# Rayleigh, the Unit for Light Radiance

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A 0.7% accurate formula is derived for the easy conversion of power spectral radiance $L_\lambda$ in W cm$^{-2}$ sr$^{-1}$ $\mu$m$^{-1}$ to rayleigh spectral radiance $R_\lambda$ in rayleigh/$\mu$m, $R_\lambda = 2\pi\lambda L_\lambda \times 10^{13}$, where the wavelength $\lambda$ is in $\mu$m. The rationale for the rayleigh unit is discussed in terms of a photon rate factor and a solid angle factor. The latter is developed in terms of an equivalence theorem about optical receivers and extended sources, and the concept is extended to the computation of photon volume emission rates from altitude profiles of zenith radiance.

## Introduction

The rayleigh, named in honor of the fourth Baron Rayleigh (R. J. Strutt, 1875–1947) who made the first measurement of the absolute intensity of the night airglow,[1] is a unit of measure for the absolute angular surface brightness of spatially extended light-emitting sources. (The rayleigh, like the watt, does not depend upon the color response of the human eye.) In the score of years since its introduction by Hunten *et al.*[2] the rayleigh, kilorayleigh, and megarayleigh have come into widespread use among aeronomists. However, the meaning and practicality of rayleigh remains unfamiliar to many workers in applied optics and, in particular, to much of the infrared community. The purpose of this paper is to make available to them an easily remembered conversion formula, and to give the rationale behind the unit.

## Conversion Formula

Pragmatically stated, the rayleigh is simply a unit for expressing radiance or spectral radiance in terms of rate of photon emission per unit of surface area taking into account a solid angle rationalization factor. To convert from power spectral radiance, which has the symbol $L_\lambda$ (or $N_\lambda$), in watt centimeter$^{-2}$ steradian$^{-1}$ micrometer$^{-1}$ (W cm$^{-2}$ sr$^{-1}$ $\mu$m$^{-1}$) to rayleigh spectral radiance $R_\lambda$ in rayleighs per micrometer (R/$\mu$m) one can use the relation

$$R_\lambda = 1.986486\pi\lambda L_\lambda \times 10^{13} \ \mathrm{R}/\mu\mathrm{m}, \qquad (1a)$$

where the wavelength $\lambda$ is expressed in $\mu$m. (In many quarters the term steradiance is coming into usage in lieu of radiance thereby giving emphasis to the solid angle normalization inherent in the entity.)

The coefficient may be rounded off to 2 giving the following more easily remembered formula which is accurate to better than 0.7%:

$$\boxed{R_\lambda = 2\pi\lambda L_\lambda \times 10^{13} \ \mathrm{R}/\mu\mathrm{m}.} \qquad (1b)$$

The inverse formula is

$$L_\lambda = (R_\lambda/2\pi\lambda) \times 10^{-13} \ \mathrm{W\ cm^{-2}\ sr^{-1}\ \mu m^{-1}}. \qquad (2)$$

These formulas are all that are needed to convert from or into rayleighs from units about which the user is more familiar. For those with a more abiding interest, a discussion will now be given about the rationale behind use of the rayleigh and the simple derivation of the conversion formulas.

## Rationale

### Photon Rate Factor

Conceptually, the aeronomists desired to have a radiance unit that would be a measure of the rate at which photons coming down from a patch of the sky would strike each square centimeter of normal area. The reason for using photons per second instead of watts is that in gaseous photochemistry (the word photophysics may be more appropriate) the photon is conveniently treated statistically along with the concentrations of the other particles of the medium. That is to say, in the statistical model it is easier to use photons/sec and molecules/cm$^3$ than it is to use watts and g/cm$^3$. The factor for conversion of photons per second into watts of light energy is simply $hc/\lambda$ times the photon rate, where the constants have their usual meanings.

