Assignment - 02

A-2) $ay^2 = x^3 \Rightarrow a = x^3/y^2$

Diff' wrt x,

$$a(2y)\frac{dy}{dx} = 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2y^2}{2yx^3}$$

$$\frac{dy}{dx} = \frac{3y}{2x}$$

$$-\frac{dx}{dy} = \frac{3y}{2x}$$

$$\int 2x \cdot dx = \int -3y \cdot dy$$

$$2\frac{x^2}{2} = -3\frac{y^2}{2} + c_1$$

$$2x^2 = -3y^2 + c \quad \therefore\ 2c_1 = c$$

A-5). $r = a(1-\cos\theta)$

$$\frac{dr}{d\theta} = -a(-\sin\theta)$$

$$\frac{dr}{d\theta} = a\sin\theta$$

$$\frac{dr}{d\theta} = \frac{r\sin\theta}{(1-\cos\theta)}$$

Replace $\frac{dr}{d\theta}$ by $-r^2 \cdot \frac{d\theta}{dr}$

$$= r^2\frac{d\theta}{dr} = \frac{r\sin\theta}{1-\cos\theta}$$

$$= r\frac{d\theta}{dr} = \frac{\sin\theta}{1-\cos\theta}$$

$$\int \frac{1-\cos\theta}{\sin\theta} \cdot d\theta = -\int \frac{dr}{r}$$

$$\int (\operatorname{cosec}\theta - \cot\theta)\, d\theta = -\ln r + c$$

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$$\log|\operatorname{cosec}\theta - \cot\theta| - |\log|\sin\theta|| = -\ln r + \ln c$$

$$\ln\left|\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta}\right| = \ln\left(\frac{c}{r}\right)$$

$$\frac{\operatorname{cosec}\theta - \cot\theta}{\sin\theta} = \frac{c}{r}$$

$$\operatorname{cosec}^2\theta - \cot\theta\operatorname{cosec}\theta = \frac{c}{r}.$$

B-1)

$\theta_0 = 30°C$

$t = 0 \longrightarrow \theta = 100°C$

$t = 15 \longrightarrow \theta = 70°C$

$t = ? \longrightarrow \theta = 40°C.$

According to Newton's Law of cooling.

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int_{100}^{70} \frac{d\theta}{(\theta - \theta_0)} = \int_0^{15} -k \cdot dt$$

$$[\ln(\theta - \theta_0)]_{100}^{70} = -k[t]_0^{15}$$

$$\ln(70-30) - \ln(100-30) = -k(15)$$

$$\ln\left(\frac{40}{70}\right) = -15k$$

$$\frac{\ln\left(\frac{7}{4}\right)}{15} = k$$

Slly,

$$[\ln(\theta - 30)]_{100}^{40} = -k(t)_0^t$$

$$\ln\left(\frac{10}{70}\right) = -kt$$

$$\frac{\ln(7)}{k} = t \Rightarrow t = \frac{\ln(7) \times 15}{\ln\left(\frac{7}{4}\right)}$$

$\therefore$ $\underline{t = 42.11 \text{ minutes}}$

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$$\ln\left(\frac{180}{\theta-20}\right) = \ln\left(\frac{9}{5}\right)^{1.5}$$

$$\frac{180}{\theta-20} = 2.414$$

$$\frac{180}{2.414} = \theta - 20$$

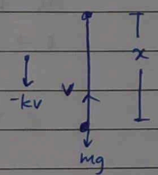
$$74.56 + 20 = \theta$$

$$\therefore \boxed{\theta = 94.56^\circ C}$$

C-1). The forces acting on body are

i). Gravitational force $= mg$

ii). Air resistance $(\propto v) = +kv.$



By De-Alemberts' Principle,

Net force = mass × acceleration

$$-kv - mg = ma$$

$$-(mg + kv) = m \cdot v \cdot \frac{dv}{dx}$$

$$-\int dx = \int \frac{m \cdot v \cdot dv}{(mg + kv)} \quad \text{---- (V.S.F.)}$$

$$-x = \frac{m}{k} \log(mg + kv) + c$$

At $t = 0$, $v = v$, $x = 0$

$$0 = \frac{m}{k} \log(mg) + \cancel{kv} + c$$

$$-x = \frac{m}{k}(\log(mg + kv)) - \frac{m}{k} \log mg$$

$$-x = \frac{m}{k} \log\left(\frac{mg + kv}{mg}\right) \quad \text{— ①}$$

B-6)

B-6). $\theta_0 = 20°C$.

