Lamb waves propagation Plotting the dispersion curves

Conference Paper · November 2016

Salah Nissabouri
Faculté des Sciences et Techniques SETTAT

Mhammed El Allami
Centre Régional des Métiers de l'Education et de la Formation de la Région de Set…

Lamb waves propagation

Plotting the dispersion curves

Majda Bakhcha, Salah Nissabouri
Department of physics applied, FST
Settat Morocco
bbakhcha@gmail.com, nissabouri.salah@gmail.com

El Hassan Boutyour, Ahmed Errkik
Labo LMEET, Department of applied physics, FST,
Settat, Morocco
Boutyour.elhassan@gmail.com, ahmed.errkik@uhp.ac.ma

Mhammed El Allami
Labo LMEET, Department of applied physics, FST,
Settat, Morocco
CRMEF Settat
m.elallami@gmail.com

***Abstract* — in this paper, we studied the ultrasonic Lamb waves theory in a thin and isotropic plate. The two equations of the symmetrical and anti-symmetrical modes are computed numerically by Matlab program using the Bisection method in order to plot the dispersion curves: phase velocity, wave number and group velocity versus the product frequency – thickness (f.e), two cases of plates are considered, a steel and aluminum plate.**

Key words: lamb waves, dispersion curves, bisection

I. INTRODUCTION

Non Destructive testing (NDT) is a technique used in industry to evaluate material properties without causing damage. NDT is deployed for materials to control defects, corrosion, welding and other. They are used to detect various forms of defects, in various positions. In our study, we use the ultrasonic Lamb waves.

The Lamb waves are elastic disturbances being propagated in plates of which the thickness is the same order of the wavelength magnitude. These waves are dispersive and they have the ability to put in vibration the totality of the plate, enable to detect internal defects whatever their depth, able to propagate without too much loss of energy.

Lamb waves have also been widely used to detect corrosion [1], to detect defects in composite materials [2,3,4,5], in aluminum [6,7], in railways [8], in welded tubes [9], in roughness solid plates [10] and in the multilayer boards[11].

II. THEORY OF LAMB WAVES

Let's take an isotropic, thin homogeneous plate with thickness (e =2d), placed in the vacuum in the direction of X positive. If the thickness of the plate is lower or equal to the wavelengths of the volume waves, the two waves of Rayleigh will couple and give rise to the Lamb waves, which will put moving the totality of the plate.

A. Rayleigh-Lamb equations

The boundary conditions for the constraints in Z = ±d, lead to the equations of Rayleigh-Lamb [12]:

$$(k^2+s^2)^2 \cosh qd \sinh sd + 4k^2 qs \sinh qd \cosh sd = 0 \quad (1)$$
$$(k^2+s^2)^2 \sinh qd \cosh sd + 4k^2 qs \cosh qd \sinh sd = 0 \quad (2)$$
$$\text{With } s^2 = k^2 - {k_T}^2 \quad (3)$$
$$q^2 = k^2 - {k_L}^2 \quad (4)$$

k is the wave number, k_L is the longitudinal wave number, k_T is the transversal wave number.

The equations (1) and (2) characterize respectively the symmetrical and anti-symmetrical Lamb waves modes.

III. DISPERSION CURVES

Dispersion curves represent the equations of Lamb waves (1) and (2).

In fact, the equations can be plotted with using the wave number k, the phase velocity V_p or the group velocity.

$$k = \frac{\omega}{V_p} \quad (5)$$

With $\omega = 2\pi f$ is the angular frequency, k and V_p depend to the frequency f which gives the Lamb waves a dispersive nature. The knowledge of the dispersion curves is indispensable for analyze the propagating modes in the controlled material.

In order to plot the dispersion curves, there are three ways: the first one is based on the numerical simulations, the second uses the data extracted after an experience, and the third requires a specific code program like Matlab.

In this paper, to compute the dispersion curves, we wrote Matlab code to plot the dispersion curves for steel and aluminum plate.

