Let $f(x) = \cosh x$

Calculate revolution surface area on $[0,1]$

$$A(f) = 2\pi \int_0^1 f(x)\sqrt{1+(f'(x))^2}\,dx = 2\pi \int_0^1 f^2(x)\,dx = 2\pi \int_0^1 \frac{e^{2x}+2+e^{-2x}}{4}\,dx =$$

$$= \frac{\pi}{2}\left(\frac{e^{2x}}{2} + 2x - \frac{e^{-2x}}{2}\right)\Bigg|_0^1 = \frac{\pi}{2}\left(\frac{e^2}{2} + 2 - \frac{e^{-2}}{2}\right)$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2} = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{n^2}{n^2+k^2} = \lim_{\lambda(P)\to 0} S(f,P,C)$$

$$f(c_k) = f\left(\frac{k}{n}\right) = \frac{n^2}{n^2+k^2} = \frac{1}{1+\frac{k^2}{n^2}} = \frac{1}{1+\left(\frac{k}{n}\right)^2}$$

$$\implies f(x) = \frac{1}{1+x^2}$$

$$\implies \lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2} = \int_0^1 f(x)\,dx = \arctan(x)\Big|_0^1 = \frac{\pi}{4}$$

$$\sum_{k=1}^{n} \frac{k}{n^2} \sin\left(\frac{k}{n}\right) = \sum_{i=1}^{n} \frac{1}{n} \cdot \frac{k}{n} \sin\left(\frac{n}{k}\right) = \int_0^1 x \sin x\,dx = -x\cos x\Big|_0^1 + \int_0^1 \cos x\,dx =$$

$$= -\cos 1 + \sin 1$$

$$\sum_{k=1}^{n} \frac{k}{n^2} \sqrt[n]{e^k} = \int_0^1 x e^x\,dx = (x-1)e^x\Big|_0^1 = 1$$

$$\left(\int_{\alpha(x)}^{\beta(x)} f(t)\,dt\right)' = f(\beta(x))\beta'(x) - f(\alpha(x))\alpha'(x)$$

$$\int_x^{x^3} \sin(t)\cos(t^2)e^t\,dt = \sin(x^3)\cos(x^6)e^{x^3}3x^2 - \sin(x)\cos(x^2)e^x$$

$$\lim_{x\to 0} \frac{\left(\int_0^{x^2} \sin(t^2)\,dt\right)}{x^6} \overset{L}{=} \lim_{x\to 0} \frac{2x\sin(x^4)}{6x^5} = \frac{1}{3} \lim_{x\to 0} \frac{\sin(x^4)}{x^4} = \frac{1}{3}$$

$$\int_{e^{-1}}^{e} \frac{\ln(x)\sin(\ln(x)+1)}{x}\,dx = \left\{\begin{matrix} t = \ln x + 1 \implies dt = \frac{dx}{x} \\ t(e) = 2 \\ t(e^{-1}) = 0 \end{matrix}\right\} =$$

$$= \int_0^2 (t-1)\sin t dt\,dx = \left\{\begin{matrix} f(t) = t-1 \implies f'(t) = 1 \\ g'(t) = \sin t \implies g(t) = -\cos t \end{matrix}\right\} =$$

$$= (1-t)\cos t\Big|_0^2 + \sin t\Big|_0^2 = -\cos(2) - 1 + \sin(2)$$

$$m \leq f(x) \leq M$$

$$m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$$

$$f(c) = \frac{\left(\int_a^b f(x)\,dx\right)}{b-a}$$

Prove that the following equation has a unique solution on $[-1,1]$

$$x = \int_0^x \sin^{100} t\,dt$$

$$g(x) = x - \int_0^x \sin^{100} t\,dt$$

$$g(-1) = -1 + \underbrace{\int_{-1}^0 \sin^{100} t\,dt}_{\leq 1} \leq 0$$

$$g(1) = 1 - \underbrace{\int_0^1 \sin^{100} t\,dt}_{\geq -1} \geq 0$$

$$\implies \exists c \in [-1,1] : g(c) = 0$$

$$g'(x) = 1 - \sin^{100} x > 0 \implies \exists! c \in [-1,1] : g(c) = 0$$