$t=0 \rightarrow \theta = 200°C$

$t=10 \rightarrow \theta = 120°C$

$t=15 \rightarrow \theta = ?$

By Newton's Law of Cooling,

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\int_{200}^{120} \frac{d\theta}{(\theta-\theta_0)} = \int_0^{10} -k \cdot dt$$

$$\left[\ln(\theta-\theta_0)\right]_{200}^{120} = -k(t)_0^{10}$$

$$\left[\ln(\theta-20)\right]_{200}^{120} = -k(10)$$

$$\ln(120-20) - \ln(200-20) = -10k.$$

$$\ln 100 - \ln 180 = -10k$$

$$\ln\left(\frac{100}{180}\right) = -10k$$

$$\ln\left(\frac{5}{9}\right) = -10k$$

$$k = -\frac{\ln(5/9)}{10}.$$

$$k = \frac{\ln(9/5)}{10}$$

S'lly, $\left[\ln(\theta-20)\right]_{200}^{\theta} = -k(t)_0^{15}$

$$\ln(\theta-20) - \ln(180) = -k(15)$$

$$\ln\left(\frac{\theta-20}{180}\right) = -k(15)$$

$$\ln\left(\frac{180}{\theta-20}\right) = 15k$$

$$\ln\left(\frac{180}{\theta-20}\right) = \frac{15}{10}\ln\left(\frac{9}{5}\right)$$

$$-(mg + kV) = m \cdot \frac{dv}{dt}$$

$$-dt = \left(\frac{m}{mg + kv}\right) dv$$

$$\int -dt = \int \frac{m}{(mg + kV)} dv$$

$$-t = \frac{m}{k} \ln (mg + kV) + c$$

At $t = 0$, $v = 0$

$$0 = \frac{m}{k} \ln (mg) + c$$

$$c = -\frac{m}{k} \ln (mg)$$

$$-t = \frac{m}{k} \ln (mg + kv) - \frac{m}{k} \ln (mg)$$

$$t = \frac{m}{k} \ln \left(\frac{mg + kv}{mg}\right)$$

$$\boxed{t = \frac{m}{k} \ln \left(\cancel{mg}\, 1 + \frac{kV}{mg}\right)} \quad -②$$

From ① & ②

$$\boxed{x = t}$$

C-4). Forces acting on body,

i). Gravitational force $= mg$

ii). Air resistance $= -kv^2$.

By De-Alembert's Principle,

Net force = mass × acceleration

$$mg - kv^2 = ma$$

$$mg - kv^2 = m \frac{dv}{dt}$$

$$dt = \frac{m}{mg - kv^2} \cdot dv$$

$$\int dt = \frac{1}{k} \int \frac{m \cdot dv}{\frac{mg}{k} - v^2}$$

$$\int dt = \frac{m}{k} \int \frac{dv}{\left(\sqrt{\frac{mg}{k}}\right)^2 - v^2}$$

$$\therefore \quad t = \frac{m}{k} \left(\frac{\sqrt{k}}{2\sqrt{mg}} \right) \ln \left(\frac{\sqrt{mg/k} + v}{\sqrt{\frac{mg}{k}} - v} \right) + c$$

$$\therefore \quad t = \frac{1}{2} \sqrt{\frac{m}{kg}} \ \ln \left(\frac{\sqrt{mg/k} + v}{\sqrt{mg/k} - v} \right) + c$$

Now at $t = 0$, $v = 0$

$$0 = \frac{1}{2} \sqrt{\frac{m}{kg}} \ \ln (1) + c$$

$\therefore$ $\underline{c = 0}$ $\quad \ldots \ \underline{\ln(1) = 0}$

$$\therefore \quad 2t \sqrt{\frac{kg}{m}} = \ln \Big($$

Q-9).

