n^2+k^2}=rac{1}{1+rac{k^2}{n^2}}=rac{1}{1+\left(rac{k}{n}
ight)^2}\ &\Longrightarrow f(x)=rac{1}{1+x^2}\ \Longrightarrow \lim_{n o\infty}\sum_{k=1}^nrac{n}{n^2+k^2}=\int_0^1f(x)\,dx=rctan(x)\int\limits_0^1=rac{\pi}{4} \end{aligned}$$

$$\sum_{k=1}^n rac{k}{n^2} \mathrm{sin}\left(rac{k}{n}
ight) = \sum_{i=1}^n rac{1}{n} \cdot rac{k}{n} \mathrm{sin}\left(rac{n}{k}
ight) = \int_0^1 x \sin x \, dx = -x \cos x rac{1}{0} + \int_0^1 \cos x \, dx = -\cos 1 + \sin 1$$

$$\sum_{k=1}^n rac{k}{n^2} \sqrt[n]{e^k} = \int_0^1 x e^x \, dx = (x-1)e^x egin{array}{c} 1 \ 0 \end{array}$$

$$\left(\int_{lpha(x)}^{eta(x)}f(t)\,dt
ight)'=f(eta(x))eta'(x)-f(lpha(x))lpha'(x)$$

$$\int_{x}^{x^3} \sin(t) \cos(t^2) e^t \, dt = \sin(x^3) \cos(x^6) e^{x^3} 3x^2 - \sin(x) \cos(x^2) e^x$$

$$\lim_{x o 0}rac{\left(\int_0^{x^2}\sin(t^2)\,dt
ight)}{x^6}\stackrel{L}{=}\lim_{x o 0}rac{2x\sin(x^4)}{6x^5}=rac{1}{3}\lim_{x o 0}rac{\sin(x^4)}{x^4}=rac{1}{3}$$

$$\int_{e^{-1}}^{e} rac{\ln(x)\sin(\ln(x)+1)}{x} \, dx = egin{cases} t = \ln x + 1 & \Longrightarrow dt = rac{dx}{x} \ t(e) = 2 \ t(e^{-1}) = 0 \end{cases} = \ = \int_{0}^{2} (t-1)\sin t dt \, dx = egin{cases} f(t) = t - 1 & \Longrightarrow f'(t) = 1 \ g'(t) = \sin t & \Longrightarrow g(t) = -\cos t \end{pmatrix} = \ = (1-t)\cos t \, + \sin t \, = -\cos(2) - 1 + \sin(2) \end{cases}$$

$$m \leq f(x) \leq M$$
 $m(b-a) \leq \int_a^b f(x) \, dx \leq M(b-a)$ $f(c) = rac{\left(\int_a^b f(x) \, dx
ight)}{b-a}$

Prove that the following equation has a unique solution on [-1,1]

$$egin{aligned} x &= \int_0^x \sin^{100}t \, dt \ g(x) &= x - \int_0^x \sin^{100}t \, dt \ g(-1) &= -1 + \underbrace{\int_{-1}^0 \sin^{100}t \, dt} \leq 0 \ g(1) &= 1 - \underbrace{\int_0^1 \sin^{100}t \, dt} \geq 0 \ &\Longrightarrow \ \exists c \in [-1,1] : g(c) = 0 \ g'(x) &= 1 - \sin^{100}x > 0 \implies \exists ! c \in [-1,1] : g(c) = 0 \end{aligned}$$