Stefan Sorohan [13] presents a method, for obtaining all the range of dispersion curves, by numerical simulation only, via common commercial finite element codes. Essentially, the method consists in a few series of modal analyses for a representative part of the inspected structure.
P. Hora [14] reports on methods for determination of Lamb wave dispersion curves by means of Fourier transform (FT). Propagating Lamb waves are sinusoidal in both the frequency domain and the spatial domain. Therefore, the temporal FT may be carried out to go from the time to the frequency domain, and then the spatial FT may be carried out to go to the frequency–wave-number domain, where the amplitudes and the wave-numbers of individual modes may be measured.
F. Schopfer [15] introduces a method which aims at extracting the dispersion curves from laser vibrometer measurement data. This method works by Fourier transforming the measurement data into the wavenumber domain and then applying the matrix pencil method.
M.S. Harb, [16] presents , fully non-contact, hybrid system which encompasses an Air-Coupled Transducer (ACT) and a

 ISSN: 2509-2014

Laser Doppler Vibrometer (LDV) for profiling A0 Lamb wave dispersion of an isotropic aluminum plate. The ACT generates ultrasonic pressure incident upon the surface of the plate. The pressure waves are partially refracted into the plate. The LDV is employed to measure the out-of-plane velocity of the excited Lamb wave mode at some distances where the Lamb waves are formed in the plate.
Pawel Packo [17] presents a method for dispersion curve calculation and analysis of numerical models for guided waves. The proposed approach utilizes the wave equation and through-thickness-only discretization of anisotropic, layered plates to obtain the Lamb wave characteristics.
Farhang Honarvar [18] proposes an alternative method which extracts the solution of the frequency equation in the form of dispersion curves from the three-dimensional illustration of the frequency equation. For this purpose, a three-dimensional representation of the real roots of the frequency equation is first plotted. The dispersion curves, which are the numerical solutions of the frequency equation, are then obtained by a suitable cut in the velocity–frequency plane.

To solve the frequency equations, many researchers use iterative techniques, such as linear [19, 20] or quadratic [21] interpolation [22, 23] or extrapolation [24] algorithms, which are very fast on a single root. However, when two roots are in close proximity, for example near the crossing points of longitudinal mode dispersion curves, the function changes sign twice and such schemes become unstable. Alternatively, the frequency equations may be solved by slower but safer iteration techniques such as Newton–Raphson, Bisection, and Mueller [25]. However, because of the high variety and number of operations, particularly in multilayered media, these methods are difficult, very slow, and time consuming [18].

IV. CASE STUDY

We will now plot the dispersion curves for steel and aluminum plate. Longitudinal velocity of steel: 6144 m/s, transverse velocity of steel: 3095 m/s, longitudinal velocity of aluminum: 6420 m/s, transverse velocity of aluminum: 3040m/s, plate thickness e = 6 mm.

To plot the dispersion curves: phase velocity vs. frequency - thickness, we have developed a Matlab program. In this program we used the Bisection method to find the zero of each propagation equations (symmetric and anti-symmetric). The bisection method is a root-finding method that repeatedly bisects an interval and then selects a subinterval in which a root must lie for further processing. It is a very simple and robust method, but it is also relatively slow. Because of this, it is often used to obtain a rough approximation to a solution. We started firstly by plotting phase velocity curves. These curves are independent of the thickness of the plate. To do this we used the algorithm (Figure 1). The curves representing the wave number are then deduced using the equation (5). As far as the group velocity curves are concerned, they are plotted using the relation:

