Uniform convergence of power series #lemma

$$\sum_{n=0}^{\infty} a_n(x-a)^n \text{ converges at least pointwise on } (a-R, a+R)$$

$$\text{By Weierstrass M-test, absolute convergence implies uniform convergence}$$

$$\implies \forall [b,c] \subset (a-R, a+R) : \sum_{n=0}^{\infty} a_n(x-a)^n \text{ converges uniformly on } [b,c]$$

Differentiation and integration of power series #lemma

$$\text{Differentiation and integration of power series does not affect the convergence radius}$$

$$\sum_{n=0}^{\infty} \frac{n}{2^n} = \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\implies \sum_{n=0}^{\infty} nx^{n-1} = \frac{1}{(1-x)^2}$$

$$\implies \sum_{n=0}^{\infty} nx^n = \frac{x}{(1-x)^2}$$

$$\implies \sum_{n=0}^{\infty} n\left(\frac{1}{2}\right)^n = \frac{\frac{1}{2}}{\left(1-\left(\frac{1}{2}\right)\right)^2} = 2$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad x \in (-1,1)$$

$$\implies \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = -\ln|1-x|$$

$$\text{Series converges for } x=-1$$

$$\text{But does it converge to } -\ln|1-x|?$$

$$\text{Turns out yes!}$$

$$S(-1) = \lim_{x\to -1} S(x) = \lim_{x\to -1} -\ln|1-x| = -\ln(2)$$

Uniform convergence on convergence interval edges #lemma

$$\text{Power series converges uniformly on edges of convergence interval too!}$$

Taylor series

$$\text{Equality "series = function" allows us to calculate sums}$$

For example

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad x \in \mathbb{R}$$

$$e = \sum_{n=0}^{\infty} \frac{1}{n!}$$