The number of photons per second per square centimeter per steradian from the quiet night airglow in the visible region of the spectrum is of the order of a million per angstrom. Thus, to render the unit of one rayleigh into the ballpark of the spectral radiances seen by aeronomists, the rayleigh has been defined in terms of megaphotons. The intensities of airglow phenomena are of the order of rayleighs, au-

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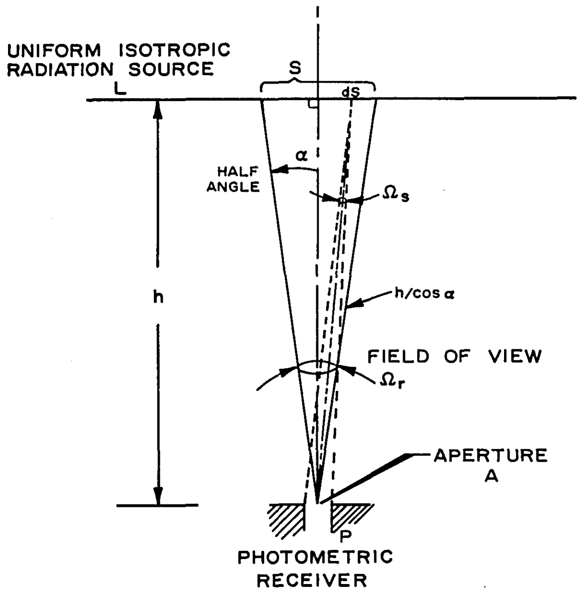


Fig. 1. Geometry for uniform extended radiation source viewed by photometric receiver with an ideal field of view.

roral emissions are generally on the order of kilorayleighs, and thermal radiances are of the order of megarayleighs.

## Solid Angle Factor

The second, more subtle, conceptual reason leading to the adoption of the rayleigh involves an equivalence theorem which might be stated as follows for light receivers looking at extended sources:

*Equivalence Theorem: If the field of view of a photometric instrument is completely and uniformly filled by an extended isotropic radiation source, the power received by the instrument is the triple product of the radiance seen, the aperture area of the instrument, and the field of view solid angle of the instrument, regardless of the distance of the external source or of the relative orientation of instrument and source.*

The situation is depicted in Fig. 1 for the case of no extinction between an emitting source of radiance $L$ in W cm$^{-2}$ sr$^{-1}$ and a receiver with an entrance aperture $A$ in cm$^2$ and field of view $\Omega_r$ in steradians (sr). Even though the $L$ of the source is defined per solid angle away from the source, the power $P$ in watts accepted by the receiver may be computed using the instrument field of view solid angle that extends in the opposite direction.

This equivalence is readily demonstrated as follows for an ideal field of view. The viewing field is assumed to be symmetrical and with infinitely sharp cutoffs of half angle $\alpha$. That is, the responsivity of the instrument is constant at full value for viewing angles less than $\alpha$ and is zero for greater angles.

The power accepted through the receiver aperture $A$ from that area $S$ of the uniform extended source $L$ that is in the field of view is

$$P = L \int_S \Omega_s dS \quad \text{(W)}, \tag{3}$$

where (see Fig. 1)

$$\Omega_s = \frac{A\cos\alpha}{(h/\cos\alpha)^2} = \frac{A\cos^3\alpha}{h^2} \tag{4}$$

is the solid angle subtended by the aperture at the location of the incremental emitting surface area

$$dS = 2\pi r dr = 2\pi(h\tan\alpha)(h d\alpha/\cos^2\alpha)$$
$$= 2\pi h^2(\sin\alpha/\cos^3\alpha)d\alpha. \tag{5}$$

Therefore,

$$P = L\int_0^\alpha \frac{A\cos^3\alpha}{h^2} 2\pi h^2 \frac{\sin\alpha}{\cos^3\alpha} d\alpha$$
$$= LA\, 2\pi \int_0^\alpha \sin\alpha\, d\alpha = LA\, 2\pi(1-\cos\alpha). \tag{6}$$