$$V_g = \frac{d\omega}{dk} \qquad (6)$$

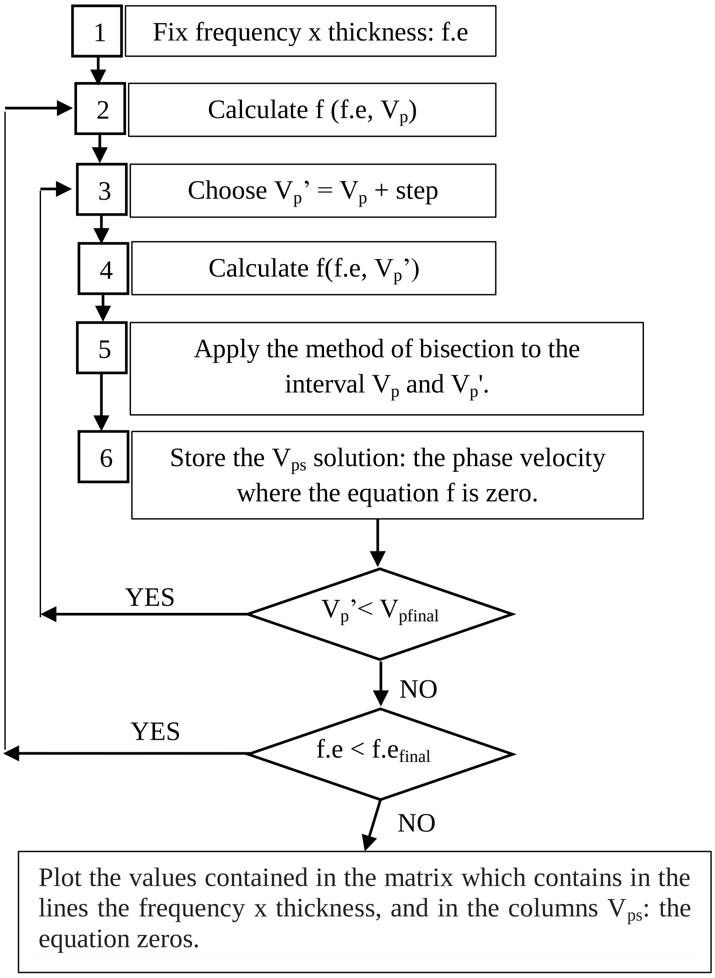

Figure 1. Algorithm to calculate the dispersion curves

With $f.e_{final}$ and V_{pfinal} are values that delimit the area of dispersion curves. In our study, we take: $f.e_{final}$ = 15000 Khz.mm and V_{pfinal} = 15000 m /s. The f (f.e, V_p) represents the two equations (1) and (2).

A. *Results :*

Note that we consider here only the real roots of the dispersion relations that correspond to the propagating modes of Lamb. The non- propagating modes (pure imaginary wave number) or attenuated (complex wave number) are not taken into account [26].

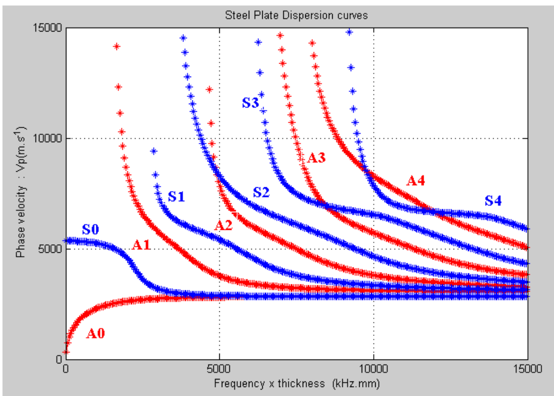

Figure 2. Dispersion curves for steel plate (phase velocity vs. 'f.e')

ISSN: 2509-2014

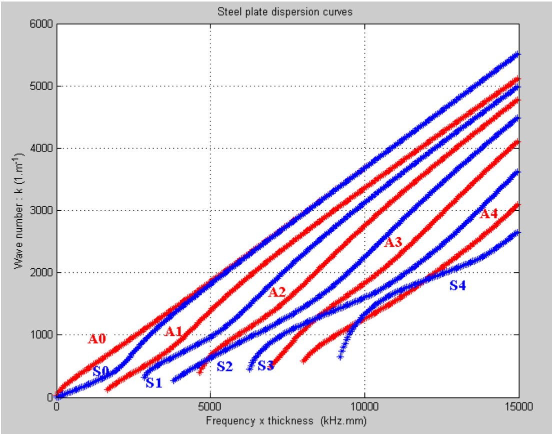

Figure 3. Dispersion curves for steel plate (wave number vs. 'f.e')

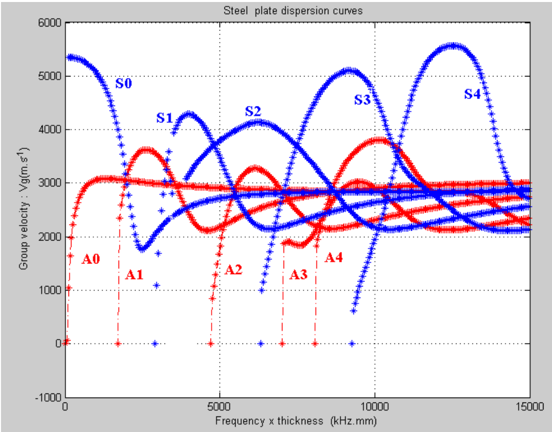

Figure 4. Dispersion curves for steel plate (group velocity vs. 'f.e')

