$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$$

$$\lim_{n\to\infty} \frac{\frac{2^{n+1}x^{n+1}}{(n+1)!}}{\frac{2^n x^n}{n!}} = \lim_{n\to\infty} \frac{2x}{n+1} = 0$$

$$\forall x \in \mathbb{R} : \sum_{n=0}^{\infty} \frac{2^n x^n}{n!} \to S(x)$$

$$\sum_{n=1}^{\infty} \frac{\sin(n!x)}{n^3+n+1}, x \in [-2\pi, 2]$$

$$\left|\frac{\sin(n!x)}{n^3+n+1}\right| \le \frac{1}{n^3+n+1} \le \frac{1}{n^3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \to M \implies \sum_{n=1}^{\infty} \frac{\sin(n!x)}{n^3+n+1} \rightrightarrows S(x)$$

$$\sum_{n=2}^{\infty} \frac{x^4+x^2}{n\ln^2(n)}, x \in [-7, 2]$$

$$\left|\frac{x^4+x^2}{n\ln^2(n)}\right| \le \frac{7^4+7^2}{n\ln^2(n)}$$

$$\sum_{n=2}^{\infty} \frac{1}{n\ln^2(n)} \to M \text{ by the Cauchy's condensation test}$$

$$\implies \sum_{n=2}^{\infty} \frac{x^4+x^2}{n\ln^2(n)} \rightrightarrows S(x)$$

$$\sum_{n=0}^{\infty} \frac{\sin(x)}{(1+x)^n}$$

$$\sum_{n=0}^{\infty} \frac{\sin(x)}{(1+x)^n} = \sin(x) \cdot \sum_{n=0}^{\infty} t^n = \frac{1}{1-\frac{1}{1+x}} = \sin(x) \cdot \frac{1+x}{x}$$

$$\text{Let } f(x) = \begin{cases} \sin(x) \cdot \frac{1+x}{x} & x \ne 0 \\ 0 & x = 0 \end{cases}$$

$$f \text{ is not continuous} \implies x \in [0, \pi) \implies \sum_{n=0}^{\infty} \frac{\sin(x)}{(1+x)^n} \not\rightrightarrows f(x)$$

$$\text{Let } d_N = \sup_{x\in(0,\pi)} |S_N(x) - f(x)|$$

$$d'_N = \sup_{x\in[0,\pi)} |S_N(x) - f(x)| = \max\{d_N, |S_N(0) - f(0)|\} = d_N$$

$$d'_n \not\to 0 \implies d_n \not\to 0 \implies x \in (0, \pi) \implies \sum_{n=0}^{\infty} \frac{\sin(x)}{(1+x)^n} \not\rightrightarrows f(x)$$

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$$

$$\text{Let } x = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \cdot x^n = \sum_{n=1}^{\infty} \int_0^x t^{n-1}\, dt \underset{\sum_{n=1}^{\infty} t^{n-1} \rightrightarrows S(x)}{\overset{t\in[-\frac{1}{2},\frac{1}{2}]}{=}} \int_0^x \sum_{n=1}^{\infty} t^{n-1}\, dt = \int_0^x \frac{1}{1-t}\, dt = -\ln|1-x|$$

$$\implies \sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n} = \ln\left(\frac{3}{2}\right)$$

$$\sum_{n=1}^{\infty} nx^n$$

$$\sum_{n=1}^{\infty} nx^n = x \cdot \sum_{n=1}^{\infty} nx^{n-1} = \frac{x}{(1-x)^2}$$