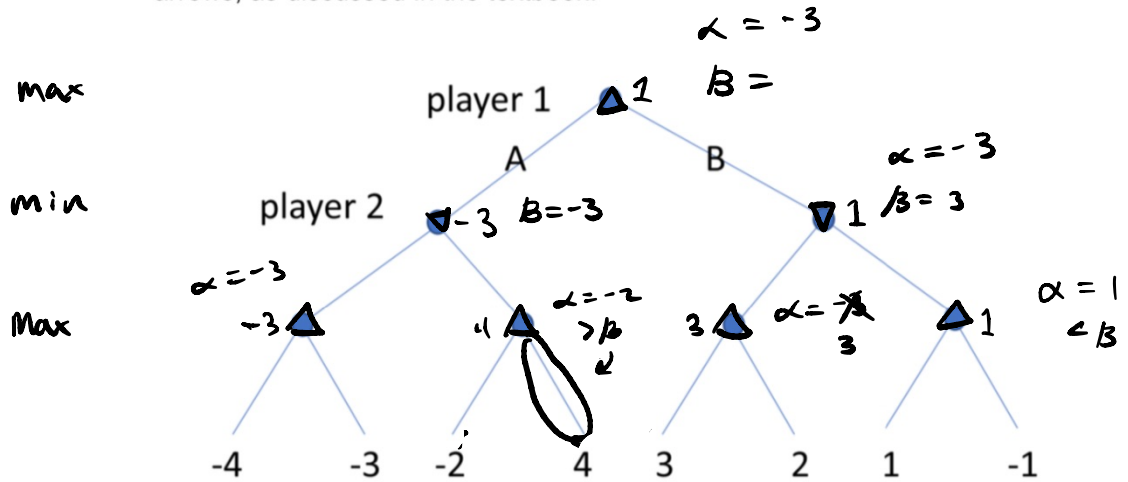1. Given the simple game tree (binary, depth 3) below, label the nodes with up or down arrows, as discussed in the textbook.

max

min

Max

player 1 $\triangle$ 1 $\alpha = -3$ $B =$

A   B

player 2 $\triangledown$ -3 B=-3 $\triangledown$ 1 $\alpha = -3$ $\beta = 3$

$\alpha = -3$
-3 $\triangle$ $\triangle$ $\alpha = -2$ 3 $\triangle$ $\alpha = \frac{-3}{3}$ $\triangle$ 1 $\alpha = 1$
                $> \beta$ $\leq \beta$

-4   -3   -2   4   3   2   1   -1

Compute the *minimax* values at the internal nodes (write the values next each node).
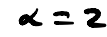
Should the player 1 take action A or B at the root?

Action B

What is the expected outcome (payoff at the end of the game)?

1

Which branches would be pruned by alpha-beta pruning? (circle them)

How could the leaves be relabeled to maximize the number of nodes pruned? (you can move the utilities around arbitrarily to other leaves, but you still have to use -4,-3,-2,-1,+1,+2,+3,+4)

$\alpha = 3$

max

player 1

A   B

$\alpha = 3$ $\alpha > \beta$

min

player 2 $\beta = 2$

$\alpha = 3$
$\beta = -4$

$\alpha = 2$

$\alpha = 3$
$> \beta$

$\alpha = 3$
$\beta = -4$

max

-4̶ 1̶   -3̶ 2   -2̶ 3   -4̶ 4   -4̶ -4   -3̶ -3   -4̶ -2   -1

How could the leaves be relabeled to eliminate pruning?

$\alpha = 2$

player 1

A   B

$\alpha = 2$
$\beta = 4$

player 2 $\beta = -1$

$\alpha = -3$
$\beta = -1$

$\alpha = 4$

$\alpha = 2$
$\beta = 4$

$\alpha = -1$

-4̶ -1   -8̶ -2   -2̶ -3   -4̶ -4   -8̶ 4   -2̶ 3   -4̶ 2   -4̶ 1

2. In a simple binary game tree of depth 4 (each player gets 2 moves), suppose all the leaves have utility 0 except one winning state (+1000) and one loosing state (-1000).

a) Could the player at the root force a win?

b) Does it matter where the 2 non-zero states are located in the tree? (e.g. adjacent or far apart)

c) If this question was changed to have a different depth, would it change the answers to the two questions above? If yes, how do the answers change? If no, explain why no change would happen.
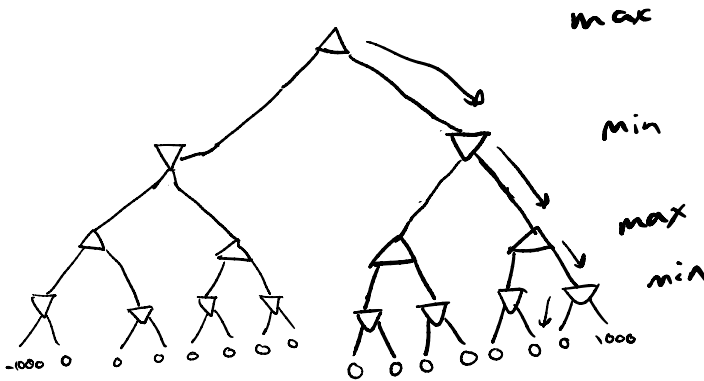
a) No, the player at the root (Player A) cannot force a win. Player B has the last move in a tree of depth 4. There are 4 possible ending move possibilities:

1. 0 and 1000 as options
2. 0 and -1000 as options
3. 0 or 0 . . .
4. 1000 or -1000

Because player B will always try to minimize (-1000), there is no case for which player A can force a win.

b) In this case, no. A win cannot be forced for any combination of moves, only a draw.

c) Yes. If the depth is an odd number, than player A will have both the first and last move. Thus, if A's first move makes an winning state possible & B's moves don't block A from reaching that, A can force a win.