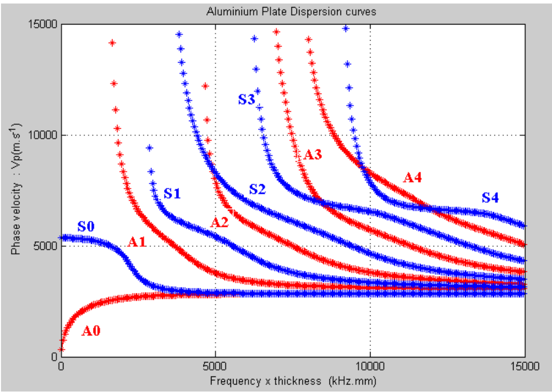

Figure 5. Dispersion curves for aluminum plate (phase velocity vs. 'f.e')

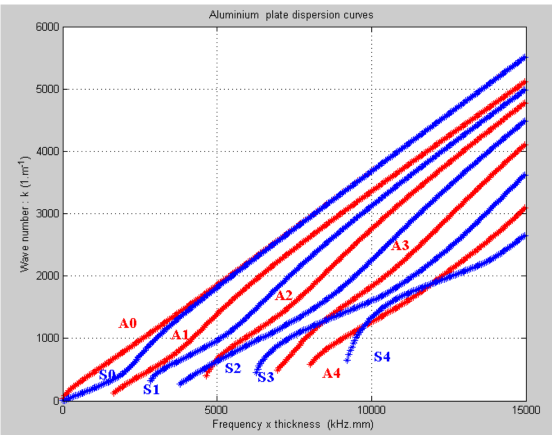

Figure 6. Dispersion curves for aluminum plate (wave number vs. 'f.e')

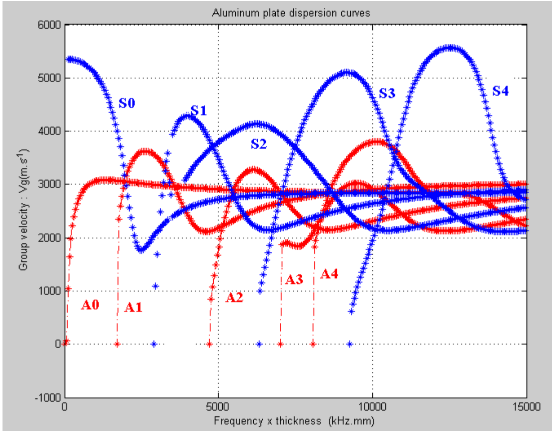

Figure 7. Dispersion curves for aluminum plate(group velocity vs. 'f.e')

The curves Figure 2 and Figure 5 show the phase velocity evolution of propagating modes versus the product frequency - thickness. The phase velocity corresponds to the propagating speed of the wavefronts, at given frequency, within a wave packet.

The curves Figure 3 and Figure 6 show the wave number evolution versus the product frequency - thickness. It is important to note that certain modes appear only after a certain values of (f.e) called product cutoff frequency - thickness. Below this, these waves do not exist as propagating mode.

The curves Figure 4 and Figure 7 show the group velocity evolution of the propagating modes versus the product frequency - thickness. The group velocity is the speed at which the wave packet propagates along the plate with the central pulse ω. It also corresponds to the propagating energy speed carried by the wave along the plate.

V. CONCLUSION

In this paper, we presented the numerical bisection method to extracts the dispersion curves for a steel and aluminum plates. The program plots the phase velocity dispersion curves, the wave number dispersion curves and the group velocity dispersion curves. The results show a very good agreement between the results obtained by the program developed using the bisection method and those obtained by using the others methods. In addition the bisection method is simple and the program does not need 3 or 4 minutes. However, this method cannot be used for composite or multi layers materials. We hope to resolve this problem and publish the results in a future communication.

REFERENCES

[1] E. CRESCENZO, ''Utilisation des ondes guidées exploitant des transducteurs à effet magnétostrictif pour la détection de corrosion'', Dijon France 2006.