The factor $2\pi(1-\cos\alpha)$ computed above just happens to be the conical solid angle

$$\Omega_r = \frac{S}{h^2} = \int_0^{2\pi} d\phi \int_0^\alpha \sin\alpha\, d\alpha$$
$$= 2\pi(1-\cos\alpha) \quad \text{(sr)} \tag{7}$$

about the optical axis of the instrument defined by the maximum acceptance angle $\alpha$ of a circularly symmetrical ideal field of view. [The more familiar field of view formula

$$\Omega_r = \pi\sin^2\alpha$$

is an approximation for small angles (within 1% for $\alpha$ <11°).] Consequently, the received power is

$$P = LA\Omega_r \quad \text{(W)}. \tag{8}$$

Thus, the received power may be computed as the

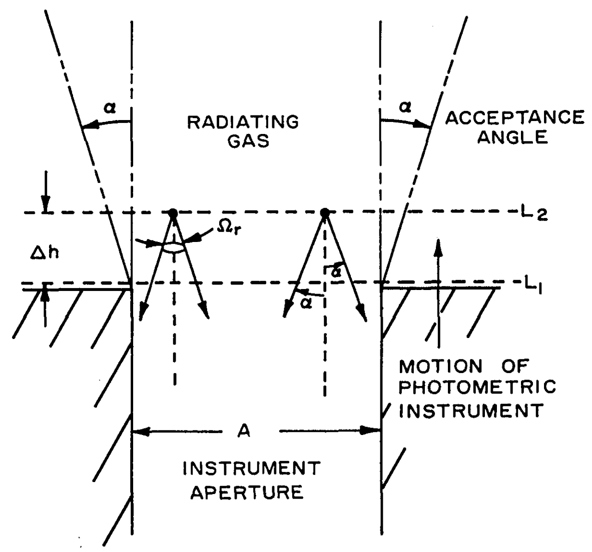


Fig. 2. Close-up geometry of an instrument acceptance aperture moving through a radiating gas.

product of the source radiance and the optical throughput $A\Omega_r$ of the instrument.

### Volume Emission Rates

Using the equivalence theorem, we can derive the volume photon-emission rate vs distance from the changing radiance seen by a photometric instrument as it moves through a radiating gas. Thereby we discover another important computational utility of the rayleigh unit.

The geometry of Fig. 1 is depicted on a much closer scale in Fig. 2. By the previously stated equivalence theorem the power received by the detector of a photometric instrument is independent of the distance of the isotropic radiant source observed to uniformly fill the field of view. Therefore, we will consider the emissions to be originating from immediately outside the input aperture $A$. Thus, we can neglect the divergence of the field of view of the instrument off into space. That is to say, the small fringing effect around the edges of the instrument aperture in Fig. 2 may be ignored.

Notice that the notion of acceptance angle very succinctly characterizes the concept of field of view. In the ideal sharp field case, only those photons with the acceptance angle $\alpha$ or less make it through the field of view determining components (field stop and baffles) all the way to the detector.

Referring again to Fig. 2 and using Eq. (8), the power being received when the instrument is at position 1 is $L_1A\Omega_r$. After the instrument has moved a distance $\Delta h$ to position 2 the power being received has changed to $L_2A\Omega_r$. For an optically thin medium $L_1 > L_2$. Thus, the change of received power per incremental volume swept out by the moving aperture is

$$\frac{\Delta P}{\Delta V} = \frac{L_2 A\Omega_r - L_1 A\Omega_r}{A\,\Delta h}$$

$$= \frac{\Omega_r}{\Delta h}(L_2 - L_1) = -\Omega_r\frac{\Delta L}{\Delta h}\ (\mathrm{W/cm^3}). \qquad (9)$$

Now if the radiating gas is optically thin (i.e., if extinction and/or absorption and reemission can be ignored), then the ratio of the power radiated isotropically from the molecules in the volume $\Delta V = A\Delta h$ to that within the acceptance solid angle $\Omega_r$ of the instrument is $4\pi(\mathrm{sr})/\Omega_r$. Consequently, the total power per incremental volume is

$$u = \frac{\Delta P_t}{\Delta V} = -\frac{4\pi}{\Omega_r}\Omega_r\frac{\Delta L}{\Delta h} = -4\pi\frac{\Delta L}{\Delta h}\ (\mathrm{W/cm^3}). \qquad (10)$$

Note that although this relation looks like it still has an inverse steradian in it, it actually does not as the $4\pi$ carries a steradian with it.

