$$a_n: \mathbb{N} o \mathbb{R} \ f_n: \mathbb{N} o \mathbb{R}^D$$

## Pointwise convergence

$$egin{aligned} f_n & o f \ orall x \in D: orall arepsilon > 0: \exists N_{x,arepsilon}: orall n \geq N_{x,arepsilon}: |f_n(x) - f(x)| < arepsilon \end{aligned}$$

$$f_n(x)=n(\sqrt[n]{x}-1) \ \lim_{n o\infty}f_n(x)=\lim_{n o\infty}rac{\sqrt[n]{x}-1}{rac{1}{n}}\stackrel{t=rac{1}{n}}{=}\lim_{t o0}rac{x^t-1}{t}=\ln(x)$$

$$f_n(x)=rac{n^2}{1+n^2x^2} \ \lim_{n o\infty}f_n(x)=\lim_{n o\infty}rac{1}{rac{1}{n^2}+x^2}-=rac{1}{x^2}$$

 $ext{Let } \{f_n(x)\} o f(x) ext{ on } [a,b] \ ext{Let } orall n, x \in [a,b]: f_n(x) ext{ is bounded} \ ext{Then not necessarily } f ext{ is bounded}$ 

## **Uniform convergence**

$$egin{aligned} f_n &
ightrightarrows f \ orall arepsilon > 0: \exists N_arepsilon: orall n \geq N_arepsilon: orall x \in D: |f_n(x) - f(x)| < arepsilon \ \end{aligned}$$
 Let  $d_n = \sup_{x \in D} |f_n(x) - f(x)| \ f_n 
ightharpoonup f \iff d_n 
ightharpoonup 0$ 

 $f_n 
ightrightarrows f$  and  $f_n$  is continuous  $\implies f$  is continuous

$$ext{Let } f_n 
ightharpoonup f ext{ and } f_n(x) ext{ be bounded} \ 
ightharpoonup f ext{ is bounded} \ ext{And exists } M ext{ that bounds } f_n ext{ for every } n \ ext{} orall n \geq N: |f_n| \leq |f_n - f| + |f - f_N| + |f_N| < 2 + M_N \ 
ightharpoonup orall n \in \mathbb{N}: |f_n| \leq \max\{M_1, M_2, \ldots, M_{N-1}, 2 + M_N\} \$$

$$f_n(x) = rac{x}{1+n^2x^2} \ f_n o 0 \ d_n = \sup_{x \in [0,1]} rac{x}{1+n^2x^2} = \max_{x \in [0,1]} rac{x}{1+n^2x^2} \ x = 0 \implies d_n = 0 \ x = 1 \implies d_n = rac{1}{1+n^2} \ 0 = \left(rac{x}{1+n^2x^2}
ight)' = rac{(1+n^2x^2)-2n^2x^2}{(1+n^2x^2)^2} = rac{1-n^2x^2}{(1+n^2x^2)^2} \ \implies x^2 = rac{1}{n^2} \implies x = rac{1}{n} \ f_n\left(rac{1}{n}
ight) = rac{1}{2n} \ \implies d_n = rac{1}{2n} o 0 \ \implies f_n \Rightarrow 0 \ \implies f_n \Rightarrow 0$$