$$\int \frac{x^3}{x^2-3x+3}dx = \int (x+3)dx + 3\int \frac{2x-3}{x^2-3x+3}dx =$$
$$= \frac{x^2}{2} + 3x + 3\ln|x^2-3x+3| + C$$

$$\int \frac{x}{x^2-4x+8}dx = \frac{1}{2}\int \frac{2x-4}{x^2-4x+8}dx + \frac{1}{2}\int \frac{4}{x^2-4x+8}dx =$$
$$= \frac{1}{2}\ln|x^2-4x+8| + 2\int \frac{1}{(x-2)^2+2^2}dx =$$
$$= \frac{1}{2}\ln|x^2-4x+8| + \arctan\left(\frac{x-2}{2}\right) + C$$

$$\int \frac{x+3}{x^2-3x-40}dx = \int \frac{x+3}{(x+5)(x-8)}dx =$$
$$= \int \frac{A}{x+5}dx + \int \frac{B}{x-8}dx =$$
$$\begin{cases} A+B=1 \\ -8A+5B=3 \end{cases} \implies \begin{cases} A=\frac{2}{13} \\ B=\frac{11}{13} \end{cases}$$
$$\implies \int \frac{x+3}{x^2-3x-40}dx = \frac{2}{13}\ln|x+5| + \frac{11}{13}\ln|x-8| + C$$

$$\int \frac{x^3-2}{x^4-x}dx = \int \frac{x^3-2}{x(x^3-1)}dx = \int \frac{x^3-2}{x(x-1)(x^2+x+1)}dx$$
$$\frac{1}{x(x-1)(x^2+x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{Cx+D}{x^2+x+1}$$
$$x^3-2 = A(x^3-1) + B(x^3+x^2+x) + Cx^3 - Cx^2 + Dx^2 - Dx$$
$$\implies \begin{cases} A+B+C=1 \\ B-C+D=0 \\ B-D=0 \\ A=2 \end{cases} \implies \begin{cases} A=2 \\ B=-\frac{1}{3} \\ C=-\frac{2}{3} \\ D=-\frac{1}{3} \end{cases}$$
$$\implies \int \frac{x^3-2}{x^4-x}dx = 2\int \frac{1}{x}dx - \frac{1}{3}\int \frac{1}{x-1}dx - \frac{1}{3}\int \frac{2x+1}{x^2+x+1}dx =$$
$$= 2\ln|x| - \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x^2+x+1| + C$$

$$\int \frac{x^6+x+1}{x^4+5x^2+4}dx = \cdots = \int x^2 - 5 + \frac{21x^2+x+21}{x^4+5x^2+4}dx =$$
$$= \int (x^2-5)dx + \int \frac{21x^2+x+21}{(x^2+1)(x^2+4)}dx = \ldots$$

$$\begin{array}{c|c} \sqrt{a^2-x^2} & x = a\cdot\sin t \\ \sqrt{a^2+x^2} & x = a\cdot\tan t \\ \sqrt{x^2-a^2} & x = \frac{a}{\cos^2 t} \end{array}$$

For example:

$$\int \frac{1}{x^2\sqrt{4-x^2}}dx$$

$$x = 2\sin t \implies dx = 2\cos t dt$$

$$\int \frac{1}{x^2\sqrt{4-x^2}}dx = \int \frac{2\cos t}{4\sin^2 t\sqrt{4\cos^2 t}}dt = \frac{1}{4}\int \frac{1}{\sin^2 t}dt = -\frac{1}{4}\cot t + C$$

$$\sin^2 t = \frac{x^2}{4} = 1-\cos^2 t \implies \cos t = \frac{\sqrt{1-x^2}}{2}$$

$$\implies \int \frac{1}{x^2\sqrt{4-x^2}}dx = \frac{-\sqrt{1-x^2}}{4x} + C$$

$$\int \frac{1}{\sin x}dx = \left\{ \begin{aligned} t &= \tan\tfrac{x}{2} \\ dx &= \tfrac{2}{1+t^2}dt \\ \sin x &= \tfrac{2t}{1+t^2} \end{aligned} \right\} = \int \frac{1+t^2}{2t}\frac{2}{1+t^2}dt = \int \frac{1}{t}dt =$$

$$= \ln\left|\tan\frac{x}{2}\right| + C$$

$$\int \frac{1}{\cos x}dx = \left\{ \begin{aligned} t &= \tan\tfrac{x}{2} \\ dx &= \tfrac{2}{1+t^2}dt \\ \cos x &= \tfrac{1-t^2}{1+t^2} \end{aligned} \right\} = \int \frac{1+t^2}{1-t^2}\frac{2}{1+t^2}dt = \int \frac{2}{1-t^2}dt =$$

$$= \ln|1-t| + \ln|1+t| + C$$