[2] C. Potel , S. Baly, 'Utilisation des ondes de Lamb pour détecter des défauts dans des matériaux composites' ,1997.

[3] M.S. Harb, F.G. Yuan , 'Non-contact ultrasonic technique for Lamb wave characterization in composite plates', 2016.

[4] W.H. Ong a, N. Rajic b, W.K. Chiu a, C. Rosalie, 'Determination of the elastic properties of woven composite panels for Lamb wave studies', 2016.

[5] L Hoskoti, 'Numerical Modelling of Propagation of the Fundamental Symmetric Lamb Mode through a Metal Composite Adhesive Joint', 2016.

ISSN: 2509-2014

[6] C Gauthier, ‘Acoustic Characterization of an Aluminum Plate with Corrugated Surface’, 2015.

[7] Ye Lu a , Lin Ye a, “Quantitative assessment of through-thickness crack size based on Lamb wave scattering in aluminium plates”, 2007.

[8] J Zhang , H Ma , W Yan , Z Li, ‘Defect detection and location in switch rails by acoustic emission and Lamb wave analysis: A feasibility study’, 2016.

[9] B Vogelaar a, M Golombok, ‘Dispersion and attenuation by transmission, reflection, and mode conversion in welded pipes’, 2016.

[10] T. VALIER-BRASIER, C. POTEL, M. BRUNEAU, ‘Propagation guidée d'ondes de cisaillement horizontales dans des plaques solides rugueuses’,2009.

[11] De-an Ta a, Zhen-qing Liu b, Xiao Liu b,’ Combined spectral estimator for phase velocities of multimode Lamb waves in multilayer plates’, 2006.

[12] M El Allami ,”Propagation des ondes de Lamb.Modélisation par éléments finis et traitement par les transformées de fourier et en ondelettes ’’université chouaib doukkali ,Mars 2011.

[13] S.Sorohan, ‘‘Extraction of dispersion curves for waves propagating in free complex waveguides by standard finite element codes’’, Ultrasonics 51 (2011) 503–515.

[14] P.Hora, ‘‘Determination of Lamb wave dispersion curves by means of Fourier transform’’, Applied and Computational Mechanics 6 (2012) 5–16.

[15] F. Schopfer, ‘‘Accurate determination of dispersion curves of guided wavesin plates by applying the matrix pencil method to laser vibrometer measurement data’’, CEAS Aeronaut JDOI 10.1007/s13272-012-0055-7.

[16] M.S. Harb, ‘‘A rapid, fully non-contact, hybrid system for generating Lamb wave dispersion curves’’, Ultrasonics (2015).

[17] P.Packo, ‘‘Generalized semi-analytical finite difference method for dispersion curves calculation and numerical dispersion analysis for Lamb waves’’, 2014 Acoustical Society of America.

[18] F.Honarvar, ‘‘an alternative method for plotting dispersion curves’’, Ultrasonics 49 (2009) 15–18.

[19] F.A. Schwab, Surface-wave dispersion computations: Knopoff’s method, Bull.Seism. Soc. Am. 60 (1970) 1491–1520.

[20] A.K. Mal, ‘‘Guided waves in layered solids with interface zones’’, Int. J. Eng. Sci. 26 (1988) 873–881.

[21] F.A. Schwab, L. Knopoff, ‘‘Surface-wave dispersion computations’’, Bull. Seism. Soc. Am. 60 (1970) 321–344.

[22] N.A. Haskell, ‘‘The dispersion of surface-waves in multilayered media’’, Bull.Seism. Soc. Am. 43 (1953) 17–34.

[23] F. Press, D. Harkrider, C.A. Seafeldt, A fast convenient program for computationof surface-wave dispersion curves in multilayered media, Bull. Seism. Soc. Am.51 (1961) 495–502.

[24] M.J.S. Lowe, Matrix techniques for modeling ultrasonic waves in multilayeredmedia, IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control42 (1995) 525–542.

[25] W.H. Press, B.P. Flannery, S.A. Teukolsky, W.T. Vettering, Numerical Recipes: ‘‘The art of scientific computing’’, Cambridge University Press, New York, 1986.

[26] E. Le Clézio, “ Diffraction des ondes de lamb par des fissures verticales ” ,Thèse de doctorat, Université de bordeaux I (2001).

 ISSN: 2509-2014