# Derivation of Geographic Coordinate System equations

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## 1 dome-master position equations in the paper

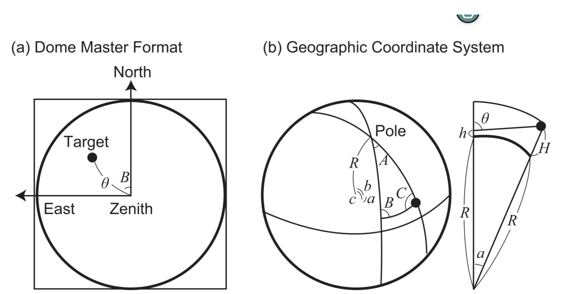



**Fig. 3.** Dome-master format **(a)** and corresponding geographic coordinate system **(b)**, where $\theta$ is the zenith angle, $B$ is the azimuth angle from the north, and $A$ is the longitude between the target and observer. See the text for details.

A is the longitude between the target and observer, B is the azimuth angle from the north of the target, R is the Earth radius, h is the altitude of observer, H is the altitude of the target, a is the angle between two points and the center of the earth, b is co-latitude of the target, c is the co-latitude of the observation point.

unkowns:$\theta$,B,a knowns:R,H,h,b,c,A

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{1}$$

$$\cos B = \frac{(\cos b \sin c - \sin b \cos c \cos A)}{\sin a} \tag{2}$$

$$\tan\theta = \frac{(R+H)\sin a}{(R+H)\cos a - (R+h)} \tag{3}$$

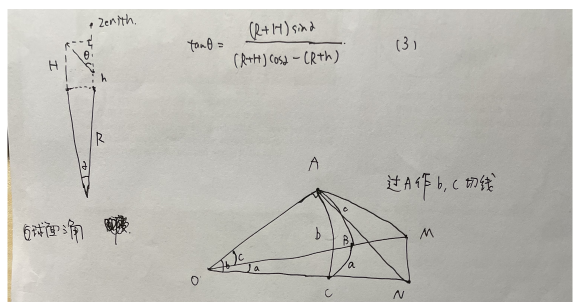
unkowns:b,A,a knowns:$\theta$,B,R,H,h,c

$$\cos b = \cos c \cos a + \sin c \sin a \cos B \tag{4}$$

$$\sin A = \frac{\sin B \sin a}{\sin b} \tag{5}$$

$$\sin(\theta - a) = \frac{\sin\theta(R+h)}{R+H} \tag{6}$$

## 2 proof




It can be seen clearly in the picture that equation(3) is easy to prove by using trigonometric formula.

for equation(1) and (4):

In the botom picture, we have:

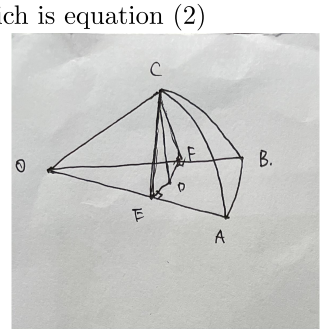
$$MN^2 = OM^2 + ON^2 - 2OMON\cos a$$
$$MN^2 = AM^2 + AN^2 - 2AMAN\cos A$$
$$2OMON\cos a = (ON^2 - AN^2) + (OM^2 - AM^2) +$$
$$2AMAN\cos A$$
$$= OA^2 + OA^2 + 2AMAN\cos A$$
$$\cos a = \frac{OA}{ON}\frac{OA}{OM} + \frac{AN}{ON}\frac{AM}{OM}\cos A$$
$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

which is equation (1) and (4)

use the definition of polar triangle(A,B,C is the three angles) and the above results

$$
\begin{aligned}
&\cos A = -\cos B\cos C + \sin B\sin C\cos A\\
&\cos a = \cos b\cos c + \sin b\sin c\cos A\\
&\cos b = \cos c\cos a + \sin c\sin a\cos B\\
&\sin c\sin a\cos B =\\
&\cos b - \cos c(\cos b\cos c + \sin b\sin c\cos A)\\
&\quad = \cos b - \cos b(\cos c)^2 - \sin b\sin c\cos c\cos A\\
&\quad = \cos b(\sin c)^2 - \sin b\sin c\cos C\cos A\\
&\sin a\cos B = \cos b\sin c - \sin b\cos c\cos A\\
&\cos B = \frac{\cos b\sin c - \sin b\cos c\cos A}{\sin a}
\end{aligned}
$$

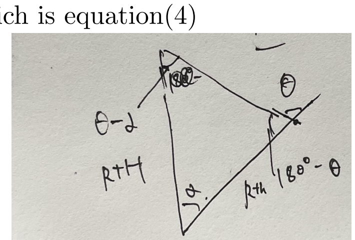
which is equation (2)




From this picture, CED=A,CFD=B

$$
\begin{aligned}
&\frac{\sin a}{\sin A} = \frac{CF/OC}{CD/CE}\\
&\frac{\sin b}{\sin B} = \frac{CE/OC}{CD/CF}\\
&= \frac{CF\cdot CE}{OC\cdot CD}\\
&\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B}
\end{aligned}
$$

which is equation(4)




notice from the above picture that:

$$
\begin{aligned}
&\sin(\theta) = \sin(180-\theta)\\
&\theta - a = 180 - a - (180-\theta)\\
&sinformula\\
&\frac{\sin(\theta - a)}{\sin\theta} = \frac{R+h}{R+H}
\end{aligned}
$$

which is equation(6)