Thus, if the radiance function $L(h)$ is experimentally observed, one can compute the volume emission rate function $u(h)$ simply by differentiating and multiplying by $4\pi$,

$$u(h) = -4\pi(dL(h)/dh)\ \ (\mathrm{W/cm^3}). \qquad (11)$$

If the $4\pi$ coefficient is absorbed into the unit for rayleigh radiance $R$, then

$$u(h) = -(dR(h)/dh)\ (\text{megaphotons sec}^{-1}\ \text{cm}^{-3}). \qquad (12)$$

This rationalization is incorporated in the synthesis of the rayleigh unit.

A concluding observation with regard to viewing uniform extended sources with specific field of view instruments is suggested by Fig. 2. One could think of projecting the aperture area $A$ of the instrument all the way into the sky and equivalently treat the radiation as all coming down parallel from a column of constant cross section through the atmosphere. The units would be W cm$^{-2}$ (column) or megaphotons sec$^{-1}$ cm$^{-2}$ (column). Such an equivalent, but ofttimes confusing, way of stating the unit set fits nicely into the scheme of Eqs. (11) and (12) for computing the altitude profile of volume emission rate from a measurement of zenith (overhead) radiance vs altitude for horizontally uniform (stratified) emission layers. An example is given in Fig. 3.[4] Differentiate the vertical radiance profile in megaphotons sec$^{-1}$ cm$^{-2}$ (column), that is, in rayleighs, with respect to altitude and you have the volume emission rate directly in megaphotons sec$^{-1}$ cm$^{-3}$. Since the slope of the radiance profile is negative, a positive volume emission rate results from Eq. (12).

### Derivation

With the foregoing rationale in mind, it is now a simple matter to derive the conversion formulas [Eqs. (1) and (2)] for converting between power spectral radiance $L_\lambda$ and rayleigh spectral radiance $R_\lambda$. This proceeds as follows. The units are given with each equation to clarify the meaning of each entity.

The photon spectral radiance $F_\lambda$ properly has units of photons sec$^{-1}$ cm$^{-2}$ sr$^{-1}$ $\mu$m$^{-1}$. Therefore the megaphoton spectral radiance $I_\lambda$, as used by Chamberlain,[3] is

$$I_\lambda = 10^{-6}F_\lambda\ \ (\text{megaphotons sec}^{-1}\text{cm}^{-2}\text{sr}^{-1}\mu\text{m}^{-1}). \qquad (13)$$

The rayleigh spectral radiance $R_\lambda$, taking into account rationalization of the $4\pi$ factor from Eq. (11) to get Eq. (12) for volume emission rate, is

$$R_\lambda = 4\pi I_\lambda = 4\pi\times10^{-6}F_\lambda\ \ (\mathrm{R}/\mu\mathrm{m}). \qquad (14)$$

The relation for the energy of a photon of wavelength $\lambda$ in $\mu$m is

$$E_\lambda = \frac{hc}{(\lambda\times10^{-4})}\ (\mathrm{J}), \qquad (15)$$

where $h$ is Planck's constant,[5]