3. Hiking Philosophers. 🚶🚶🚶 Three philosophers, Alex (A), Bob (B), and Charlie (C), are going on a hike and need to decide the order in which they will hike. Alex and Charlie have PhDs, while Bob has a MS degree. Adjacent hikers in the sequence have to have different degrees. Finally, Charlie does not want to be last.

a) Show how to set this up as a Constraint Satisfaction Problem. (what needs to be defined?)

$\underline{vars} = \{Alex, Bob, Charlie\} = \{A, B, C\}$

$\underline{domains}: dom(s) = \{1, 2, 3\} \rightarrow$ referring to hike order
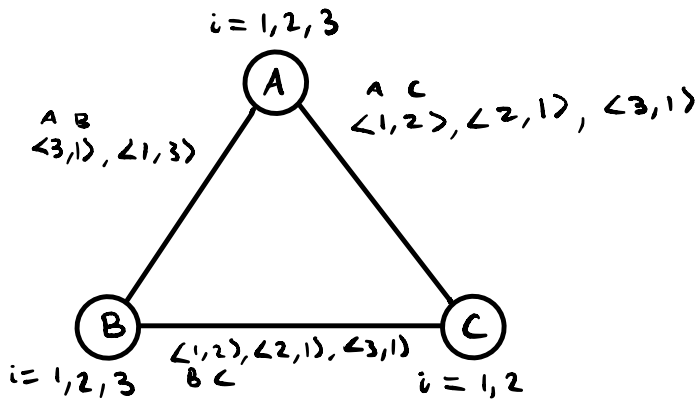
$D_i =$ Degree of hiker in position $i = 1, 2, 3$

$i_x =$ Position of hiker x

$\underline{constraints}:$

1) $D_i \neq D_{i-1}$ and $D_i \neq D_{i+1}$

2) $i_c \neq 3$

3) $i_A \neq i_B \neq i_C$

b) Draw the Constraint Graph (label all nodes and edges)

$i = 1, 2, 3$

(A)

A C
$\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 3, 1\rangle$

A B
$\langle 3, 1\rangle, \langle 1, 3\rangle$

(B)   (C)

$\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 3, 1\rangle$
B C

$i = 1, 2, 3$ $i = 1, 2$

c) <u>Trace</u> how plain <u>Backtracking</u> (BT) (with no heuristics) would solve this problem, assuming values are processed in *alphanumeric* order. Identify instances where back-tracking happens.

$vars = \{A, B, C\}$, state representation $= \langle i_A, i_B, i_C \rangle$
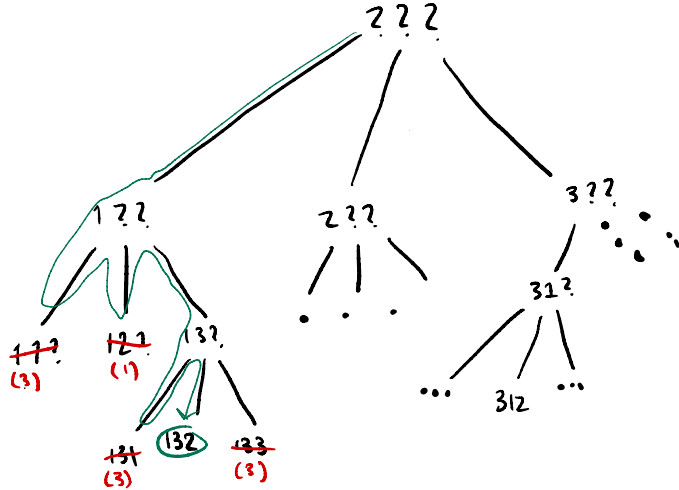
$$D_A = PhD = D_B, \quad D_C = MS$$

constraints:

1) $D_i \neq D_{i-1}$ and $D_i \neq D_{i+1}$

2) $i_C \neq 3$

3) $i_A \neq i_B \neq i_C$

Trace

Try $i_A = 1$

    Try $i_B = 1$ ✗ (3)

       backtrack

    Try $i_B = 2$ ✗ (1)

       backtrack

    Try $i_B = 3$

       Try $i_C = 1$ ✗ (3)

          backtrack

       Try $i_C = 2$ ✓

d) <u>Trace</u> how BT would solve this problem using the <u>MRV heuristic</u>.

<u>constraints:</u>

1) $D_i \neq D_{i-1}$ and $D_i \neq D_{i+1}$ $\Rightarrow$ This means $i_A, i_B \neq 2$
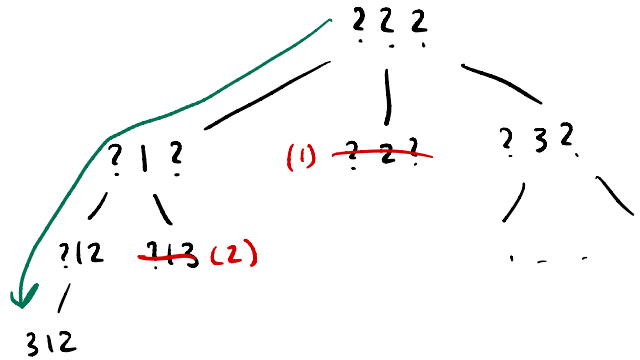
2) $i_c \neq 3$

State representation $= \langle i_A, i_B, i_c \rangle$

3) $i_A \neq i_B \neq i_c$

Try i_B first as it is the most constrained

Then try i_c.

Remove used numbers f

? ? ?

? 1 ?   (1) ~~? 2 ?~~   ? 3 2

?|2   ~~2|3~~ (2)

312

<u>Trace</u>

Try $i_B = 1$

  Try $i_c = 2$

    Try $i_A = 3$ ✓