$$egin{aligned} \int_0^1 x\,dx &= \left\{rac{\Delta x_i = rac{1}{m}}{x_i = rac{i}{m}}
ight\} = \lim_{m o\infty} \sum_{i=1}^m rac{f\left(rac{i}{m}
ight)}{m} = \lim_{m o\infty} \sum_{i=1}^m rac{i}{m^2} = \lim_{m o\infty} rac{1}{m^2} \sum_{i=1}^m i = \lim_{m o\infty} rac{1}{m^2} \cdot rac{m(m+1)}{2} = \lim_{m o\infty} rac{m^2+m}{2m^2} = rac{1}{2} \end{aligned}$$

$$egin{split} \int_0^5 (5-x)\,d &= \left\{ egin{aligned} \delta x_i &= rac{5}{m} \ x_i &= rac{5i}{m} \end{aligned}
ight\} = \lim_{m o\infty} \sum_{i=1}^m \left(5-rac{5i}{m}
ight) \cdot rac{5}{m} = \ &= \lim_{m o\infty} \sum_{i=1}^m rac{25}{m} - \sum_{i=1}^m rac{25i}{m^2} = 25 - \lim_{m o\infty} rac{25}{m^2} \sum_{i=1}^m i = rac{25}{2} \end{split}$$

$$egin{aligned} \int_0^1 x^2 \, dx &= egin{cases} \Delta x_i = rac{1}{m} \ x_i = rac{i}{m} \end{pmatrix} = \lim_{m o \infty} \sum_{i=1}^m rac{i^2}{m^3} = \lim_{m o \infty} rac{1}{m^3} \sum_{i=1}^m i^2 = \ &= \lim_{m o \infty} rac{m(m+1)(2m+1)}{6m^3} = \lim_{m o \infty} rac{2m^3 + 3m^2 + m}{m^3} = rac{1}{3} \end{aligned}$$

$$\int_3^5 x \, d = \left\{ rac{\Delta x_i = 3 + rac{2}{m}}{x_i = rac{2i}{m}}
ight\} = \lim_{m o \infty} \sum_{i=1}^m rac{6}{m} + rac{4i}{m^2} = \lim_{m o \infty} 6 \sum_{i=1}^m rac{1}{m} + rac{4m^2 + 4m}{2m^2} = = 6 + 2 = 8$$

$$\int_0^1 \sqrt{x} \, dx = \left\{ egin{aligned} \Delta x_i &= rac{1}{m} \ x_i &= rac{i}{m} \end{aligned}
ight\} = \lim_{m o \infty} \sum_{i=1}^m \sqrt{rac{i}{m}} rac{1}{m} = \lim_{m o \infty} rac{1}{m\sqrt{m}} \sum_{i=1}^m \sqrt{i} = ??? \ \int_0^1 \sqrt{x} \, dx &= \left\{ egin{aligned} \Delta x_i &= rac{2i-1}{m^2} \ x_i &= rac{i^2}{m^2} \end{aligned}
ight\} = \lim_{m o \infty} \sum_{i=1}^m rac{i(2i-1)}{m^3} = \lim_{m o \infty} rac{2}{m^3} \sum_{i=1}^m i^2 - rac{1}{m^3} \sum_{i=1}^m i = \lim_{m o \infty} rac{2(2m^3 + 3m^2 + m)}{6m^3} - rac{m^2 + m}{2m^3} = \lim_{m o \infty} rac{4m^3 + 3m^2 - m}{6m^3} = rac{2}{3}$$

$$egin{aligned} D(x) &= egin{cases} 0 & x \in \mathbb{Q} \ 1 & x
otin \mathbb{Q} \end{cases} \ & x_i \in \mathbb{Q} \implies \sum_{i=1}^m rac{1}{m} \cdot 0
ightarrow 0 \ & x_i
otin \mathbb{Q} \implies \sum_{i=1}^m rac{1}{m} \cdot 1
ightarrow 1 \end{aligned}$$

 $\implies D(x)$ is not Riemann-integrable

$$f(x) = egin{cases} rac{1}{x^2} & x>0 \ 0 & x=0 \end{cases} ext{on } [0,1]$$

f is not bounded at $0 \implies f$ is not integrable on [0,1]

$$egin{split} \int_{1}^{4}rac{1}{x^{3}}\,dx &= \left\{ egin{aligned} \Delta x_{i} = 4^{i/n}\left(1 - rac{1}{4^{1/n}}
ight)
ight\} = \lim_{n o \infty} \sum_{i=1}^{n}rac{1}{4^{3i/n}} \cdot 4^{i/n}\left(1 - rac{1}{4^{1/n}}
ight) = \ &= \lim_{n o \infty}\left(1 - rac{1}{4^{1/n}}
ight) \cdot \sum_{i=1}^{n}rac{1}{4^{2i/n}} = \lim_{n o \infty}\left(1 - rac{1}{4^{1/n}}
ight) \cdot rac{1}{4^{2/n}} \cdot rac{\left(rac{1}{4^{2}} - 1
ight)}{rac{1}{4^{2/n}} - 1} = \ &\lim_{n o \infty}rac{rac{1}{4^{2/n}}\left(1 - rac{1}{16}
ight)}{1 + 4^{1/n}} = rac{15}{32} \end{split}$$