$$h = 6.626196\times10^{-34}\ \mathrm{J\text{-}sec}, \qquad (16)$$

and $c$ is the speed of light in a vacuum

$$c = 2.9979250\times10^{10}\ \mathrm{cm/sec}. \qquad (17)$$

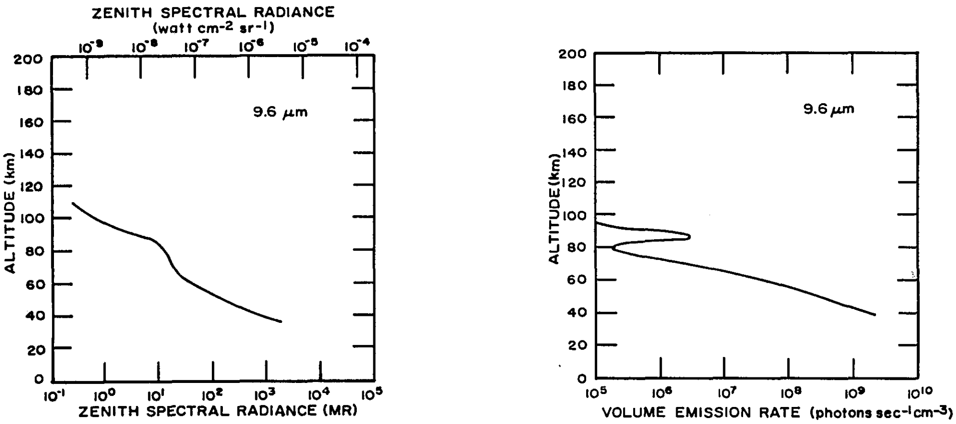


Fig. 3. Example of the results of a computation (assuming optically thin horizontally uniform conditions) of the volume emission rate altitude distribution (right-hand figure) computed from a zenith spectral radiance profile (left-hand figure) experimentally measured from a rocket.

Therefore, the power spectral radiance $L_\lambda$ in W cm$^{-2}$ sr$^{-1}$ $\mu$m$^{-1}$ is, from Eqs. (13) and (14),

$$L_\lambda = \frac{hc}{\lambda \times 10^{-4}} F_\lambda = \frac{hc}{\lambda \times 10^{-4}} \frac{R_\lambda}{4\pi \times 10^{-6}}$$
$$= \frac{(6.626196 \times 10^{-34})(2.9979250 \times 10^{10})}{(10^{-4})(4\pi \times 10^{-6})} \frac{R_\lambda}{\lambda} \quad (18)$$
$$\cong \frac{R_\lambda}{2\pi\lambda} \times 10^{-13} \text{ (W cm}^{-2}\text{sr}^{-1}\mu\text{m}^{-1}).$$

Inversely,

$$R_\lambda = 2\pi\lambda L_\lambda \times 10^{13} \text{ (R}/\mu\text{m)}. \quad (19)$$

If the spectral radiance is expressed in terms of frequency (vacuum wavenumber), Eq. (19) becomes

$$R_\nu = (2\pi/\nu) L_\nu \times 10^{9} \text{ (R/cm}^{-1}), \quad (20)$$

where the wavenumber $\nu$ is expressed in reciprocal centimeters (cm$^{-1}$).

## Monochromatic and Nonmonochromatic Radiation

A final observation is that for a band $\Delta\lambda$ of radiation which is reasonably monochromatic, then the rayleigh radiance

$$R = R_\lambda \Delta\lambda \text{ (R)} \quad (21)$$

can be converted directly from power radiance as given by

$$L = L_\lambda \Delta\lambda \text{ (W cm}^{-2}\text{sr}^{-1}). \quad (22)$$

The conversion formula to rayleighs is simply

$$R = 2\pi\lambda L \times 10^{13} \text{ (R)}, \quad (23)$$

and inversely

$$L = \frac{R}{2\pi\lambda} \times 10^{-13} \text{ (W cm}^{-2}\text{sr}^{-1}), \quad (24)$$

where $\lambda$ in $\mu$m is taken as the wavelength at the center of the spectral band $\Delta\lambda$.

Even if the radiation is not quasimonochromatic, however, the rayleigh radiance in units of $4\pi$ megaphotons sec$^{-1}$ cm$^{-2}$ sr$^{-1}$ is still valid. In other words, rayleigh has meaning for nonmonochromatic radiation, but the conversion formulae [Eqs. (23) and (24)] break down. In this situation conversion Formula (24) would necessarily be replaced by a summation or an integral form, such as

$$L = \sum_{i=1}^{n} \frac{R_{\lambda_i}\Delta\lambda_i}{2\pi\lambda_i} \times 10^{-13}$$
$$= \int_{\lambda_0}^{\lambda_n} \frac{R_\lambda}{2\pi\lambda} \times 10^{-13} d\lambda \text{ (W cm}^{-2}\text{sr}^{-1}). \quad (25)$$

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