The forces acting on body are

i). Gravitational force $= mg$

ii). Air resistance $(\propto v^2) = -kv^2$.

By De-Alembert's Principle,

Net force = mass × acceleration

$mg - kv^2 = ma$

~~mg - kv~~ $\quad mg - kv^2 = m \cdot v\frac{dv}{dx}$.

$$\int dx = \frac{1}{-2}\int \frac{m \cdot (-2) v \cdot dv}{mg - kv^2} \quad \ldots (V.S.F)$$

$$x = -\frac{m}{2k} \log(mg - kv^2) + c$$

At $t = 0$, $x = 0$ & $v = 0$

$$0 = \frac{-m}{2k} \log(mg - 0) + c$$

$$c = \frac{m}{2k} \log(mg)$$

$$\therefore \quad x = \frac{-m}{2k} \log(mg - kv^2) + \frac{m}{2k} \log(mg)$$

$$x = \frac{m}{2k}\left(\log mg - \log(mg - kv^2)\right)$$

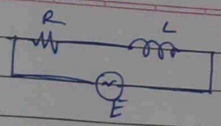
$$x = \frac{m}{2k} \log\left(\frac{mg}{mg - kv^2}\right)$$

$$\therefore \frac{2kx}{m} = \log\left(\frac{mg}{mg - kv^2}\right)$$

$$\therefore \frac{2kx}{m} = \log\left(\frac{ka^2}{\cancel{k}a^2 - kv^2}\right)$$

$$\therefore \frac{2kx}{m} = \log\left(\frac{a^2}{a^2 - v^2}\right)$$

Q.3)



∴ By KVL,

$$IR + L\frac{dI}{dt} = E$$

$$\frac{dI}{dt} + \frac{IR}{L} = \frac{E}{L}$$

∴ $I.F. = e^{\int R/L \cdot dt} = e^{Rt/L}$

∴ General sol^n is

$$I.(e^{Rt/L}) = \int \frac{E}{L}.e^{Rt/L}.dt + c$$

$$I.\ e^{Rt/L} = \frac{E}{\cancel{L}}\,\frac{e^{Rt/L}}{\left(\frac{R}{\cancel{L}}\right)} + c$$

$$I.\ e^{Rt/L} = \frac{E.\ e^{Rt/L}}{R} + c$$

At $t=0$, $I=0$.

$$0 = \frac{E}{R}(1) + c$$

$$c = -E/R$$

∴ $I.\ e^{Rt/L} = \frac{E}{R}\left(e^{Rt/L} - 1\right)$

∴ $I = \frac{E}{R}\left(1 - e^{-Rt/L}\right)$

~~Put t =~~ Hen, $I_{max} = \frac{E}{R}$ — (1)

Put $t = \frac{L\log 2}{R}$, $I = \frac{E}{R}\left(1 - e^{-\cancel{R}\times\frac{\cancel{L}\log 2}{\cancel{R}\times\cancel{L}}}\right)$

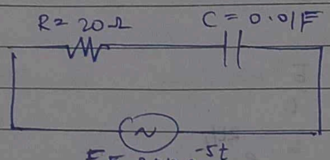
$$I = \frac{E}{R}\left(1 - e^{\log(2)^{-1}}\right)$$

$$I = E/R\left(1 - \frac{1}{2}\right) = \frac{1}{2}\frac{E}{R} \quad — (2)$$

$\therefore$ From ① & ②

$$\boxed{I = \frac{I_{max}}{2}}$$

D-10).



According to KVL,

$$IR + \frac{q}{C} = E$$

$$R\frac{dq}{dt} + \frac{q}{C} = E$$

$$\frac{dq}{dt} + \frac{q}{CR} = \frac{E}{R} \quad \text{--- Linear D.E.}$$

$$\therefore \quad I.F. = e^{\int 1/RC \, dt} = e^{t/RC}.$$

$$\therefore \quad q\left(e^{t/RC}\right) = \int \frac{E}{R} e^{t/RC} . dt . + c$$

$$q\left(e^{t/RC}\right) = \frac{E}{R} \frac{e^{t/RC}}{\left(\frac{1}{RC}\right)} + c'$$

$$q\left(e^{t/RC}\right) = E.C \; e^{t/RC} + c'$$

$$q = E.C + ce^{-t/RC}.$$

At $t=0, \; q=0$

$$0 = EC + c'$$

$$\therefore \quad c' = -EC.$$

$$\therefore \quad q = EC\left(1 - e^{-t/RC}\right)$$

Diff' wrt t

$$\frac{dq}{dt} = -E\cancel{C} \; e^{-t/RC} \left(\frac{-1}{R\cancel{C}}\right)$$

$$\boxed{I = \frac{E}{R} e^{-t/RC}}$$

E-2)

$d_1 = 10\,cm$, $r_1 = 5\,cm$, $k = 0.0015$.
$T = 200^\circ c$

$d_2 = 15\,cm \Rightarrow T = 50^\circ c$.

$$\int_5^{15} q\frac{dx}{x} = -2\pi k \int_{200}^{50} \cancel{T}\,.\,dT$$

$$q\left[\ln|x|\right]_5^{15} = -2\pi k\,(T)_{200}^{50}$$

$$q(\ln|15| - \ln|5|) = -2\pi k(50-200)$$

$$q\ln(3) = 2\pi k(150)$$

$$\cancel{q = \frac{\ln(3)}{2\pi k(150)}}$$

$$q = \frac{300\pi k}{\ln 3}$$

$$q\int_5^{7.5}\frac{dx}{x} = -2\pi k\int_{200}^{x} dT$$

$$q\left(\ln\left(\frac{7.5}{5}\right)\right) = -2\pi k(x-200)$$

$$\frac{300\cancel{\pi k}}{\ln 3}\ln\left(\frac{7.5}{5}\right) = (200-x)\cancel{2\pi k}$$

$$\frac{150 \times \ln(1.5)}{\ln(3)} = 200 - x$$

$$55.36 = 200 - x$$

$$x = 144.64^\circ c$$

E-3)

~~$r_1 - 2 - 1$~~ $r_1 = 5\,cm$, $T = 150^\circ c$

$r_2 = 10\,cm$, $T = 30^\circ c$

$k = 0.002$, $x = 8.5$

$$q\int_5^{10}\frac{dx}{x} = -2k\pi(2000)\int_{150}^{30} T$$

$$q\left[\ln(x)\right]_5^{10} = 4000\pi k(120)$$

$$q = \frac{4000\pi k(120)}{\ln(2)}$$

$q = 4351.06$ cal/sec

$$q \int_{5}^{8.5} \frac{dx}{x} = -4000\pi k \int_{150}^{x} T$$

$$q \ln(1.7) = 4000\pi k (150 - x)$$

$$x = 150 - \frac{4351.06 \ln(1.7)}{4000\pi k}$$

$$x = 58.1359°C$$

Q.6) $h = 5 cm$, $T = 100°c$

$r_2 = 15 cm$

$k = 0.0006$, $T = 30°c$

q per hour   length = 1m.

$$q \int_{5}^{15} \frac{dx}{x} = -2\pi k (100) \int_{100}^{30} dT$$

$$q \ln(3) = 200\pi k (70)$$

$$q = \frac{200 \times 0.0006 \times 70}{\ln(3)}$$

$q = 7.646$ cal/sec $\times 3600$

$q = 1651538.053$ cal/hr